THE QED PHOTON IN A CURVED GRAVITATIONAL FIELD

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The classical photon and curved spacetime

- Maxwell's equations

$$D_\mu F^{\mu\nu} = 0, \qquad D_{[\mu} F_{\nu\lambda]} = 0$$

- Geometric optics approximation

$$A_\mu = \left(A_\mu + i\varepsilon B_\mu + \ldots\right) e^{i\frac{\theta}{\varepsilon}}, \qquad k_\mu = \partial\theta, \qquad A_\mu = A a_\mu$$

- Solving Maxwell equation, we find

$$O\left(\varepsilon^{-1}\right): \qquad k^2 = 0 \Rightarrow k^\mu D_\mu k^\nu = 0, \qquad \frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{ds} \frac{dx^\lambda}{ds} = 0$$

$$O(1): \qquad \nabla_k (\ln A) = -\frac{1}{2} k^\mu{}_{;\mu}, \qquad \nabla_k a^\mu = 0$$

- Bianchi identity

$$D_{[\lambda} F_{\mu\nu\,]} = 0 \qquad \Rightarrow \qquad f_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu .$$

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The QED photon in a fixed background curved spacetime

- The effects of vacuum polarization on photon propagation were originally discussed in a paper by Drummond and Hathrell. The effective action is

$$\Gamma = \int dx\sqrt{-g}\left[-\frac{1}{4}F_{\mu\upsilon}F^{\mu\upsilon} + \frac{1}{m^2}\left(aRF_{\mu\upsilon}F^{\mu\upsilon} + bR_{\mu\nu}F^{\mu\lambda}F^{\nu}{}_{\lambda} + cR_{\mu\nu\lambda\rho}F^{\mu\nu}F^{\lambda\rho}\right)\right]$$

$a = -\frac{1}{144}\frac{\alpha}{\pi}, \qquad b = \frac{13}{144}\frac{\alpha}{\pi}, \qquad c = -\frac{1}{360}\frac{\alpha}{\pi}, \qquad \alpha =$ fine structure constant

- The equation of motion are given by

$$D_\mu F^{\mu\nu} + \frac{1}{m^2}\left[2bR^{\mu}{}_{\lambda}D_\mu F^{\lambda\nu} + 4cR^{\mu\nu}{}_{\lambda\rho}D_\mu F^{\lambda\rho}\right] = 0$$

- In geometric Optics approximation and by using Bianchi identity we have the modified light cone equation in the form of

$$k^2 a^\nu + \frac{2b}{m^2}\left[R^{\mu}{}_{\lambda}\left(k_\mu k^\lambda a^\nu - k_\mu k^\nu a^\lambda\right)\right] + \frac{8c}{m^2}\left(R^{\mu\nu}{}_{\lambda\rho}k_\mu k^\lambda a^\rho\right) = 0.$$

- In terms of the Weyl tensor and for transverse normalized polarization vector, it can be re-expressed as

$$k^2 - \frac{(2b+4c)}{m^2}R_{\mu\lambda}k^\mu k^\lambda + \frac{8c}{m^2}\left(C_{\mu\nu\lambda\rho}k^\mu k^\lambda a^\rho a^\nu\right) = 0.$$

The Characteristics of Quantum Gravitational Optics Theory

- Local Lorentz invariance
- Violation of strong equivalence principle
- Modification to the light cone condition

$$\left[\eta^{(a)(b)} + \alpha\sigma^{(a)(b)}(R)\right] \; k_{(a)}k_{(b)} = 0$$

- Quantum gravitational optics as a biometric theory

$$k^2 - \frac{(2b+4c)}{m^2} R_{\mu\lambda} k^\mu k^\lambda + \frac{8c}{m^2}\left(C_{\mu\nu\lambda\rho} k^\mu k^\lambda a^\rho a^\nu\right) = 0.$$

The effective metric $G_{\mu\nu}$ determines "physical null cones" which are distinct from the "geometrical null cones".

- superluminal photons
- Gravitational birefringence
- No necessity for causal violation

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Characteristics of a bundle of ray

in a background gravitational field

- The physical light cones are determined by the effective metric $G_{\mu\nu}$ and are distinct from the geometric null cones which are fixed by the spacetime metric $g_{\mu\nu}$. The physical light cones are the integral curves of the wave vector

$$k^{\nu} = l^{\nu} + \frac{1}{m^{2}}\left[(2b+4c)R^{\mu\nu}l_{\mu} - 8cC^{\nu\lambda\mu\rho}l_{\mu}a_{\lambda}a_{\rho}\right]$$

- The connecting vector field q^{μ}, represents the displacement to an infinitesimally nearby geodesic and satisfy the Jacobi equation

$$\nabla_{k}\nabla_{k}q^{\mu} = \ldots = R^{\mu}{}_{\nu\rho\sigma}k^{\nu}k^{\sigma}q^{\rho}$$

Note that k^{μ} is null w.r.t the metric $G_{\mu\nu}$, but the curvature components w.r.t. $g_{\mu\nu}$ characterize the acceleration toward or away from each other. The vector field q^{μ} is Lie transported along the ray by the vector field k^{μ}.

$L_{K}q^{\mu} = 0$ that is $\nabla_{q}k^{\mu} = \nabla_{k}q^{\mu}$

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- In studying the bundle, we concentrate on the evolution of q^{μ} and follow the calculations using the Newman-Penrose formalism. The first step is the construction of a basis of null vectors $l^{\mu}, n^{\mu}, m^{\mu}$ and $\bar{m}^{\mu}$, such that

$$e_{(1)} = e^{(2)} = l, \qquad e_{(2)} = e^{(1)} = n, \qquad e_{(3)} = -e^{(4)} = m, \qquad e_{(4)} = -e^{(3)} = \bar{m}.$$

$$l.n = -m.\bar{m} = 1 \qquad l.l = n.n = m.m = \bar{m}.\bar{m} = 0, \qquad l.m = l.\bar{m} = n.m = n.\bar{m} = 0,$$

- Directional derivatives are defined as

$$\gamma_{(c)(a)(b)} = e^{\mu}_{(c)} e_{(a)\mu;\nu} e^{\nu}_{(b)}$$

$$\gamma_{(c)(a)(b)} = -\gamma_{(a)(c)(b)}$$

- Due to the freedom in choosing the basis, as a result of Lorentz transformation, many of our quantities are of the same nature as gauge quantities. The allowed gauge transformations:

Class I: $l \mapsto l, \qquad m \mapsto m + al, \qquad \bar{m} \mapsto \bar{m} + a^{*}l, \qquad n \mapsto n + a^{*} + a\bar{m} + aa^{*}l,$

Class II: $l \mapsto A^{-1}l, \qquad n \mapsto An, \qquad m \mapsto e^{i\theta}m, \qquad \bar{m} \mapsto e^{-i\theta}\bar{m}.$

- Fixing the gauge is possible through choosing the parallel propagating tetrads

$$\nabla_{l} l^{\nu} = \nabla_{l} n^{\nu} = \nabla_{l} m^{\nu} = \nabla_{l} \bar{m}^{\nu} = 0$$

such that l be the velocity vector along the geometric ray.

- The connecting vector in terms of tetrad components are defined as

$$q^{\mu} = gl^{\mu} + \zeta \bar{m}^{\mu} + \bar{\zeta} m^{\mu} + hn^{\mu}$$

- The equation governing the propagation of q^{μ} along the vector field k^{μ},

$$\nabla_k q^{(c)} = \left[\eta^{(a)(c)} + \frac{1}{m^2} A^{(a)(c)} \right] q^{(b)} \gamma_{(a)(1)(b)} + q_{(a)} \gamma^{(a)(c)(b)} k_{(b)},$$

where $A^{(a)(c)}$ is a symmetric tensor made up of Ricci and Weyl tensor components. It is easily seen that under the class I or II transformations, most of the terms in this equation are not changing invariantly.

- Demanding the invariance, we impose the condition that the neighboring pair of rays to satisfy the condition

$$q^{\mu} k_{\mu} = 0$$

and call them abreast. For abreast rays, we have $\nabla_k \left(q^{\mu} k_{\mu} \right) = 0$ and

$$\nabla_k h = 0.$$

$$\nabla_k \zeta = -\left(\bar{\zeta} \sigma + \zeta \rho \right) - \frac{1}{m^2} \left[R^{(3)(4)} \left(\bar{\zeta} \sigma + \zeta \rho \right) + R^{(4)(4)} \left(\bar{\zeta} \sigma^* + \zeta \rho^* \right) \right],$$

- Since the transverse distances from the specified ray are (frame) observer-independent, we can study the changing pattern of the intersection of physical bundle of ray with the spatial 2-plane spanned with m^{μ} and $\bar{m}^{\mu}$ and specify the optical scalars namely the expansion, shear and twist of this pattern.

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- To study the expansion, we consider the propagation along k of the area of a small triangle formed by the points $0, \zeta_1$ and ζ_2 .

$$\nabla_k \delta A = \nabla_k \left[\frac{i}{2} \left(\zeta_1 \bar{\zeta}_2 - \zeta_2 \bar{\zeta}_1 \right) \right] = -\mathrm{Re}\left[\rho + \frac{1}{m^2} \left(R_{(3)(4)} \rho + R_{(3)(3)} \sigma^* \right) \right] \delta A$$

We define the effective expansion as

$$\theta_{eff} := -\mathrm{Re}\left[\rho + \frac{1}{m^2} \left(R_{(3)(4)} \rho + R_{(3)(3)} \sigma^* \right) \right]$$

- Setting θ_{eff} and the $\bar{\varsigma}$ coefficient equal to zero, we obtain

$$\nabla_k \varsigma = -i\,\mathrm{Im}\left[\rho + \frac{1}{m^2} \left(R_{(3)(4)} \rho + R_{(3)(3)} \sigma^* \right) \right] \varsigma$$

So it could be claimed that $\mathrm{Im}\left[\rho + \frac{1}{m^2} \left(R_{(3)(4)} \rho + R_{(3)(3)} \sigma^* \right) \right]$ measures the twist in the bundle cross section and called the effective twist

$$\omega_{eff} := -\mathrm{Im}\left[\rho + \frac{1}{m^2} \left(R_{(3)(4)} \rho + R_{(3)(3)} \sigma^* \right) \right].$$

- If $\theta_{eff} = \omega_{eff} = 0$, the remaining part i.e, $\sigma + \frac{1}{m^2} \left(R_{(3)(4)} \sigma + R_{(3)(3)} \rho^* \right)$ measures the distortion in the shape of the bundle cross section.

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- The effective scalar which is responsible for shearing is

$$\sigma_{eff} := \sigma + \frac{1}{m^2}\left(R_{(3)(4)}\sigma + R_{(3)(3)}\rho^*\right).$$

- The equations of evolution for the connecting vector components can be shown as matrix form

$$\nabla_k Z = PZ, \qquad z = \begin{pmatrix} \varsigma \\ \bar{\varsigma} \end{pmatrix} \qquad P = \begin{pmatrix} \theta_{eff} - i\omega_{eff} & \sigma_{eff} \\ \sigma^*_{eff} & \theta^*_{eff} + i\omega^*_{eff} \end{pmatrix}$$

- For abreast rays, the deviation vector satisfy $\nabla_k \nabla_k Z = 0$ and the variation of optical scalars along the physical ray is given by

$$\nabla_k P = P^2.$$

- The equations governing the propagation of the optical scalars or the modified Raychaudhuri equation are

$$\nabla_k \theta_{eff} = \omega^2_{eff} - \theta^2_{eff} - \left|\sigma_{eff}\right|^2$$
$$\nabla_k \omega_{eff} = -2\theta_{eff}\,\omega_{eff}$$
$$\nabla_k \sigma_{eff} = -2\theta_{eff}\,\sigma_{eff}.$$

- This work can be found in arXiv:gr-qc/0605009.

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- The modified equations in QGO are

$$\frac{d^2x^\mu}{ds^2}+\Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{ds}\frac{dx^\lambda}{ds}-\frac{1}{m^2}D^\mu\left[4cR_{\lambda\sigma\rho\tau}a^\sigma a^\tau\frac{dx^\lambda}{ds}\frac{dx^\rho}{ds}\right]=0$$

$$k^\nu=k^{(0)\nu}+\frac{1}{m^2}\left[-8cC^{\nu\lambda\mu\rho}k_\mu a_\lambda a_\rho\right]$$

- The above interactions result in a phase velocity,

$$k^\nu\to k^{(0)\nu}+\partial^\nu\Phi,\qquad \Phi\equiv\frac{1}{m^2}R^{\nu\lambda\mu\rho}k_\mu k_\nu a_\lambda a_\rho$$

- It can be shown that $\nabla_k\Phi=0$ and Φ is a local phase. Furthermore, the relative phase of the wave function at k and $k+dk$ is zero and this correction term result in a slight deviation of the wave-front from the zero order direction.
- The polarization dependent ray shift is related to the uncertainty in defining the ray trajectory.
- If there are interactions which the induced phase does not parallely propagate, in other words, the integral of the wave vector projection onto the ray is not zero, the acquired phase causes the rotation of the polarization plane of wave. This phase would be non-local with topological source which can substantially distort the wave-front.

Thank You