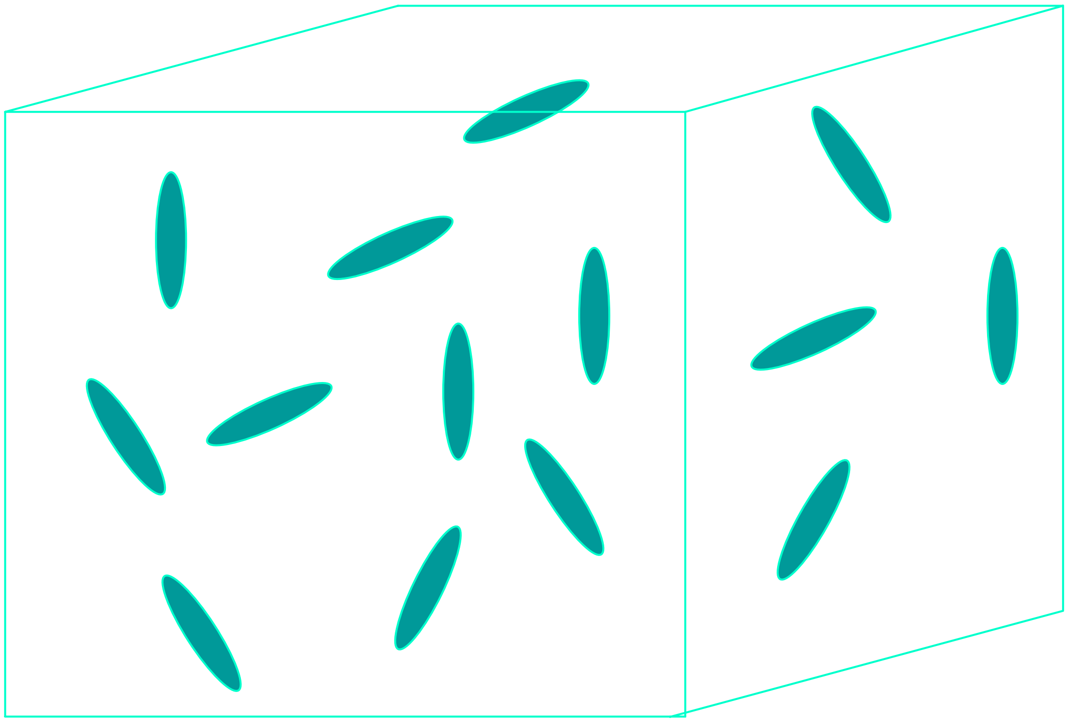
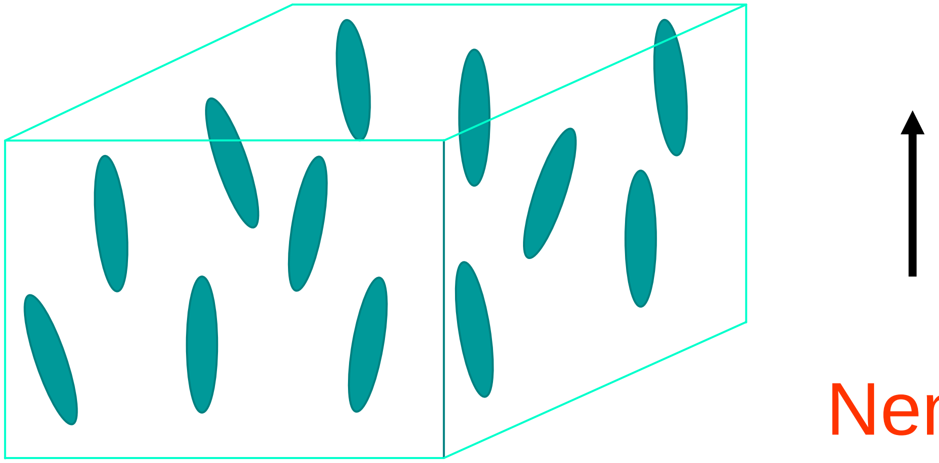
Liquid Crystals & Casimir Effect



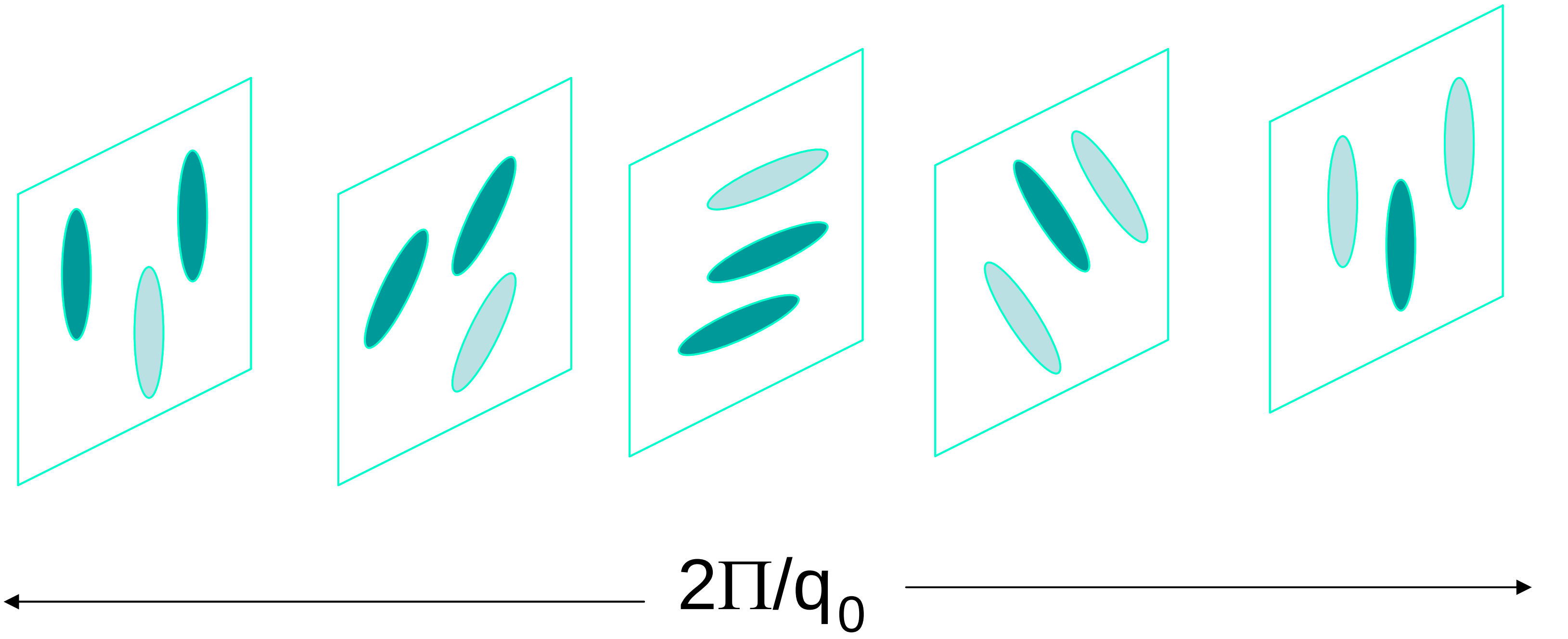
High T   isotropic phase

Low T



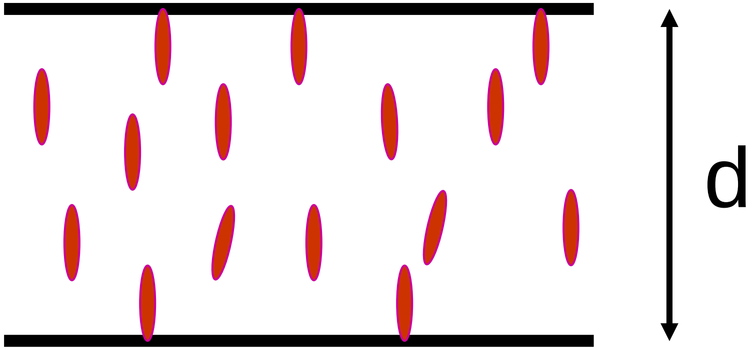
$$Q_{ij} = s(T)(n_i n_j - \tfrac{1}{3}\delta_{ij})$$

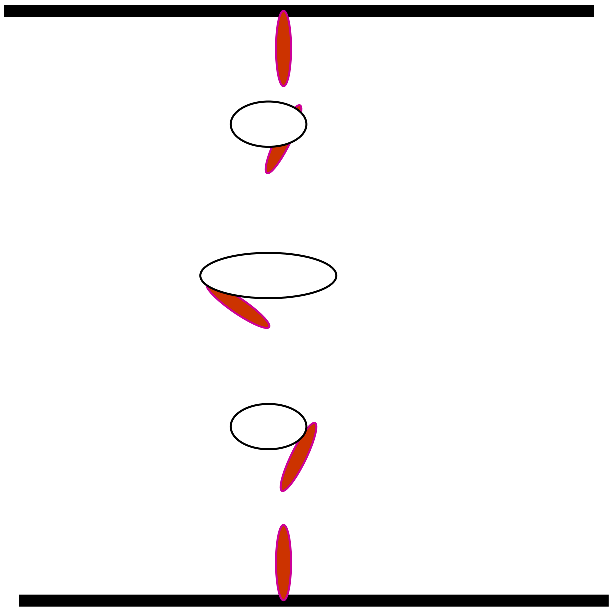
$$s(T) = \tfrac{1}{2} < 3\cos^2\theta - 1 >$$



Chiral nematic

Frank elastic free energy





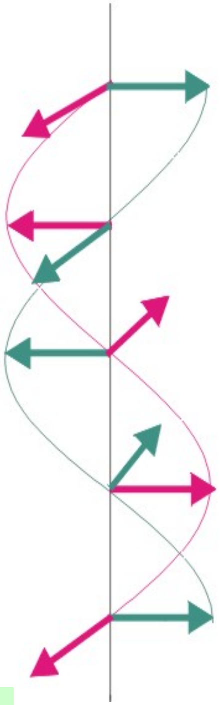
$$F_{\mathrm{el}} = \tfrac{1}{2}\int \mathrm{d}^3x \, K_{ijkl} \, \partial_i n_j \, \partial_k n_l$$

$$F_{\mathrm{el}} = \frac{1}{2}\int_V \mathrm{d}^3x \left[K_1(\boldsymbol{\nabla}\cdot\mathbf{n})^2 + K_2(\mathbf{n}\cdot\boldsymbol{\nabla}\times\mathbf{n}+q_0)^2 + K_3(\mathbf{n}\times\boldsymbol{\nabla}\times\mathbf{n})^2\right]$$

$$K_1 = K_2 = K_3 = K$$

n(r)=Z+δn(r)

$$\boxed{\delta\mathbf{n}(\mathbf{r}) = n_x(\mathbf{r})\hat{\mathbf{x}} + n_y(\mathbf{r})\hat{\mathbf{y}}}$$



$$\begin{aligned}\phi_1 &= \cos(q_0 z)n_x + \sin(q_0 z)n_y \\ \phi_2 &= -\sin(q_0 z)n_x + \cos(q_0 z)n_y\end{aligned}$$

Normal modes bear the signature of the intrinsic helical distortion present in the bulk.

In lateral Fourier space

$$\boxed{H = \frac{KA}{2}\sum_{\mathrm{p}}\sum_{w=1,2}\int_0^d \left[(\partial_z\phi_w)^2 + \left(p^2 - q_0^2\right)\phi_w^2\right]\,\mathrm{d}z}$$

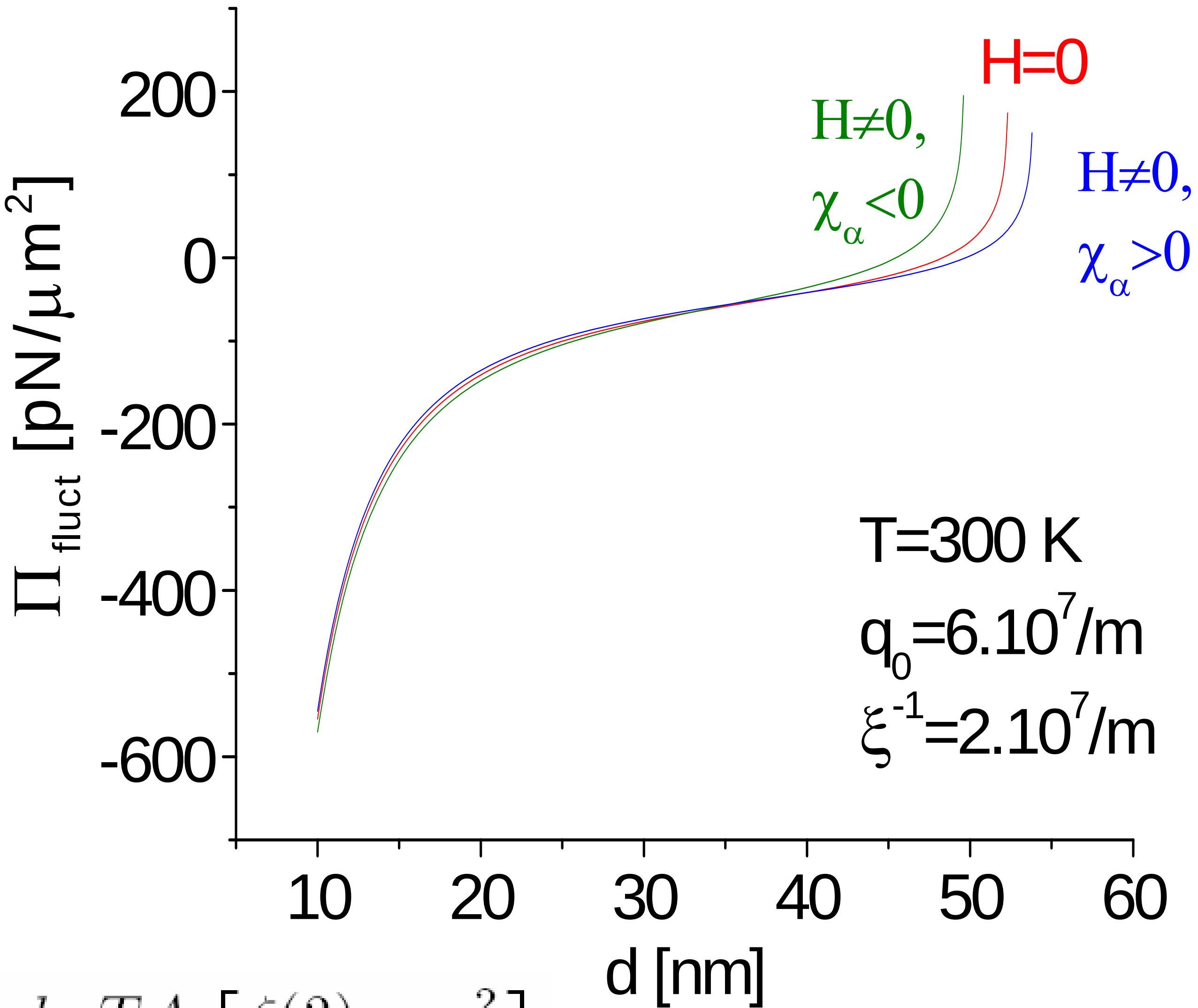
Strong anchoring

$$Z \propto \prod_{p<q_0} \left[\sin\left(\sqrt{q_0^2 - p^2 d} \right) \right]^{-1} \prod_{p>q_0} \left[\sinh\left(\sqrt{p^2 - q_0^2 d} \right) \right]^{-1}$$

$$F = -k_B T \ln Z$$

$$F = A(F_{\text{surf}}^{z=0} + F_{\text{surf}}^{z=d}) + \delta F(d) + V F_{\text{bulk}}$$

$$\mathcal{F} = -\partial F / \partial d$$



$$\mathcal{F}_{fluct}(q_0 d \ll 1) \approx -\frac{k_B T A}{4\pi}\left[\frac{\zeta(3)}{d^3} + \frac{q_0^2}{d}\right]$$

F. K.P. Haddadan, D.W. Allender, and S. Zumer, PRE, 64, 061701 (2001).

$$\mathcal{F}_{fluct}(q_0 d \to \pi) \approx -\frac{k_B T A q_0^3}{2\pi^2} \ln\left(2\sin(q_0 d)\right)$$

Magnetic field H

$$\xi = \sqrt{K/|\chi_a| H^2} \quad \text{magnetic coherence length}$$

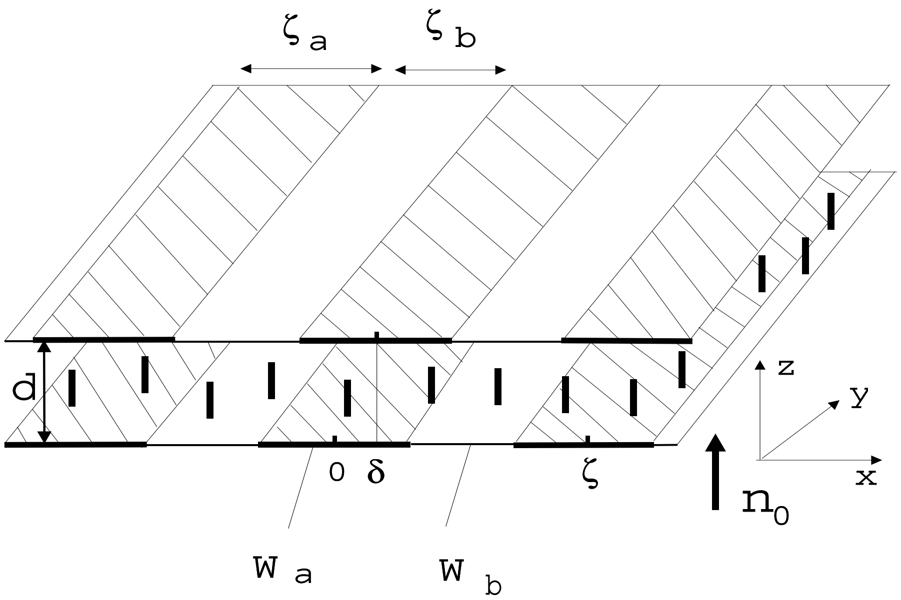
H strong and the positive anisotropy of the diamagnetic susceptibility χ_a

$$\mathcal{F}_{\mathrm{fluct}}(d \gg \xi_{eff}) = -\frac{k_B T A}{2\pi \xi_{eff}^2 d} \exp(-2d/\xi_{eff})$$

$$\xi_{eff}^{-1} = \sqrt{\xi^{-2} - q_0^2}$$

$$F_s = -\frac{1}{2}\int_A \mathrm{d}^2x\, W(\mathbf{x})\,(\mathbf{n}\cdot\mathbf{e})^2$$

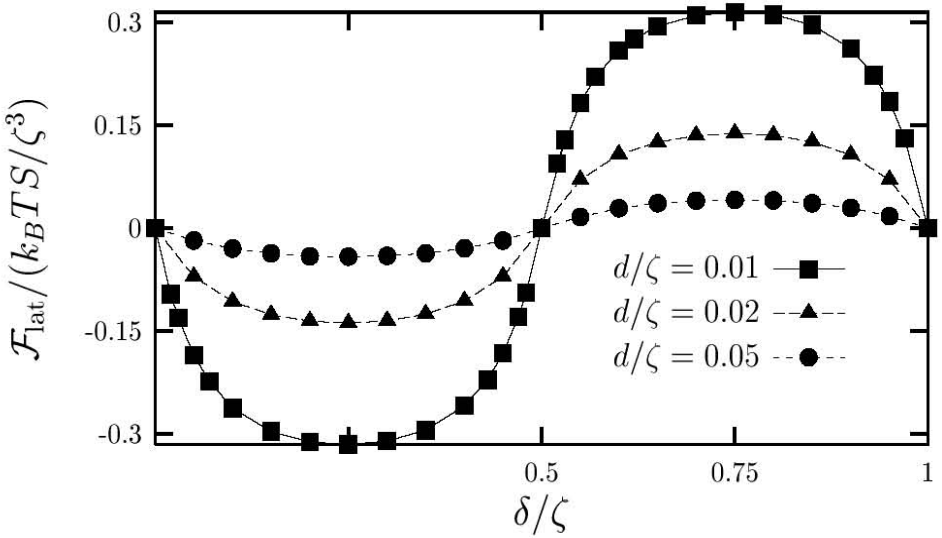
$$\lambda_{a(b)} = K/W_{a(b)}$$



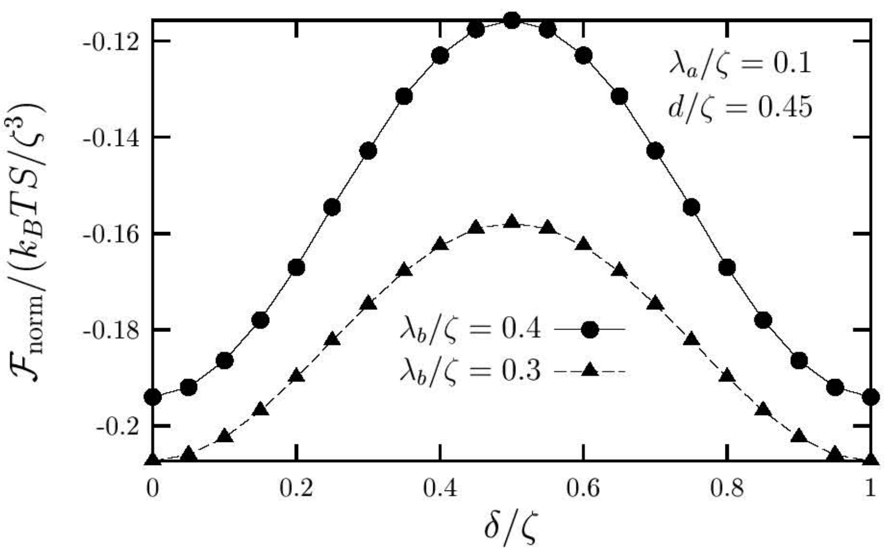
Lateral force

$$\boxed{\begin{aligned}\frac{\mathcal{F}_{\text{lat}}(d \gg \zeta)}{k_B T S} &= \frac{8(\lambda_b - \lambda_a)^2}{\pi\zeta^3\lambda_a^2} e^{-2\pi d/\zeta} \sin\left(\frac{2\pi\delta}{\zeta}\right) \\ &\times f(d/\zeta, \lambda_b/\zeta) + O\Big(\Big(\frac{\lambda_b - \lambda_a}{\lambda_a}\Big)^3\Big)\end{aligned}}$$

F.K.P. Haddadan and S. Dietrich, to be published in PRE



0.3
0.15
0
-0.15
-0.3
0.5
0.75
1
$\mathcal{F}_{\mathrm{lat}}/(k_BTS/\zeta^3)$
δ/ζ
$d/\zeta = 0.01$
$d/\zeta = 0.02$
$d/\zeta = 0.05$



-0.12
-0.14
-0.16
-0.18
-0.2
0
0.2
0.4
0.6
0.8
1
$\mathcal{F}_{\mathrm{norm}}/(k_BTS/\zeta^3)$
δ/ζ
$\lambda_a/\zeta = 0.1$
$d/\zeta = 0.45$
$\lambda_b/\zeta = 0.4$
$\lambda_b/\zeta = 0.3$

Conclusion

- Theoretical aspects of the liquid-crystalline fluctuation-induced force
- Electro-optic technology of microconfined LC
- Pattern formation of thin LC dewetting films