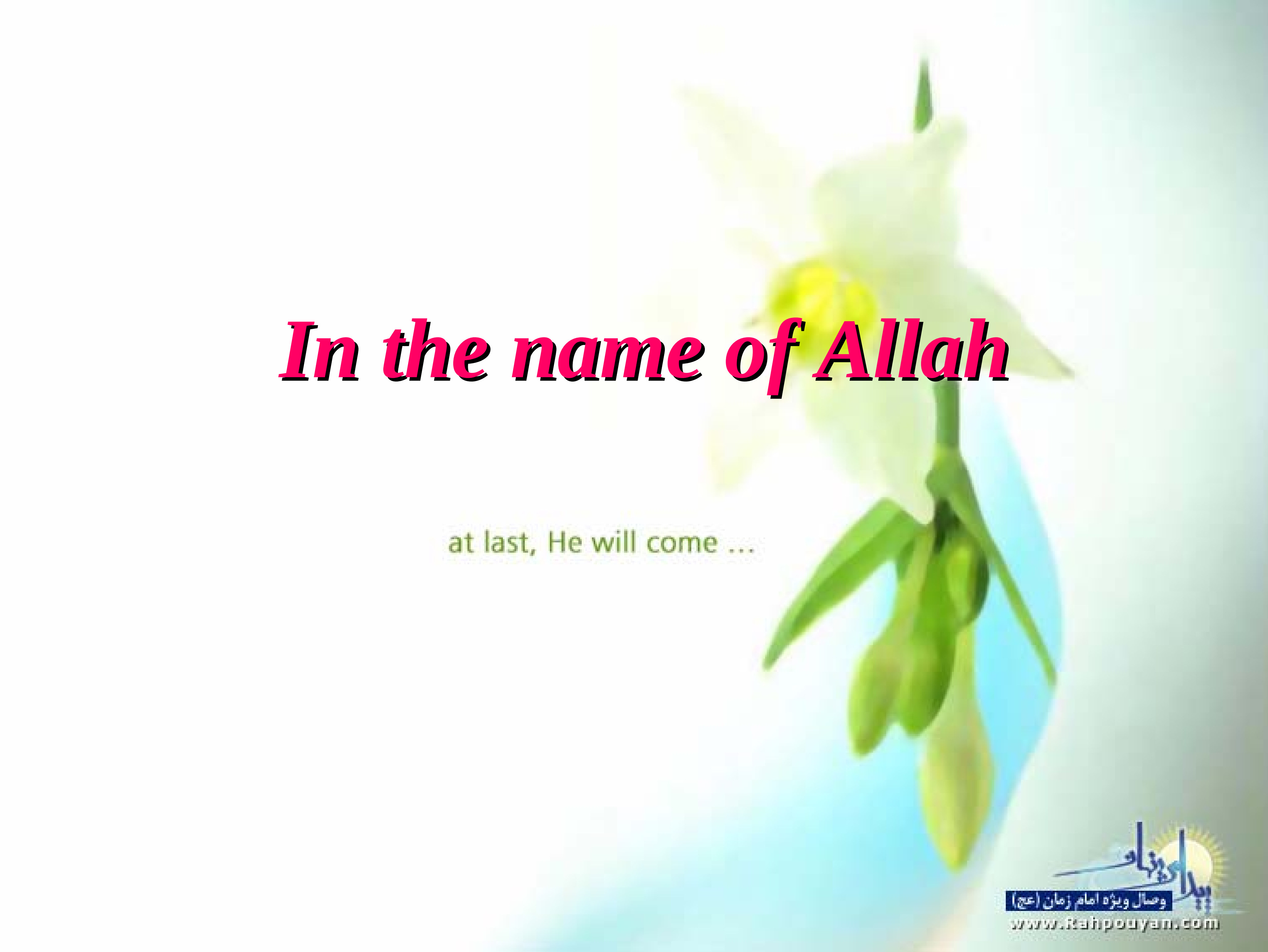
In the name of Allah



Exact rotating solutions in (n+1)-dimensional EMd gravity

by

A. Sheykhi, N. Riazi,
M. H. Dehghani and J. Pakravan
Department of Physics, Shiraz University

Contents

I. Introduction

- It seems likely that gravity is not given by the Einstein action, at least at sufficiently high energy scales. The most promising alternative seems to be that offered by string theory, where the gravity becomes scalar-tensor in nature. In the low energy limit of the string theory, one recovers Einstein gravity along with a scalar dilaton field which is non-minimally coupled to the gravity.

- The Einstein action can be modified to include a dilaton term in addition to the familiar Maxwell term, which again gets modified by a dilaton coupling.

- Thus, it is worth to find exact solutions of EMd gravity and investigate how the properties of black holes/branes are modified when a dilaton is present.

- Exact solutions for charged dilaton black hole/string have been constructed by many authors.

- The exact solutions are all static. Unfortunately, exact rotating solutions to the Einstein equation coupled to matter fields are difficult to find except in a limited number of cases (some limited values of the coupling constant , small angular momentum or small charge).

- Recently, magnetic (electric) charged rotating black string solutions in four-dimensional EMd gravity with Liouville-type potential has been constructed by Dehghani (Dehghani and Farhangkhah).
- Till now, charged rotating dilaton black hole/brane solutions for an arbitrary coupling constant in more than four dimensions has not been constructed.
- Our aim here is to construct exact, rotating charged dilaton black brane in (n+1)-dimensions for an arbitrary value of coupling constant and investigate their properties.

II. Field Equations and solutions

- The action of n+1 dimensional EMd gravity with one scalar field can be written as

$$I_G = -\frac{1}{16\pi}\int_{\mathcal{M}} d^{n+1}x\sqrt{-g}\left(\mathcal{R} - \frac{4}{n-1}(\nabla\Phi)^2 - V(\Phi) - e^{\frac{-4\alpha\Phi}{n-1}}F_{\mu\nu}F^{\mu\nu}\right) + \frac{1}{8\pi}\int_{\partial\mathcal{M}} d^n x\sqrt{-\gamma}\Theta(\gamma),$$

$$V(\Phi) = 2\Lambda e^{2\beta\Phi},$$

- One may refer to Λ as the cosmological constant, since in the absence of the dilaton field the action reduces to the action of EM gravity with cosmological constant.

The equations of motion can be obtained by varying the action with respect to the electromagnetic field , the metric and the dilaton field which yields the following field equations

$$\mathcal{R}_{\mu\nu} = \frac{4}{n-1}\left(\partial_\mu \Phi \partial_\nu \Phi + \frac{1}{4} g_{\mu\nu} V(\Phi)\right) + 2e^{\frac{-4\alpha\Phi}{n-1}}\left(F_{\mu\eta}F_\nu{}^\eta - \frac{1}{2(n-1)} g_{\mu\nu} F_{\lambda\eta}F^{\lambda\eta}\right),$$

$$\nabla^2\Phi = \frac{n-1}{8}\frac{\partial V}{\partial \Phi} - \frac{\alpha}{2} e^{-\frac{4\alpha\Phi}{n-1}} F_{\lambda\eta}F^{\lambda\eta},$$

$$\partial_\mu\left(\sqrt{-g}\, e^{\frac{-4\alpha\Phi}{n-1}} F^{\mu\nu}\right) = 0,$$

Our aim here is to construct the n+1 dimensional rotating solutions of the above field equations.

- The metric of n+1 dimensional rotating solution with cylindrical or toroidal horizons and k rotation parameters can be written as (Awad 2003):

$$ds^2 = -f(r)\left(\Xi dt - \sum_{i=1}^{k} a_i d\phi_i\right)^2 + \frac{r^2}{l^4}R^2(r)\sum_{i=1}^{k}\left(a_i dt - \Xi l^2 d\phi_i\right)^2$$
$$-\frac{r^2}{l^2}R^2(r)\sum_{i<j}^{k}(a_i d\phi_j - a_j d\phi_i)^2 + \frac{dr^2}{f(r)} + \frac{r^2}{l^2}R^2(r)dX^2,$$
$$\Xi^2 = 1 + \sum_{i=1}^{k}\frac{a_i^2}{l^2},$$

- where a_{i}'s are k=[n/2] rotation parameters.
- The functions f(r) and R(r) should be determined.
- The angular coordinates are in the range 0 < φ _{i} <2 π .
- dX^2 is the Euclidean metric on the (n-k-1)-dimensional submanifold with volume Σ _{n-k-1}.

- The Maxwell equation can be integrated immediately to give

$$F_{tr} = \frac{q\Xi e^{\frac{4\alpha\Phi}{n-1}}}{(rR)^{n-1}}$$

$$F_{\phi_i r} = -\frac{a_i}{\Xi} F_{tr}.$$

- where q, is an integration constant related to the electric charge of the brane.
- In order to solve the system of equations for three unknown functions f(r), R(r) and Φ(r), we make the anzats

$$R(r) = e^{\frac{2\alpha\Phi}{n-1}}$$

- One can easily show that above field equations have solutions of the form.

$$f(r) = \frac{2\Lambda(\alpha^2+1)^2 b^{2\gamma}}{(n-1)(\alpha^2-n)} r^{2(1-\gamma)} - \frac{m}{r^{(n-1)(1-\gamma)-1}} + \frac{2q^2(\alpha^2+1)^2 b^{-2(n-2)\gamma}}{(n-1)(\alpha^2+n-2)} r^{2(n-2)(\gamma-1)},$$

$$\Phi(r) = \frac{(n-1)\alpha}{2(1+\alpha^2)} \ln\left(\frac{b}{r}\right),$$

- where b and m are arbitrary constant and $\gamma = \frac{\alpha^2}{\alpha^2+1}$.
- In order to fully satisfy the system of equations, we must have

$$\beta = \frac{2}{n-1}\alpha.$$

Properties of solutions

- $\alpha=0$: the solution reduces to asymptotically (A)dS charged rotating black branes.
- $n=3$: the solution reduces to the four-dimensional charged rotating dilaton black strings.
- The spacetime is neither asymptotically flat nor (A)dS.
- The Kretschmann and Ricci scalars diverge at $r=0$, they are finite for $r \neq 0$ and go to zero at infinity. Thus, there is an essential singularity located at $r=0$.
- $\alpha = \sqrt{n}$: we have no solution.

- $\alpha > \sqrt{n}$: at infinity the dominant term is the second term, and therefore the spacetime has a cosmological horizon for positive values of the mass parameter m, despite the sign of the cosmological constant Λ.

- $\alpha < \sqrt{n}$: at infinity the dominant term is the first term and we have a cosmological horizon if $\Lambda>0$ and no cosmological horizon if $\Lambda<0$.

- $\alpha < \sqrt{n}$ and $\Lambda<0$: we have black brane solutions.

- One can obtain the casual structure by finding the roots of $f(r)=0$. Unfortunately, it is not possible to find explicitly the location of horizons for an arbitrary value of α.

III. Thermodynamics of black brane

- The temperature and i'th component of angular velocity of the horizon are

$$T_+ = \frac{f'(r_+)}{4\pi\Xi} = \frac{1}{4\pi\Xi}\left(\frac{(n-\alpha^2)m}{\alpha^2+1}r_+{}^{(n-1)(\gamma-1)} - \frac{4q^2(\alpha^2+1)b^{-2(n-2)\gamma}}{(\alpha^2+n-2)r_+{}^{\gamma}}r_+{}^{(2n-3)(\gamma-1)}\right)$$

$$= -\frac{2(1+\alpha^2)}{4\pi\Xi(n-1)}\left(\Lambda b^{2\gamma}r_+^{1-2\gamma} + \frac{q^2b^{-2(n-2)\gamma}}{r_+{}^{\gamma}}r_+{}^{(2n-3)(\gamma-1)}\right),$$

$$\Omega_i = \frac{a_i}{\Xi l^2}.$$

- The temperature is negative for the two cases:
- i) $\alpha > \sqrt{n}$ despite the sign of Λ.
- ii) $\Lambda > 0$ despite the value of α.

- The finite stress-energy tensor in (n+1)-dimensional Einstein-dilaton gravity with Liouville-type potential may be written as

$$T^{ab} = \frac{1}{8\pi}\left[\Theta^{ab} - \Theta\gamma^{ab} + \frac{n-1}{l_{\mathrm{eff}}}\gamma^{ab}\right],$$

$$l_{\mathrm{eff}}^2 = \frac{(n-1)(\alpha^2 - n)}{2\Lambda} e^{\frac{-4\alpha\phi}{n-1}}.$$

- For α=0 , the 1_{eff} reduces to 1^2=-n(n-1)/2Λ of the (A)dS spacetimes.
- The first two terms is the variation of the action with respect to γ ^{ab}, and the last term is the counterterm which removes the divergences.

- To compute the conserved charges of the spacetime, one should choose a spacelike surface B in ∂M with metric $\sigma\{ij\}$.
- Then, the quasilocal mass and angular momentum can be written as

$$
\begin{aligned}
M &= \int_B d^{n-1}\varphi \sqrt{\sigma} T_{ab} n^a \xi^b, \\
J &= \int_B d^{n-1}\varphi \sqrt{\sigma} T_{ab} n^a \varsigma^b,
\end{aligned}
$$

- where σ is the determinant of the metric $\sigma\{ij\}$.
- n^{a} is the unit normal vector, ($\xi = \partial / \partial t$) and ($\varsigma = \partial / \partial \phi$) are timelike and rotational Killing vector fields on the boundary B .

- Denoting the volume of the hypersurface boundary at constant t and r by $V_{n-1}=(2\pi)^{k}\Sigma_{n-k-1}$, the mass and angular momentum per unit volume V_{n-1} of the black branes ($\alpha < \sqrt{n}$) can be calculated

$$M=\frac{b^{(n-1)\gamma}}{16\pi l^{n-2}}\left(\frac{(n-\alpha^2)\Xi^2+\alpha^2-1}{1+\alpha^2}\right)m, \qquad J_i=\frac{b^{(n-1)\gamma}}{16\pi l^{n-2}}\left(\frac{\alpha^2-n}{1+\alpha^2}\right)\Xi m a_i.$$

- For $a_{i}=0$, the angular momentum per unit volume vanishes, and therefore a_{i}'s are the rotational parameters of the spacetime.

- Black hole/brane entropy typically satisfies the so called area law, which states that the entropy of the black hole equals one- quarter of the area of its horizon. the entropy per unit volume V_{n-1} of the black brane is

$$S = \frac{\Xi b^{(n-1)\gamma} r_+^{(n-1)(1-\gamma)}}{4 l^{n-2}}$$

- Next, we calculate the electric charge of the solutions. To determine the electric field we should consider the projections of the electromagnetic field tensor on special hypersurfaces. Then the electric field is

$$E^{\mu} = g^{\mu\rho} e^{\frac{-4\alpha\phi}{n-1}} F_{\rho\nu} u^{\nu},$$

- and the electric charge per unit volume V_{n-1} can be found by calculating the flux of the electric field at infinity, yielding

$$Q = \frac{\Xi q}{4\pi l^{n-2}}.$$

- The electric potential U, measured at infinity with respect to the horizon, is defined by

$$U = A_\mu \chi^\mu \big|_{r\to\infty} - A_\mu \chi^\mu \big|_{r=r_+},$$

- where χ is the null generators of the event horizon given

$$\chi = \partial_t + \sum_{i=1}^{k} \Omega_i \partial_{\phi_i},$$

The vector potential A_{μ } corresponding to electromagnetic tensor, can be written as

$$A_\mu = \frac{q b^{(3-n)\gamma}}{\Gamma r^\Gamma} \left(\Xi \delta_\mu^t - a_i \delta_\mu^i \right) \quad \text{(no sum on i)}.$$

$$\Gamma = (n-3)(1-\gamma) + 1$$

- Therefore the electric potential may be obtained as

$$U = \frac{qb^{(3-n)\gamma}}{\Xi\Gamma {r_+}^{\Gamma}}.$$

Finally, we consider the first law of thermodynamics for the black brane. Although it is difficult to obtain the mass M as a function of the extensive quantities S, J and Q for an arbitrary values of α, but one can show numerically that the thermodynamic quantities calculated in this section satisfy the first law of thermodynamics

$$dM = TdS + \sum_{i=1}^{k} \Omega_i dJ_i + UdQ,$$

Conclusions

- Unfortunately, exact rotating solutions to the Einstein equation coupled to matter fields are difficult to find except in a limited number of cases.
- Indeed, no explicit rotating charged black hole solutions have been found except for some dilaton coupling such as $\alpha = \sqrt{3}$ or $\alpha = 1$.
- For general dilaton coupling, the properties of charged dilaton black holes have been investigated only for rotating solutions with infinitesimally small angular momentum or small charge.
- Till now, charged rotating dilaton black hole/brane solutions for an arbitrary coupling constant in more than four dimensions has not been constructed.

- We constructed a class of (n+1)-dmensional charged rotating dilaton black brane solutions with Liouville-type potentials .
- These solutions are neither asymptotically flat nor (A)dS.
- In the presence of Liouville-type potential, we obtained exact solutions provided $\alpha \neq \sqrt{n}$ and $\beta = 2\alpha/(n-1)$.
- We also computed the conserved and thermodynamics quantities of the (n+1)-dimensional rotating charged black brane, and found that they satisfy the first law of thermodynamics

Current research

- Investigation of the stability of the above solutions.
- Magnetic rotating solutions in n+1 dimensional EMd gravity.
- Exact rotating solutions in n+1 dimensional Gauss-Bonnet dilaton gravity

and

Thank you very much for your attention

at last, He will come …