An exploration into the nature of using valon model for calculating polarized parton distribution functions and hadron structure function

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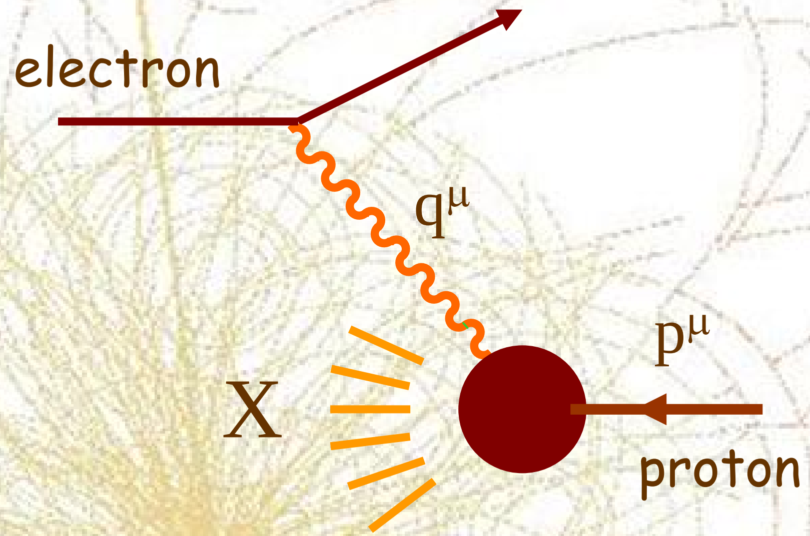
OutLine:

- Study of nucleon structure function
  - Using of Valon model for study of nucleon structure functions and pdf
    - Polarized nucleon structure function in valon model
      - Some notes about polarized sea quark
        - My Results

DIS experiment



$$Q^2 = -q^2 = -(k - k')^2$$

$$x = \frac{Q^2}{2P.q} = \frac{Q^2}{2M_N \nu}$$

$$\nu = \frac{P.q}{M_N} = E - E'$$

$$\frac{d^2\sigma}{dE'd\Omega} = 4E'\frac{\alpha^2}{Q^2}[2\sin^2(\frac{\theta}{2})W_1(Q^2,\nu) + \cos^2(\frac{\theta}{2})W_2(Q^2,\nu)]$$

With define:

$$M_N W_1(Q^2,\nu) = F_1(Q^2.x)$$

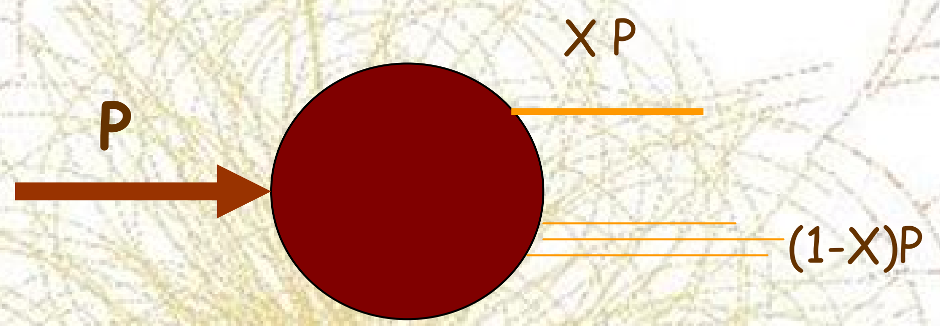
$$\nu W_2(Q^2,\nu) = F_2(Q^2.x)$$

$$y = \frac{\nu}{E} = \frac{E - E'}{E} = 1 - \frac{E'}{E}$$

Now,We can get F1 & F2 from experiment…

Quark-Parton Model

$$f_i(x) = \frac{dP_i}{dx}$$



In parton model we have :

$$F_1(x) = \frac{1}{2}\sum_i f_i(x) q_i^2$$

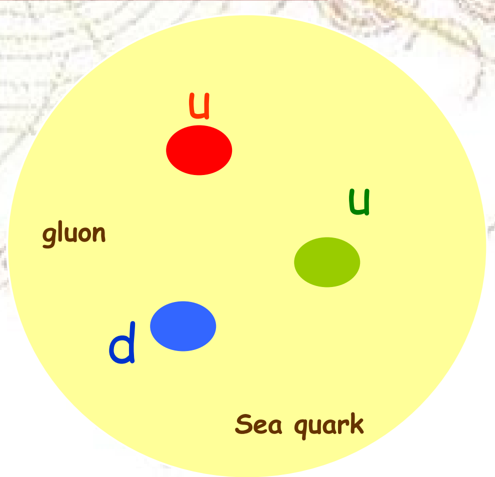
$$F_2(x) = \sum_i f_i(x) q_i^2 x$$

So we have:

$$F_2(x) = x\{\frac{4}{9}[u(x) + \bar{u}(x)] + \frac{1}{9}[d(x) + \bar{d}(x) + s(x) + \bar{s}(x)]\}$$

With define: $u(x) = u_v(x) + \bar{u}(x)$ $d(x) = d_v(x) + \bar{d}(x)$

We have: $\int_0^1 (u(x) - \bar{u}(x))dx = 2$ $\int_0^1 (d(x) - \bar{d}(x))dx = 1$



Glap equations give quark and gluon evolution:

$$\frac{dq_i(x,Q^2)}{dLnQ^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{d\xi}{\xi}[q_i(\xi,Q^2)P_{qq}(\frac{x}{\xi})+g(\xi,Q^2)P_{qg}(\frac{x}{\xi})]$$

$$\frac{dg(x,Q^2)}{dLnQ^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{d\xi}{\xi}[\sum q_i(\xi,Q^2)P_{gq}(\frac{x}{\xi})+g(\xi,Q^2)P_{gg}(\frac{x}{\xi})]$$

Mellin transformation:

$$M(n,Q^2)=\int_0^1 dx x^{n-1} f(x,Q^2)$$

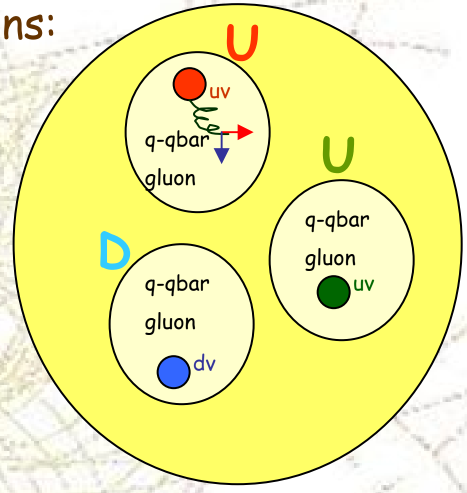
With use Mellin transformation we have:

$$\frac{d}{dt}M_{qv}(n,Q^2)=A_{qq}(n)M_{qv}(n,Q^2)$$

So: we can derive parton distribution in proton by solving GLAP. eq in moment space and an inverse Mellin transformation.

Nucleon structure function in valon framework

Proton



The structure function of a hadron is convolution of two distributions:

- valon distributions in proton
- quark distributions in a valon.

In an unpolarized situation we can write:

$$F_2^p(x, Q^2) = \sum_v \int_x^1 dy G_{v/p}(y) \mathcal{F}_2^v\left(\frac{x}{y}, Q^2\right) ,$$

Proton structure function

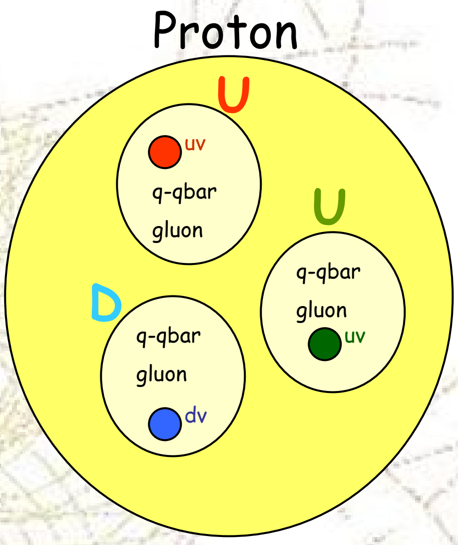
Summation is over the three valons

Describes the valon distribution in a proton. It independs on Q^2.

Structure function of a v valon. It depends on Q^2 and the nature of the probe.

Parton distribution function in valon model:PDF

If you know PDF in a valon,you can get PDF in proton as:



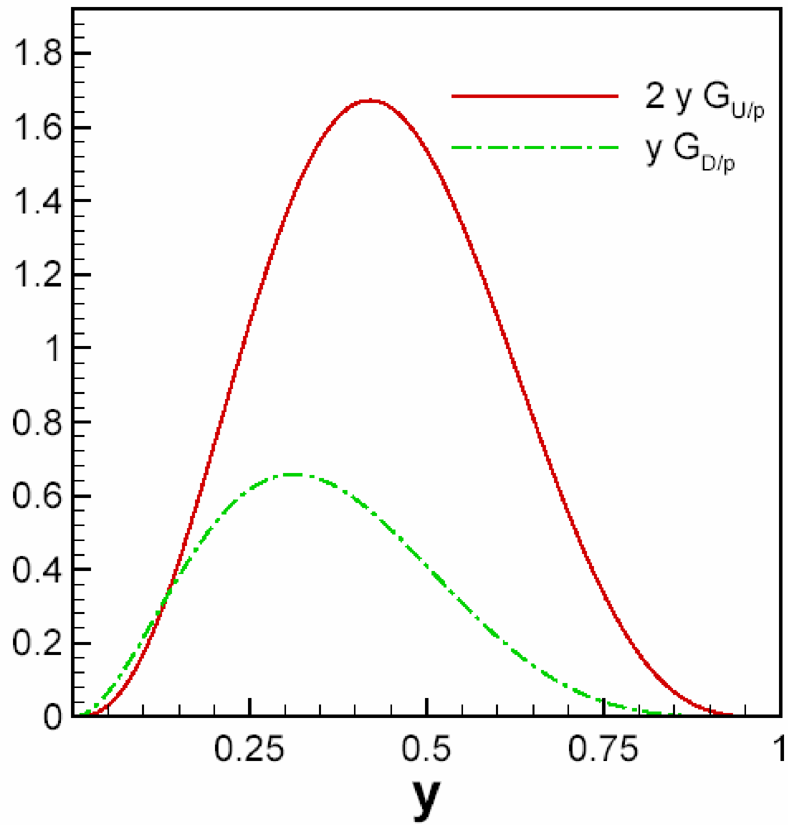
$$q_{i/p}(x,Q^2)=\sum_j\int_x^1 q_{i/j}(\frac{x}{y},Q^2)G_{j/p}(y)\frac{dy}{y}$$

PDF in proton

Summation is over the three valons

PDF in a valon. It comes from solving GLAP.eq in a valon

Valon distribution in proton

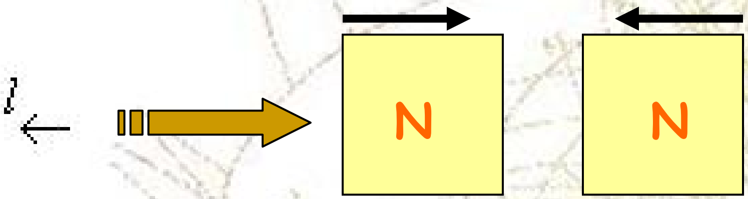


Unpolarized valon distributions as function of *y*.

R. C. Hwa and C. B. Yang, *Phys. Rev.* C 66 (2002)

Firooz Arash & Ali.N.Khorramian-Physical Review C 67,045201 (2003)

Polarized Nucleon Structure functions in valon framework:



$$\sigma_{\rightrightarrows}^{\leftarrow} - \sigma_{\leftleftarrows}^{\leftarrow} \approx g_1$$

Where:

$$g_1(x,Q^2) = \sum_q e_q{}^2(\delta q(x,Q^2) + \overline{\delta q}(x,Q^2))$$

$$\delta q(x) = q\uparrow(x) - q\downarrow(x)$$

Probability to find parton with spin aligned/anti-aligned to proton spin

Analogy with un polarized case:

$$F_1(x,Q^2) = \frac{1}{2}\sum_q e_q{}^2(q(x,Q^2) + \bar{q}(x,Q^2))$$

$$g_1(x,Q^2) = \frac{1}{2}\sum_q e_q{}^2(\delta q(x,Q^2) + \overline{\delta q}(x,Q^2))$$

$$q(x) = q\uparrow(x) + q\downarrow(x)$$

$$\delta q(x) = q\uparrow(x) - q\downarrow(x)$$

The first moment is a measure for the quark contribution to the proton spin:

$$\Delta q = \int_0^1 \delta q(x, Q^2)\, dx$$

Now we use valon model derive polarized parton distribution (PPDF) and polarized structure function:

$$\delta q_{i/p}(x, Q^2) = \sum_j \int_x^1 \delta q_{i/j}\left(\frac{x}{y}, Q^2\right) \delta G_{j/p}(y) \frac{dy}{y}$$

Polarized PDF in proton

Summation is over the three valons

Polarized PDF in a valon

Valon helicity distribution in proton

Polarized hadron structre function in valon framework is :

$$g_1^p(x,Q^2)=\sum_{\nu}\int_x^1\frac{dy}{y}\delta G_{\frac{\nu}{p}}(y)f_1^{\nu}(\frac{x}{y},Q^2),$$

Polarized structure function of proton

Summation is over the three valons

Valon helicity distribution in proton

Polarized structure function of a ν valon. It depends on Q^2 and the nature of the probe.

Now, we construct valon model. Let's go to calculation of ppdf & polarized structure function with this model…

DGlAP eq. in mellin space at NLO approximation gives ppdf in a valon :

Spin physics and polarized structure function/Reya &Lampe(hep-ph/9810270)

$$\delta q_{NS}^{n}(Q^2) = \{1+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{2\pi}(\frac{-2}{\beta_0})(\delta P_{NS}^{(1)n}-\frac{\beta_1}{\beta_0}\delta P_{qq}^{(0)n})\}L^{\frac{-2}{\beta_0}\delta P_{qq}^{(0)n}}$$

$$\begin{pmatrix}\delta\Sigma^{n}(Q^2)\\ \delta g^{n}(Q^2)\end{pmatrix} = \{L^{\frac{-2}{\beta_0}\hat{P}^{(0)n}} + \frac{\alpha_s(Q^2)}{2\pi}\hat{U}L^{\frac{-2}{\beta_0}\hat{P}^{(0)n}} - \frac{\alpha_s(Q_0^2)}{2\pi}L^{\frac{-2}{\beta_0}\hat{P}^{(0)n}}\hat{U}\}\begin{pmatrix}1\\1\end{pmatrix}$$

With solving above equation, We can derive ppdf in a valon

$$\delta q_{NS} \approx \delta q - \overline{\delta q} \approx \delta q_{Valence}$$

gives Valence quark distribution

$$\delta\Sigma = \sum_q (\delta q + \overline{\delta q})$$

gives Valence & sea quark distribution

$$\frac{\alpha_s(Q^2)}{4\pi} \simeq \frac{1}{\beta_0 \ln Q^2/\Lambda^2} - \frac{\beta_1}{\beta_0^3}\frac{\ln\ln Q^2/\Lambda^2}{(\ln Q^2/\Lambda^2)^2}$$

$$Q_0^2 = 1Gev^2$$

$$\Lambda_{QCD}^2 = 235Gev^2$$

qbar polarization is very small

$$\Delta\Sigma^{n=1} = \int_0^1 \Sigma(z)dz = 0.9825$$

$$\Delta q_{NS}^{n=1} = \int_0^1 qvalence(z)dz = 1.01$$

$$\overline{\Delta q}_{total}^{valon} \cong -0.01$$

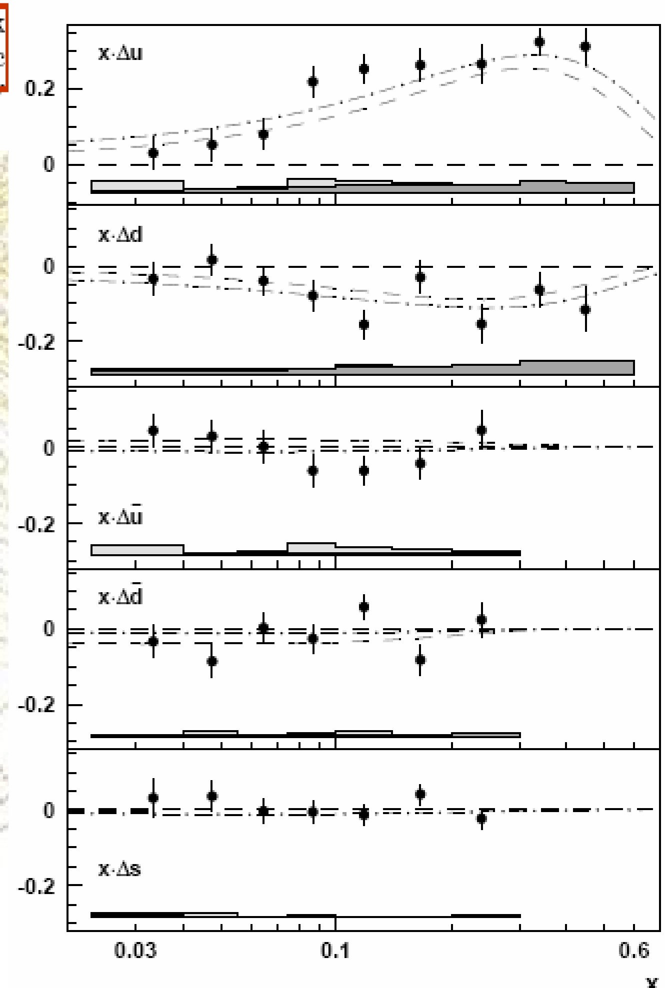
Quark helicity distributions in the nucleon for *up*, *down*, and *strange* quarks from semi–inclusive deep–inelastic scattering

Polarized deep–inelastic scattering data on longitudinally polarized hydrogen and deuterium targets have been used to determine double spin asymmetries of cross sections. Inclusive and semi–inclusive asymmetries for the production of positive and negative pions from hydrogen were obtained in a re–analysis of previously published data. Inclusive and semi–inclusive asymmetries for the production of negative and positive pions and kaons were measured on a polarized deuterium target. The separate helicity densities for the up and down quarks and the anti–up, anti–down, and strange sea quarks were computed from these asymmetries in a "leading order" QCD analysis. The polarization of the up–quark is positive and that of the down–quark is negative. All extracted sea quark polarizations are consistent with zero, and the light quark sea helicity densities are flavor symmetric within the experimental uncertainties. First and second moments of the extracted quark helicity densities in the measured range are consistent with fits of inclusive data.

The HERMES Collaboration (2004)

hep-ex/0307064

hep-ex/0407032



In the above calculation , we neglected **sea quarks polarization. They are zero!**

Structure function of hadron :

$$\boxed{g_1^n(Q^2) = \frac{1}{2}\sum_q e_q^2\{(1+\frac{\alpha_s}{2\pi}\delta C_q^n)[\delta q^n(Q^2)+\delta\bar{q}^n(Q^2)]+\frac{\alpha_s}{2\pi}2\delta C_g^n\delta g^n(Q^2)\}}$$

$$\boxed{g_1^{U(n)} = (1+\frac{\alpha_s(Q^2)}{2\pi}\delta C_q^n)\frac{2}{9}\delta q_{NS}^n(Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\frac{2}{3}\delta C_g^n\delta g^n(Q^2)}$$

U valon structure function

$$\boxed{g_1^{D(n)} = (1+\frac{\alpha_s(Q^2)}{2\pi}\delta C_q^n)\frac{1}{18}\delta q_{NS}^n(Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\frac{2}{3}\delta C_g^n\delta g^n(Q^2)}$$

D valon structure function

$$\boxed{g_1^p = 2\delta G^{U(n)} g_1^{U(n)} + \delta G^{D(n)} g_1^{D(n)}}$$

Proton structure function

$$\boxed{\delta u_v(n,Q^2) = 2\delta q_{NS}^n(Q^2)\delta G^{U(n)},}$$

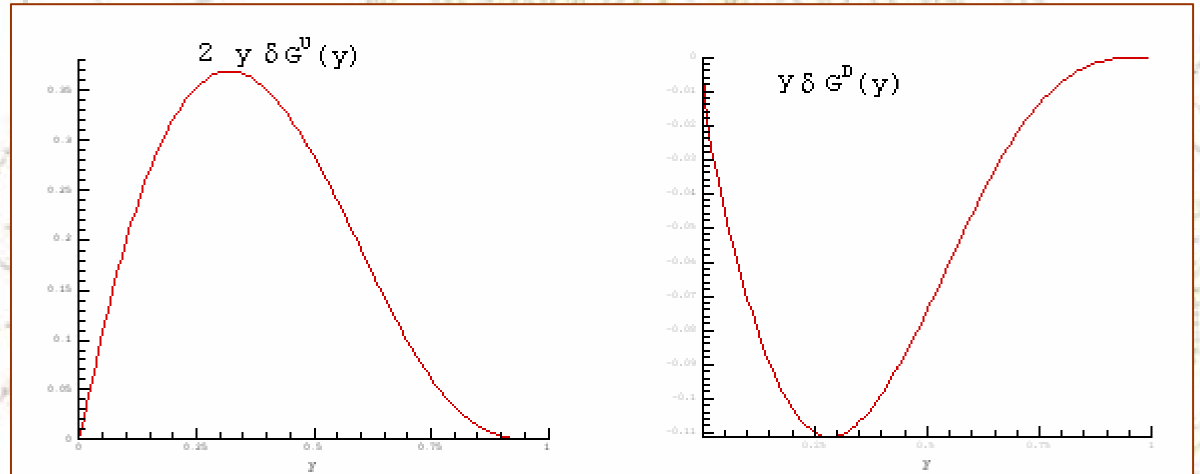
$$\boxed{\delta d_v(n,Q^2) = \delta q_{NS}^n(Q^2)\delta G^{D(n)},}$$

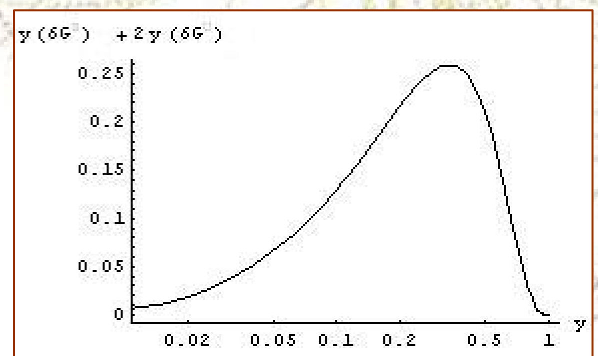
$$\boxed{\delta g(n,Q^2) = \delta g^n(Q^2)(2\delta G^{U(n)} + \delta G^{D(n)})}$$

ppdf in proton

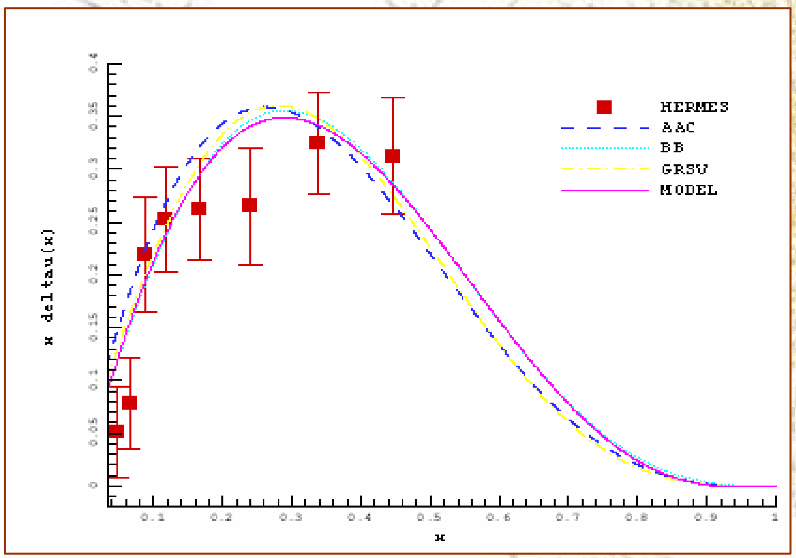
Polarized valon distributions

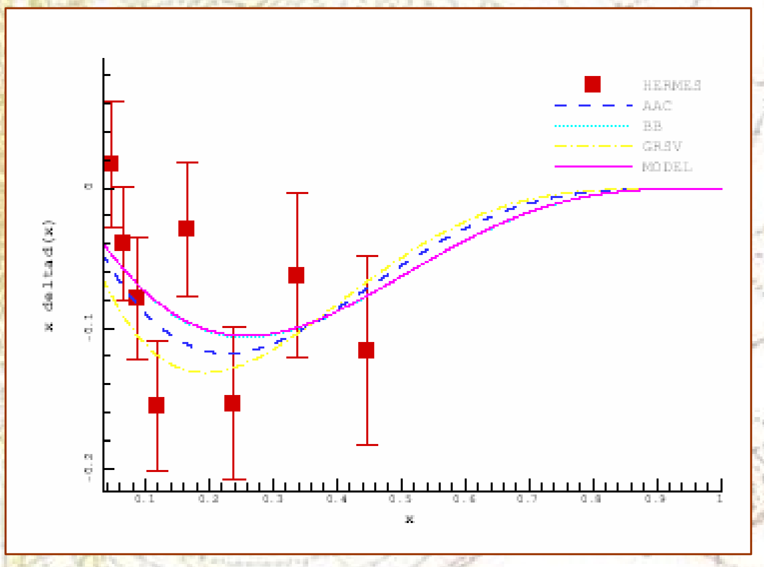
I extract them from fit to BB model.

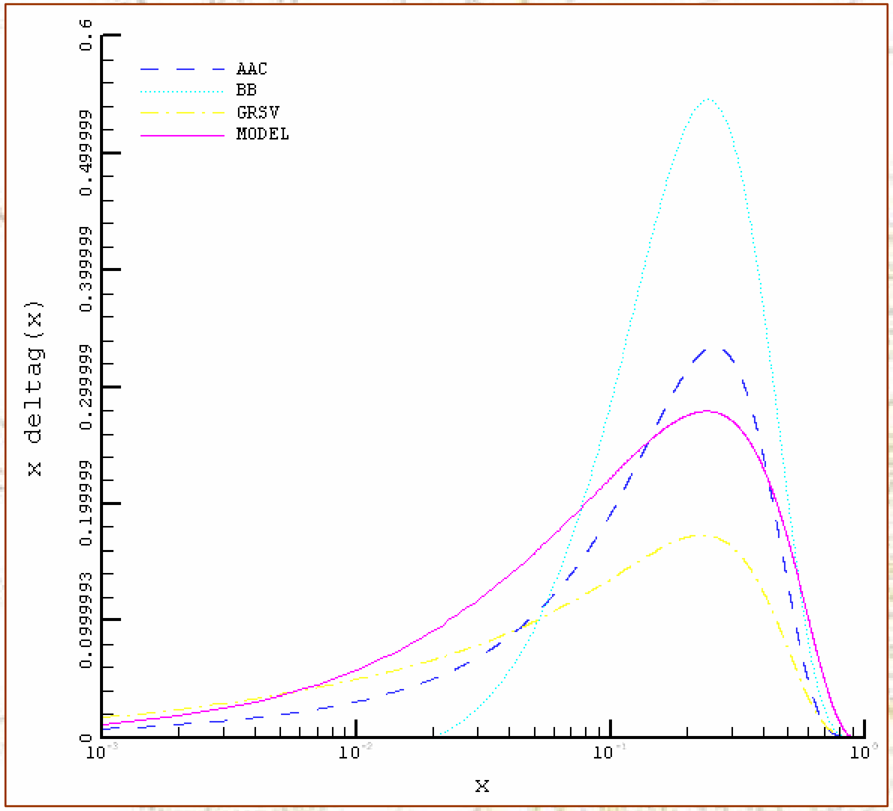




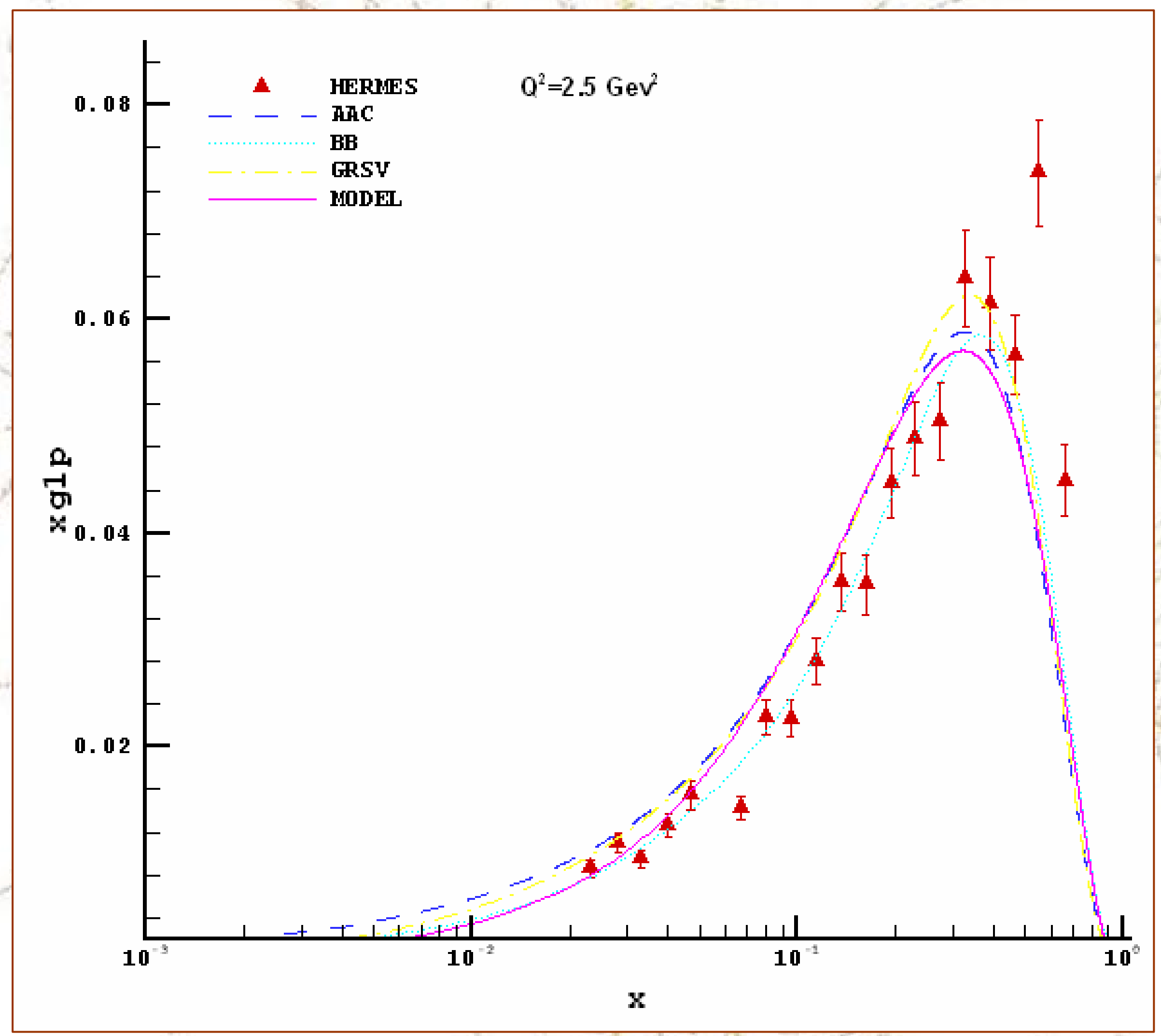
PPDF in proton :



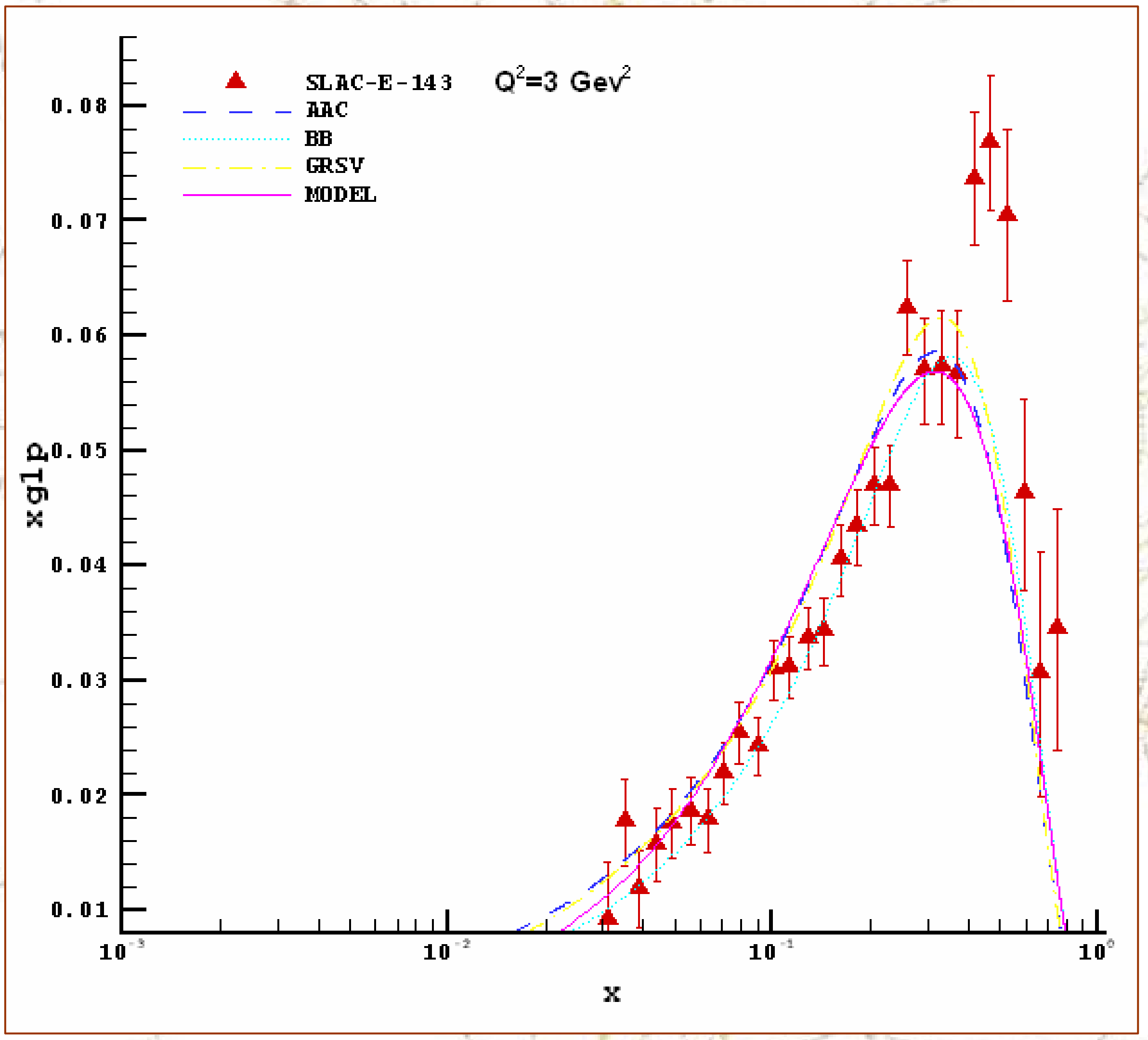




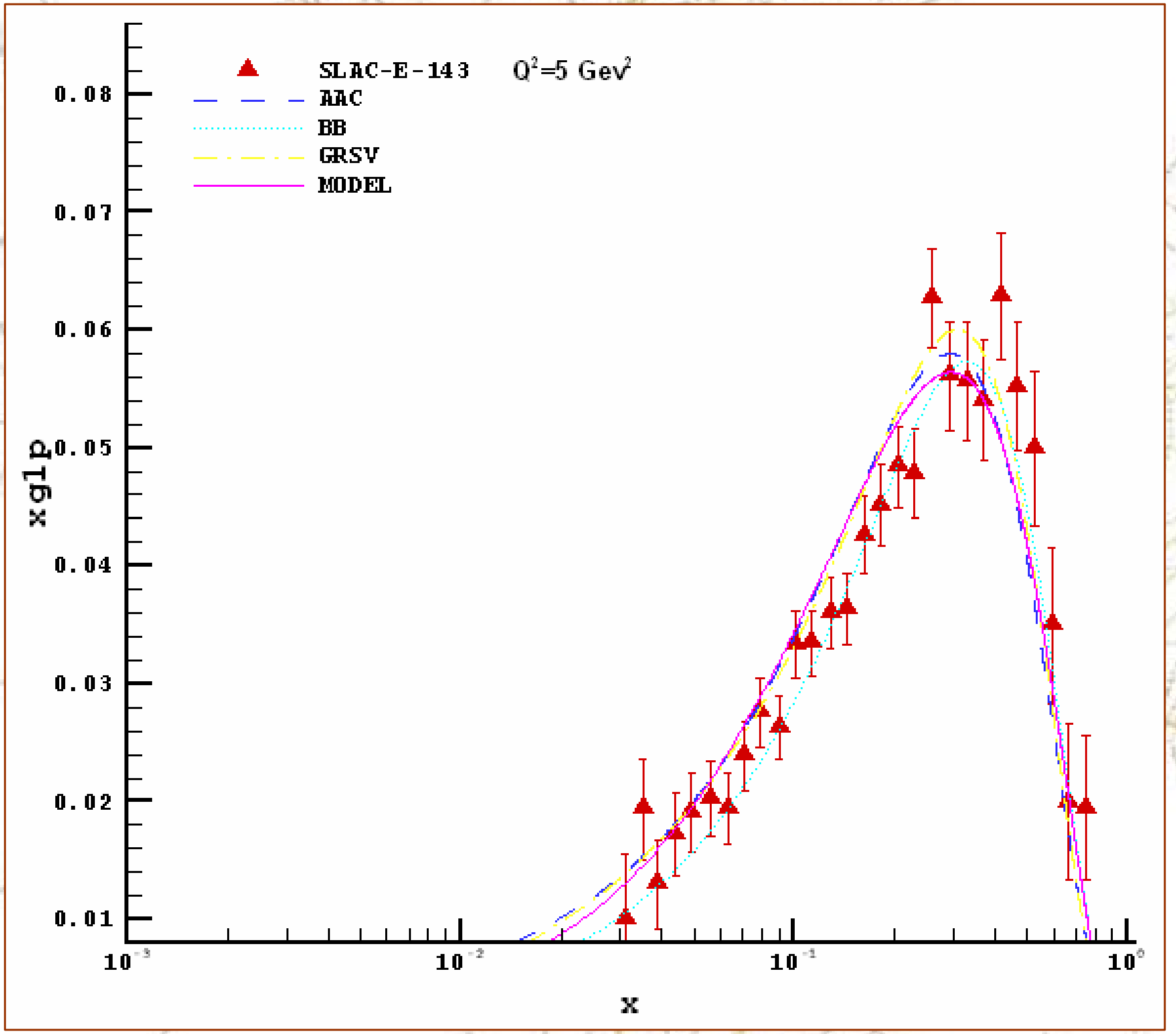
Gluon polarization



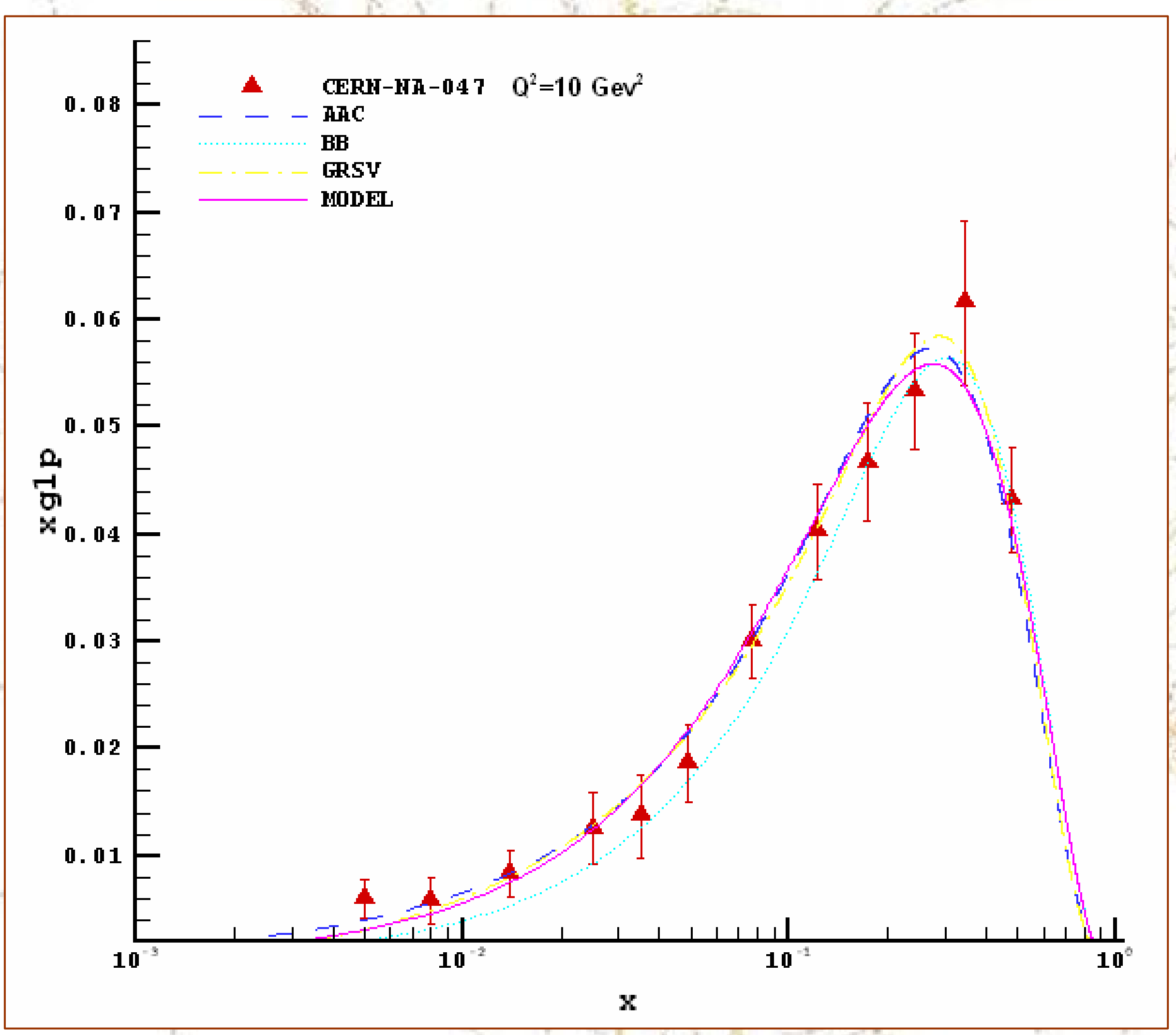
HERMES
AAC
BB
GRSV
MODEL
Q²=2.5 Gev²
x g1p
0.08
0.06
0.04
0.02
10⁻³
10⁻²
10⁻¹
10⁰
x



SLAC-E-143
Q^2=3 Gev^2
AAC
BB
GRSV
MODEL
x g1 p
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
10^{-3}
10^{-2}
10^{-1}
10^{0}
x



SLAC-E-143 Q^2=5 Gev^2
AAC
BB
GRSV
MODEL
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
xg1p
10^{-3}
10^{-2}
10^{-1}
10^{0}
x



CERN-NA-047 Q^2=10 Gev^2
AAC
BB
GRSV
MODEL
xg1p
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
10^{-3}
10^{-2}
10^{-1}
10^{0}
x

$$\Gamma_1^p(Q^2) = \int_0^1 g_1(x,Q^2)dx$$

Experimental results:

Experiment	$\Gamma_1(Q^2/GeV^2)$
EMC,SLAC	$\Gamma_1^p(10.7) = 0.126 \pm 0.010 \pm 0.015$
SMC	$\Gamma_1^p(10) = 0.136 \pm 0.013 \pm 0.011$
E143	$\Gamma_1^p(3) = 0.127 \pm 0.004 \pm 0.010$

My results:

$Q^2(Gev^2)$	Γ_1^p
2	0.1204
2.5	0.1238
3	0.1259
5	0.1310
10	0.1356

It seems the results have good agreement with experimental results.


Thank you for your attention…