Instantaneous Thermalization in Holographic Plasmas

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based on arxiv:1010.5443

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May 18, 2011
Outline

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A d-dim CFT is dual to a (d+1)-dim gravity theory.
The d-dim CFT lives on the boundary of the bulk gravity theory (asymptotically $AdS_{d+1}$ background).
Different asymptotically AdS spacetimes manifest themselves by different states in the boundary field theory.
- Vacuum state in the CFT $\leftrightarrow$ pure AdS spacetime.
- Thermal state in the gauge theory $\leftrightarrow$ AdS black hole (late-time configuration a generic state evolves to).

$\Rightarrow$ The dual gravitational description of strongly coupled gauge theories provides an efficient way to study the thermodynamic properties of gauge theories.
The dynamics of a system in the boundary field theory is described by the dynamics of the gravitational theory.

We might use AdS/CFT techniques to study out-of-equilibrium evolutions in strongly coupled gauge theories (QGP).

- Small deviation of global equilibrium $\rightarrow$ Linear Response Theory (Brownian Motion)

\[ H \rightarrow H + H_{\text{pert}} ; \quad H_{\text{pert}} = -AF(t) \]

where $|F(t)| \sim A << 1$

- Long-Wavelength limit $\rightarrow$ Fluid dynamics (hydrodynamics), $L >> l_{\text{mfp}}$

- non-linear regime in amplitude and frequency $\rightarrow$ Black hole formation
Brownian Motion

- The external quark as the end point of a string on the boundary (J. de Boera, V. Hubenyb, M. Rangamanib, M. Shigemoria, ’08)
- The other end of the string is attached to the horizon.
- The fluctuation of the transverse modes are explained by the Hawking radiation of the black hole. (A. Lawrence, E. Martinec (95))
Brownian Motion
Rapid Thermalization

- Thermalization in the QFT corresponds to black hole formation in the bulk. (S. Bhattacharyya, S. Minwalla (09))

\[ ds^2 = -\left(\frac{r^2}{l^2} - M(v)\right)dv^2 + 2dvdr + \frac{r^2}{l^2}dX^2 \]

\[
M(v) = \begin{cases} 
0, & v < 0 \\
M, & v > 0
\end{cases}
\]
far from equilibrium process

System equilibrates at the time scale of $\frac{1}{T_H}$.

instant thermalization of local operators

Non-local observables distinguish the out of equilibrium situation.
Formation of 3-dim planar BTZ black hole from $AdS_3$, by collapse of a null shell (Vaidya)
String stretched along the r-direction and its fluctuations are in the transverse direction X.

\[ ds^2 = -(\frac{r^2}{l^2} - M(v))dv^2 + 2dvdr + \frac{r^2}{l^2}dX^2 \]

\[ = g_{\mu\nu}(x)dx^\mu dx^\nu + G(x)dX^2 \]

null coordinates:

\[ v = t + r^* \; ; \quad u = t - r^* \]

The string \( \sigma \)-model action

\[ S_{NG} = -\frac{1}{2\pi\alpha'} \int d^2x \sqrt{-\det \gamma_{\mu\nu}} \]

\[ \approx -\frac{1}{4\pi\alpha'} \int d^2x \sqrt{-g}g^{\mu\nu}G\partial_\mu X \partial_\nu X \]
(v, r) well-defined coordinates

\[
\begin{align*}
\text{AdS}_3 & : \quad r^* = -\frac{l^2}{r} \\
\text{BTZ} & : \quad r^* = \frac{l^2}{2r_H} \ln\left(\frac{r - r_H}{r + r_H}\right)
\end{align*}
\]

\(\omega\) and \(u\) are the outgoing null coordinate in \(\text{AdS}_3\) and BTZ, respectively

\[
\omega = \frac{2l^2}{r_H} \tanh\left(\frac{r_H}{2l^2} u\right).
\]
AdS\(_3\) Mode Expansion

\[ X(r, t) = \int_0^\infty dk \left[ a_k u_k(t, r) + a_k^\dagger u_k^*(t, r) \right] \]

where

\[ u_k(t, r) = \frac{1}{k l} \sqrt{\frac{2\alpha'}{\omega}} e^{i k r_c^*} \left[ \sin(k(r^* - r_c^*)) - k r_c^* \cos(k(r^* - r_c^*)) \right] e^{-i \omega t} \]
BTZ Mode Expansion

\[
X(r, t) = \int_{0}^{\infty} dk \left[ b_k u_k(t, r) + b_k^\dagger u_k^*(t, r) \right]
\]

where

\[
u_k = \sqrt{\frac{2\alpha'}{\omega}} \frac{e^{ikr_c^*} e^{-i\omega t}}{r(r_H + ikl^2)(r_H^2 - ikl^2 r_c)} \left[ (r_H^2 r + k^2 l^4 r_c) \cos k(r^* - r_c^*) - kl^2 (r_H^2 - r_c r) \sin k(r^* - r_c^*) \right]
\]
Two-Point Functions

- **Green’s Function**

\[
\langle 0 | [X(t, r), X(t', r')] | 0 \rangle = 2i \int_0^\infty dk \text{Im}(u_k(t, r)u_k^*(t', r'))
\]

- **Hadamard Function**

\[
\langle 0 | \{X(t, r), X(t', r')\} | 0 \rangle = 2 \int_0^\infty dk \left[ \text{Re}(u_k(t, r)u_k^*(t', r')) + \int_0^\infty dk' \left( u_k(t, r)u_{k'}^*(t', r')\langle 0 | a_k^\dagger a_{k'} | 0 \rangle + u_k^*(t, r)u_{k'}(t', r')\langle 0 | a_k^\dagger a_{k'} | 0 \rangle \right) \right]
\]
AdS$_3$ Two-point Functions

- **Green’s Function**

  \[ G(r, t; r', t') = \frac{-\alpha' \pi}{2l^2} (r^{*2} + r'^{2} - \Delta t^2) \]

- **Hadamard Function**

  \[
  g(t, r; t', r') = \frac{\alpha'}{2l^2} \left( \left(12r_c^2 - 4r_c(r^* + r'^*) - 4r^* r'^* \right) + (\Delta t^2 - (r^{*2} + r'^{2})) \ln|\Delta t^2 - (r^* - r'^*)^2| \right. \\
  - (\Delta t^2 - (r^{*2} + r'^{2})) \ln|(-2r_c^* + r^* + r'^*)^2 - \Delta t^2| \\
  - 4r_c^*(r^*_c + \Delta t) \ln| -2r_c^* + r^* + r'^* - \Delta t| \\
  - 4r_c^*(r^*_c - \Delta t) \ln| -2r_c^* + r^* + r'^* + \Delta t| \right)
  \]
Green’s Function

\[ G(r, r', \Delta t) = \frac{-2\pi\alpha' l^2}{r_H^2} \left[ 1 - \cosh\left( \frac{r_H \Delta t}{l^2} \right) \sech\left( \frac{r_H r^*}{l^2} \right) \sech\left( \frac{r_H r'^*}{l^2} \right) \right] \]

Hadamard Function (in the limit \( r_c^* \to 0 \))

\[ g(t, r; t', r') = \frac{\alpha' r_H^4}{6/10} r^*3 r'^*3 \frac{1}{\sinh^4 \frac{r_H \Delta t}{2l^2}} \]

where \( \beta = \frac{1}{T_H} = \frac{2\pi l^2}{r_H} \).

The Hadamard function is periodic in imaginary time (the vacuum is thermal)
There is no global time-like killing vector in Vaidya background.

Bogoliubov coefficients $\rightarrow$ calculated using geometric optics approximation, in the late time limit

An elegant way used to calculate the on-set of Hawking radiation is trace anomaly method. (C. Callan, S. Giddings, J. Harvey, A. Strominger (91))
Time-Dependent Hawking Radiation

Our method:

- In general a solution to the equation of motion, $\phi(u, v)$, with the source $J(u, v)$ satisfies the equation

$$\partial_v \partial_u \phi + \frac{r_H^2}{2l^4} \frac{1}{\sinh^2(\frac{r_Hr^*}{l^2})} \phi = J(u, v)$$

and can be written as

$$\phi(u, v) = \int du' dv' \ G(u, v; u', v')J(u', v')$$

- $\phi(u, v)$ plays the role of Hadamard function after the shock wave.

- One can assume that the source gets the form

$$J(u, v) = \partial_u \phi_0(u, v) \ \delta(v).$$
• Propagation of the first point to the boundary

\[ g_{\text{AdS}_3;\text{BTZ}} = \int_0^\infty du'_1 \Theta(u'_1 - v_1)\Theta(u_1 - u'_1) \]

\[ G(u_1, v_1; u'_1, v'_1 = 0) \partial_{u'_1} g(u'_1, v'_1 = 0; u'_2, v'_2) \]

where \((u_1, v_1)\) is in BTZ space and \((u_2, v_2)\) is in AdS_3 space.

• Propagating the second point to the boundary

\[ g_{\text{BTZ}} = \frac{\alpha' r_H^4}{6l^{10}} r_1^* r_2^* 3 \frac{1}{\sinh^4 \left( \frac{r_H(t_1-t_2)}{2l^2} \right)} \]

This implies instantaneous thermalization on the boundary of the 3-dimensional Vaidya spacetime for non-local operators like the Hadamard function.
Future Direction

- Generalization to higher dimensions
- Considering other non-local operators

Thank you