On Applications of AdS/CFT correspondence

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Forces

1. Gravity
2. Electroweak
3. Strong interaction

How to describe?

1. Classical theory
2. Quantum Field theory (weakly or strongly coupled)

AdS/CFT correspondence

Basically AdS/CFT correspondence is a duality or a relation between two theories one with a gravity and the other without gravity.

The gravitational theory is usually defined in higher dimension.

Well developed case is the one where the gravity is defined on an AdS geometry where the dual theory is a CFT living in the conformal boundary of AdS space.

Classical gravity on AdS_{d+1} background is dual to d-dimensional “Large N” strongly coupled CFT on its boundary.

AdS_{d+1} metric in Poincare coordinates

$$ds^2 = \frac{r^2}{R^2}(-dt^2 + d\vec{x}^2) + \frac{R^2}{r^2}dr^2$$

AdS_{d+1} metric in global coordinates

$$ds^2 = -(1 + \frac{r^2}{R^2})dt^2 + \frac{dr^2}{1 + \frac{r^2}{R^2}} + r^2 d\Omega_{d-1}^2$$

Here boundary is at $r \to \infty$

Consider a 4-dimensional flat space $\{X_1, \cdots, X_4\}$. The distance between two near by points is

$$ds^2 = dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2$$

An sphere with radius R is defined

$$X_1^2 + X_2^2 + X_3^2 + X_4^2 = R^2$$

So that

$$X_1 = R\cos\theta, \quad X_2 = R\sin\theta\cos\phi, \quad X_3 = \sin\theta\sin\phi\cos\psi, \quad X_3 = \sin\theta\sin\phi\sin\psi$$

Distance between two near by points on the sphere

$$ds^2 = R^2(d\theta^2 + \sin^2\theta d\phi^2 + \sin^2\theta\sin^2\phi d\psi^2) = R^2 d\Omega_3^2$$

Consider a 6-dimensional flat space with two time-like coordinates. The distance between two near by points is

$$ds^2 = -dX_1^2 - dX_2^2 + dX_3^2 + dX_4^2 + dX_5^2 + dX_6^2$$

An sphere in this space may be defined

$$-X_1^2 - X_2^2 + X_3^2 + X_4^2 + X_5^2 + X_6^2 = -R^2$$

So that

$$X_1^2 + X_2^2 = R^2 + r^2, \qquad X_3^2 + X_4^2 + X_5^2 + X_6^2 = r^2$$

Distance between two near by points is given by

$$ds^2 = -(1 + \frac{r^2}{R^2})d\tau^2 + \frac{dr^2}{1 + \frac{r^2}{R^2}} + r^2 d\Omega_3^2$$

There is one to one correspondence between objects in CFT and those in the gravitational theory on AdS space.

$$\{r, t, \vec{x}\} \longleftrightarrow \{\text{scale of energy}, t, \vec{x}\}$$

$$\text{Symmetries} \longleftrightarrow \text{Symmetries}$$

$$\text{Fields } \Phi(r, t, \vec{x}) \longleftrightarrow \text{Operators } \mathcal{O}(t, \vec{x})$$

$$\text{On shell action} \longleftrightarrow \text{Expectation values}$$

$$\text{UV}, \text{IR} \longleftrightarrow \text{near boundary}, \text{near horizon}$$

- Suppose $\mathcal{O}(t,\vec{x})$ on the boundary corresponds to field $\Phi(r,t,\vec{x})$ in the bulk.

- Suppose $\mathcal{O}(t,\vec{x})$ has dimension Δ:

$$\mathcal{O}(\lambda x) = \lambda^{\Delta}\mathcal{O}(x)$$

- One can solve equations of motion of $\Phi(r,t,\vec{x})$ in the bulk.

- The asymptotic expansion is

$$\lim_{r\to 0}\Phi(r,t,x) \sim r^{d-\Delta}\varphi(t,x) + r^{\Delta}\phi(t,x)$$

Δ is given in terms of mass, dimension,....

- $\varphi(t,x)$ is source and $\phi(t,x)$ is response.

- $\varphi(t,x)$ is source:

$$\int d^dx\, \mathcal{L}_{\mathsf{CFT}} + \int d^dx\, \varphi(x)\mathcal{O}(x)$$

- $\phi(t,x)$ is response:

$$\langle \mathcal{O} \rangle = \frac{1}{\sqrt{g_{(0)}}} \frac{\delta S}{\delta \varphi} \sim \phi[\varphi(x)] + \text{local terms}$$

- Retarded Green's function

$$G_R(\omega,k) = \frac{\phi(k,\omega)}{\varphi(k,\omega)}$$

Applications

- Non-Fermi Liquid.
- High T$_c$ superconductor.
- Quantum Critical points
- QGP and AdS/QCD

Fermi Liquid

1. The ground state of an interacting fermionic system is characterized by a Fermi surface in momentum space at $k = k_F$.

2. Despite (possibly strong) interactions among fundamental fermions, the *low energy* excitations near the Fermi surface nevertheless behave like weakly interacting particles and holes, which are called quasi-particles and quasi-holes.

$$\epsilon(k) = v_F(k - k_F) + \ldots \qquad v_F = \frac{k_F}{m_*} \; .$$

The propagator describing the propagation of a particle of energy $\epsilon(k)$:

$$G_R(t, \vec{k}) \sim \theta(t) e^{-i\epsilon(k)t}$$

Fourier transforming

$$G_R(\omega, \vec{k}) = \frac{1}{\omega - \epsilon(k)}$$

When turning on interactions, a particle (or hole) can now decay into another particle plus a number of particle-hole pairs. Thus the above equation should be modified to

$$G_R(t, \vec{k}) \sim e^{-i\epsilon(k)t - \frac{\Gamma}{2}t}$$

Γ is the decay rate. Near Fermi surface

$$G_R(\omega, \vec{k}) = \frac{1}{\omega - v_F(k - k_F) + \Sigma(k, \omega)}, \qquad \Sigma = \frac{i\Gamma}{2} \sim i\omega^2$$

Within this model, for Fermi liquid one finds

$$\rho = \rho_0 + \rho_1 T^2, \qquad \chi \sim T$$

The theory has been tremendously successful in explaining *almost* all metallic states in nature.

Since the 1980's, there has been an accumulation of metallic materials whose thermodynamic and transport properties differ significantly from those predicted by Fermi liquid theory .

$$\rho \sim T, \qquad \chi \sim T \ln T$$

One prominent class of examples of these so-called non-Fermi liquids is the strange metal phase of the cuprate superconductors, which refers to the metallic state above the superconducting transition temperature T_c.

The immediate question is whether one or both of the postulates stated earlier break down.

For high T_c cuprates in the strange metal region, ARPES experiments indicate that a Fermi surface still exists, but excitations exhibit a much broader peak than that for a Fermi liquid. The experimental results can be fit well to the following expression, postulated as "Marginal Fermi liquid" (MFL) in

$$G_R(\omega, k) = \frac{h}{\omega - v_F(k - k_F) + \Sigma(\omega, k)}$$

with the self-energy $\Sigma(\omega, k)$ given by

$$\Sigma(\omega) \approx c\omega \log \omega + d\omega, \qquad c \text{ real}, \quad d \text{ complex} .$$

How to construct gravity dual?

1. Strongly coupled field theory $\longrightarrow$ Gravitational theory

2. Finite density $\longrightarrow$ Gauge field

3. Cooper pair $\longrightarrow$ Scalar field

4. Finite Temperature $\longrightarrow$ Black hole

5. Fermionic system $\longrightarrow$ Fermion

The Minimal model

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{\kappa}(R^2 - \Lambda) - \frac{1}{g_{YM}^2}F^2 - \frac{1}{2}|\nabla\phi|^2 + V(\phi) - \bar{\psi}i\Gamma^a D_a\psi - m\bar{\psi}\psi\Big]$$

1. In normal state $\phi = 0$

2. Consider fermions as probe $\longrightarrow$ Gravity+Maxwell

3. Charged black hole

$$ds^2 = r^2(-fdt^2 + d\vec{x}^2) + \frac{dr^2}{r^2 f}, \qquad A_t = \mu(1 - \frac{r_0}{r})$$

$$f = 1 + \frac{Q}{r^4} - \frac{M}{r^3}$$

Setting $\psi = (-gg^{rr})^{-1/4} e^{-i\omega t + ik_1 x^1} \Psi$ the equation of motion becomes

$$\sqrt{\frac{g_{ii}}{g_{rr}}}(\Gamma^r \partial_r - m\sqrt{g_r r})\Psi + iK_\mu \Gamma^\mu \Psi = 0$$

where $K_\mu = (-u, k_1)$ with

$$u = \sqrt{\frac{g_{ii}}{-g_{tt}}}(\omega + qA_t)$$

Let us decompose the fermion Ψ as follows

$$\Psi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

then near the boundary one finds

$$\chi_\alpha \sim a_\alpha r^{mR} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_\alpha r^{-mR} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \text{for } r \to \infty$$

So that the Retarded Green's function reads

$$G_R^\alpha(\omega, k) = \frac{b_\alpha(\omega, k)}{a_{\alpha(\omega,k)}}$$

At low energy (IR) we have $\omega \ll \mu$. In small ω limit and at zero temperature one finds

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + \mathcal{O}(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \mathcal{O}(\omega)\right)}{a_+^{(0)} + \omega a_+^{(1)} + \mathcal{O}(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \mathcal{O}(\omega)\right)}$$

where $\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$ with

$$\nu_k = \sqrt{m^2 - q^2 + \frac{k^2}{\mu^2}}$$

If $a_k^{(0)}|_{k_F} = 0$ so that $a_+^{(0)} \approx \partial_k a_+^{(0)}(k_F)(k - k_F)$, then the above equation at leading order reads

$$G_R(\omega, k) \approx \frac{h}{\omega - v_F(k - k_F) + \Sigma}, \qquad \Sigma \sim \mathcal{G}_k(\omega)$$

The dispersion relation of small excitations near fermi surface is given by

$$\omega - v_F(k - k_F) + hc(k)\omega^{2\nu} = 0$$

1. $\nu > \frac{1}{2}$ $\qquad \omega = v_F(k - k_F), \qquad \Gamma \sim \omega^{2\nu}$

2. $\nu < \frac{1}{2}$ $\qquad \omega = v_F(k - k_F)^{1/2\nu}, \qquad \Gamma \sim \omega$

3. $\nu = \frac{1}{2}$

$$G_R(\omega, k) \approx \frac{h}{\omega - v_F(k - k_F) + c\omega \log \omega + d\omega}$$

Conductor/Superconductor phase transition

As we change the temperature, the scalar field could be non-zero leading to a hairy black hole

In dual theory the coopar pair gets non-zero expectation value leading to a second order phase transition

The resultant phase is a superconductor phase

References

1. N. Iqbal, H. Liu and M. Mezei, arXiv:1110.3814

2. T. Nishioka, S. Ryu, and T. Takayanagi, arXiv: 0905.0932

3. S. A. Hartnoll, arXiv:0903.3246

Quantum critical points (at $T = 0$)

There is no classical order parameter, symmetry bracking....

Is there a good order parameter?

Entanglement entropy!