Effective Field Theory of Multi-Field Inflation a la Weinberg

Nima Khosravi

Cosmology Group, AIMS

JCAP, arXiv:1203.2266

map:

- keywords: Effective Field Theory, Inflation

single field	**C. Cheung et al.** **JHEP, arXiv:0709.0293** **The EFT of Inflation**	**S. Weinberg** **PRD, arXiv:0804.4291** **EFT for Inflation**
multi-field	L. Senatore and M. Zaldarriaga JHEP, arXiv:1009.2093 The EFT of Multifield Inflation	JCAP, arXiv:1203.2266 EFT of Multi-Field Inflation a la Weinberg

Effective Field Theory

- an *effective* theory:
- is true for a certain domain of energy.

- two cases:
  - as a part of a true theory for whole energy scales
    - using EFT to simplify calculations!
  - in lack of a complete theory for the energy scales of interests
    - using EFT since there is no other choice!
    - constructing EFT, by considering e.g. symmetry properties of the model.

EFT for Multi-Field Inflation

- the most general form of Lagrangian up to the 4th order derivatives:

after simplifications:

$$\Big\{ \; b_1^{IJ}(\vec{\varphi})\Box\varphi_I\Box\varphi_J + b_2^{IJK}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\Box\varphi_K + b_3^{IJKL}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J\nabla_\nu\varphi_K\nabla^\nu\varphi_L + b_4^{IJ}(\vec{\varphi})\nabla_\mu\varphi_I\nabla^\mu\varphi_J \qquad \text{(A1)}$$

$$+ \; b_5(\vec{\varphi}) + b_6^{IJ}(\vec{\varphi})(\nabla^\mu\varphi_I)(\Box\nabla_\mu\varphi_J) + b_7^I(\vec{\varphi})(\nabla_\mu\nabla_\nu\varphi_I)^2 + b_8^{IJK}(\vec{\varphi})(\nabla_\mu\varphi_I)(\nabla_\nu\varphi_J)(\nabla^\mu\nabla^\nu\varphi_K) + b_9^I(\vec{\varphi})\nabla^\mu\Box\nabla_\mu\varphi_I$$

$$+ \; b_{10}^I(\vec{\varphi})\Box^2\varphi_I + b_{11}^I(\vec{\varphi})(\nabla^\mu)\nabla_\mu\Box\varphi_I + c_1^{IJ}(\vec{\varphi})R\nabla_\mu\varphi_I\nabla^\mu\varphi_J + c_2^{IJ}(\vec{\varphi})R^{\mu\nu}\nabla_\mu\varphi_I\nabla_\nu\varphi_J + c_3^I(\vec{\varphi})R\Box\varphi_I$$

$$+ \; c_4^I(\vec{\varphi})(\nabla^\mu R)(\nabla_\mu\varphi_I) + c_5(\vec{\varphi})\Box R + c_6^I(\vec{\varphi})R_{\mu\nu}\nabla^\mu\nabla^\nu\varphi_I + a_1(\vec{\varphi})R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + a_2(\vec{\varphi})R_{\mu\nu}R^{\mu\nu} + a_3(\vec{\varphi})R^2 + a_4(\vec{\varphi})R \Big\}$$

before simplifications!

perturbations:

- perturbations in single field model (Weinberg's paper)

$$\begin{aligned}\mathcal{L} &= \sqrt{g}\Big[-\frac{M^2}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - M_P^2U(\varphi) + f(\varphi)\Big(g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}\Big)^2\Big] \\ &= \bar{\mathcal{L}} - \frac{1}{2}a^3\Big(M^2 + 4f(\bar\varphi)\dot{\bar\varphi}^2\Big)\times\Big(-\dot{\delta\varphi}^2 + a^{-2}(\vec\nabla\delta\varphi)^2\Big) \\ &+ 4a^3f(\bar\varphi)\dot{\bar\varphi}^2\Big(\dot{\delta\varphi}^2 + \dot{\delta\varphi}^3/\dot{\bar\varphi} - a^{-2}\dot{\delta\varphi}(\vec\nabla\delta\varphi)^2/\dot{\bar\varphi} + \frac{1}{4}\dot{\delta\varphi}^4/\dot{\bar\varphi}^2 - \frac{1}{2}a^{-2}\dot{\delta\varphi}^2(\vec\nabla\delta\varphi)^2/\dot{\bar\varphi}^2 + \frac{1}{4}a^{-4}(\vec\nabla\delta\varphi)^4/\dot{\bar\varphi}^2\Big)\end{aligned}$$

-- it is up to 4^{th} order of perturbations automatically.

-- speed of sound ≠ 1

-- large non-Gaussianity?

-- speed of sound is constrained by validity of EFT!

perturbations:

- multi-field case:

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(2)} &= b_3^{IJKL}(\bar\varphi)\Big\{\dot{\bar\varphi}_I\dot{\bar\varphi}_J\delta\dot\varphi_K\delta\dot\varphi_L + \dot{\bar\varphi}_I\dot{\bar\varphi}_K\delta\dot\varphi_J\delta\dot\varphi_L + \dot{\bar\varphi}_I\dot{\bar\varphi}_L\delta\dot\varphi_K\delta\dot\varphi_J + \dot{\bar\varphi}_K\dot{\bar\varphi}_J\delta\dot\varphi_I\delta\dot\varphi_L + \dot{\bar\varphi}_L\dot{\bar\varphi}_J\delta\dot\varphi_I\delta\dot\varphi_L + \dot{\bar\varphi}_K\dot{\bar\varphi}_L\delta\dot\varphi_I\delta\dot\varphi_J \\
&- a^{-2}\dot{\bar\varphi}_I\dot{\bar\varphi}_J\partial_i\delta\varphi_K\partial^i\delta\varphi_L - a^{-2}\dot{\bar\varphi}_K\dot{\bar\varphi}_L\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big\} - \frac{M^2}{2}\delta^{IJ}\Big[-\delta\dot\varphi_I\delta\dot\varphi_J + a^{-2}\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big],
\end{aligned}
$$

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(3)} &= b_3^{IJKL}(\bar\varphi)\Big\{\dot{\bar\varphi}_I\delta\dot\varphi_J\delta\dot\varphi_K\delta\dot\varphi_L + \dot{\bar\varphi}_J\delta\dot\varphi_I\delta\dot\varphi_K\delta\dot\varphi_L + \dot{\bar\varphi}_K\delta\dot\varphi_L\delta\dot\varphi_I\delta\dot\varphi_J + \dot{\bar\varphi}_L\delta\dot\varphi_K\delta\dot\varphi_I\delta\dot\varphi_J \\
&- a^{-2}\Big(\dot{\bar\varphi}_I\delta\dot\varphi_J\partial_i\delta\varphi_K\partial^i\delta\varphi_L + \dot{\bar\varphi}_J\delta\dot\varphi_I\partial_i\delta\varphi_K\partial^i\delta\varphi_L + \dot{\bar\varphi}_K\delta\dot\varphi_L\partial_i\delta\varphi_I\partial^i\delta\varphi_J + \dot{\bar\varphi}_L\delta\dot\varphi_K\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(4)} = b_3^{IJKL}(\bar\varphi)\Big\{\delta\dot\varphi_I\delta\dot\varphi_J\delta\dot\varphi_K\delta\dot\varphi_L \; &- \; a^{-2}\Big(\delta\dot\varphi_I\delta\dot\varphi_J\partial_i\delta\varphi_K\partial^i\delta\varphi_L + \delta\dot\varphi_K\delta\dot\varphi_L\partial_i\delta\varphi_I\partial^i\delta\varphi_J\Big) \\
&+ \; a^{-4}\partial_i\delta\varphi_I\partial^i\delta\varphi_J\partial_j\delta\varphi_K\partial^j\delta\varphi_L\Big\}.
\end{aligned}
$$

perturbations!

$$\mathcal{L} = -a^3 \Big\{ -\frac{M_1^2}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{M_2^2}{2}\partial_\mu\chi\partial^\mu\chi - M_P^2 U(\varphi,\chi) + g_1(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)^2 + g_2(\varphi,\chi)\big(\partial_\mu\chi\partial^\mu\chi\big)^2$$

$$+ g_3(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)\big(\partial_\nu\varphi\partial^\nu\chi\big) + g_4(\varphi,\chi)\big(\partial_\mu\chi\partial^\mu\chi\big)\big(\partial_\nu\chi\partial^\nu\varphi\big)$$

$$+ g_5(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\varphi\big)\big(\partial_\nu\chi\partial^\nu\chi\big) + g_6(\varphi,\chi)\big(\partial_\mu\varphi\partial^\mu\chi\big)\big(\partial_\nu\varphi\partial^\nu\chi\big)\Big\}$$

perturbations:

- two-field case:

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(2)} &= \dot{\delta\varphi}^2\left[\frac{M_1^2}{2}+6g_1\dot{\bar\varphi}^2+3g_3\dot{\bar\varphi}\dot{\bar\chi}+(g_5+g_6)\dot{\bar\chi}^2\right]+\dot{\delta\chi}^2\left[\frac{M_2^2}{2}+6g_2\dot{\bar\chi}^2+3g_4\dot{\bar\varphi}\dot{\bar\chi}+(g_5+g_6)\dot{\bar\varphi}^2\right]\\
&+ \dot{\delta\varphi}\dot{\delta\chi}\left[3g_3\dot{\bar\varphi}^2+3g_4\dot{\bar\chi}^2+4(g_5+g_6)\dot{\bar\varphi}\dot{\bar\chi}\right]\\
&- a^{-2}\Big(\partial_i\delta\varphi\partial^i\delta\varphi\left[\frac{M_1}{2}+2g_1\dot{\bar\varphi}^2+g_3\dot{\bar\varphi}\dot{\bar\chi}+g_5\dot{\bar\chi}^2\right]+\partial_i\delta\chi\partial^i\delta\chi\left[\frac{M_2}{2}+2g_2\dot{\bar\chi}^2+g_4\dot{\bar\varphi}\dot{\bar\chi}+g_5\dot{\bar\varphi}^2\right]\\
&\qquad +\partial_i\delta\varphi\partial^i\delta\chi\left[g_3\dot{\bar\varphi}^2+g_4\dot{\bar\chi}^2+2g_6\dot{\bar\varphi}\dot{\bar\chi}\right]\Big),
\end{aligned}
$$

second order perturbations!

…. and cubic and quartic terms!

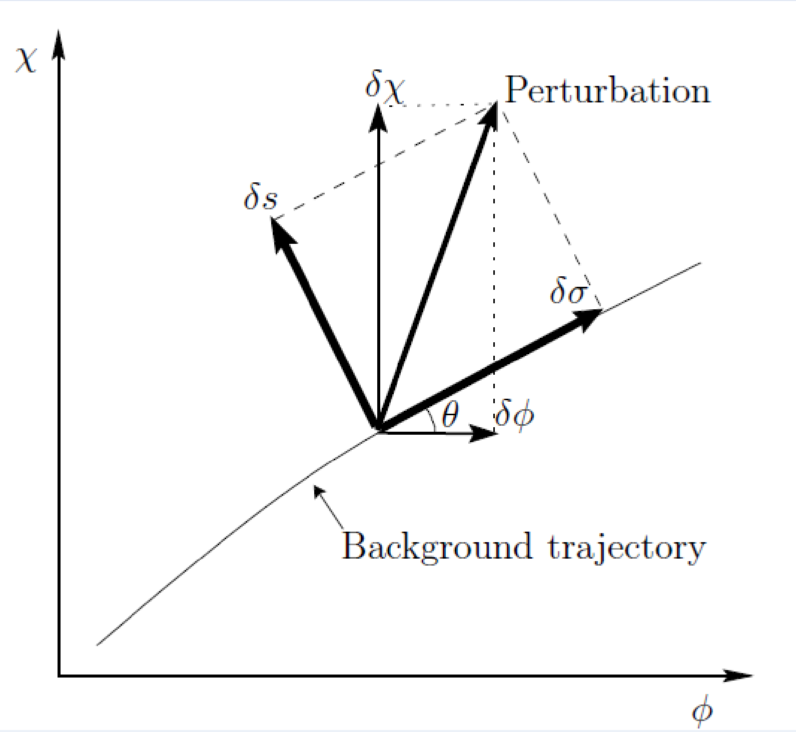
. . . transition to adiabatic and entropy curvature perturbations!

perturbations: adiabatic & entropy modes

- adiabatic mode: entropy mode:

$$\delta\sigma \equiv \vec{T}.\vec{\delta}, \qquad \delta s \equiv \vec{N}.\vec{\delta}$$

$$\vec{\delta} \equiv (\delta\varphi, \delta\chi)\,, \qquad \vec{T} = (\cos\theta, \sin\theta) \equiv (\dot{\varphi}/\dot{\sigma}, \dot{\chi}/\dot{\sigma})\,, \qquad \vec{N} \equiv (\sin\theta, -\cos\theta)$$



Gordon et al. [arXiv:astro-ph/0009131]

perturbations: adiabatic & entropy modes

$$4\dot\sigma\left(\vec{T}.\dot{\vec{\delta}}\right)^3 \times \left[g_1 \cos^4\theta + g_2 \sin^4\theta + g_3 \cos^3\theta \sin\theta + g_4 \cos\theta \sin^3\theta + (g_5+g_6)\cos^2\theta \sin^2\theta\right]$$

$$+\ \dot\sigma\left(\vec{N}.\dot{\vec{\delta}}\right)^3 \times$$

$$\left[+g_4 \cos^4\theta + g_3 \sin^4\theta - 2\Big(2g_2 - (g_5+g_6)\Big)\cos^3\theta \sin\theta + 2\Big(2g_1 - (g_5+g_6)\Big)\cos\theta\sin^3\theta + 3(g_4-g_3)\cos^2\theta\sin^2\theta\right]$$

$$+\ 3\dot\sigma\left(\vec{T}.\dot{\vec{\delta}}\right)^2\left(\vec{N}.\dot{\vec{\delta}}\right) \times$$

$$\left[+g_3 \cos^4\theta + g_4 \sin^4\theta + 2\Big(2g_1 - (g_5+g_6)\Big)\cos^3\theta \sin\theta - 2\Big(2g_2 - (g_5+g_6)\Big)\cos\theta\sin^3\theta + 3(g_3-g_4)\cos^2\theta\sin^2\theta\right]$$

$$+\ 2\dot\sigma\left(\vec{T}.\dot{\vec{\delta}}\right)\left(\vec{N}.\dot{\vec{\delta}}\right)^2 \times$$

$$\left[(g_5+g_6)\Big(\cos^4\theta + \sin^4\theta\Big) + 3(g_4-g_3)\Big(\cos^3\theta\sin\theta - \cos\theta\sin^3\theta\Big) + \Big(3(g_1+g_2) - 2(g_5+g_6)\Big)\cos^2\theta\sin^2\theta\right]$$

$$(\dot{\delta\sigma} - \dot\theta\,\delta s)$$

$$(\dot{\delta s} + \dot\theta\,\delta\sigma)$$

note that just these two combinations appear in this formalism!

shape of non-Gaussianity

- due to previous slide: for example:

$$(\vec{T}.\dot{\vec{\delta}})^3 = \dot{\delta\sigma}^3 - 3\dot{\theta}\dot{\delta\sigma}^2\delta s + 3\dot{\theta}^2\dot{\delta\sigma}\delta s^2 - \dot{\theta}^3\delta s^3$$

equilateral NG → local NG

in adiabatic mode → in entropy mode

--- in this formalism

the “Cosine” between different kinds of NG is fixed!

amplitude of NG

$$\frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}} = \frac{\{H^3, H^2\dot\theta, H\dot\theta^2, \dot\theta^3\}}{\{H^2, H\dot\theta, \dot\theta^2\}} \times \left(\frac{f(g_i)}{M^2}\dot\sigma^2\right) \times \frac{\delta\sigma}{\dot\sigma} = \frac{\{H^3, H^2\dot\theta, H\dot\theta^2, \dot\theta^3\}}{H \times \{H^2, H\dot\theta, \dot\theta^2\}} \times \left(\frac{f(g_i)}{M^2}\dot\sigma^2\right) \times \zeta$$

$\dot\theta >> H$

$$f_{NL} \sim \frac{\mathcal{L}^{(3)}}{\mathcal{L}^{(2)}}\zeta^{-1} \sim \frac{\dot\theta}{H} \times \left(\frac{f(g_i)}{M^2}\dot\sigma^2\right)$$

-- validity condition of EFT i.e. $\frac{f(g_i)}{M^2}\dot\sigma^2 < 1$ constrains the amplitude of NG!

-- except if the curvature of classical (background) path be large!

-- or: if by a mechanism (e.g. Vainshtein) one can modify the validity condition of EFT!

compare with Senatore & Zaldarriaga

-- Senatore & Zaldarriaga model is based on Cheung et al.'s work!

-- in Cheung's work, EFT is constructed on perturbations' level!

  -- since their model is single field, the perturbation is associated to adiabatic mode!

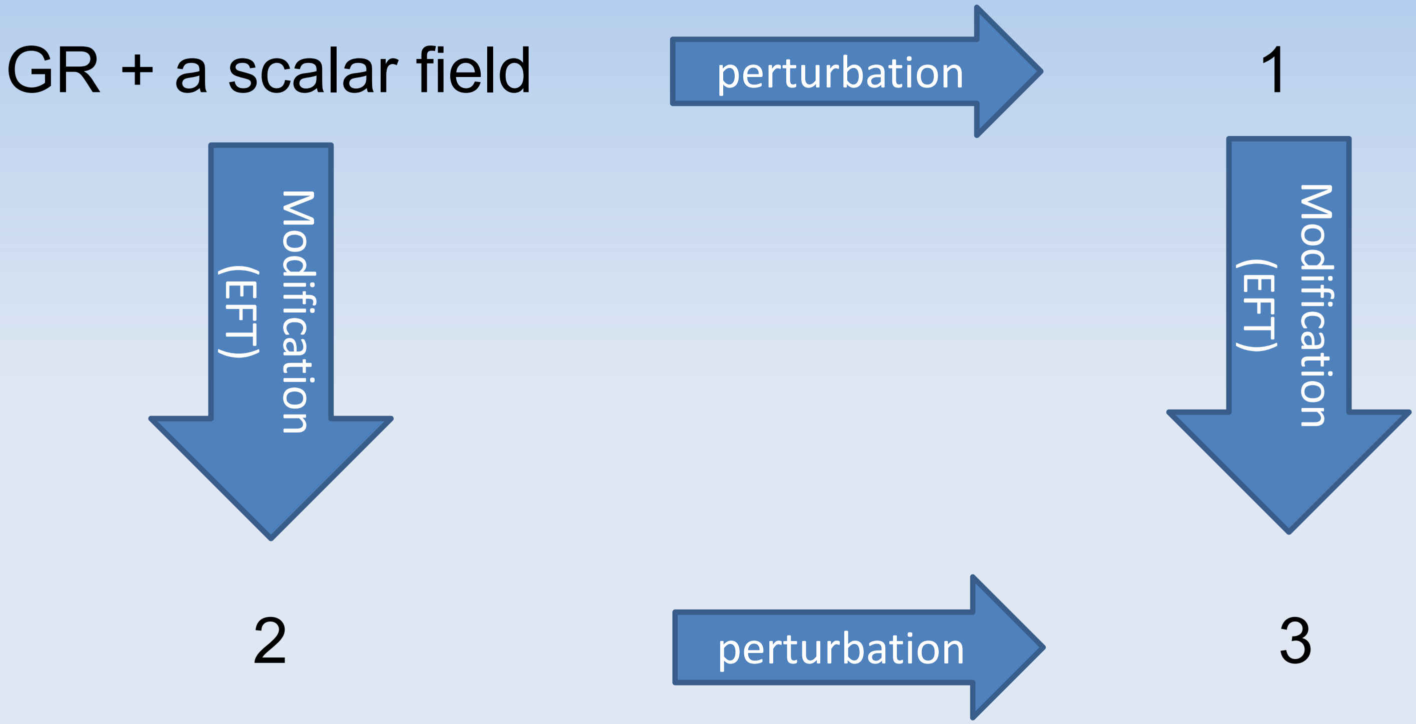
-- so in Senatore & Z., the entropy modes are added into a base with already known adiabatic mode!

but

-- in our case we started with zeroth order term of perturbations.

  -- then defined the perturbations without any distinguishability between adiabatic and entropy modes!

compare with Senatore & Zaldarriaga



Cheung et al. and Senatore & Z.: 0 → 1 → 3

Weinberg and this talk: 0 → 2 → 3

compare with Senatore & Zaldarriaga

- so in Senatore & Zaldarriaga, the shift symmetry results in a Lagrangian similar to

$$
\begin{aligned}
a^{-3}\Delta\mathcal{L}^{(2)} &= \dot{\delta\varphi}^2\Big[\frac{M_1^2}{2} + 6g_1\dot{\bar\varphi}^2 + 3g_3\dot{\bar\varphi}\dot{\bar\chi} + (g_5+g_6)\dot{\bar\chi}^2\Big] + \dot{\delta\chi}^2\Big[\frac{M_2^2}{2} + 6g_2\dot{\bar\chi}^2 + 3g_4\dot{\bar\varphi}\dot{\bar\chi} + (g_5+g_6)\dot{\bar\varphi}^2\Big] \\
&+ \dot{\delta\varphi}\dot{\delta\chi}\Big[3g_3\dot{\bar\varphi}^2 + 3g_4\dot{\bar\chi}^2 + 4(g_5+g_6)\dot{\bar\varphi}\dot{\bar\chi}\Big] \\
&- a^{-2}\Big(\partial_i\delta\varphi\partial^i\delta\varphi\Big[\frac{M_1}{2} + 2g_1\dot{\bar\varphi}^2 + g_3\dot{\bar\varphi}\dot{\bar\chi} + g_5\dot{\bar\chi}^2\Big] + \partial_i\delta\chi\partial^i\delta\chi\Big[\frac{M_2}{2} + 2g_2\dot{\bar\chi}^2 + g_4\dot{\bar\varphi}\dot{\bar\chi} + g_5\dot{\bar\varphi}^2\Big] \\
&\qquad + \partial_i\delta\varphi\partial^i\delta\chi\Big[g_3\dot{\bar\varphi}^2 + g_4\dot{\bar\chi}^2 + 2g_6\dot{\bar\varphi}\dot{\bar\chi}\Big]\Big),
\end{aligned}
$$

$$\delta\varphi \to \delta\varphi + c_1 \ \text{ and } \ \delta\chi \to \delta\chi + c_2$$

i.e. there are just *derivatives* of adiabatic and entropy perturbations!

but in our model the case is different!

compare with Senatore & Zaldarriaga

shift symmetry: $\delta\varphi \to \delta\varphi + c_1 \text{ and } \delta\chi \to \delta\chi + c_2$

due to

$$\delta\sigma \equiv \vec{T}.\vec{\delta}, \qquad \delta s \equiv \vec{N}.\vec{\delta}$$

$$\vec{\delta} \equiv (\delta\varphi, \delta\chi)\,, \qquad \vec{T} = (\cos\theta, \sin\theta) \equiv (\dot{\varphi}/\dot{\sigma}, \dot{\chi}/\dot{\sigma})\,, \qquad \vec{N} \equiv (\sin\theta, -\cos\theta)$$

results in

$$\delta\sigma \to \delta\sigma + (c_1 \cos\theta + c_2 \sin\theta)$$
$$\delta s \to \delta s + (c_1 \sin\theta - c_2 \cos\theta)$$

which causes a new symmetry for adiabatic and entropy modes:

$$\dot{\delta\sigma} - \dot{\theta}\delta s \to \dot{\delta\sigma} - \dot{\theta}\delta s \quad \leftarrow \quad \vec{T}.\dot{\vec{\delta}}$$
$$\dot{\delta s} + \dot{\theta}\delta\sigma \to \dot{\delta s} + \dot{\theta}\delta\sigma \quad \leftarrow \quad \vec{N}.\dot{\vec{\delta}}$$

conclusions

-- this model does not predict a large non-Gaussianity except:

-- for a highly curved classical path in the phase-space!

-- or if a shielding mechanism allows large first correction term in EFT.

-- different shapes of non-Gaussianity are correlated!

-- in contrast to Senatore & Zaldarriaga, we suggest EFT for multi-filed inflation should be constructed as

$$\Delta\mathcal{L} \propto \sum c_{n_0,n_1,\ldots,n_N}\left(\vec{T}.\dot{\vec{\delta}}\right)^{n_0}\left(\vec{N}_1.\dot{\vec{\delta}}\right)^{n_1}\left(\vec{N}_2.\dot{\vec{\delta}}\right)^{n_2}\ldots\left(\vec{N}_N.\dot{\vec{\delta}}\right)^{n_N}$$

TED.com