A simple model to calculate the full transverse spin structure function

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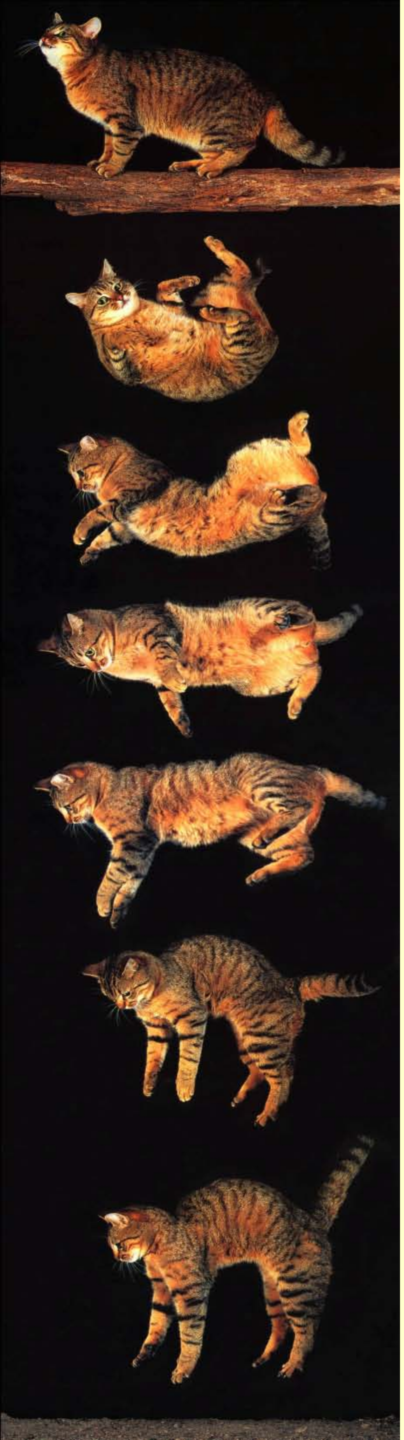
Outline

Polarized hadron structure

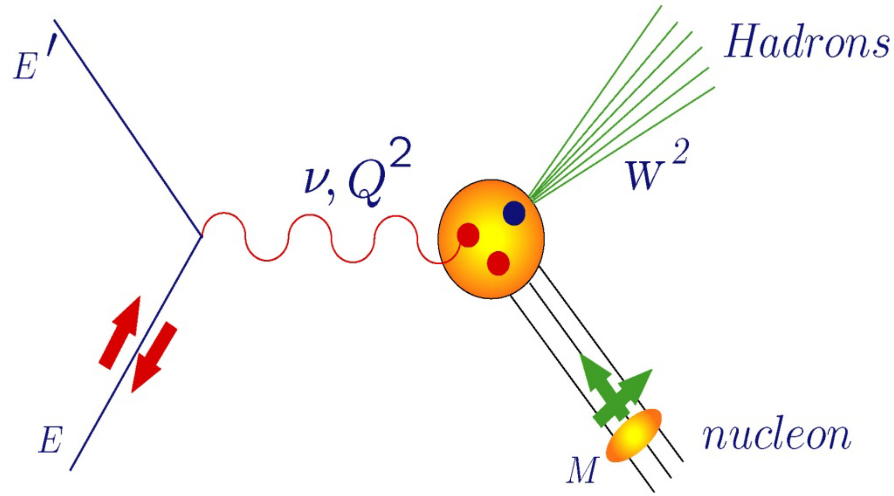
Valon model for extacting hadron structure

Transverse spin structure function

Spin is everywhere…



Polarized Deep Inelastic Electron Scattering



$x = \dfrac{Q^2}{2M\nu}$ Fraction of nucleon momentum carried by the struck quark

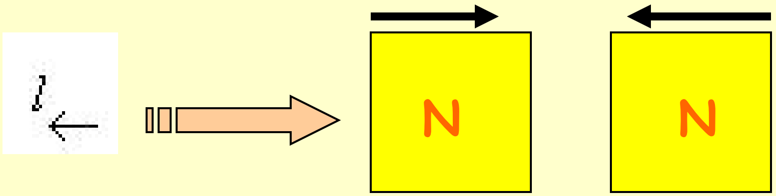
Q^2 = 4-momentum transfer of the virtual photon, ν = energy transfer, θ = scattering angle

- All information about the nucleon vertex is contained in

F_2 and F_1 the unpolarized (spin averaged) structure functions,

and

g_1 and g_2 the spin dependent structure functions



$$\sigma^{\leftarrow}_{\Rightarrow} - \sigma^{\leftarrow}_{\Leftarrow} \approx g_1$$

Where: $$g_1(x,Q^2) = \sum e_q^2 (\delta q(x,Q^2) + \overline{\delta q}(x,Q^2))$$

$$\delta q(x) = q\uparrow(x) - q\downarrow(x)$$

Probability to find parton with spin aligned/anti-aligned to proton spin

The first moment is a measure for the quark contribution to the proton spin.

$$\Delta q = \int_0^1 \delta q(x,Q^2)\,dx$$

Analogy with un polarized case:

$$F_1(x,Q^2) = \frac{1}{2}\sum_q e_q{}^2 (q(x,Q^2) + \bar{q}(x,Q^2))$$

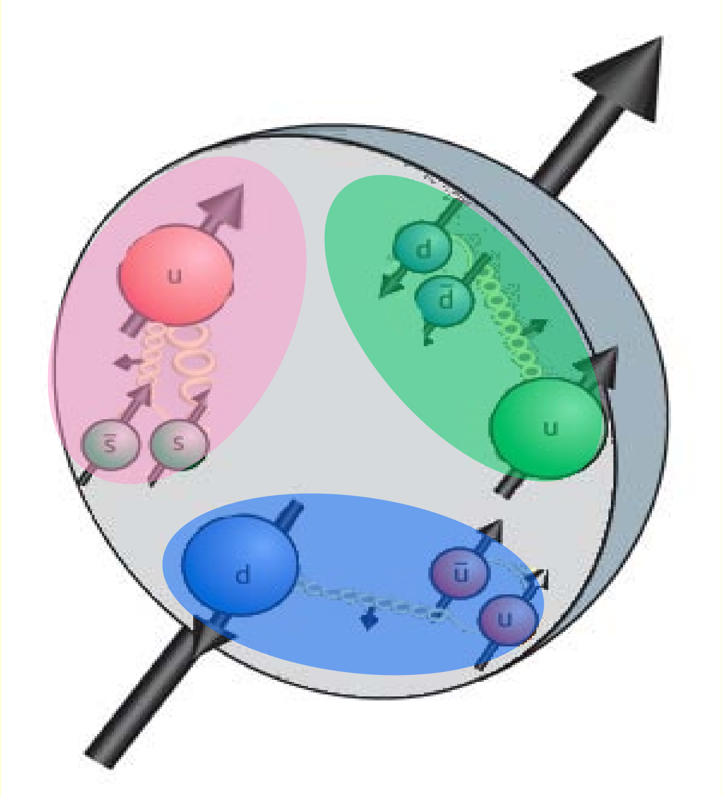
$$g_1(x,Q^2) = \sum e_q^2 (\delta q(x,Q^2) + \overline{\delta q}(x,Q^2))$$

$$q(x) = q\uparrow(x) + q\downarrow(x)$$

$$\delta q(x) = q\uparrow(x) - q\downarrow(x)$$

Study of Nucleon Structure function in the Valon model

- Valon : valence quark and its associated sea quarks and gluons.
- The structure of a valon arises from the perturbative dressing of the valence quark in QCD.
- The valons carry all the momentum of nucleon and the quantum number of valon is the quantum number of its valence quark.
- They play a role in scattering problems as the constituent quarks do in bound-state problems.
- At sufficiently low value of Q^2 the internal structure of a valon cannot be resolved.



The existence of the valon can be inferred from the measurement of the Natchmann moments of the proton structure functions at **Jefferson laboratory**. They point to the existence of a new scaling that can be interpreted as a constituent form factor consistent with the elastic nucleon data. They suggest that there exist extended objects inside the proton and the size of these constituents are 0.2-0.3 fm.(hep-ph/0301206v2)

The structure function of a hadron is convolution of two distributions:

- valon distributions in proton.
- Structure function of a valon.

In an unpolarized situation we can write:

$$\boxed{F_2^p(x,Q^2) = \sum_v \int_x^1 dy G_{v/p}(y) \mathcal{F}_2^v(\frac{x}{y}, Q^2) ,}$$

Proton structure function

Summation is over the three valons

Describes the valon distribution in a proton. It independs on Q^2.

Structure function of a *v* valon. It depends on Q^2 and the nature of the probe.

If you know PDF in a valon, you can get PDF in proton as:

$$q_{i/p}(x,Q^2)=\sum_j\int_x^1 q_{i/j}(\frac{x}{y},Q^2)G_{j/p}(y)\frac{dy}{y}$$

PDF in proton

PDF in a valon.
It comes from solving GLAP.eq in a valon

Valon distribution in proton

Un-polarized structure of nucleon in the framework of valon model had been studied in :

Firooz Arash & Ali.N.Khorramian-Physical Review C 67,045201 (2003)

Polarized Nucleon Structure in valon framework:

PPDF in valon model

$$\delta q_{i/p}(x,Q^2)=\sum_j \int_x^1 \delta q_{i/j}\left(\frac{x}{y},Q^2\right)\delta G_{j/p}(y)\frac{dy}{y}$$

PPDF in proton

Summation is over the three valons

PPDF in a valon. It comes from solving the DGLAP eq.

Valon helicity distribution in proton

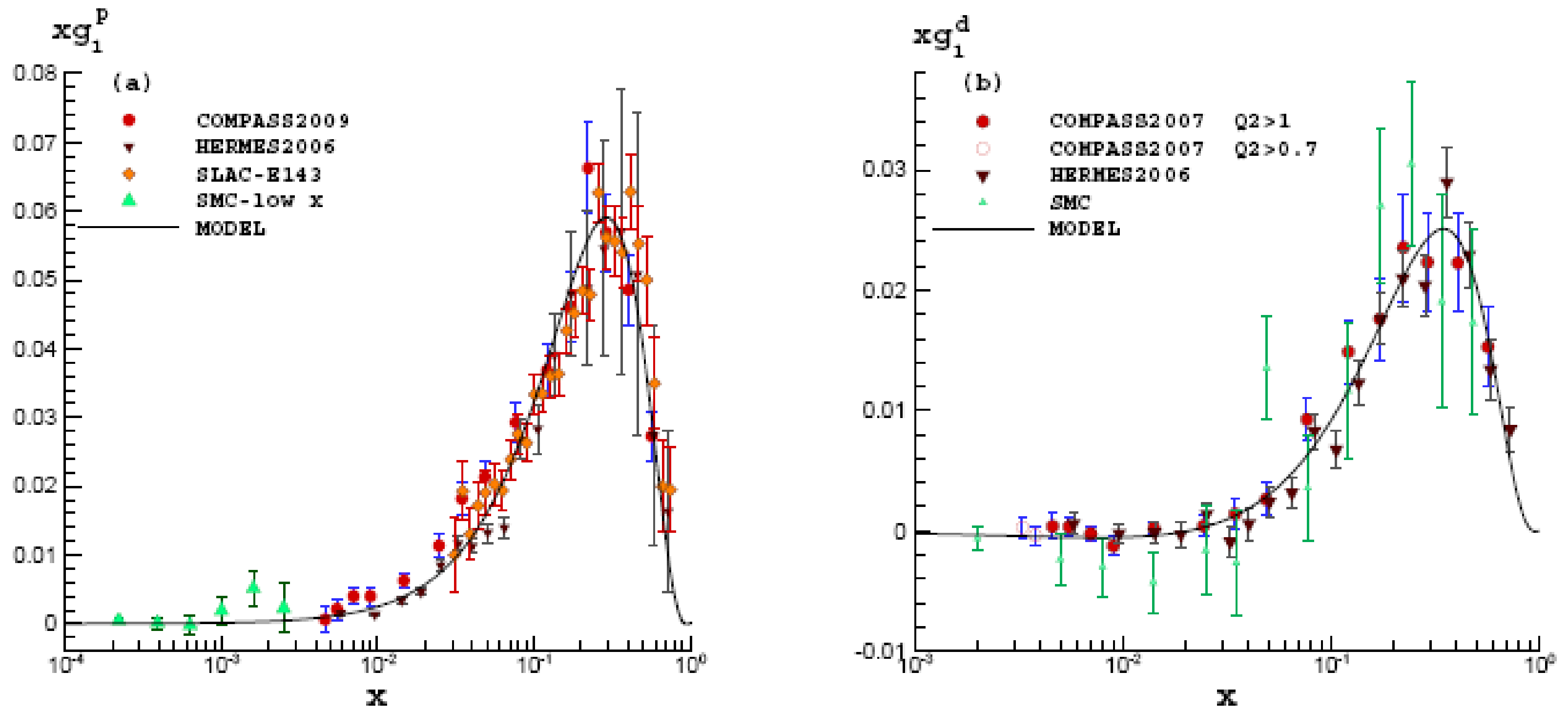
Polarized hadron structure in valon model

$$g_1^p(x,Q^2)=\sum_\nu \int_x^1 \frac{dy}{y}\delta G_{\frac{\nu}{p}}(y) f_1^\nu\left(\frac{x}{y},Q^2\right),$$

Polarized structure function of proton

Valon helicity distribution in proton

Polarized structure function of a ν valon. It depends on Q^2 and the nature of the probe.



F.Arash, F.Taghavi Shahri, JHEP07(2007)071

Transverse structure function is made of two components: a twist-2 part and a mixed twist part:

$$g_2(x,Q^2) = g_2^{ww}(x,Q^2) + \bar{g}_2(x,Q^2)$$

$$\bar{g}_2(x,Q^2) = -\int_x^1 \frac{\partial}{\partial y}\left(\frac{m}{M}h_T(y,Q^2) + \xi(y,Q^2)\right)\frac{dy}{y}.$$

$$g_2^{ww}(x,Q^2) = -g_1(x,Q^2) + \int_0^1 g_1(x,Q^2)\frac{dy}{y}$$

The twist-3 term represents qgq correlations. Therefore, any non-zero result for this term at a given Q^2 will reflect a departure from the non-interacting partonic regime

K.Slifer et.al(RSS Collaboration), Phys.Rev.Lett 105,101601(2010)

Only these two groups calculated the twist-3 part:

X.Song, Phys.Rev.D **54**,1955(1996)

X.Song, Phys.Rev.D **63**,094019(2001)

M. Wakamatsu, Phys.Lett. B**487** (2000) 118-124

We have these sum rules :

There are two important and well known sum rules regarding $g_1(x,Q^2)$ and $g_2(x,Q^2)$.

The first one is OPE sum rule:

$$\Gamma_1^n = \int_0^1 x^n g_1(x,Q^2)dx = \frac{a_n}{2}, n = 0, 2, 4, ...$$

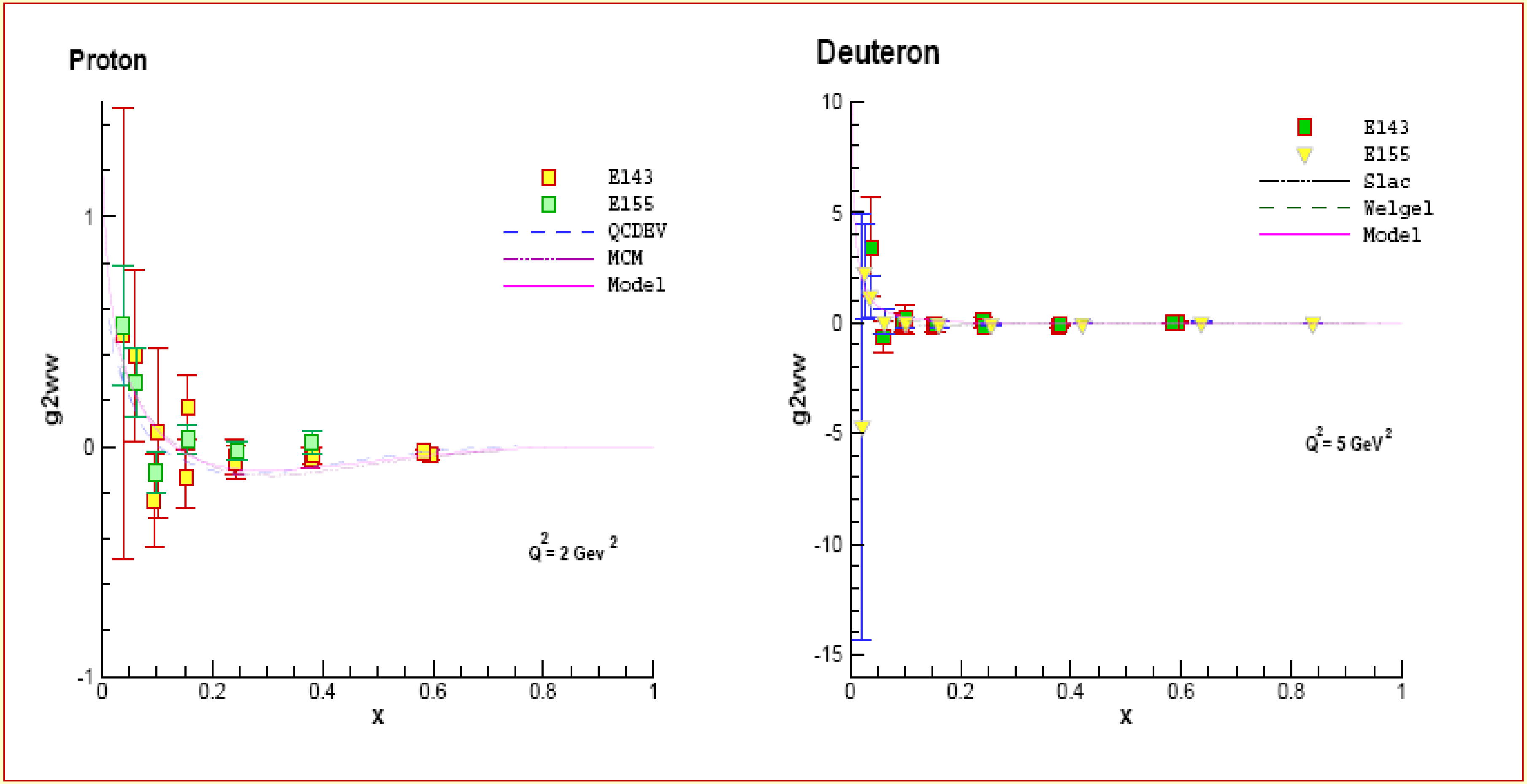
$$\Gamma_2^n = \int_0^1 x^n g_2(x,Q^2)dx = \frac{1}{2}\frac{n}{n+1}(d_n - a_n), n = 2, 4, ...$$

The second one is Burkhardt-Cottingham sum rule

$$\int_0^1 g_2(x,Q^2)dx = 0$$

$$\boxed{\begin{aligned} \int_0^1 g_2^{ww}(x,Q^2)dx &= 0 \\ \int_0^1 x^2 g_2^{ww}(x,Q^2)dx &= -\frac{1}{3}a_2 \\ \int_0^1 x^2 \bar{g}_2(x,Q^2)dx &= \frac{1}{3}d_2 \end{aligned}}$$

Calculation of the twist-2 term, $g_2^{ww}(x, Q^2)$



Calculating the twist-3 term, $\bar{g}_2(x, Q^2)$

$$\bar{g}_2(x, Q^2) = -\int_x^1 \frac{\partial}{\partial y}\left(\frac{m}{M} h_T(y, Q^2) + \xi(y, Q^2)\right)\frac{dy}{y}$$

$$\bar{g}_2(n, Q^2) = L^{\frac{\gamma_n^g}{2b_0}} \bar{g}_2(n, Q_0^2)$$

This part is important

$$\bar{g}_2(n, Q^2) = \int_0^1 x^{n-1} g_2(x, Q^2) dx,$$

$$L \equiv \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)},$$

$$b_0 = \frac{11}{3} N_c - \frac{2}{3} N_f$$

$$\gamma_n^g = 2N_c\left(S_{n-1} - \frac{1}{4} + \frac{1}{2n}\right)$$

$$S_{n-1} = \sum \frac{1}{j}$$

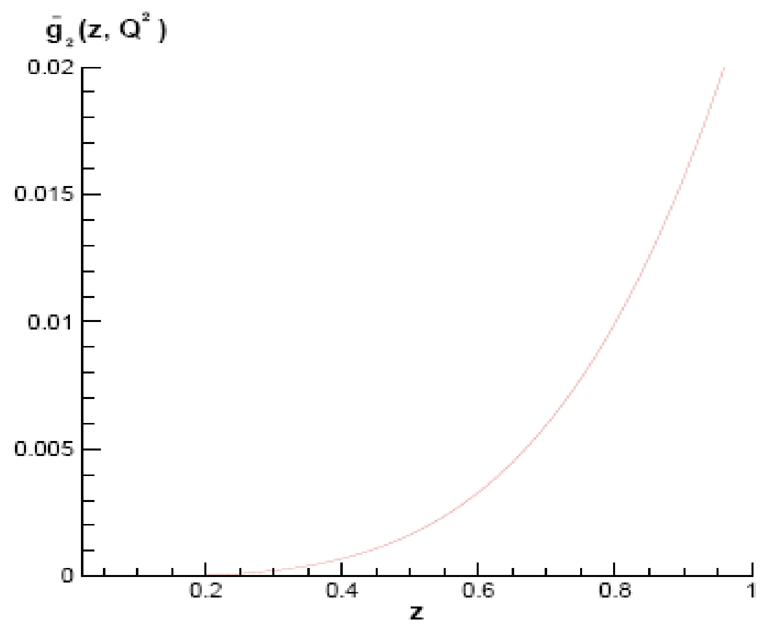
A.Ali, V.M.Brauun, G.Hiller, Phys.Lett.B **266**(1991)117

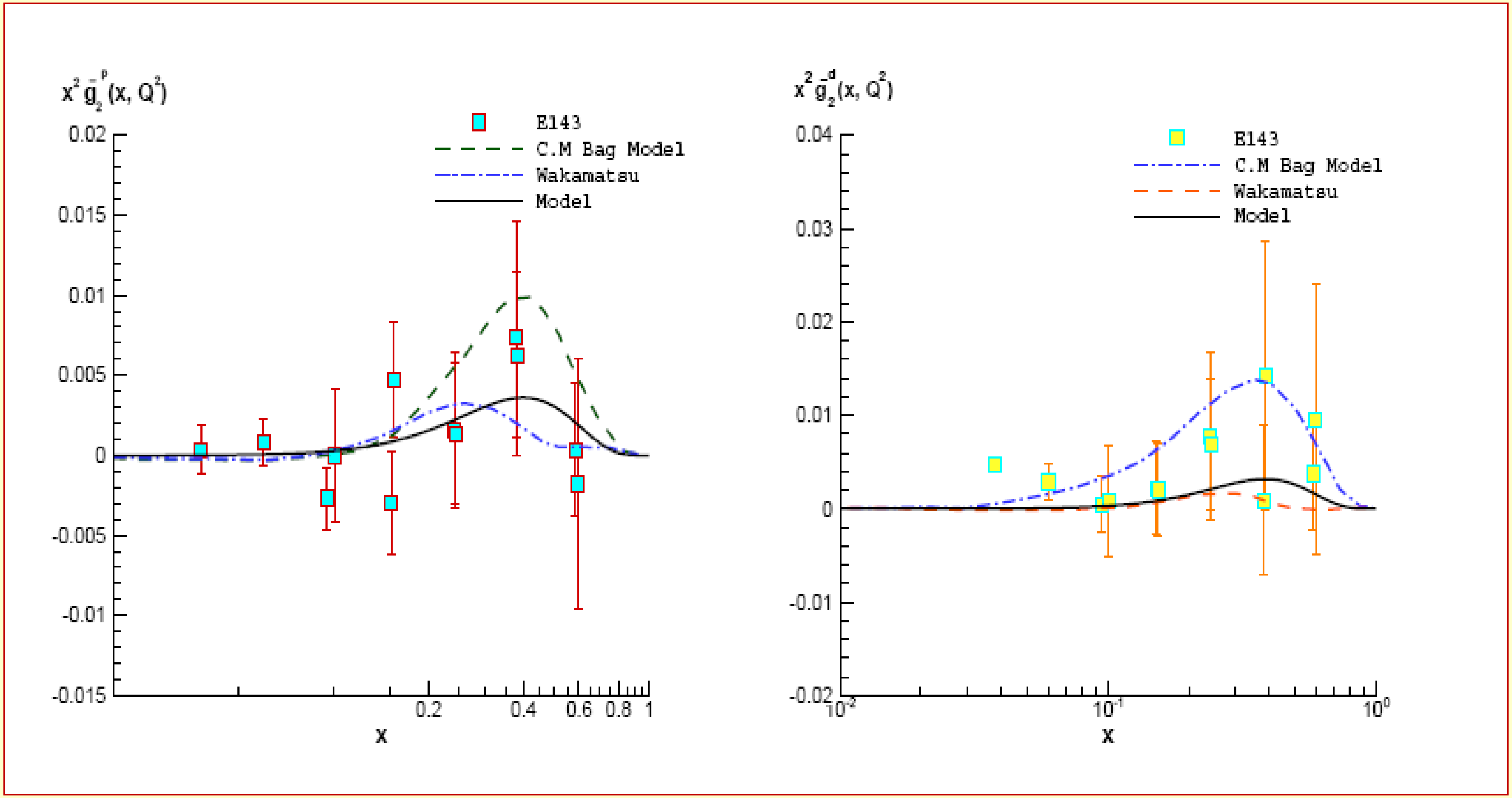
$$\bar{g}_2^{valon}(z,Q_0^2) = A\delta(z-1)$$

$$g_2(n,Q_0^2) = A \times 1$$

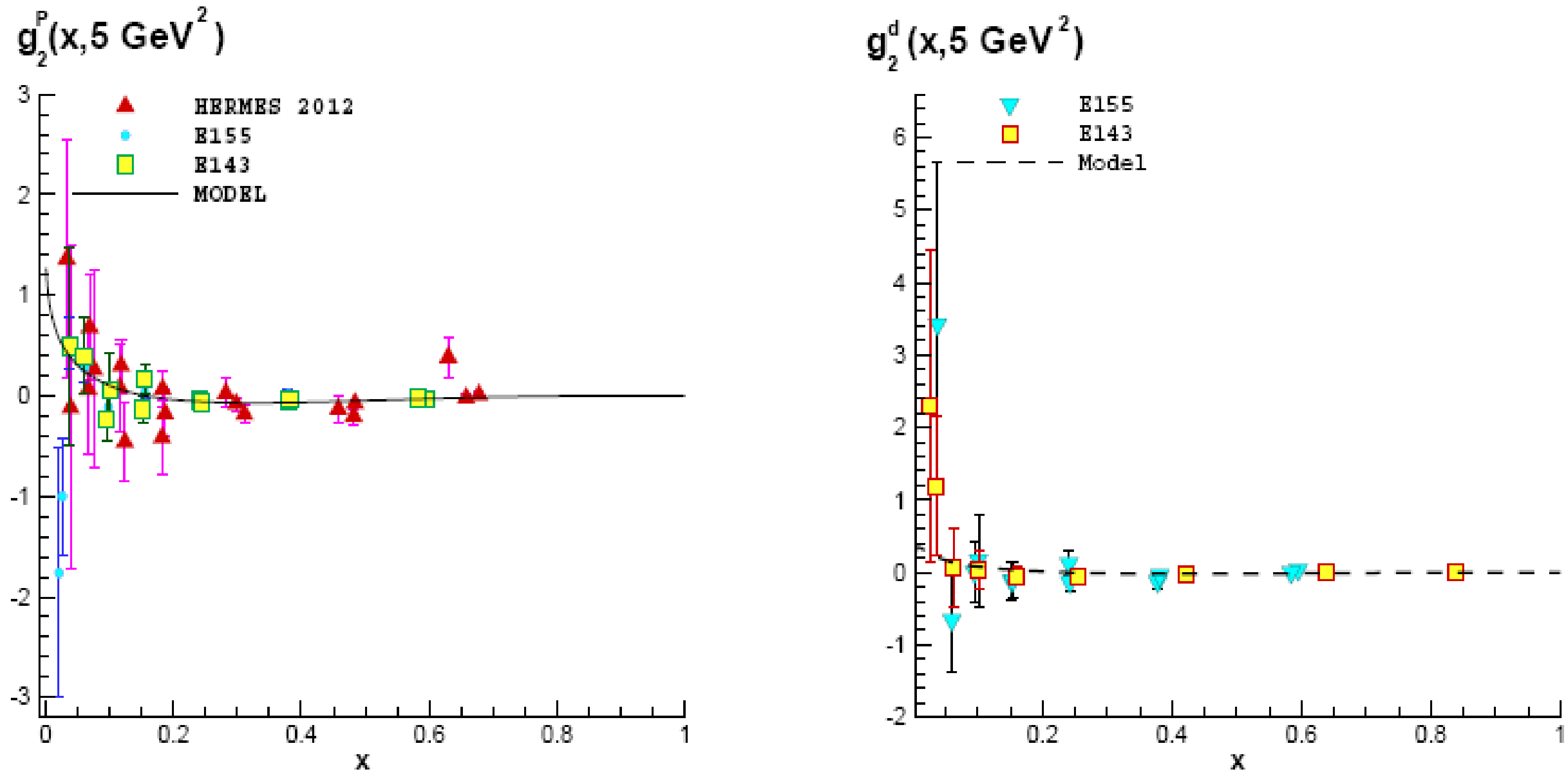
$$\bar{g}_2^{valon}(z,Q^2) = f(Q^2)\bar{g}_2(z,Q_0^2) = f(Q^2)A\delta(z-1)$$

$$\bar{g}_2^{valon}(z,Q^2=5GeV^2) = 0.0233z^{3.827}\delta(z-1)$$





$x^2 \bar{g}_2^p(x, Q^2)$
E143
C.M Bag Model
Wakamatsu
Model
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0.2
0.4
0.6
0.8
1
x
$x^2 \bar{g}_2^d(x, Q^2)$
E143
C.M Bag Model
Wakamatsu
Model
0.04
0.03
0.02
0.01
0
-0.01
-0.02
10^{-2}
10^{-1}
10^{0}
x



$g_2^p(x,5\ \mathrm{GeV}^2)$
HERMES 2012
E155
E143
MODEL
x
$g_2^d(x,5\ \mathrm{GeV}^2)$
E155
E143
Model
x

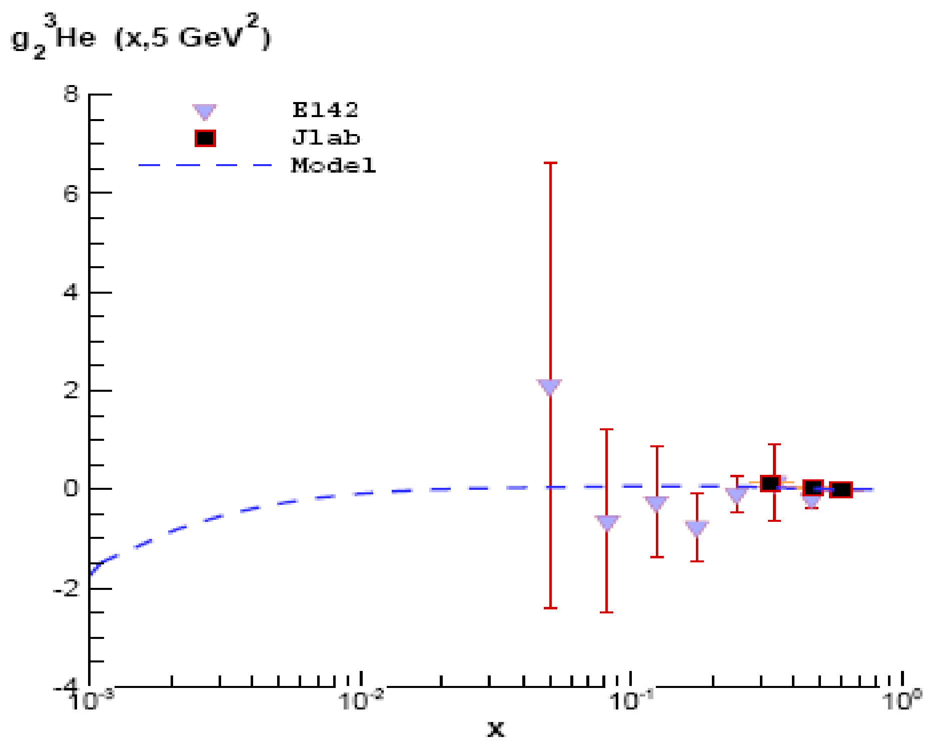
$$g_1^{^3\mathrm{He}}(x,Q^2) = \int_x^3 \frac{dy}{y}\Delta f^n_{^3\mathrm{He}}(y) g_1^n(x/y,Q^2) + 2\int_x^3 \frac{dy}{y}\Delta f^p_{^3\mathrm{He}}(y) g_1^p(x/y,Q^2)$$

$$\Delta f^p_{^3\mathrm{He}}(y) \quad = \quad \frac{ap + cp\ y + ep\ y^2 + gp\ y^3 + ip\ y^4}{1 + bp\ y + dp\ y^2 + fp\ y^3 + hp\ y^4}$$

Nuclear Physics A 831 (2009) 243–262.

$$\Delta f^n_{^3\mathrm{He}}(y) \quad = \quad (\frac{an + cn\ x}{1 + bn\ x + dn\ x^2})^2$$

$\Delta f^p_{^3\mathrm{He}}(y)$		$\Delta f^n_{^3\mathrm{He}}(y)$	
ap	0.00203	an	0.03682
bp	-4.01660	bn	-1.99756
cp	-0.01385	cn	-0.01201
dp	6.06288	dn	1.00815
ep	0.02688		
fp	-4.07592		
gp	-0.02057		
hp	1.02974		
ip	0.00550		



	a_2^p	d_2^p	a_2^d	d_2^d
Valon model	0.0224	0.0042	0.010	0.0037
MIT bag model [28, 35]	–	0.01	–	0.005
QCD sum rule [36]	–	$-(0.6 \pm 0.3)10^{-2}$	–	-0.017
QCD sum rule [37]	–	$-(0.3 \pm 0.3)10^{-2}$	–	-0.013
Lattice QCD [38]	$(3 \pm 0.64)10^{-2}$	$-(4.8 \pm 0.5)10^{-2}$	$(13.8 \pm 5.2)10^{-3}$	-0.022
CM bag model by Song [28]	0.0210	0.0174	0.0087	0.0067
E143 [1]	$(2.42 \pm 0.20)10^{-2}$	$(0.54 \pm 0.5)10^{-2}$	$(8.0 \pm 0.16)10^{-3}$	$(3.9 \pm 9.2)10^{-3}$

The twist-2 and twist-3 matrix elemet operators *,a2 and d2, for the proton and* the deuteron, calculated in the valon model. Also included the experimental data and the results from other theoretical investigations.

The results for the Burkhardt-Cottingham sum rule.

$$\int_0^1 g_2(x, Q^2)dx = 0$$

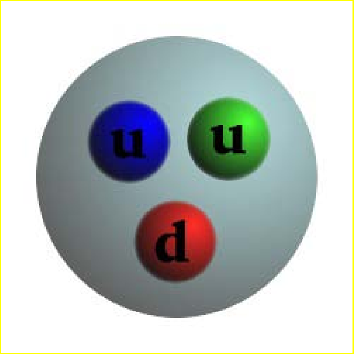
	bag model by Song[28]	E143[1]	E155[3]	HERMES 2012[5]	Valon model
$\int g_2^p(x, Q^2)dx$	-0.0016	-0.014 ± 0.028	-0.022 ± 0.071	$0.006 \pm 0.024 \pm 0.017$	-0.004
$\int g_2^d(x, Q^2)dx$	-0.00287	-0.034 ± 0.082	0.023 ± 0.044	-	0.010

Conclusions

We have used the valon model and calculated the transverse spin structure function for nucleon and deuteron. We offer a simple approach for calculating the twist-3 part of the transverse spin structure function. It is evident that our results for both the twist-2 part and for the full transverse spin structure functions are in good agreements with the experimental data. We have also provided a comparison of our results with other works.


Thank you for your attention

Backup



$$p^{\uparrow} = \sqrt{\frac{2}{3}}(u^{\uparrow}u^{\uparrow})d^{\downarrow} + \sqrt{\frac{1}{3}}(u^{\uparrow}u^{\downarrow})d^{\uparrow} \tag{1}$$

$$u^{\uparrow} = 5/3 \quad u^{\downarrow} = 1/3 \quad d^{\uparrow} = 1/3 \quad d^{\downarrow} = 2/3. \tag{2}$$

and so

$$\begin{array}{llll} u & \equiv u^{\uparrow} + u^{\downarrow} & \equiv \int dx u(x) & = 2 \\ d & \equiv d^{\uparrow} + d^{\downarrow} & \equiv \int dx d(x) & = 1 \\ \Delta u & \equiv u^{\uparrow} - u^{\downarrow} & \equiv \int dx \Delta u(x) & = 4/3 \\ \Delta d & \equiv d^{\uparrow} - d^{\downarrow} & \equiv \int dx \Delta d(x) & = -1/3 \end{array} \tag{3}$$

This model clearly has all of the proton's spin carried by its valence quarks

$$\Delta q \equiv \Delta u + \Delta d = 1 \tag{4}$$