Hawking radiation from cosmological black holes

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I. MARGINALLY OUTER TRAPPED SURFACE AND EMISSION SURFACE

The term cosmological black hole (CBH) is used to describe a collapsing structure within an otherwise expanding universe. This is different from the term astrophysical black holes (ABH) coined to use for the applications of static, stationary or asymptotically flat black holes in astrophysics (see for example recent papers [1, 2]. In this draft we intend to discuss the hawking radiation from CBH which is more general than ABH.

Let me take some definitions from [3] that are needed to describe the dynamical black hole boundary.

**Definition 1.** A trapping horizon $H$ is a hypersurface in a 4-dimensional spacetime that is foliated by 2-surfaces (which we will take to be of spherical topology) such that $\theta_l|_H = 0$, $\theta_n|_H \neq 0$ and $\mathcal{L}_n \theta_l|_H \neq 0$. A trapping horizon is called *outer* if $\mathcal{L}_n \theta_l|_H < 0$, *inner* if $\mathcal{L}_n \theta_l|_H > 0$, *future* if $\theta_n|_H < 0$ and *past* if $\theta_n|_H > 0$.

In this definition, and what follows, $\ell^a$ and $n^a$ are respectively the future-directed outgoing and ingoing null normals in a point and $\theta(\ell)$ and $\theta_n$ are the expansion of the congruences of curves generated by these vector fields.

**Definition 2.** A marginally trapped tube (MTT), $T$ is a hypersurface in a 4-dimensional spacetime that is foliated by two-surfaces (again assumed to be of spherical topology) such that $\theta_n|_T < 0$ and $\theta_l|_T = 0$.

Let $V^a$ be a tangential to $H$, and orthogonal to the foliation by marginally trapped surfaces. Hence it is always possible to find a function $C$ and normalization of $\ell^a$ such that $V^a = \ell^a - C n^a$. In addition, the definition of $V^a$ implies that $\mathcal{L}_V \theta_l = 0$, which gives us an expression for $C$:

$$C = \frac{\mathcal{L}_n \theta_l}{\mathcal{L}_n \theta_l}.$$  \hspace{1cm} (1)

For the future inner trapping horizon $C < 0$ and for the future outer trapping horizon $C > 0$. EMOTS defined in [1]are equivalent to the *future inner trapping horizon* and OMOTS are equivalent to *future outer trapping horizon*. In some cases there is EMOTS as is shown in Fig.(1), but the EMOTS cannot generally forms in a dynamical metric as discussed in [3]. Therefore, the emission surface is not limited to the EMOTS and as we will discuss in the next section the OMOTS which are slowly evolving horizon play an important role in the hawking radiation.

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**FIG. 1:**
For Cosmological black hole the OMOTS will form and it tend to slowly evolving horizon and at the late time it becomes isolated horizon see [4].

II. ROLE OF THE WKB APPROXIMATION ON THE RADIATION

Because of the FRW background and the matter falling in to the cosmological black hole, the cosmological black hole metric is dynamical. Hence finding the semi-classic vacuum solution for this spacetime is not easy to find the hawking temperature. To this end, tunneling method help to find the black hole temperature without calculation the vacuum at the far infinity [5–8]. The key point for this calculation is the WKB approximation must be written for the wave near the horizon [8, 9]. On the other side, this method clarify the apparent horizon has a main role in the black hole radiation [10].

To examine the WKB approximation for the waves one should calculated the redshift of the light emitted near the apparent horizon. If an emitter send a light ray to an observer with null vector $k^\mu$, the relative light redshift that is calculated by observer with 4-velocity $u^\mu$ is,

$$1 + z = \frac{(k^\mu u_\mu)_e}{(k^\mu u_\mu)_o}.$$  

(2)

It was shown that the redshift of the light emitted from the apparent horizon for the dust CBH is not infinite, but only in the case that the apparent horizon is slowly evolving horizon [4] is infinite. As a result, hawking radiation will occur for from OMOTS (slowly evolving horizon) not EMOTS for dust cosmological black hole.

III. RADIATION ESCAPE AND MASS LOSS

The basic point of the dynamical black hole collapse is the apparent horizon surface will separate form event horizon and cosmological black hole is not exceptional from this event [4, 11]. Any observer which is in the outside of the event horizon can not have any signal inside of the event horizon or around the apparent horizon in classical gravity. But can we implement this scenario in the semi-classical gravity?

There is two scenario for this case. First the emission surface for OMOTS be in the out side of the event horizon. In this case as shown in the Fig.(2), the black hole evaporate and we get the standard view of evaporation see [12].

The second scenario is that the emission surface located in the outside of OMOTS and inside the event horizon. As shown in the Fig.(3), no evaporation will occur in this scenario and Ellis [1] argument should be implicate in the evaporation process.

A. Penrose diagram for late time de Sitter background

In this subsection we assume that after some time (at a redshift) the cosmological constant becomes important and our cosmological background evolve as de Sitter space. The Penrose diagram for the case that the emission surface in outside the even horizon in depicted in Fig.(4) and for the case the emission surface in inside the even horizon in depicted in Fig.(5).

FIG. 2:


FIG. 3:
FIG. 4:

FIG. 5: