Entanglement Renormalization Group

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- A major challenge in quantum many-body problems is their exponentially-large Hilbert space.

- For many many-body systems, variational principle is used which asserts that

$$E_0 \leq \frac{\langle\psi|\widehat{H}|\psi\rangle}{\langle\psi|\psi\rangle}$$

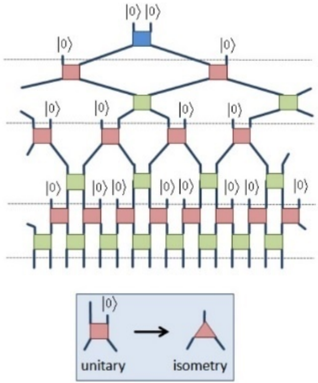
  with E_0 the ground-state energy (lowest eigenvalue).

- If we have a class of variational ansatz states $|\psi(z)\rangle$ which are parameterized by a set of parameters z, we can try to find a good approximation of the ground state of $\widehat{H}$ within this variational class by finding the parameters $z_\star$ that minimize the energy expectation value.

- In relativistic QFT and any system having a large ratio ξ/Λ, however, the variational principle has not met with the same success as in other areas of many-body physics.
- Conceptual issues standing in the way of a successful application of the variational principle in this regime where outlined by Feynman in a talk given in 1987.
  - Sensitivity to high frequencies.
  - In relativistic QFT, because of zero point energies, the prime quantity of interest for the application of the variational principle is ill-defined.
  - Lack of suitable variational ansatz does not allow for high-accuracy computations of observable quantities (just Gaussians).

- But, in comparison with a state randomly chosen from the full Hilbert state $\mathcal{H}$, the quantum vacuum states which we observe in nature are highly atypical (entropy area law ...)
- Different tools have been developed to find this physical submanifold of the full Hilbert state $\mathcal{H}$.
  - Quantum Monte Carlo approach. But for very interesting systems it is stopped, notorious sign problem and fermion doubling.
  - Wilson's numerical renormalization group (NRG) (1970s), based on Kadanoff's idea (1966), in which the eigenvectors of the Hamiltonian with lowest eigenvalues are used. It is also inefficient (1970-1992).
  - White (1992) introduced the density matrix renormalization group (DMRG). The eigenvectors of the reduced density matrix with greater eigenvalues are used.

- Nowadays, DMRG has became a cornerstone of the idea to parametrize the physical state (systems on lattices) by a variational class of states named as matrix product states (MPS) (2004).
  - Finding the physical properties of strongly interacting systems.
  - Classifying the quantum phases.
  - More interestingly, constructing new exactly solvable models.
- There are, however, a number of important cases, where MPSs either fail to approximate the solution or provide less accurate results. Critical systems.
- Vidal (2005-2007) discovered the multiscale entanglement renormalization ansatz (MERA) for describing the critical systems. MERA has also a basis in quantum information, as it can be seen as a class of quantum circuits.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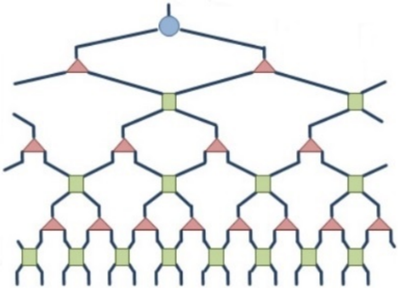
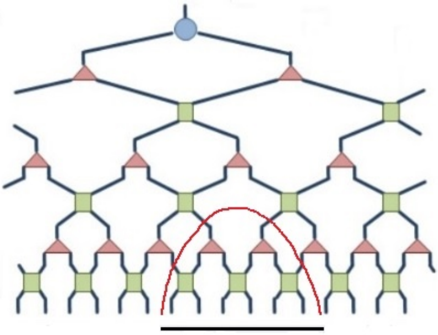


- Swingle (2009), based on Maldacena (1997),Ryu and Takayanagi (2006), noticed a connection between the MERA with completely different side of the physics, string theory and specially AdS geometry.

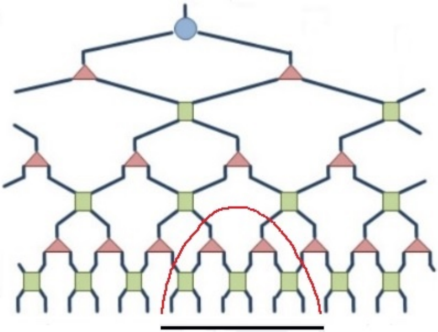


- Susskind and Maldacena (2013),Susskind (2014)

- According to the mentioned points, it is clear that we are at a very interesting crossing point where different branches of theoretical physics: quantum information theory, condensed matter physics, renormalization group theory, quantum gravity and black hole physics and even cosmology join together.

- The continuous MERA (cMERA) have just been developed for free theories based on using Gaussian ansatz in variational method. Haegeman,Osborne,Verschelde and Verstraete (2011)

- One of the open and interesting problems was trying to develop cMERA for interacting theories.



- Feynman:

"I think it should be possible some day to describe field theory in some other way than with the wave functions and amplitudes. It might be something like the density matrices..."

Thank YOU