Inflation and Primordial Universe

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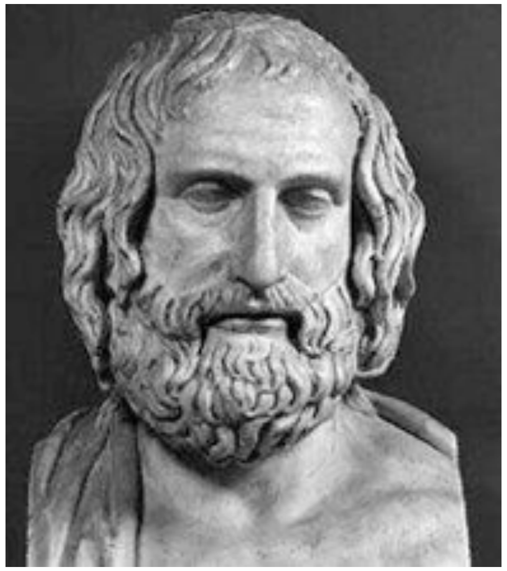
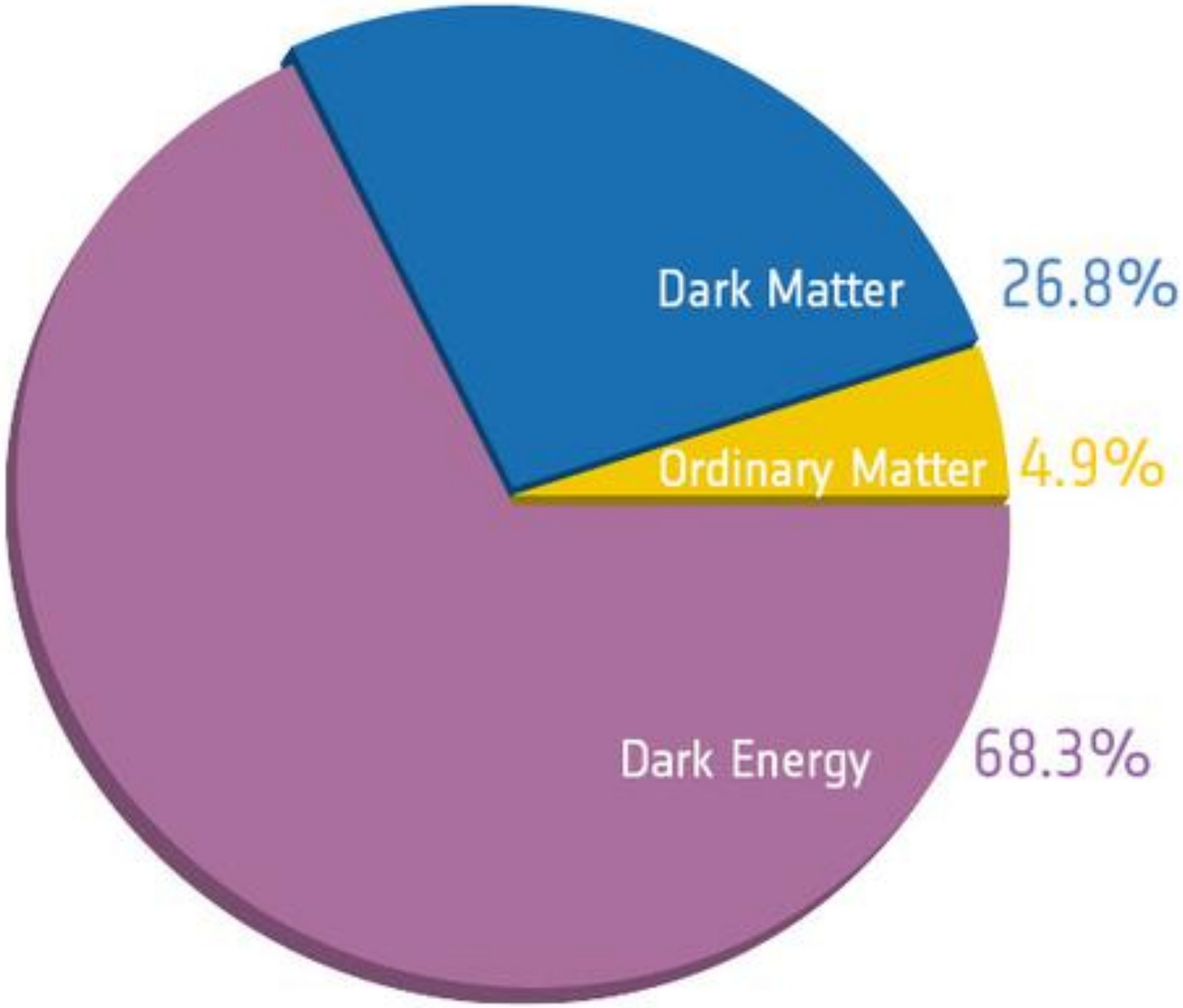
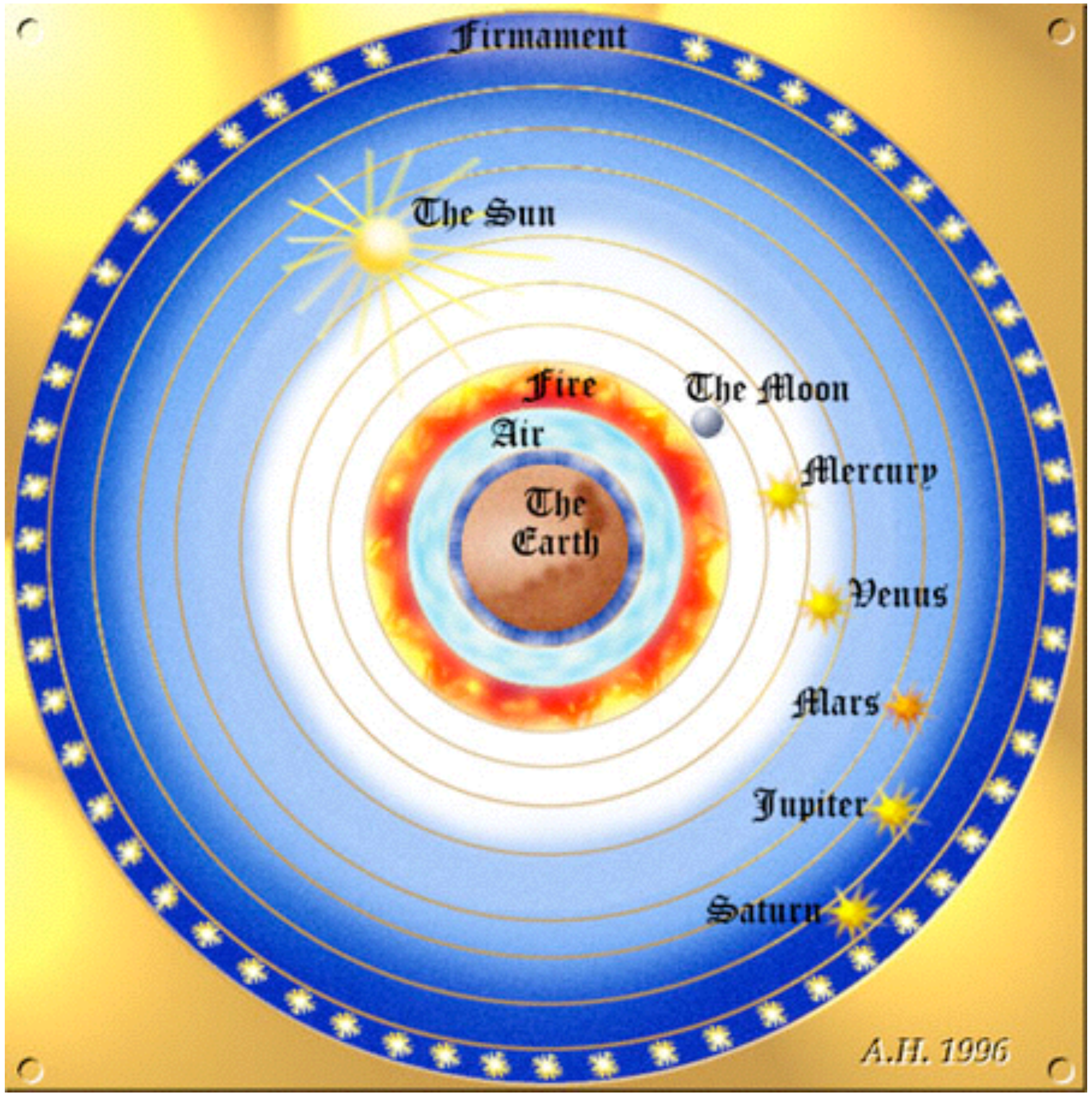
The Standard Model of Cosmology

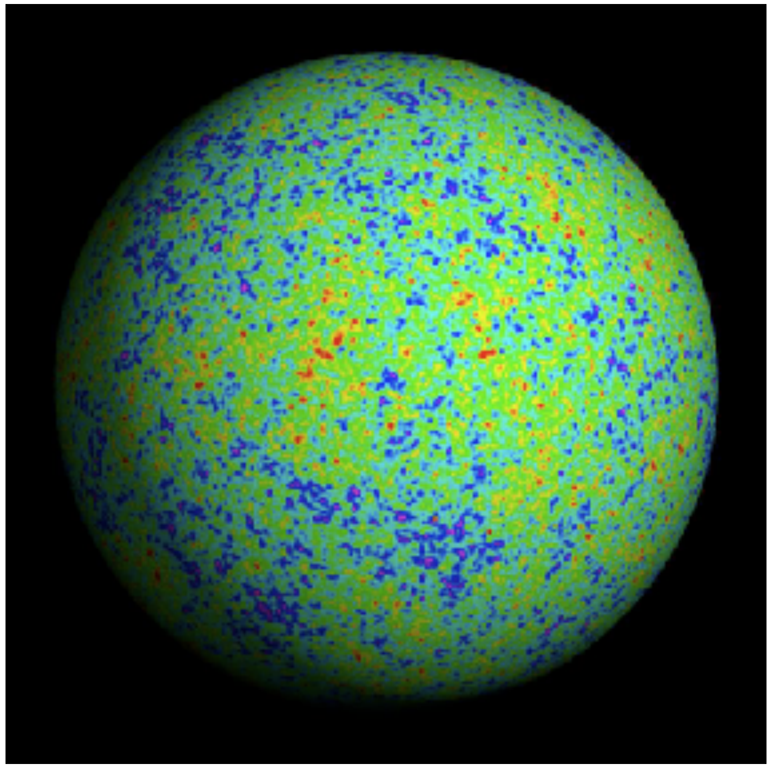


- Ordinary Atoms:(Baryons) 5%
- Dark Matter: 26%
- Dark Energy: 69%





The Initial Conditions Puzzles

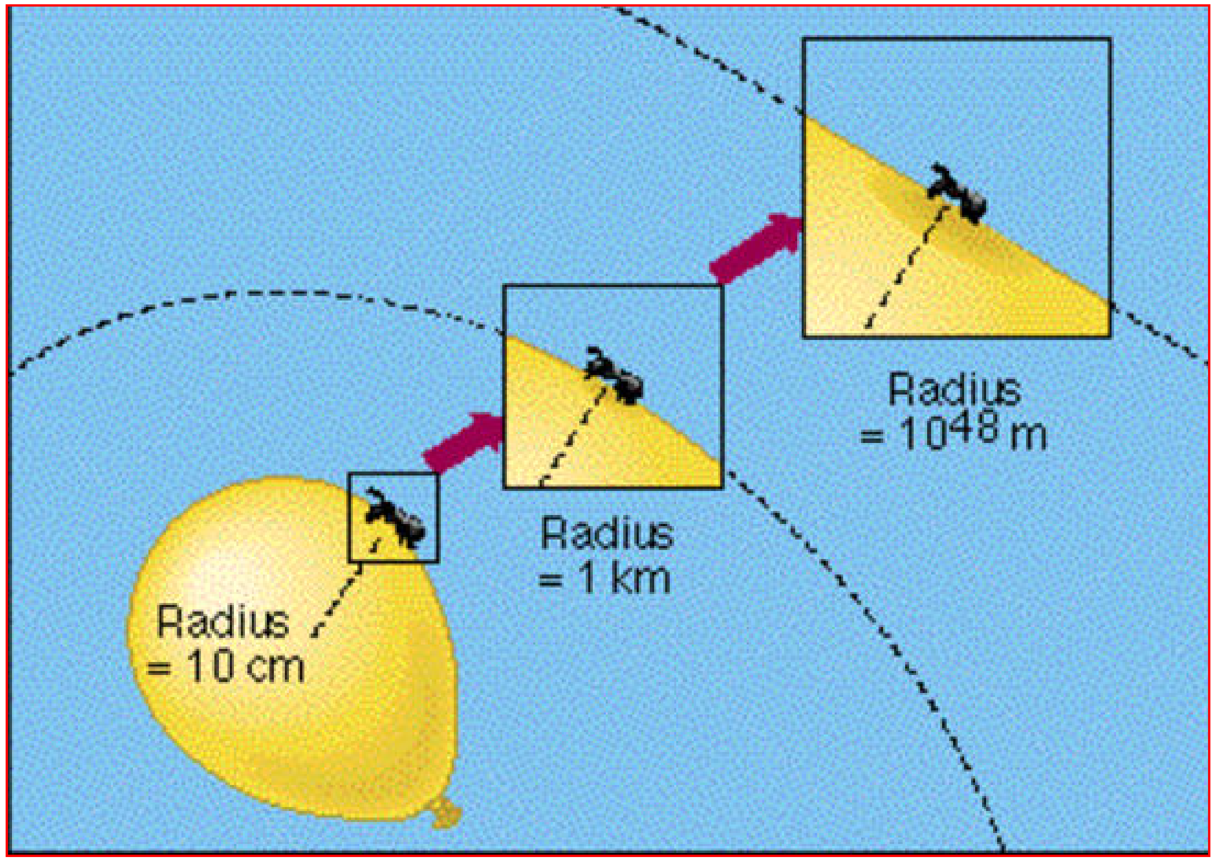


Despite the successes of the big bang cosmology, there are initial conditions problems:

- The Horizon Problem: Why is the Universe so homogeneous and isotropic? During its evolution, the Universe did not have enough time to become so isotropic and homogeneous.
- The Flatness Problem: Why is the Universe so flat? If $\Omega \sim 1$ today, then extrapolating back to very early Universe at Planck time we find $|\Omega - 1| \sim 10^{-60}$.
- There are tiny fluctuations at the level of 10^{-5} on the smooth CMB background, which are almost scale invariant, adiabatic and Gaussian. What mechanism can create these perturbations ?

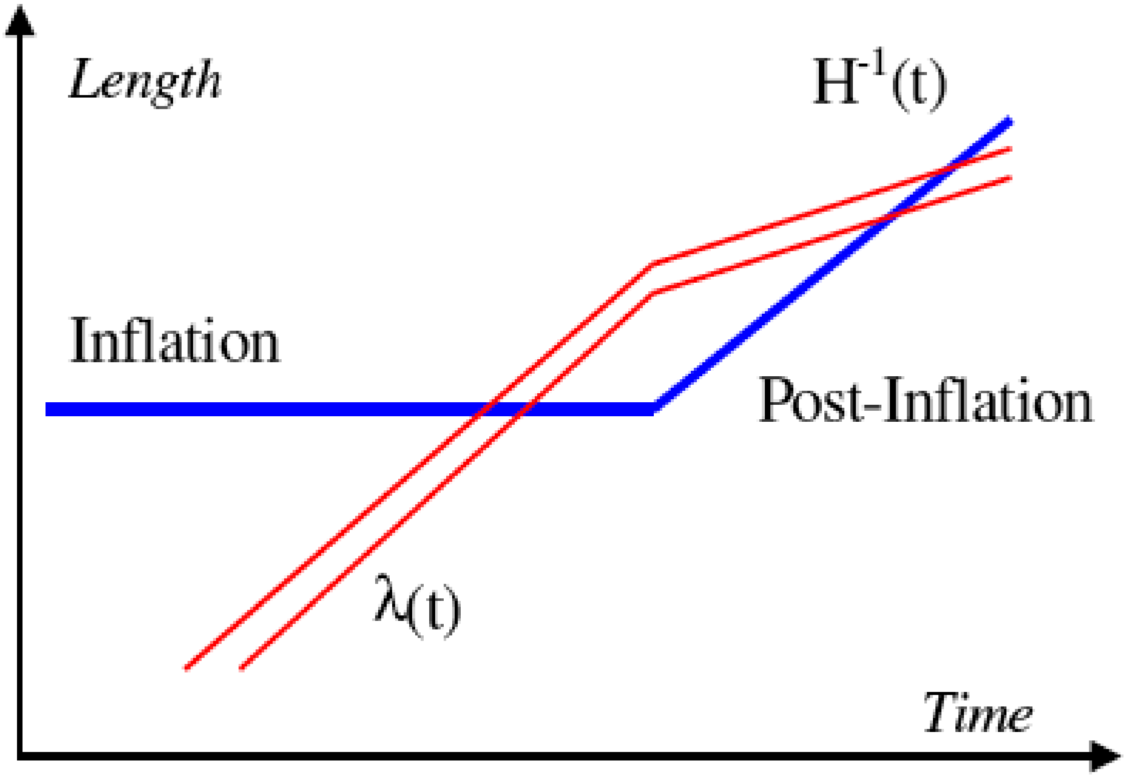
Inflation

A short period of acceleration in very early Universe will provide all these necessary initial conditions and flattens the Universe.



www.astro.princeton.edu/~tremaine/ast541/das.ppt

- Primordial quantum fluctuations during inflation seeds the observed almost scale invariant Gaussian perturbations in CMB.



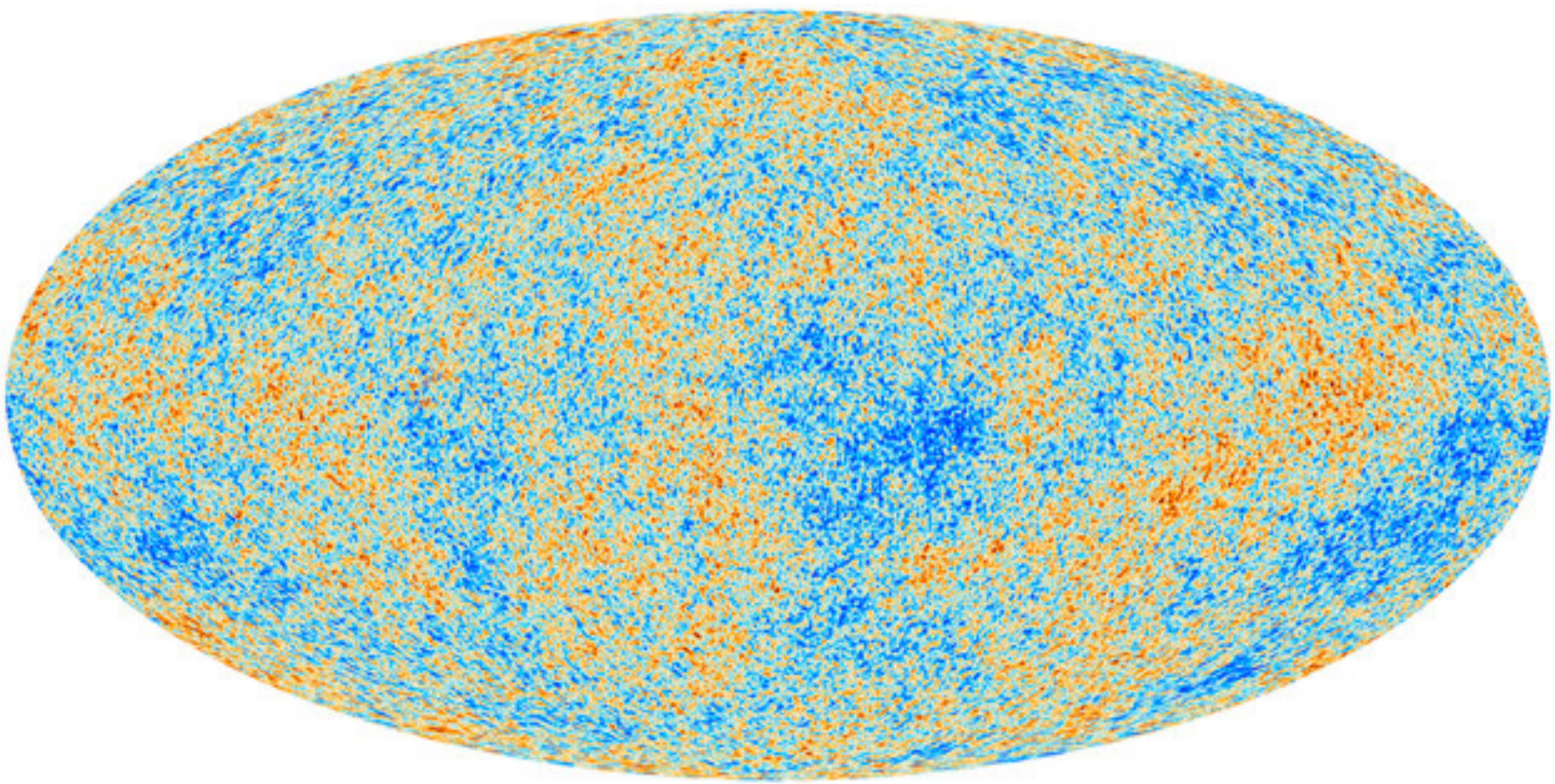
- Originally all of these modes were inside the horizon. Inflation stretches their wavelengths outside the horizon. While outside the horizon, they ``freeze out``. Later on they re-enter the horizon to form the observed structures.

Inflation and Observations

All observations (WMAP, Planck,...) strongly support inflation.

The basic predictions of inflation are that the primordial perturbations are nearly scale invariant, nearly adiabatic and nearly Gaussian.

In CMB perturbations we observe the quantum vacuum fluctuations.



Planck Observation

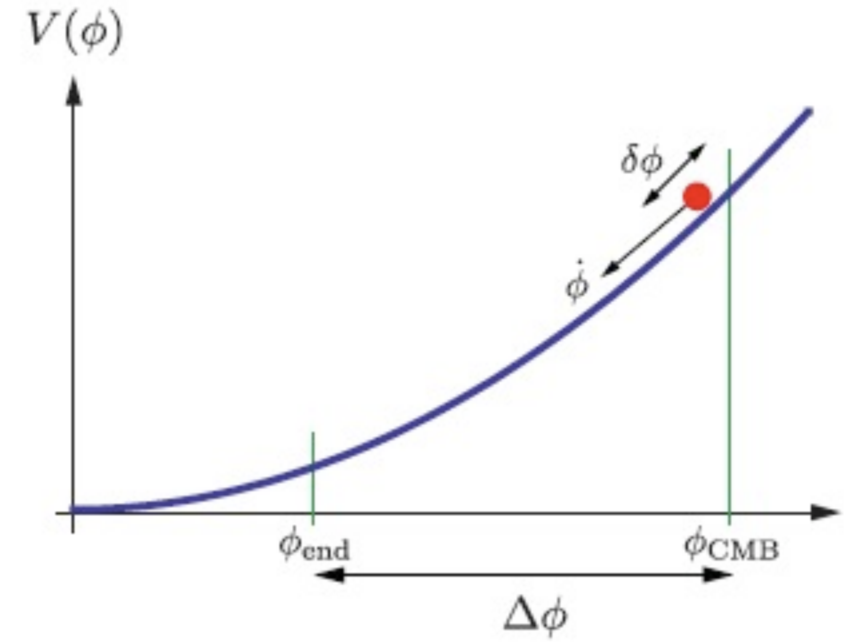
$$\left\langle \frac{\delta T^2}{T^2} \right\rangle \propto \left\langle \mathcal{R}^2 \right\rangle = \left(\frac{H^2}{2\pi\dot{\phi}} \right)^2$$

Slow Roll Inflation

The necessary condition for inflation

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0 \qquad \rightarrow \qquad p < -\frac{\rho}{3}$$

In most models, inflation is derived by a scalar field, the inflaton.



$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

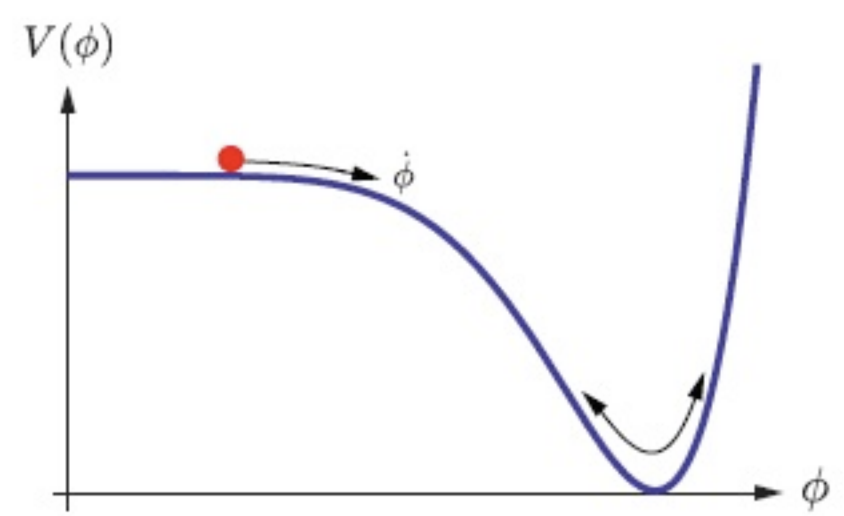
$$a(t) \sim e^{Ht} \qquad , \qquad H^2 = \frac{8\pi G}{3}V$$

Inflation ends when the field reaches near the minimum of its potential.

Baumann.

To solve the flatness and horizon problem we require N= H (t_f - t_i) = 60.

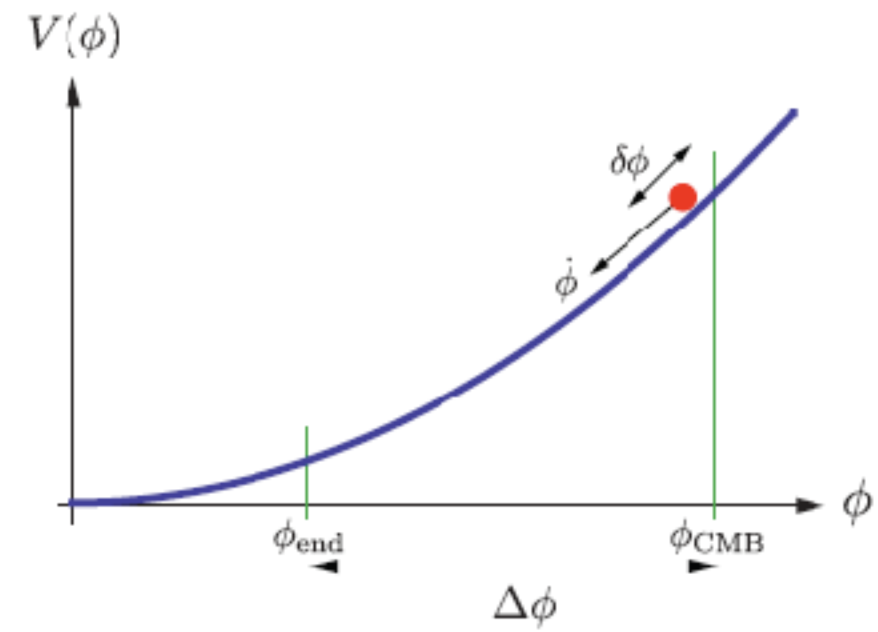
Reheating



The inflaton field oscillates around the minimum of its potential releasing its energy into the Standard Model Particle Physics

Models of Inflation

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)\right] .$$



Large field inflation:

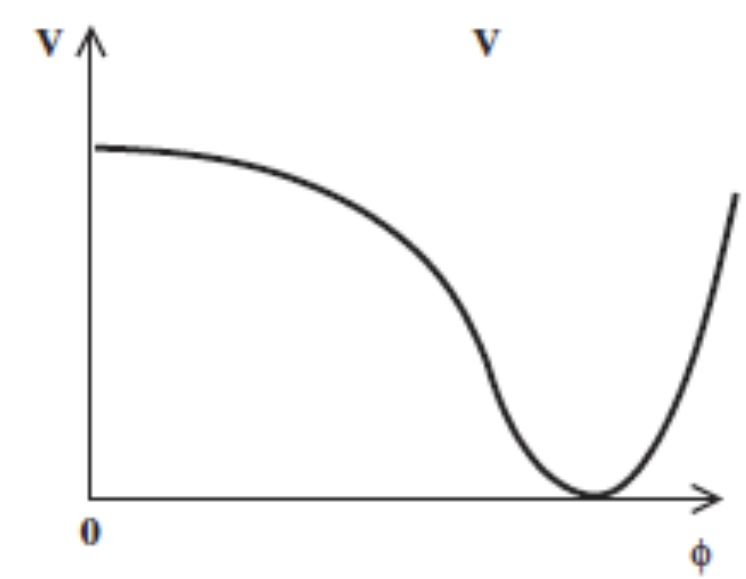
$$V = \frac{1}{2}m^2\,\phi^2 \qquad \phi_i \sim 15\,M_P$$

$$V = \frac{\lambda}{4}\,\phi^4 \qquad \lambda \sim 10^{-14} \qquad \phi_i \sim 22\,M_P$$

Small field inflation:

$$V(\phi) = V_0\left[1 - \left(\frac{\phi}{\mu}\right)^2\right]^2 .$$

$$= V_0 - \frac{1}{2}m^2\phi^2 + \ldots .$$

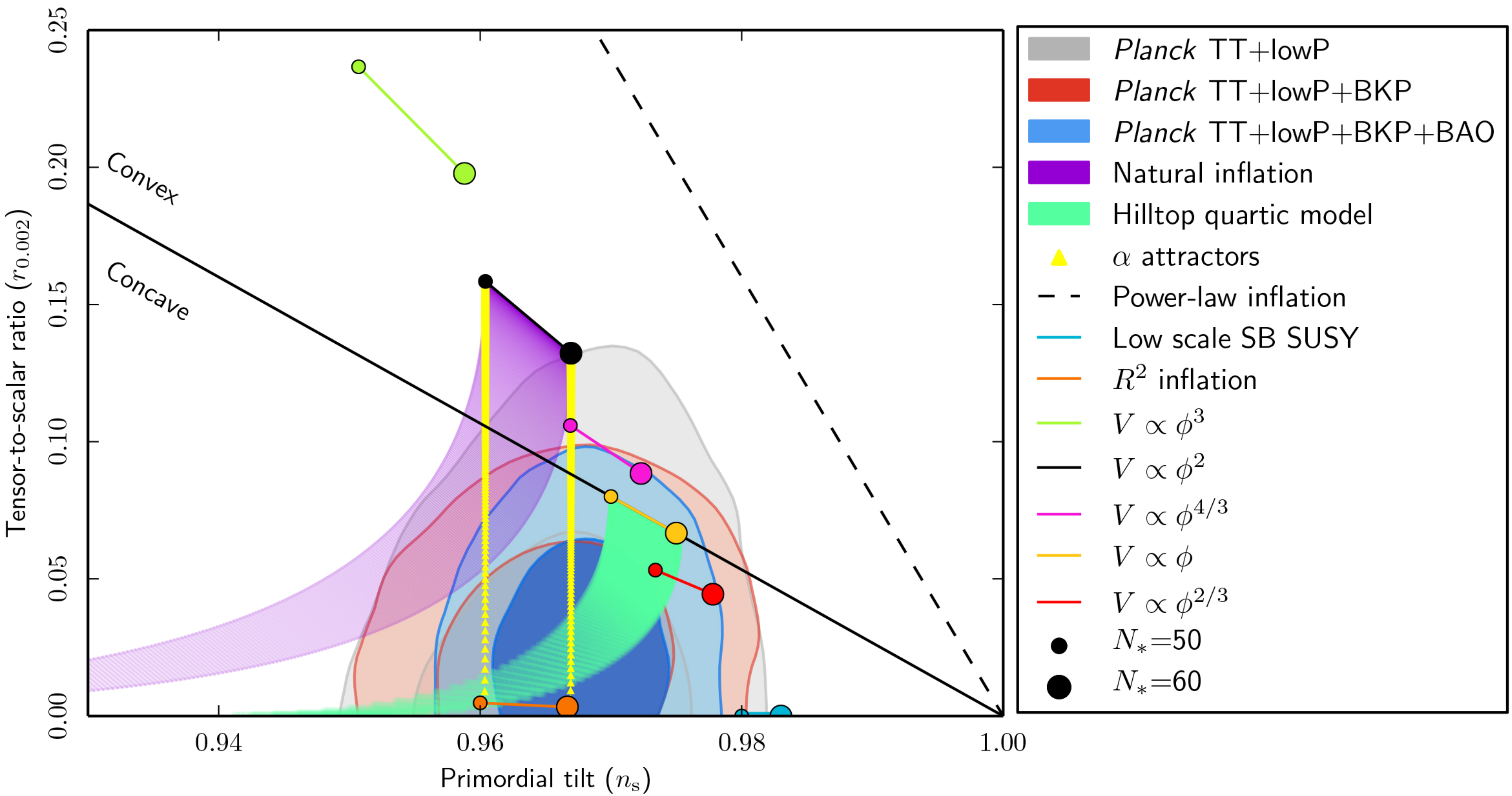


Various Potential Considered:

- Power-Law Inflation ———— $V(\phi) = \lambda M_{\rm pl}^4 \left(\frac{\phi}{M_{\rm pl}}\right)^n$,
- Hilltop models ———— $V(\phi) \approx \Lambda^4 \left(1 - \frac{\phi^p}{\mu^p} + ..\right.$
- Natural Inflation ———— $V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right]$
- D-brane Inflation ———— $V(\phi) = \Lambda^4 \left(1 - \frac{\mu^p}{\phi^p} + ...\right)$
- Exponential Potentials ———— $V(\phi) = \Lambda^4 \left(1 - e^{-q\phi/M_{\rm pl}} + ...\right)$
- Spontaneously broken SUSY ———— $V(\phi) = \Lambda^4 \left[1 + \alpha_h \log(\phi/M_{\rm pl})\right]$
- Starobinsky R^2 Inflation ———— $S = \int d^4x \sqrt{-g} \frac{M_{\rm pl}^2}{2} \left(R + \frac{R^2}{6M^2}\right)$

$$V(\phi) = \Lambda^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{\rm pl}}\right)^2$$

Constraint on Single Field Inflation



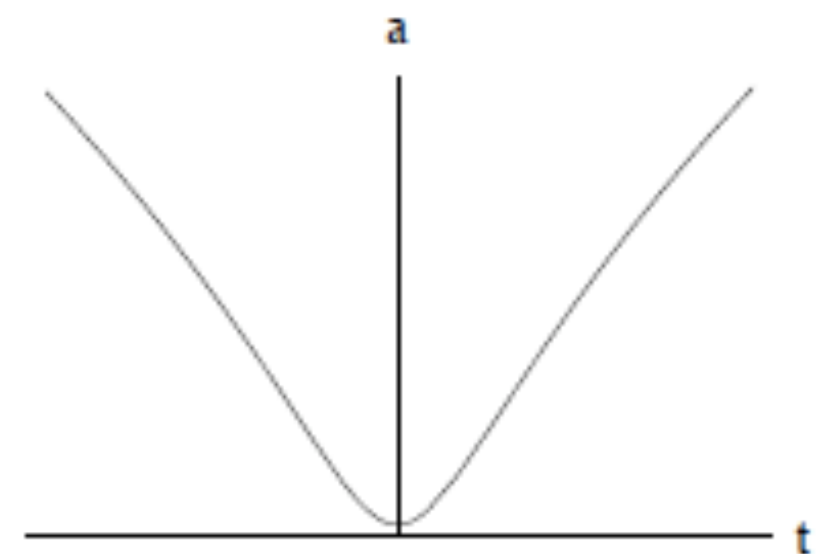
- The joint data analysis from Planck/BICEP2/Keck Array indicates $r < 0.1$.
- The data prefers concave potential with $\partial^2 V < 0$.
- Simple potential such as ϕ^2 and ϕ^4 are disfavored.

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_s}\right)^{n_s - 1}$$

Alternatives to Inflation: prospects

Alternative to inflation includes models of bounce and string gas cosmology.

Predictions of bounce scenarios:



- Produce adiabatic, almost scale invariant perturbations.
- No appreciable amount of gravitational waves.
- Significant amount of non-gaussianity.

PLANCK and upcoming observations may have a good chance to verify or rule out bounce (ekpyrotic)/inflationary scenarios.

Open questions and future directions:

How to achieve bounce or bypass NEC?

Can ekpyrotic models be embedded in high energy physics?

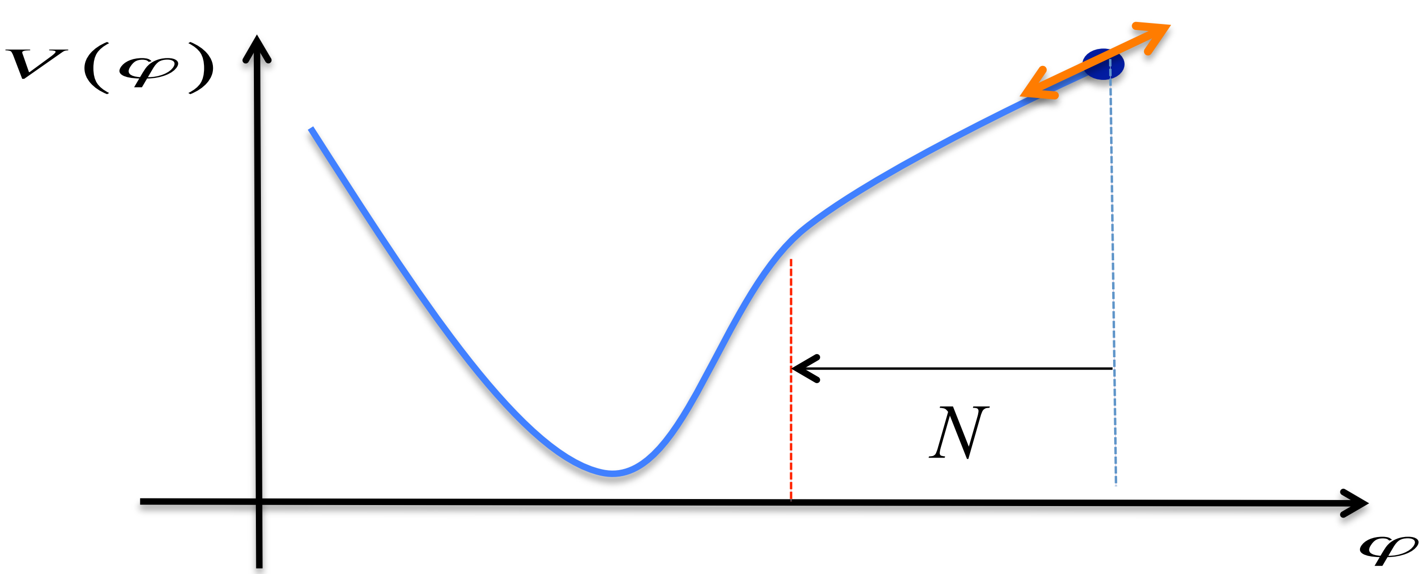
Stochastic Inflation

Stochastic inflation is an elegant approach to study primordial perturbations in inflationary backgrounds.

Quantum perturbations swept out to super-horizon scales by the background expansion act a source for small scale perturbations. This effect can be captured by a Gaussian noise $\xi(N)$.

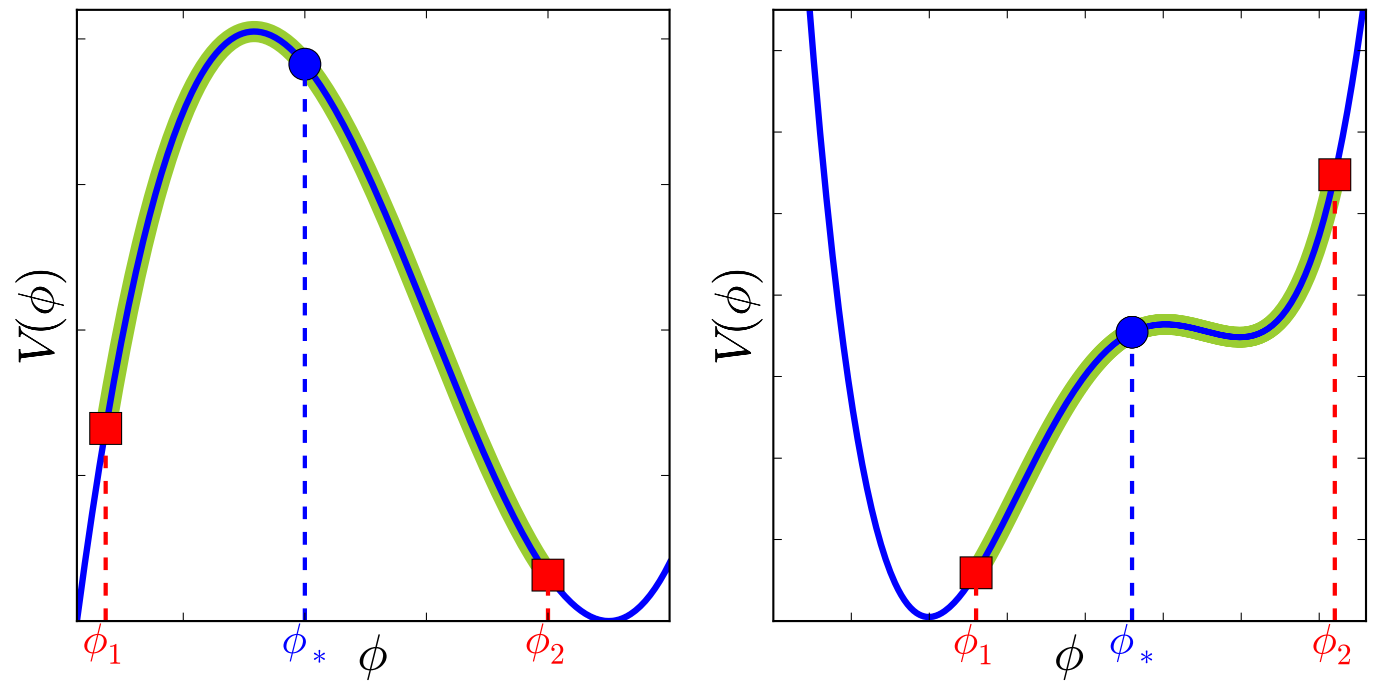
The corresponding Langevin equation for the inflaton dynamics is given by

$$\frac{d\phi}{dN} = -\frac{V_\phi}{3H^2} + \frac{H}{2\pi}\xi(N)\,, \qquad \Big\langle \xi(N)\,\xi(N')\Big\rangle = \delta(N - N')$$



D. Wands

First Passage Time



Suppose the inflaton field is initially located at $\phi = \phi_*$. Then the probability p_1 that it first reaches ϕ_1 before reaching ϕ_2 is given by

$$v\, p_1''(\phi) - \frac{v'}{v}\, p_1'(\phi) = 0\,, \qquad v \equiv \frac{V(\phi)}{24\pi^2 M_P^4}$$

This can be solved to give

$$p_1 = \frac{\int_{\phi_*}^{\phi_2} \exp\left[\frac{-1}{v(x)}\right]}{\int_{\phi_1}^{\phi_*} \exp\left[\frac{-1}{v(x)}\right]}$$

Note that $v \sim 10^{-12}$ and the integrand is exponentially sensitive to the shape of the potential.

Fall and Escape Probability

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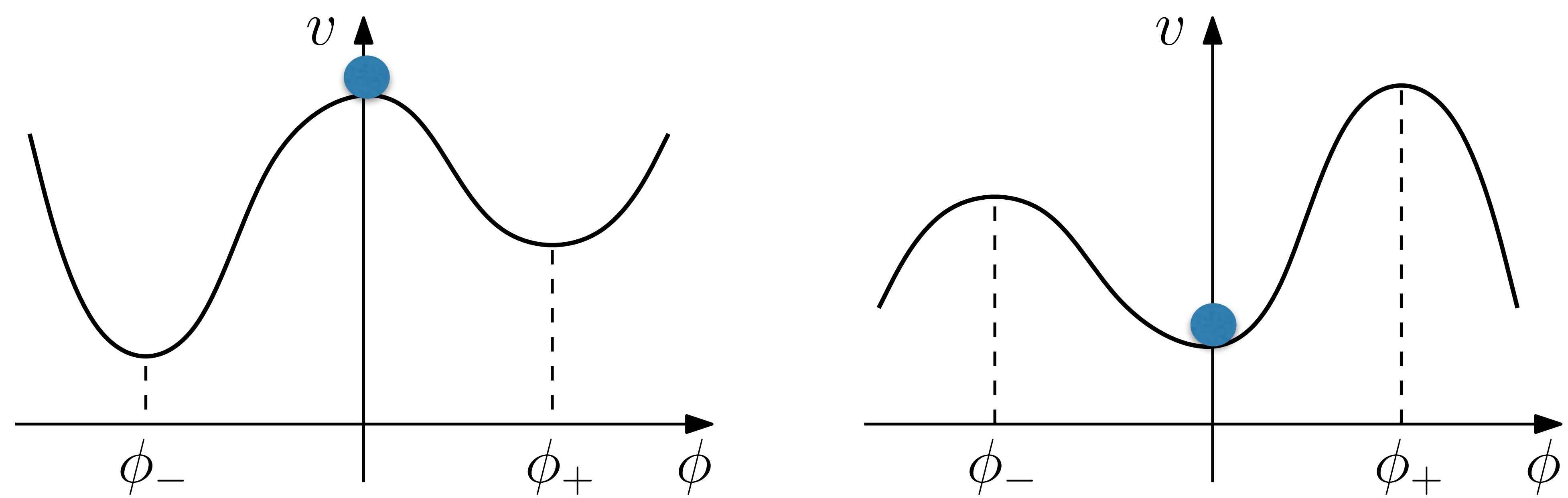
Using the first passage technique mentioned above, we can calculate the fall and escape probability in inflationary potentials.



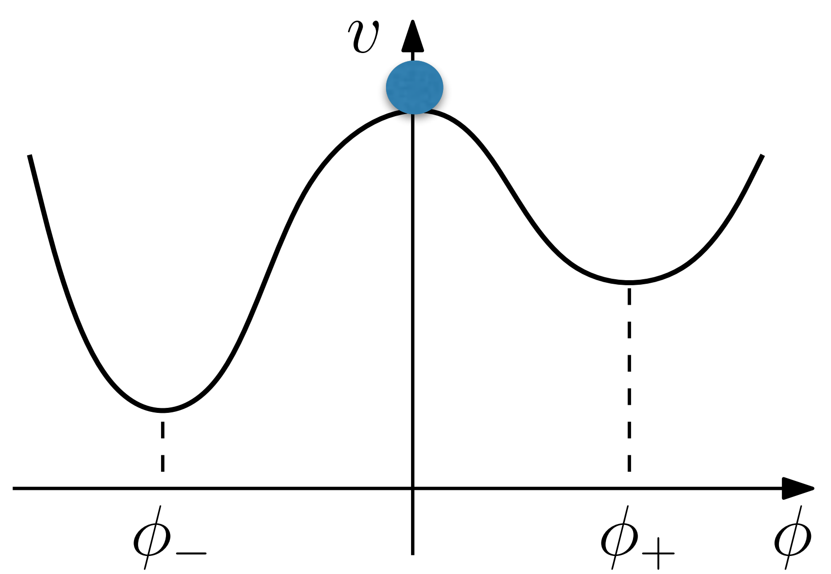
falling from the top of the potential

escaping from the top of the potential

$$R = \frac{p_+}{p_-} = \frac{\int_{\phi_-}^{0} e^{-\frac{1}{v(\phi)}} d\phi}{\int_{0}^{\phi_+} e^{-\frac{1}{v(\phi)}} d\phi} \, .$$

Falling from local maximum

Suppose the field is initially located at the maximum of the potential. We would like to calculate the probability that it falls to either of its two minima due to quantum fluctuations (quantum kicks).



Because of the exponential form of the integrand, most of the contributions to the integral comes from near the maximum:

$$\int_0^{\phi_+} \exp\left[\frac{-1}{v(\phi)}\right] d\phi = \int_0^{\phi_+} \exp\left[\frac{-1}{v(0)} + \frac{1}{2}\frac{v''(0)}{v(0)^2}\phi^2 + \frac{1}{3!}\frac{v'''(0)}{v(0)^2}\phi^3 + \dots\right] d\phi$$

For the series expansion near the maximum to be valid we require $\frac{v(0)\left|v'''(0)\right|}{|v''(0)|^{3/2}} \ll 1$.

Then the integral can be taken using the method of the steepest descent and

$$\int_0^{\phi_+} \exp\left[\frac{-1}{v(\phi)}\right] d\phi \simeq \sqrt{\frac{\pi}{2}}\frac{v(0)\exp^{-\frac{1}{v(0)}}}{\sqrt{|v''(0)|}}\left[1 + \sqrt{\frac{2}{\pi}}\frac{v(0)v'''(0)}{3\,|v''(0)|^{3/2}}\right]$$

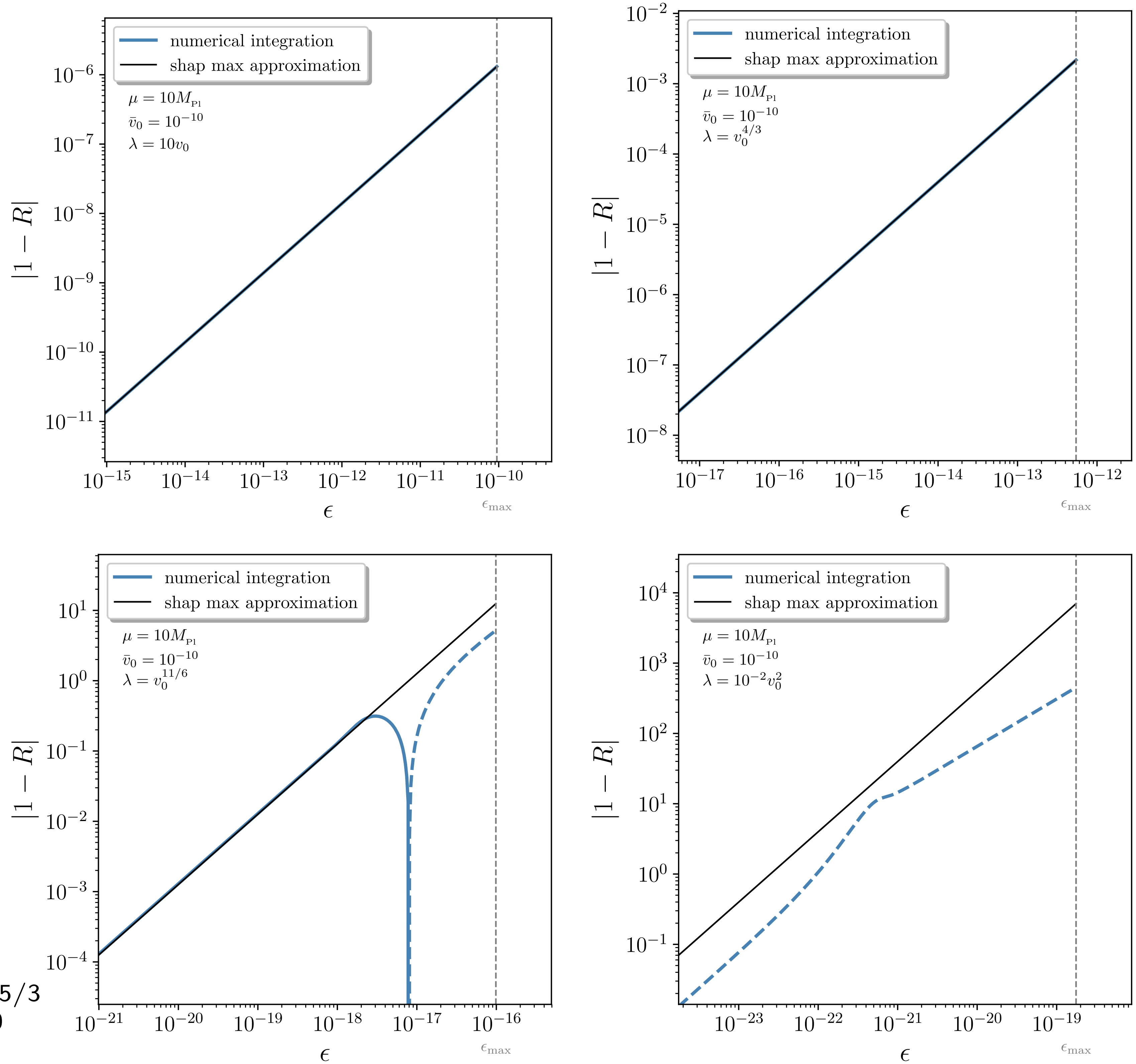
The fractional ratio of the probabilities, R, is given by

$$R = \frac{p_+}{p_-} \simeq 1 - \frac{2}{3}\sqrt{\frac{2}{\pi}}\frac{v(0)v'''(0)}{|v''(0)|^{3/2}}$$

Example: $$v(\phi) = \left\{ \lambda \left[\left(\frac{\phi}{\mu} \right)^2 - 1 \right]^2 + \bar{v}_0 \right\} + \epsilon \left(\frac{\phi}{\mu} \right)^3$$

$\lambda \gg \bar{v}_0$

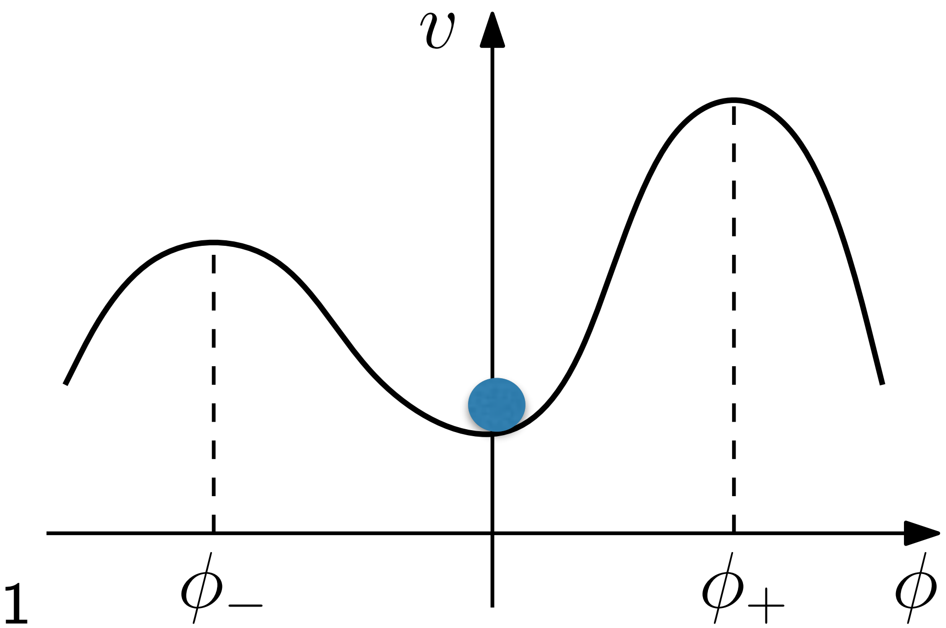
$\bar{v}_0^{5/3} \ll \lambda \ll \bar{v}_0$



$\lambda \ll \bar{v}_0^2$

$\bar{v}_0^2 \ll \lambda \ll \bar{v}_0^{5/3}$

Escaping from local minimum



Suppose the field is located at a local minimum of the potential. We would like to calculate the probability for escaping through either of barriers.

If the following conditions are met

$$\frac{v(\phi_+)\,|v'''(\phi_+)|}{|v''(\phi_+)|^{3/2}} \ll 1\,, \qquad \frac{|v''(\phi_+)|}{v(\phi_+)^2}\phi_+^2 \gg 1$$

then

$$\int_0^{\phi_+} \exp\left[\frac{-1}{v(\phi)}\right] d\phi = \sqrt{\frac{\pi}{2}}\,\frac{v(\phi_+)\,e^{-\frac{1}{v(\phi_+)}}}{\sqrt{|v''(\phi_+)|}}\,,$$

and the ratio of the two tunnelling probabilities is

$$R \simeq \sqrt{\frac{v''(\phi_+)}{v''(\phi_-)}}\,\frac{v(\phi_-)}{v(\phi_+)}\exp\left[\frac{1}{v(\phi_+)} - \frac{1}{v(\phi_-)}\right]$$

Since $v \sim 10^{-12}$ the dominant contributions come from the exponential terms and $R \sim e^{1/v(\phi_+)-1/v(\phi_-)}$.

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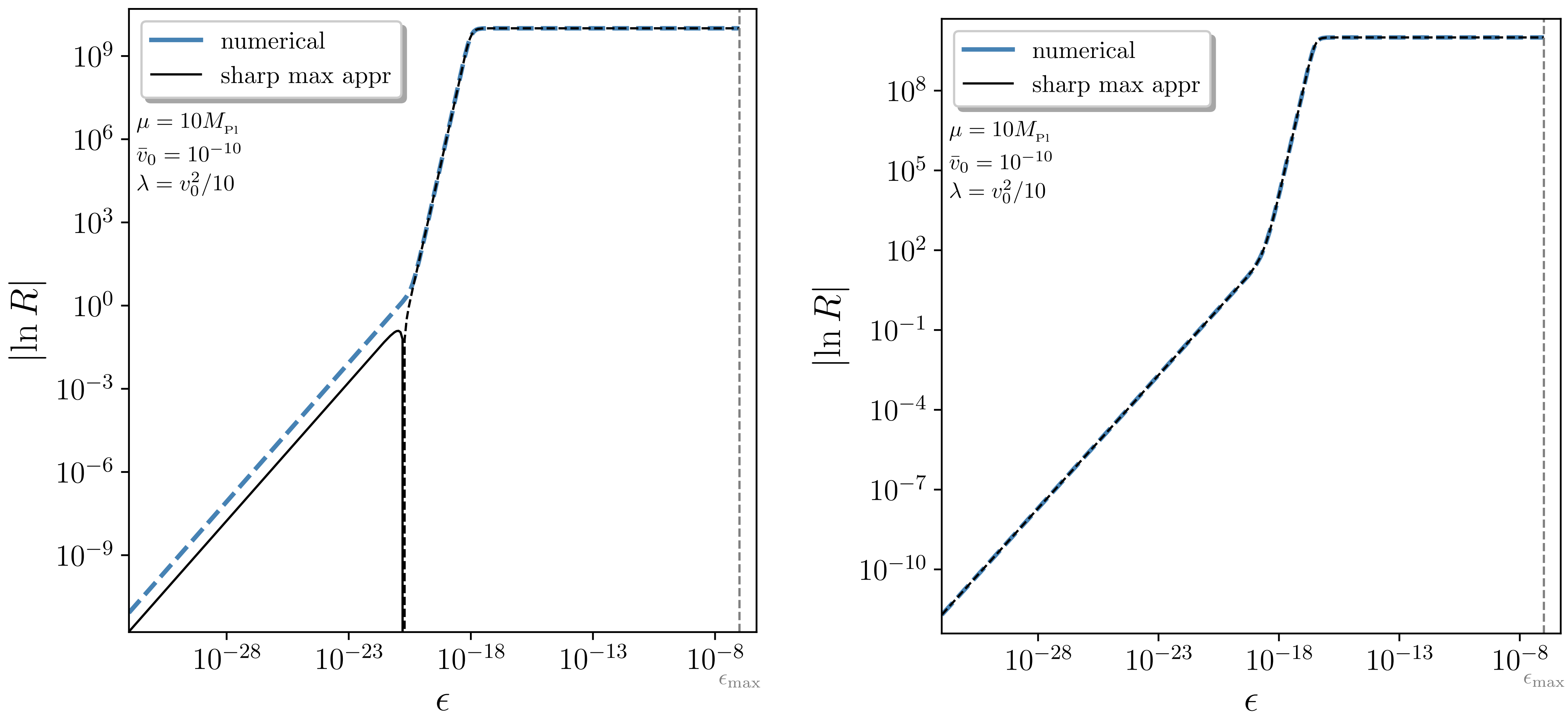
Example: $v = \left\{ -\lambda \left[\left(\frac{\phi}{\mu} \right)^2 - 1 \right]^2 + \bar{v}_0 \right\} + \epsilon \left(\frac{\phi}{\mu} \right)^3$



Left: $\lambda < \bar{v}_0^2$ and the sharp maximum approximation breaks down at $\epsilon < \bar{v}_0^2$.
Right: $\lambda > \bar{v}_0^2$ and the sharp maximum approximation always holds.

In both cases, R significantly differs from one when $\epsilon > \bar{v}_0^2$.

Tunnelling in a generic potential

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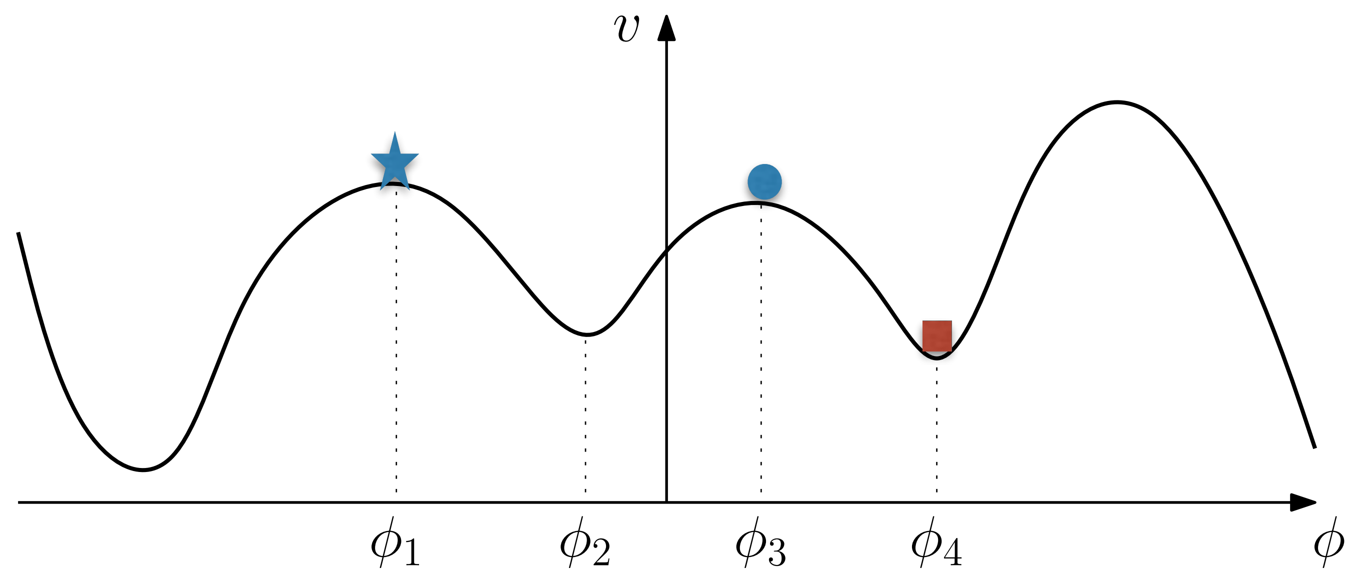
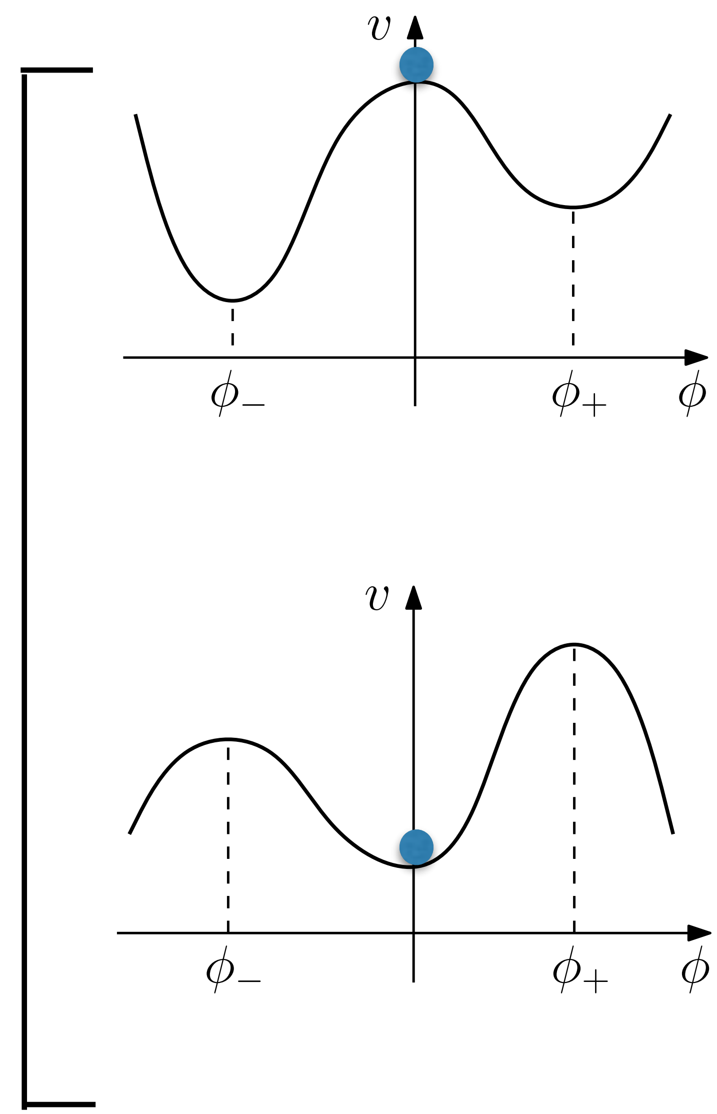
Let us consider a general potential with multiple minima and maxima.



We define p_{ijk} as the probability of reaching ϕ_i before reaching ϕ_k, starting from ϕ_j.

Example: p_+ in previous example is $p_{\phi_+ 0 \phi_-}$.



From our starting formula, we have:

$$p_{ijk} = \frac{I_j^k}{I_i^k}\,, \qquad I_i^j \equiv \int_{\phi_i}^{\phi_j} e^{-1/v}\, d\phi$$

From the integral structure of I_i^j we obtain

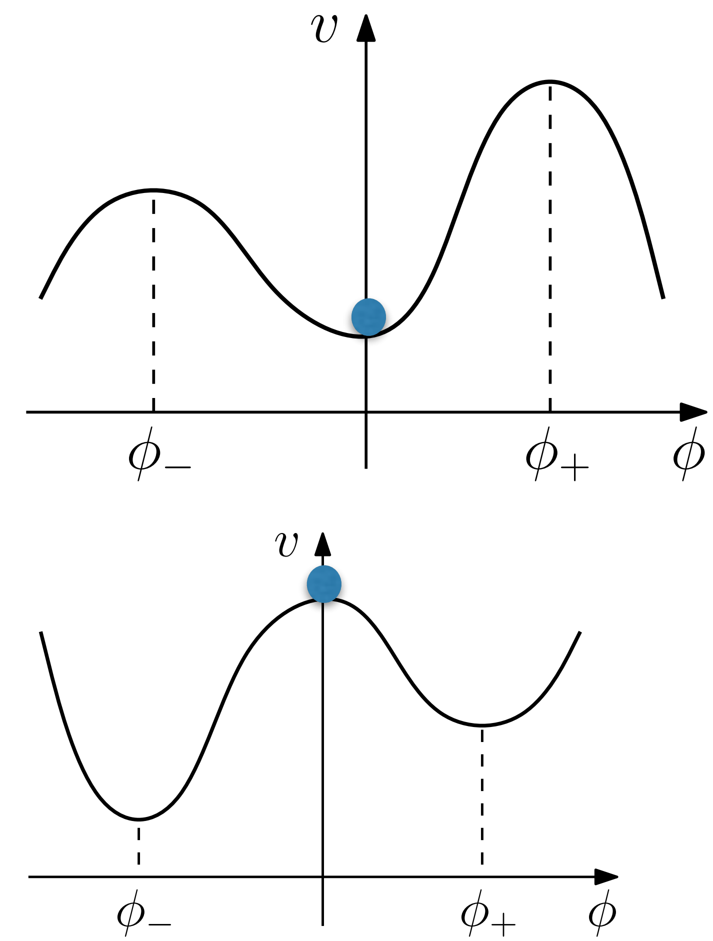
$$p_{134} = \frac{p_{123}\, p_{234}}{1 - p_{321}\, p_{234}}$$

This formula expresses p_{134} entirely in terms of building blocks "fall" (p_{234}) and "escape" (p_{123} and p_{321}) probabilities.

Conclusion

- Inflation is the leading paradigm for early Universe and for generating large scale structures.
- The basic predictions of inflation are that the primordial perturbations are nearly scale invariant, nearly adiabatic and nearly Gaussian.
- Stochastic inflation is a novel method to study primordial perturbations using stochastic formalism.
- Super-horizon perturbations act as a source of Gaussian noise for small scale perturbations inside the horizon.
- Using the first passage technique, we can calculate the falling probability from a local maximum to nearby minima and the escaping probability from a local minimum to nearby maxima.
- The fall probability and the escape probability are the building blocks for calculating the tunnelling in a generic potential with multiple maxima and minima.

Decay rate



Using the first passage technique, we can calculate the typical time it takes for falling or escaping.

The mean number of e-folds $\langle \mathcal{N} \rangle$ to reach either ϕ_- or ϕ_+, starting from the initial value ϕ, is given by

$$\langle \mathcal{N} \rangle'' - \frac{v'}{v^2} \langle \mathcal{N} \rangle' = -\frac{1}{M_P^2\, v(\phi)}$$

with boundary conditions $\langle \mathcal{N} \rangle(\phi_-) = \langle \mathcal{N} \rangle(\phi_+) = 0$.

This can be solved to obtain

$$\langle \mathcal{N} \rangle (\phi) = \int_{\phi_-}^{\phi} \frac{dx}{M_P} \int_{x}^{\bar{\phi}(\phi_-,\phi_+)} \frac{dy}{M_P} \frac{1}{v\,(y)} \exp\left[\frac{1}{v\,(y)} - \frac{1}{v\,(x)}\right]$$

where $\bar{\phi}(\phi_-, \phi_+)$ is an integration constant that is implicitly set through the boundary condition $\langle \mathcal{N} \rangle(\phi_+) = 0$.