Chemistry and holography in generalized quasi-topological gravity

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Overview

Introduction

- To construct a UV complete theory of quantum gravity.

  For example, the addition of higher derivative terms to Einstein-Hilbert action can yield a power-counting renormalizable theory [Stelle (1978)].

- In the context of the AdS/CFT correspondence [Maldacena,(1998)].

  Studying the dual theory beyond large N.

Overview

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  - ▷ Thermodynamics of charged black holes in cubic GQG
- Charged black hole solutions
  - ▷ Full theory and equations of motion
  - ▷ Asymptotic solution
  - ▷ Near horizon solution
  - ▷ Intermediate region
- Thermodynamic considerations
  - ▷ First law and Smarr relation
  - ▷ Equation of state
- Charged black holes: Canonical ensemble
  - ▷ Critical behaviour in four dimensions
  - ▷ Critical behaviour in six dimensions
- Holographic hydrodynamics
  - ▷ Computation of η/s
- Conclusion

Higher-curvature theories

Lovelock gravity

These theories are ghost-free and so are candidates for generalizations of Einstein gravity in higher dimensions.[Zwiebach(1985)]

However, Lovelock gravity that is kth order in curvature is only non-trivial for spacetime dimensions $d > 2k + 1$.[Lovelock(1971)]

Higher-curvature theories

Quasi-topological gravity

• For spherically symmetric metrics, their field equations are second-order.

• Non-trivial for any dimension $d \geq 5$.

• The linearized equations of motion of quasi-topological gravity coincide with those of Einstein gravity on maximally symmetric spacetime backgrounds [Myers-Sinha(2011)].

Higher-curvature theories

Einsteinian Cubic Gravity

• The unique cubic theory of gravity whose Lagrangian is of the same form in all dimensions and propagates only the usual massless and transverse graviton on maximally symmetric backgrounds.

• ECG is neither trivial nor topological in four dimensions. [Bueno-Cano(2016)]

Properties of ECG

• There is a single independent field equation, reducing it to a second-order differential equation determining the metric function $f(r)$.
• The black hole solutions are "non-hairy" in the sense that they are characterized by mass alone.
• Despite the lack of an analytic solution to the equations of motion, the thermodynamic properties of black holes can be studied exactly.

Higher-curvature theories

Generalized quasi-topological gravities

• More general theories of gravity in four and higher dimensions These theories, named generalized quasi-topological gravities, propagate only the usual massless transverse graviton in vacuum, admit non-hairy black hole solutions characterized by a single metric function, and allow for non-perturbative studies of black hole thermodynamics [Hennigar-Kubiznak-Mann(2017)], [Ahmed-Hennigar-Mann-Mir(2017)].

Overview

Thermodynamics of charged black holes in cubic GQG

Black hole chemistry

• The cosmological constant is promoted to a thermodynamic variable interpreted as pressure in the first law of black hole mechanics[Henneaux - Teitelboim(1985)], [Kastor-Ray- Traschen(2010)].

• Analogy between charged anti-de Sitter black holes and Van der Waals fluids [Kubiznak- Mann(2012)].

• Thermodynamic phase behaviour for black holes, including the exhibition of triple points, re-entrant phase transitions, polymer-like behaviour, and even super fluid-like phase transitions [Altamirano-Kubiznak-Mann-Sherkatghanad(2014)], [Altamirano-Kubiznak-Mann(2013)], [Dolan-Kostouki-Kubiznak- Mann(2014)], [Hennigar-Mann-Tjoa(2017)].

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Charged black hole solutions

Charged static, spherically symmetric AdS black holes in generalized quasi-topological gravity.

Full theory and equations of motion

• The most general cubic theory satisfying the condition $g_{tt}g_{rr} = -1$ ensuring dependence on a single metric function include the cubic Lovelock and quasi-topological terms, in addition to the generalized quasi-topological term.

Charged black hole solutions

In d spacetime dimensions, the action is given by

[[Hennigar-Kubiznak-Mann(2017)]]

$$\mathcal{I} = \frac{1}{16\pi G}\int d^dx\sqrt{-g}\Bigg[\frac{(d-1)(d-2)}{L^2} + R - \frac{1}{4}F_{ab}F^{ab} + \frac{12(2d-1)(d-2)\mu\mathcal{S}_{3,d}}{(d-3)(4d^4-49d^3+291d^2-514d+184)}\Bigg]$$

where the cosmological constant

$$\Lambda = -\frac{(d-1)(d-2)}{2L^2}$$

$$\mathcal{S}_{3,d} = 14R_a{}^e{}_c{}^f R^{abcd}R_{bedf} + 2R^{ab}R_a{}^{cde}R_{bcde} - \frac{4(66-35d+2d^2)}{3(d-2)(2d-1)} R_a{}^c R^{ab}R_{bc} - \frac{2(-30+9d+4d^2)}{(d-2)(2d-1)}R^{ab}R^{cd}R_{acbd} - \frac{(38-29d+4d^2)}{4(d-2)(2d-1)} RR_{abcd}R^{abcd} + \frac{(34-21d+4d^2)}{(d-2)(2d-1)}R_{ab}R^{ab}R - \frac{(30-13d+4d^2)}{12(d-2)(2d-1)}R^3$$

Charged black hole solutions

The ansatz for the metric

$$ds^2 \;=\; -N(r)^2 f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma^2_{(d-2),k} \tag{1}$$

we set $N(r) = 1$.

$d\Sigma^2_{(d-2),k}$ denotes the line element of the $(D-2)$-dimensional transverse space, which we take to be a surface of constant scalar curvature $k = +1,\ 0,\ -1$.

the metric function, for a maximally symmetric space

$$f_{\mathrm{AdS}}(r) = k + f_\infty \frac{r^2}{L^2} \tag{2}$$

Here, L is the length scale associated with the cosmological constant.

Charged black hole solutions

Introducing a Maxwell field $F_{ab} = \partial_a A_b - \partial_b A_a$, with electromagnetic one form defined as

$$A = qE(r)dt \tag{3}$$

where

$$E(r) = \sqrt{\frac{2(d-2)}{(d-3)}}\frac{1}{r^{d-3}} \tag{4}$$

The only independent field equation

$$\frac{d}{dr}F[f,f',f''] = 0 \tag{5}$$

with

$$F = r^{d-3}\left(k - f(r) + \frac{r^2}{L^2}\right) + \mu F_{\mathcal{S}_{3,d}} + r^{3-d}q^2\,. \tag{6}$$

Charged black hole solutions

The contribution from the cubic generalized quasi-topological term

$$F_{\mathcal{S}_{3,d}} = \frac{12}{(4d^4 - 49d^3 + 291d^2 - 514d + 184)} \Big[(d^2 + 5d - 15)\Big(\frac{4}{3}r^{d-4}f'^3 - 8r^{d-5}ff''\big(\frac{rf'}{2} + k - f\big) - 2r^{d-5}((d-4)f - 2k)f'^2 + 8(d-5)r^{d-6}ff'(f-k)\Big) - \frac{1}{3}(d-4)r^{d-7}(k-f)^2 \times \Big(\big(-d^4 + \frac{57}{4}d^3 - \frac{261}{4}d^2 + 312d - 489\big)f + k\big(129 - 192d + \frac{357}{4}d^2 - \frac{57}{4}d^3 + d^4\big)\Big)\Big] .$$

direct integration yields

$$F = m \tag{7}$$

where m is related to the mass of black hole.

Overview

Asymptotic solution

The asymptotic form of the metric function is

$$f(r) = k + f_\infty \frac{r^2}{L^2} - \frac{m}{r^{d-3}} + \frac{q^2}{r^{2d-6}} + \epsilon g(r) \tag{8}$$

Assuming that $\mu \neq 0$ the homogenous part of the equation at large r is

$$g_h'' - \frac{4}{r} g_h' - \gamma^2 r^{d-3} g_h = 0 \tag{9}$$

where

$$\gamma^2 = \frac{3(4d^4 - 49d^3 + 291d^2 - 514d + 184)L^2 h'(f_\infty)}{144(d-1)(d^2+5d-15) f_\infty \mu \; m} . \tag{10}$$

the case of $\gamma^2 > 0$, while $f_\infty > 0$ (for asymptotic AdS space) demands that the product $m\mu < 0$.

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Near horizon solution

expansion for the metric function near horizon:

$$f(r) = 4\pi T(r - r_+) + \sum_{i=2} a_n (r - r_+)^n \tag{11}$$

where T is Hawking temperature of the black hole,

$$T = \frac{f'(r_+)}{4\pi} . \tag{12}$$

Near horizon solution

At each order of $(r - r_+)$, from the first two orders

$$\begin{aligned}
m &= \frac{\mu r_+^{d-7}}{(4d^4 - 49d^3 + 291d^2 - 514d + 184)} \Big[256\pi^2(d^2 + 5d - 15) \\
&\quad (3k + 4\pi r_+ T) r_+^2 T^2 - (d-4)(4d^4 - 57d^3 + 357d^2 - 768d + 516) \\
&\quad k^3 \Big] + r_+^{d-3}\left(k + \frac{r_+^2}{L^2}\right) + \frac{q^2}{r_+^{d-3}}, \\
0 &= (d-3)k r_+^{d-4} + (d-1)\frac{r_+^{d-2}}{L^2} - 4\pi r_+^{d-3} T - (d-3) r_+^{2-d} q^2 \\
&\quad + \frac{\mu r_+^{d-8}}{(4d^4 - 49d^3 + 291d^2 - 514d + 184)} \Big[12\pi(d-4)(d-6) \\
&\quad (4d^3 - 33d^2 + 127d - 166) k^2 r_+ T - 512\pi^3 (d-4)(d^2 + 5d - 15) r_+^3 \\
&\quad T^3 - 768\pi^2 (d-5)(d^2 + 5d - 15) k r_+^2 T^2 \\
&\quad - (d-4)(d-7)(516 - 768d + 357d^2 - 57d^3 + 4d^4) k^3 \Big] \qquad (13)
\end{aligned}$$

Overview

Intermediate region

using numerical methods to verify that these solutions are indeed joined in the intermediate region.

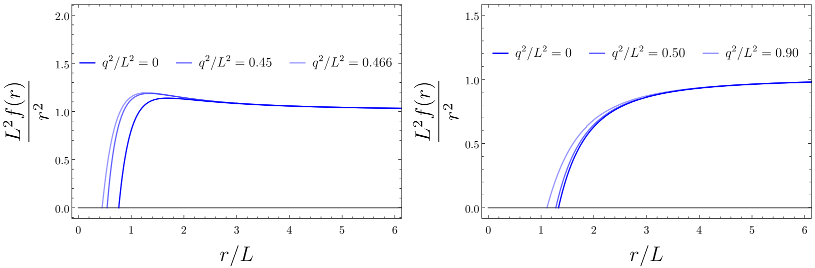

Figure: **Numerical solutions**

Overview

Thermodynamic considerations

Applying the black hole chemistry formalism, taking both Λ and μ to be thermodynamic variables[Kubiznak-Mann-Teo(2017)].

First law and Smarr relation

The Iyer-Wald formalism, to calculate the entropy[Wald(1993)]

$$S = -2\pi \oint d^{d-2}x\sqrt{\gamma}P^{abcd}\hat{\varepsilon}_{ab}\hat{\varepsilon}_{cd} \tag{14}$$

where

$$P^{abcd} = \frac{\partial \mathcal{L}}{\partial R_{abcd}} \tag{15}$$

and $\hat{\varepsilon}_{ab}$ is the binormal to the horizon, which is normalized as $\hat{\varepsilon}_{ab}\hat{\varepsilon}^{ab} = -2$.

Thermodynamic considerations

First law and Smarr relation

The pressure is defined

$$P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi L^2} \tag{16}$$

with other thermodynamic quantities, satisfy the first law of black hole thermodynamics

$$dM = TdS + VdP + \Phi dQ + \Psi_\mu d\mu \tag{17}$$

with V the thermodynamic volume conjugate to the pressure and Ψ_μ the potential conjugate to the coupling μ.
the Smarr formula

$$(d-3)M = (d-2)TS - 2PV + (d-3)\Phi Q + 4\mu\Psi_\mu \tag{18}$$

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Thermodynamic considerations

The equation of state

$$P = \frac{T}{v} - \frac{(d-3)}{\pi(d-2)}\frac{k}{v^2} + \frac{e^2}{v^{2d-4}} + (d-7)(d-4)\frac{\beta_0}{v^6} - (d-6)(d-4)\beta_1\frac{T}{v^5} + (d-5)\frac{\beta_2}{v^4}T^2 + (d-4)\frac{\beta_3}{v^3}T^3 \quad (19)$$

where we refer to v as the specific volume and the others are rescaled physical parameters. In the sequel we choose β_3 and e as the free parameters.

the explicit form of the Gibbs free energy $G = M - TS$

$$\mathcal{G} = \left[\frac{4}{d-2}\right]^{d-1}\frac{G}{\Sigma_{(d-2),k}} = \frac{v^{d-1}P}{d-1} + \frac{v^{d-3}k}{\pi(d-2)} + \frac{e^2}{(d-3)v^{d-3}} - \beta_0(d-4)v^{d-7} - \left(\frac{v^{d-2}}{d-2} - \beta_1(d-4)v^{d-6}\right)T - \beta_0\frac{48\pi^2(d-2)^2\left(d^2+5d-15\right)v^{d-5}}{4d^4-57d^3+357d^2-768d+516}T^2 - \beta_3 v^{d-4}T^3 \quad (20)$$

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Charged black holes: Canonical ensemble

- Thermodynamics in the canonical (fixed charge) ensemble.
- The equation of state in general d using the rescaled parameters

$$
\begin{aligned}
P \;=\; & \frac{T}{v} - \frac{(d-3)k}{\pi(d-2)v^2} + \frac{\beta_3(d-4)T^3}{v^3} + \frac{6\beta_3(d-5)kT^2}{\pi(d-2)v^4} + \frac{e^2}{v^{2d-4}} \\
& - \frac{3\beta_3(d-6)(d-4)k^2 T}{8\pi^2(d-2)^2} \frac{\left(4d^3-33d^2+127d-166\right)}{\left(d^2+5d-15\right)v^5} \\
& + \frac{\beta_3(d-7)(d-4)\left(4d^4-57d^3+357d^2-768d+516\right)k}{8\pi^3(d-2)^3\left(d^2+5d-15\right)v^6}
\end{aligned}
$$

- A necessary condition for a critical point to occur

$$
\frac{\partial P}{\partial v} = \frac{\partial^2 P}{\partial v^2} = 0\,. \qquad (21)
$$

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Charged black holes: Canonical ensemble

Critical behaviour in four dimensions

• In four dimensions, the field equation for cubic generalized quasi-topological gravity reduces to that of Einsteinian cubic gravity (ECG).

• The equation of state

$$P = \frac{T}{v} - \frac{k}{2\pi v^2} + \frac{e^2}{v^4} - \frac{3\beta_3 k T^2}{\pi v^4} \tag{22}$$

• Solving equation to obtain critical points

$$T_{c\pm}^2 = \frac{3\pi^3 e^2 \pm \pi\sqrt{9\pi^4 e^4 - 4k^4\beta_3}}{18\pi^2 k\beta_3}, \quad P_{c\pm} = \frac{3\pi^2 e^2 \pm \sqrt{9\pi^4 e^4 - 4k^4\beta_3}}{32k^2\beta_3},$$

$$v_{c\pm} = \frac{2k}{3\pi T_{c\pm}}$$

Charged black holes: Canonical ensemble

Critical behaviour in four dimensions

- The ratio of critical quantities

$$\frac{P_c v_c}{T_c} = \frac{3}{8} \tag{23}$$

is independent of the black hole parameters and in this sense is universal.

- The critical points are characterized by mean field theory critical exponents which, for generic values of parameters and k = 1 in the physical domain, are given by [Gunasekaran-Mann- Kubiznak(2012)]

$$\alpha = 0, \quad \beta = \frac{1}{2}, \quad \gamma = 1, \quad \delta = 3 \tag{24}$$

Critical behaviour in four dimensions

• Considering the $P - v$ graph

Two distinguishable (stable) phases for $T < T_c$. These merge at $T = T_c$ and then for $T > T_c$ they become indistinguishable, the hallmark of a standard Van-der-Waals (VdW) phase transition.

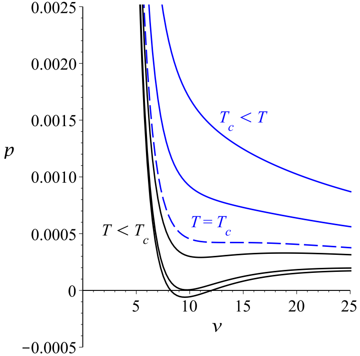


Figure: **Critical behaviour in four dimensions**

Critical behaviour in four dimensions

- Considering the $P-T$ graph

The critical points correspond to the end point of a line of first order phase transitions. This line of coexistence demarcates phases of large and small black holes.

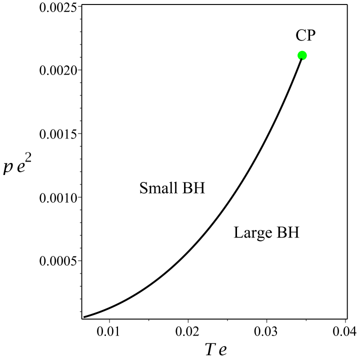
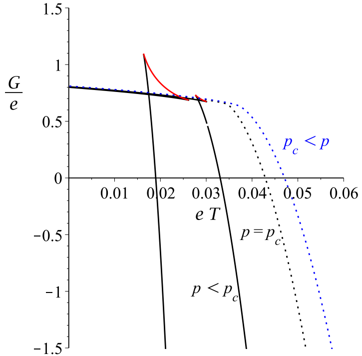


Figure: **Critical behaviour in four dimensions**

Critical behaviour in four dimensions

• Considering the $G - T$ graph
an analysis of the Gibbs free energy reveals typical van der Waals behaviour, while the concave patch of the Gibbs free energy indicates negative specific heat,

$$C_p = -T \frac{\partial^2 G}{\partial T^2} . \quad (25)$$



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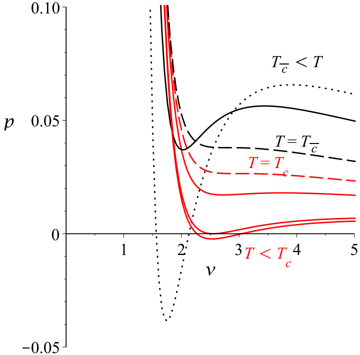
Critical behaviour in six dimensions

- The equation of state becomes

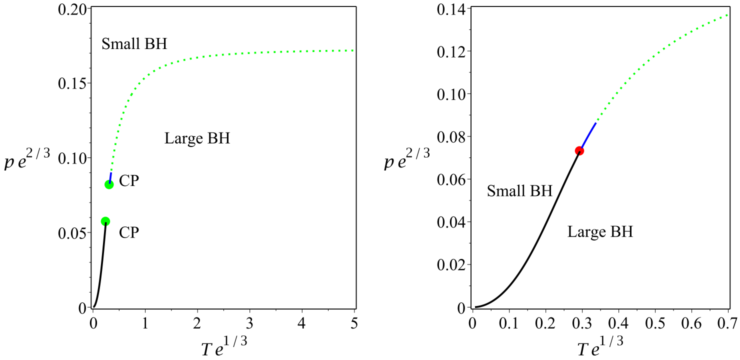
$$P=\frac{T}{v}-\frac{3k}{4\pi v^2}+\frac{2\beta_3 T^3}{v^3}+\frac{3\beta_3 k T^2}{2\pi v^4}-\frac{\beta_3 k}{8\pi^3 v^6}+\frac{e^2}{v^8}\,. \tag{26}$$

- There can be up to two critical points for the six dimensional spherical ($k=+1$) black holes.



Critical behaviour in six dimensions

• For generic values of the coupling, each of the two critical points are described by mean field theory critical exponents. One marks the end point of a first order coexistence line, while the other marks the beginning of a first order coexistence line,



Overview

Holographic hydrodynamics

- The uncharged black branes in the cubic theory

$$T_p = \frac{3(d-2)r_+}{8\pi L^2}\,, \quad \mu_p = -\frac{L^4(4d^4-49d^3+291d^2-514d+184)}{54(d-2)^3(d^2+5d-15)}\,. \tag{27}$$

these conditions also imply that the entropy of the black brane vanishes.

- The mass and entropy of the black branes is always positive in $\mu \in [\mu_p, 0]$.

Overview

Holographic hydrodynamics

Computation of η/s

• For field theories possessing Einstein gravity duals, the shear viscosity to entropy density ratio has a universal form $\eta/s = 1/(4\pi)$.

• It was conjectured by Kovtun, Son, and Starinets that this represents a universal lower bound for all substances, i.e. $\eta/s \geq 1/(4\pi)$ (the KSS bound)[[Kovtun-Son- Starinets(2005)].

• The ratio η/s takes on all real values $\eta/s \in [(4\pi)^{-1}, \infty)$ as a function of the coupling μ.

Computation of η/s

The planar class of metrics, introducing $z = 1 - r_+^2/r^2$

$$ds^2 = \frac{r_+^2}{L^2(1-z)}\left(-g(z)dt^2 + \sum_i dx_i^2\right) + \frac{L^2}{4g(z)}\frac{dz^2}{(1-z)^2} \tag{28}$$

Near the horizon, we can expand $g(z)$ as,

$$g(z) = g_0^{(1)}z + g_0^{(2)}z^2 + g_0^{(3)}z^3 + \cdots . \tag{29}$$

Perturbing the metric by the shift [Paulos(2010)]

$$dx_i \to dx_i + \epsilon e^{-i\omega t}dx_j . \tag{30}$$

The perturbed metric is substituted into the Lagrangian and a small ϵ expansion is performed.

Computation of η/s

Now, using the 'time' formula, the shear viscosity is given by

$$\eta = -8\pi T \lim_{\omega,\epsilon\to 0} \frac{\mathrm{Res}_{z=0}\sqrt{-g}\mathcal{L}}{\omega^2\epsilon^2} \tag{31}$$

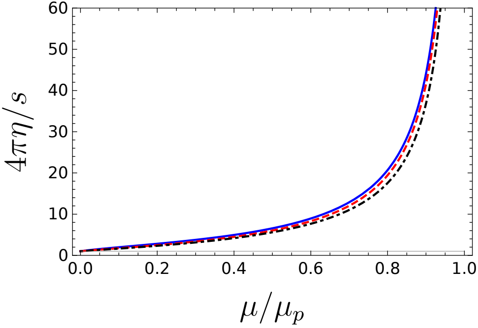
considering a small μ expansion of η/s

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{12\mu}{L^4}\frac{(d-1)^2(23d^4-83d^3-18d^2+256d-136)}{(d-3)(4d^4-49d^3+291d^2-514d+184)} + \mathcal{O}(\mu^2)\right]. \tag{32}$$

The KSS bound $\eta/s \geq 1/(4\pi)$ holds in all dimensions in the cubic generalized quasi-topological theories, at least when the coupling is small.

Computation of η/s

• To see the explicit μ dependence of η/s, we must resort to numerical techniques. In these plots, we see the same characteristic structure: for $\mu = 0$, η/s begins at $1/(4\pi)$ and then monotonically increases as $\mu \to \mu_p$. Increases the spacetime dimension shifts the curves down slightly, but the overall structure is the same in all dimensions.



Conclusion

• We have studied electrically charged static AdS black holes in cubic generalized quasi-topological gravity.
• These black holes are characterized by a single metric function, and our study considered spherical, planar, and hyperbolic base manifolds.
• We have constructed a number of asymptotic and numeric solutions to the full field equations.
• The thermodynamic properties of the solutions can be studied exactly.
• We verified the extended first law and Smarr relation for the solutions, working in the framework of black hole chemistry, treating the cosmological constant as a thermodynamic pressure.
• On the canonical (fixed charge) ensemble, we find a variety of interesting phase structure, including Van der Waals type behaviour.
• we find that in all dimensions the KSS bound is upheld in these theories.

The End