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Parity violation in an isotropic vector inflation

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Inflationary Universe with anisotropic hair

- If the fields are very heavy compared to the Hubble scale during inflation, then they are not expected to play important roles.
- If the fields are light or semi-heavy they can have non-trivial effects on cosmological observables. (X. Chen and Y. Wang, JCAP 1004, 027 (2010))
- One issue with the vector fields in background is that they have preferred directions so in general models of inflation with background vector fields are anisotropic.
- The second issue with the vector fields is that because of the conformal invariance, they are quickly diluted in an expanding background, so their effects become rapidly insignificant during inflation.
- To remedy the second issue, the gauge kinetic coupling in these models is a function of the inflaton field so the conformal invariance is broken. (M. a. Watanabe, S. Kanno and J. Soda, Phys. Rev. Lett. 102, 191302 (2009),

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

- By choosing an appropriate form of the gauge kinetic coupling, the electric field energy density becomes nearly constant so the gauge field survives the expansion till end of inflation.

Inflationary Universe with anisotropic hair

- The vector and scalar fields are given by the form: $A_\mu = (0, A_x(t), 0, 0)$ and $\phi = \phi(t)$
- The Axisymmetric Bianchi Type I is given by:

$$ds^2 = -dt^2 + e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}\left(dy^2 + dz^2\right)\right]$$

- Here, e^{α} is an isotropic scale factor and σ represents a deviation from the isotropy.
- By Assuming $f = e^{-2\alpha} = e^{2c\kappa^2 \int \frac{V}{V'} d\phi}$ the energy density of the vector field remains almost constant during the slow-roll inflation.
- Now, by considering the equation of motion for scalar and vector field, we have

$$\frac{d\phi}{d\alpha} = -\frac{V'}{\kappa^2 V} + \frac{2cp_A^2}{V'} e^{-4c\kappa^2 \int (V'/V) d\phi - 4\alpha - 4\sigma} \Longrightarrow f^2 e^{-4\alpha - 4\sigma} \simeq \frac{(c-1)V'^2}{2c^2\kappa^2 p_A^2 V}$$

- The energy density ratio between the scalar and vector fields is defined as:
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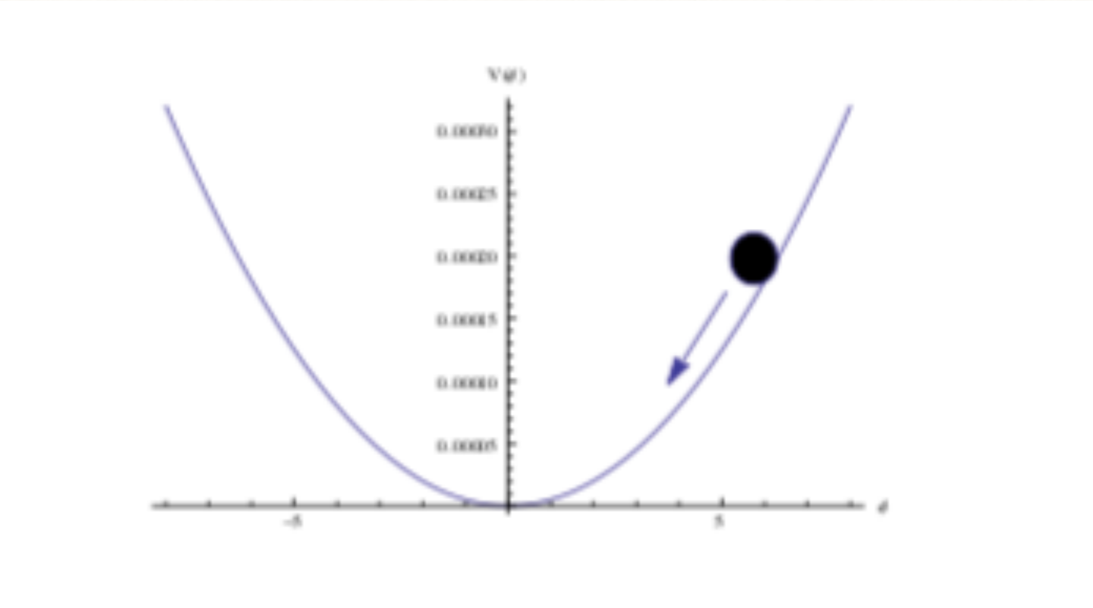
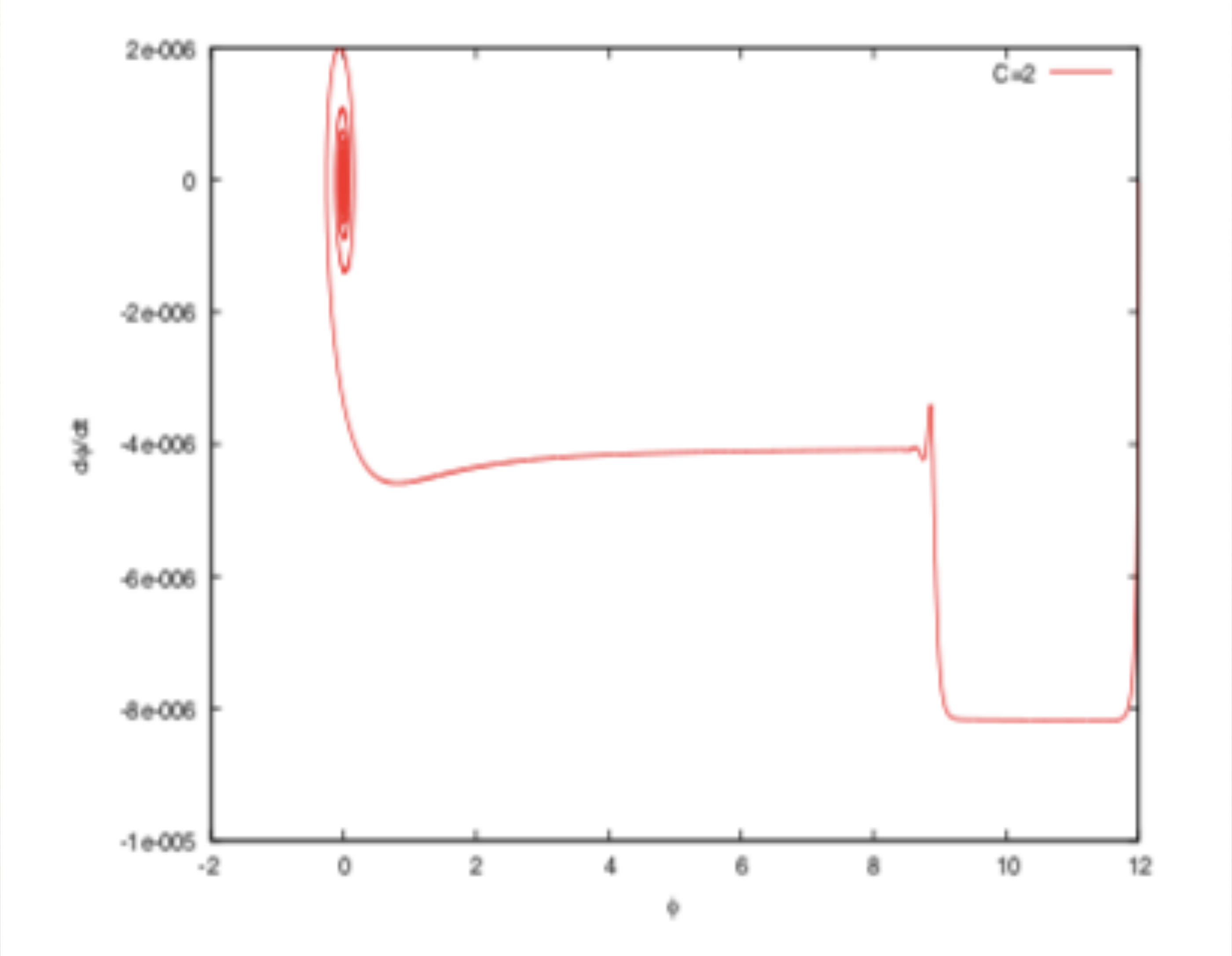
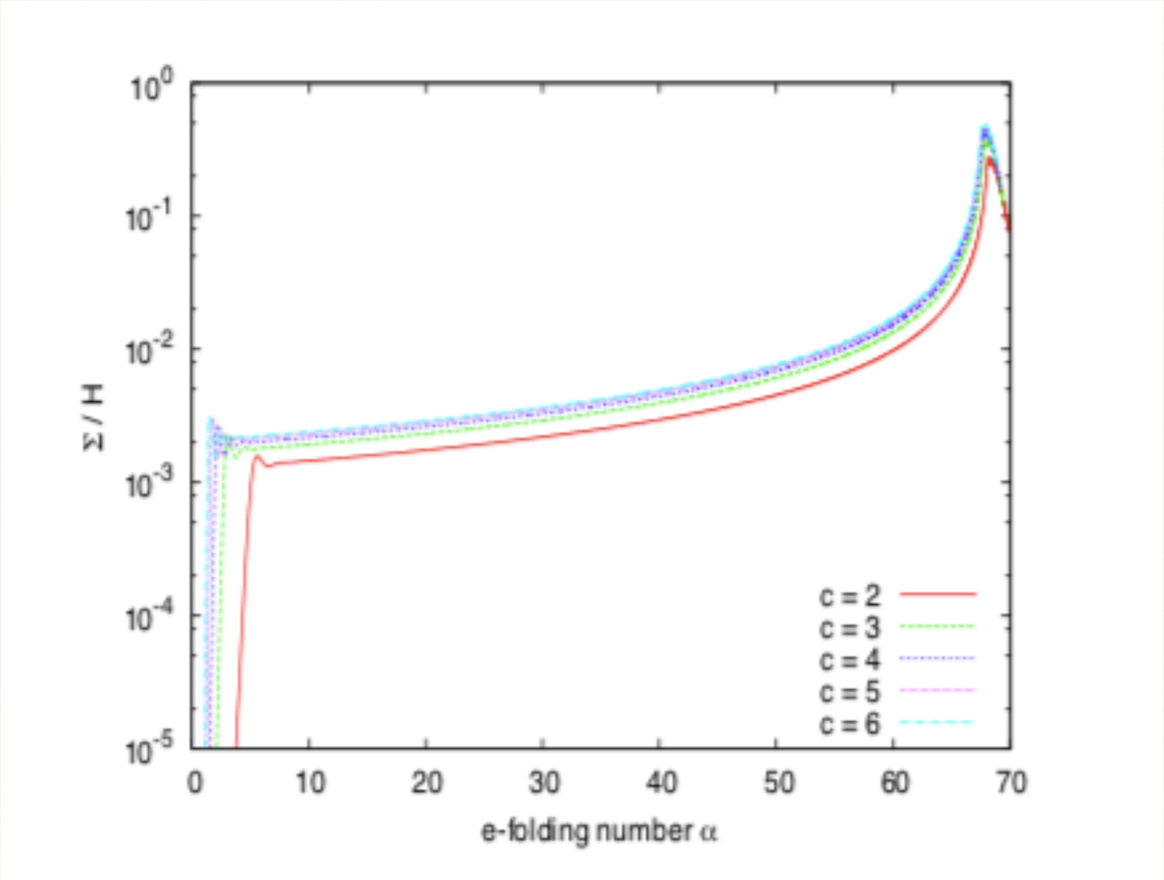
$$\mathcal{R} = \frac{\rho_A}{\rho_\phi} = \frac{p_A^2 f^{-2} e^{-4\alpha-4\sigma}}{\dot{\phi}^2 + 2V} \approx \frac{c-1}{2c}\epsilon_H$$

$$\epsilon_H = \frac{1}{c}\epsilon_V \quad ; \quad \epsilon_V \equiv \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2$$







The growth of anisotropy never exceeds that of the background, and inflation is not destroyed.

Isotropic vector inflation

- We extend the model of anisotropic inflation by considering a triplet of U(1) gauge fields coupled non-minimally to the inflaton field as (K. Yamamoto, Phys. Rev. D 85, 123504 (2012),

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{f^2(\phi)}{4}\sum_a\left(F^{(a)}_{\mu\nu}F^{\mu\nu}_{(a)}\right)\right]$$

- This action has the same form as in models of anisotropic inflation but the gauge fields here enjoy an additional internal global O(3) symmetry.
- This new symmetry the model is no longer anisotropic and this mode admits homogeneous and isotropic solution.
- Note that the above definition can be thought as the global limit of some locally gauged models with gauge symmetry homomorphic to O(3) symmetry. (M. Henneaux, J. Math. Phys. 23, 830 (1982).
- For instance, we can obtain the above model from the global limit of an SU(2) model with the gauge coupling g since the SU(2) model is homomorphic to SO(3). (J. Cervero and L. Jacobs, Phys. Lett. 78B, 427 (1978).
- This model therefore admits the isotropic flat FRW cosmological background

$$ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$$

with the ansatz

$$A^{(a)}_\mu(t) = A(t)\,\delta^a_\mu$$

- The associated Maxwell and KG equations are given by the following relations.
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$$\dot{A} = \frac{q_0}{a}f^{-2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = \frac{3q_0^2}{a^4}\left(\frac{f_{,\phi}}{f^3}\right)$$

Isotropic vector inflation

- Now, by considering the equation of motion for scalar and vector field, we have

$$\phi\frac{d\phi}{dN} = -\frac{V_{,\phi}}{V} + \frac{6q_0^2 c e^{-4c\int (V/V_{,\phi})d\phi}}{V_{,\phi}}e^{-4N}$$

$$e^{-4N}e^{-4c\int\frac{V}{V_{,\phi}}d\phi} = \frac{(c-1)V_{,\phi}^2}{6c^2q_0^2V}$$

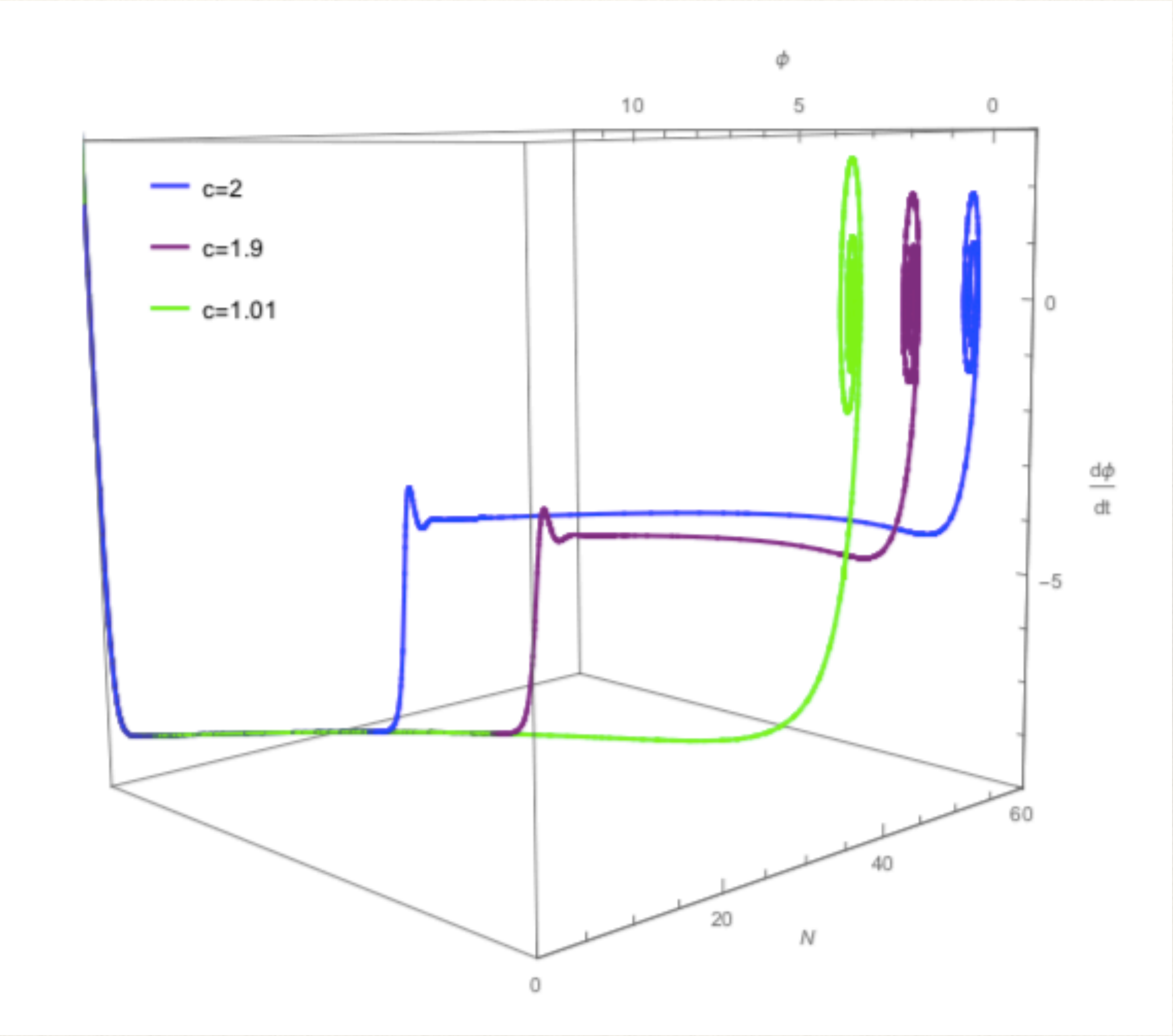
Consequently, vector energy density becomes nearly constant during the second phase of inflation

$$R \equiv \frac{\rho_A}{\rho_\phi} = \frac{3q_0^2}{2V+\dot{\phi}^2}a^{-4}f^{-2} = \frac{c-1}{4c^2}\left(\frac{V_{,\phi}}{V}\right)^2$$

$$R = \frac{c-1}{2c}\epsilon_H = \frac{I}{2}\epsilon$$

For anisotropic inflation, the amplitude of quadrupolar anisotropy g_* is given by $g_* = 24IN_e^2$ and demanding from CMB observations requires: (P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A16 (2016) .

$$|g_*| \lesssim 10^{-2} \implies I \lesssim 10^{-7}$$



Adding helicity to an isotropic vector inflation

- We consider a triplet of U(1) gauge field whose Lagrangian has the form (C. Caprini and L. Sorbo, JCAP 1410 (2014)

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{f^2(\phi)}{4}\sum_a\left(F_{\mu\nu}^{(a)}F^{\mu\nu}_{(a)}\right) + \frac{J^2(\phi)\gamma}{8}\sum_a\left(F_{\mu\nu}^{(a)}\tilde{F}^{\mu\nu}_{(a)}\right)\right]$$

- The new parity violating term allows more freedom in tuning the amplitude of the field at the end of inflation.
- Moreover, for U(1) gauge field, it leads to a helical magnetic field that is amplified at large scales by magnetohydrodynamical processes during the radiation dominated epoch (C. Caprini and L. Sorbo, JCAP 1410 (2014) .
- This model satisfies the observational lower bounds on fields in the intergalactic medium, while providing a seed for the galactic dynamo, if inflation occurs at an energy scale ranging from 10^5 to 10^{10} GeV.
- However, the gauge field is a source of tensors during inflation and generates a spectrum of gravitational waves that can give a sizable tensor to scalar ratio r = O(0.2) even if inflation occurs at low energies.
- The model predicts fully helical cosmological magnetic fields and a chiral spectrum of primordial gravitational waves.
- Moreover, since the gauge field has a definite helicity, the tensor modes pro- duced during inflation will be parity-odd, and as a consequence will generate non-vanishing ⟨TB⟩ and ⟨EB⟩ in the Cosmic Microwave Background

Cosmological Perturbations

- The perturbations of the metric is given by

$$\delta g_{00} = 2a^2\alpha \quad \delta g_{0i} = a^2(\partial_i\beta + B_i) \quad \delta g_{ij} = a^2(2\psi\delta_{ij} + 2\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + h_{ij})$$

- The vector and tensor fields are subject to transverse and traceless conditions

$$\partial_i B_i = \partial_i F_i = \partial_i h_{ij} = h_{ii} = 0$$

- The perturbations of the gauge field with global O(3) symmetry are given by (R. Emami, S. Mukohyama, R. Namba and Y. l. Zhang, JCAP 1703, no. 03, 058 (2017)

$$\delta A_0^{(a)} = Y_a + \partial_a Y \quad \delta A_i^{(a)} = \delta Q\,\delta_{ia} + \partial_i(\partial_a M + M_a) + \epsilon_{iab}(\partial_b U + U_b) + t_{ia}$$

- The gauge fields as usual enjoys the local $U(1)$ symmetry. At the level of linear perturbations we have

- $$\delta A_\mu^{(a)} \to \delta A_\mu^{(a)} - \partial_\mu\partial_a\Lambda - \partial_\mu\Lambda_a^\perp$$

$$Y \to Y - a^{-1}\Lambda' \quad M \to M - \Lambda \quad Y_a \to Y_a - a^{-1}\Lambda'^\perp_a \quad M_a \to M_a - \Lambda_a^\perp$$

- The above transformations show that one scalar mode and two vector modes are not real physical degrees of freedom and can be removed by the local gauge transformation. Without loss of generality, we choose $M = 0 \quad M_a = 0$
- For the scalar modes, we work in the spatially flat gauge in which $\psi = 0\,, E = 0$
- Note that the gauge fields enjoys global O(3) symmetry so the scalar, vector and tensor perturbations decouples at the linear order of perturbations. Moreover, since our setup is isotropic, the vector perturbations decay as usual in an expanding universe.

Scalar Perturbations

- The quadratic action in Fourier space is

$$S^{(2)} = \frac{1}{2}\int d\tau d^3k \Big\{ U_c'^2 - \Big[k^2 - \frac{1}{\tau^2}(2+6\epsilon-2\eta)\Big]U_c^2 + \delta Q_c'^2 - \Big[k^2 - \frac{1}{\tau^2}(2+6\epsilon-2\eta)\Big]\delta Q_c^2$$
$$+\delta\phi_c'^2 - \Big[k^2 - \frac{1}{\tau^2}\Big(2+4I+9\epsilon-3\eta\Big)\Big]\delta\phi_c^2 + 8\sqrt{I}\Big[\frac{1}{\tau^2}\delta\phi_c\delta Q_c + \frac{1}{\tau}\delta\phi_c'\delta Q_c\Big]$$
$$-\frac{2k\gamma}{3\pi}\Big(6+3I+4\epsilon\Big)\Big[\sqrt{I}\delta U_c\delta\phi_c + (1-I)\delta U_c\delta Q_c\Big]\Big\}$$

- where we have defined the canonically normalized fields as

$$\delta Q_c = \sqrt{2}f\delta Q \;, U_c = \sqrt{2}kfU \,, \delta\phi_c = a\delta\phi$$

- The comoving curvature perturbations R is given by

$$\mathcal{R} = \psi + H\delta u \,,$$

- where ψ measures the spatial curvature and δu is the velocity potential which is defined as $\delta T_i^0 = (\varrho + p)\partial_i \delta u$.

$$\mathcal{R} = -aH\frac{\sqrt{2}fA'\delta Q_c + a\phi'\delta\phi_c}{2f^2A'^2 + a^2\phi'^2}$$

- Now assuming that $I \ll 1$ and taking the leading contribution of I, we find

$$\mathcal{R} = -\frac{H}{\phi'}\Big[(1-I)\delta\phi_c - \sqrt{I}\delta Q_c\Big]\,.$$

Adiabatic and entropy decompositions

- The scalar modes $\delta\phi_c$ and δQ_c can be decomposed into the adiabatic and entropic components as follows (C. Gordon, D. Wands, B. A. Bassett and R. Maartens, Phys. Rev. D 63, 023506 (2001)

$$\delta\sigma_c = \cos\theta\delta\phi_c + \sin\theta\delta Q_c \quad \delta s_c = -\sin\theta\delta\phi_c + \cos\theta\delta Q_c \,,$$

$$\cos\theta \equiv \sqrt{1-I}\,, \sin\theta \equiv -\sqrt{I}\,.$$

- The comoving curvature perturbations and the associated normalized entropy perturbation are given by

$$\mathscr{R} = -\frac{H}{\dot\phi}\cos\theta\,\delta\sigma\,. \qquad \mathscr{S} \equiv -\frac{H}{\dot\phi}\cos\theta\,\delta s\,.$$

- The power spectrum of curvature perturbation at the end of inflation is given by

$$\langle \mathscr{R}^\dagger(\tau_e,\mathbf{k})\,\mathscr{R}(\tau_e,\mathbf{k}')\rangle = \left(\frac{H}{\dot\phi}\right)^2 \cos^2\theta\,\langle\delta\sigma^\dagger\delta\sigma\rangle \equiv \frac{2\pi^2}{k^3}\mathscr{P}_{\mathscr{R}}\,(2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')\,.$$

- The free wavefunction for $M_{i\mathrm{k}} = \left\{\delta\sigma_c(k), \delta s_c(k), U_c\right\}$ with the Bunch-Davies initial condition, is given by

$$M_{i\mathrm{k}} = v(k)a_{i\mathrm{k}} + v(k)^\star a^\dagger_{i\text{-}\mathrm{k}}\,; \quad v(k) = \frac{ie^{-ik\tau}}{\sqrt{2k^3\tau}}\Big(1+ik\tau\Big)$$

- We can consider the total hamiltonian as $H = H_0 + H_I$

$$H^s_1 = -\frac{8\sqrt{I}}{\tau^2}\delta\sigma_c\delta s_c \quad H^s_2 = \frac{4\sqrt{I}}{\tau}\delta\sigma_c\delta s_c{}' \quad H^s_3 = -\frac{6I}{\tau}\delta s_c^2 \quad H^s_4 = \frac{12I}{\tau^2}\delta\sigma_c^2 \quad H^s_5 = \frac{2k\gamma I}{\tau}\delta s_c U_c$$

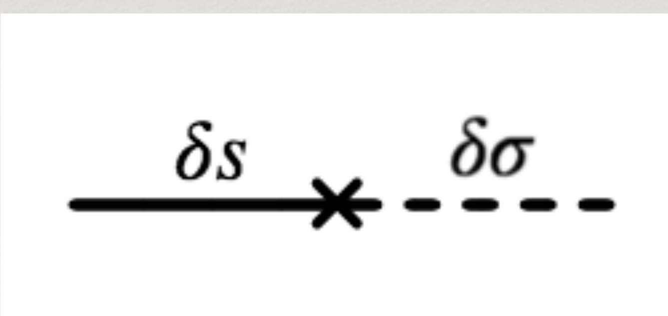
In-In formalism

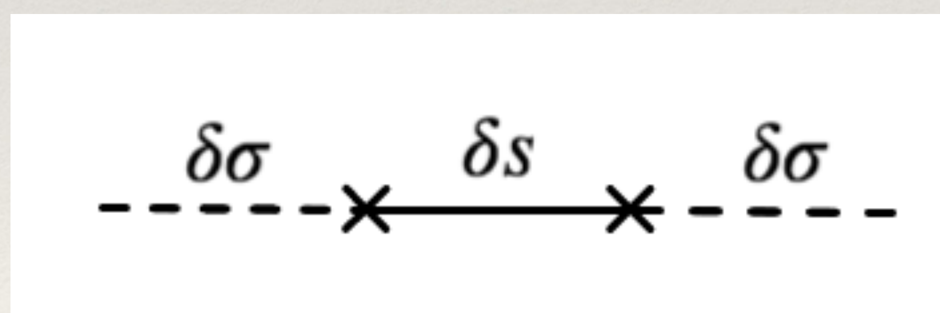
- The two-point function for the adiabatic mode is then given by (S. Weinberg, Phys. Rev. D 72, 043514 (2005),

$$\left\langle \delta\sigma^2(\tau_e) \right\rangle = \left\langle 0 \left| \left[\bar{T} \exp\left(i \int_{\tau_0}^{\tau_e} H_I(\tau'') d\tau'' \right) \right] \delta\sigma(\tau_e)^2 \left[T \exp\left(-i \int_{\tau_0}^{\tau_e} H_I(\tau') d\tau' \right) \right] \right| 0 \right\rangle$$

- $$= \langle 0 | \delta\sigma^2 | 0 \rangle + i \left\langle 0 \left| \int_{\tau_0}^{\tau_e} d\tau_1 \left[H_I(\tau_1), \delta\sigma^2(\tau_e) \right] \right| 0 \right\rangle$$

$$- \left\langle 0 \left| \int_{\tau_0}^{\tau_e} d\tau_1 \int_{\tau_0}^{\tau_1} d\tau_2 \left[H_I(\tau_2), \left[H_I(\tau_1), \delta\sigma^2(\tau_e) \right] \right] \right| 0 \right\rangle + \ldots$$





$$H_1^s = -\frac{8\sqrt{I}}{\tau^2} \delta\sigma_c \delta s_c$$

Output

- By neglecting the sub-leading IN_e contributions, the total curvature perturbation is obtained to be

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}^{(0)}_{\mathcal{R}}\left(1 + 16IN_e^2\right)$$

- Therefore, the corrections in the spectral index is given by

$$\Delta n_s = \Delta \frac{d\ln \mathcal{P}_{\mathcal{R}}}{d\ln k}\Big|_* = 32IN_e \frac{dN_e}{d\ln\, k} = 32IN_e$$

- In order to have a nearly scale invariant power spectrum we require Δn_S to be at the order of the slow-roll parameters ε and η. As a result, we conclude that $I \lesssim \epsilon/10N_e \approx 10^{-4}$

- The cross- correlation of the entropy and the curvature perturbation is given by

$$\mathcal{P}_{\mathcal{R}\mathcal{S}} = 4\mathcal{P}^{(0)}_{\mathcal{R}}\sqrt{I}N_e$$

- Correction to the power spectrum of the entropy mode is

$$\mathcal{P}_{\mathcal{S}} = \mathcal{P}^{(0)}_{\mathcal{R}}\left(1 - \frac{56}{3}IN_e\right)$$

- The cross- correlation of the entropy and the U_c is given by

$$\mathcal{P}_{\mathcal{S}\mathcal{U}} = I\pi\gamma\mathcal{P}^{(0)}_{\mathcal{R}}\left(1 - 4IN_e\right)$$

Future plans

- Tensor perturbation: the model predicts fully a chiral spectrum of primordial gravitational waves.

- Adding helicity to the Charged Vector inflation model (H. Firouzjahi, M. Gorji, S.A.H Mansoori, T. Rostami, A. Karami , arXiv: 1812.07464)

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}R - \frac{1}{2}D_\mu\phi\overline{D^\mu\phi} - V(\phi,\bar{\phi}) - \frac{f^2(\phi)}{4}\sum_a \left(F^{(a)}_{\mu\nu}F^{\mu\nu}_{(a)}\right) + \frac{J^2(\phi)\gamma}{8}\sum_a \left(F^{(a)}_{\mu\nu}\tilde{F}^{\mu\nu}_{(a)}\right)\Big]$$

- Non-Gaussianity in chiral models

Thanks for your attention