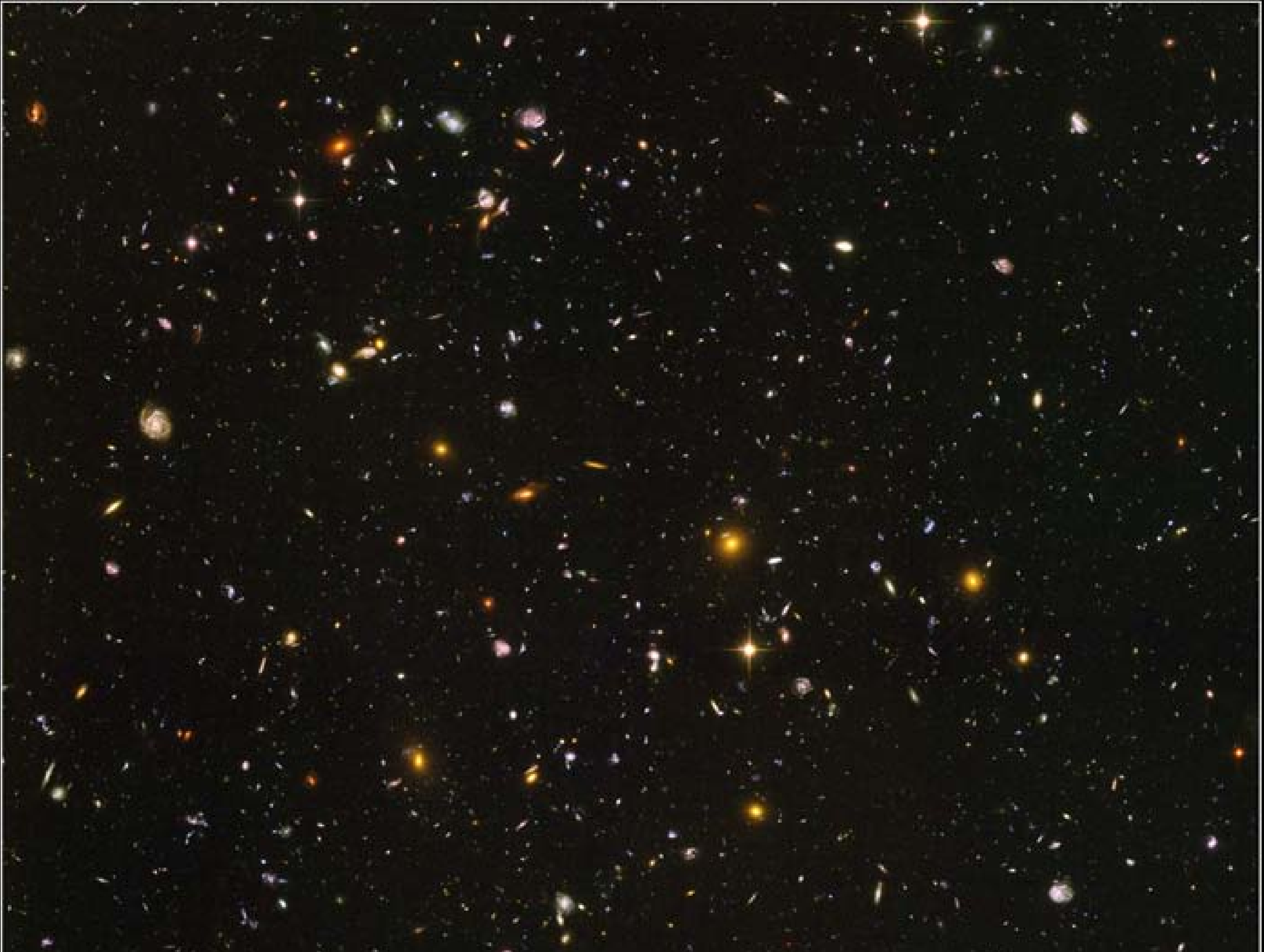
Revisiting Cosmic No-Hair Theorem for Inflationary Settings

A. Malek-Nejad
&
M. M. Sheikh-Jabbari

based on

arXiv:1203.0219

Celebrating "DBI in the Sky"   April 2012, IPM, Tehran



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a **generic initial condition**

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Cosmic No-Hair Conjecture ?

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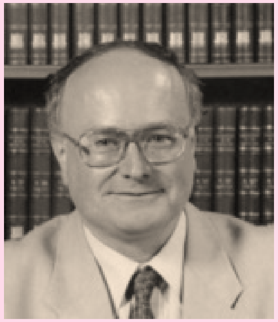
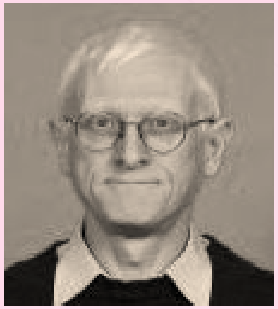
(by Gibbons & Hawking 1977, Hawking & Moss 1982)

all expanding-universe models with a **positive cosmological constant** **(Λ>0)** asymptotically approach the **de-Sitter** solution.





Cosmic No-Hair Conjecture

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The **1st** attempt to **formulate** that made by **Wald 1983.**

Cosmic No-Hair **Conjecture**

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Cosmic No-Hair **Theorem**

(by R. Wald 1983)

Wald's Cosmic No-Hair Theorem

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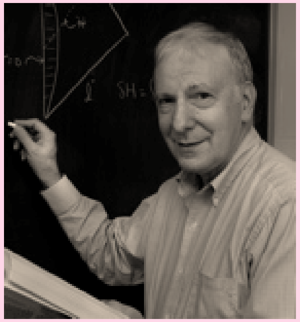

- ***Theorem:*** In the **General Relativity**, consider **Initially expanding** **Bianchi-type** (homogeneous but anisotropic) models, with the total energy-momentum tensor as

$$T_{\mu\nu} = -\Lambda_0 \, g_{\mu\nu} + \tilde{T}_{\mu\nu}$$, where

$\Lambda_0 > 0$ is a **positive** cosmological constant &

$\tilde{T}_{\mu\nu}$ satisfies **Strong** & **Dominant energy conditions**.

This system will approach **de-Sitter** space exponentially fast, with the time-scale $\sqrt{3/\Lambda_0}$.

Bianchi-type models

(*Homogenous* But **Anisotropic** space)

$$ds^2 = -dt^2 + a^2(t) e^{\beta_{ij}(t)} e^i \otimes e^j$$

$$\mathrm{tr}\,\beta_{ij} = 0$$

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FRW (Homogeneous and Isotropic Space)

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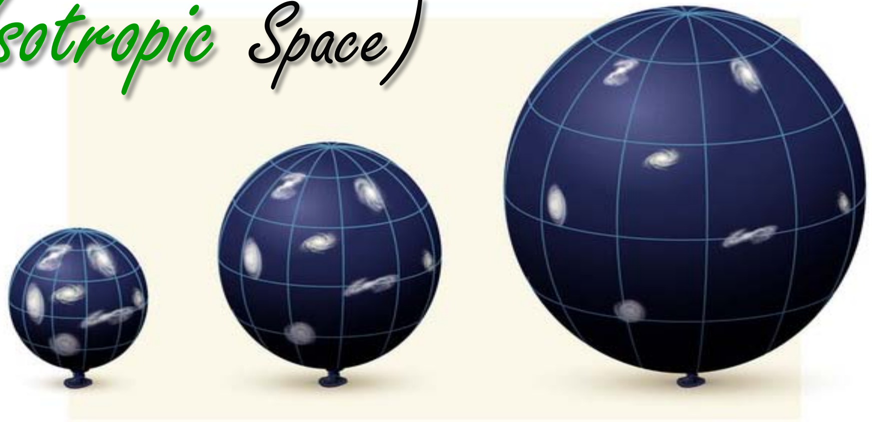

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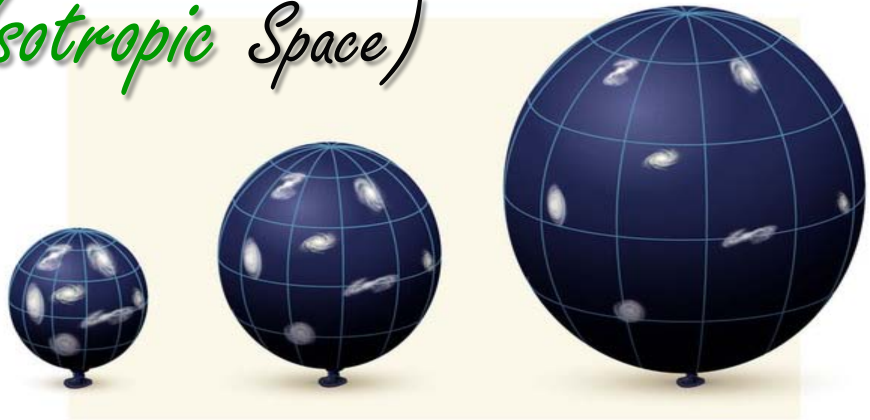

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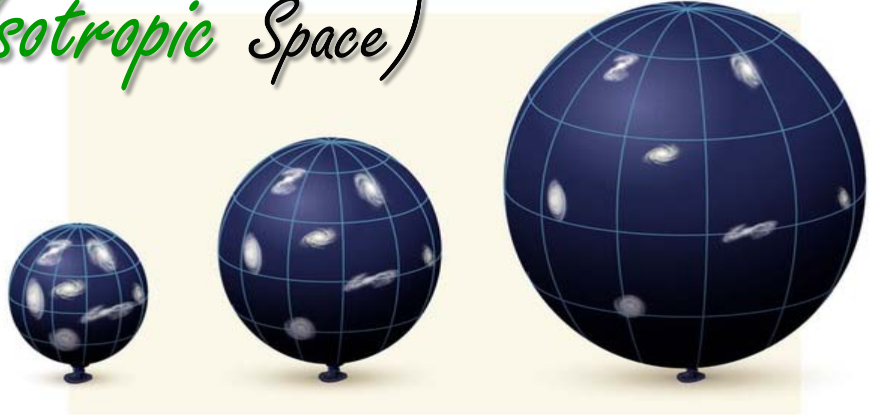

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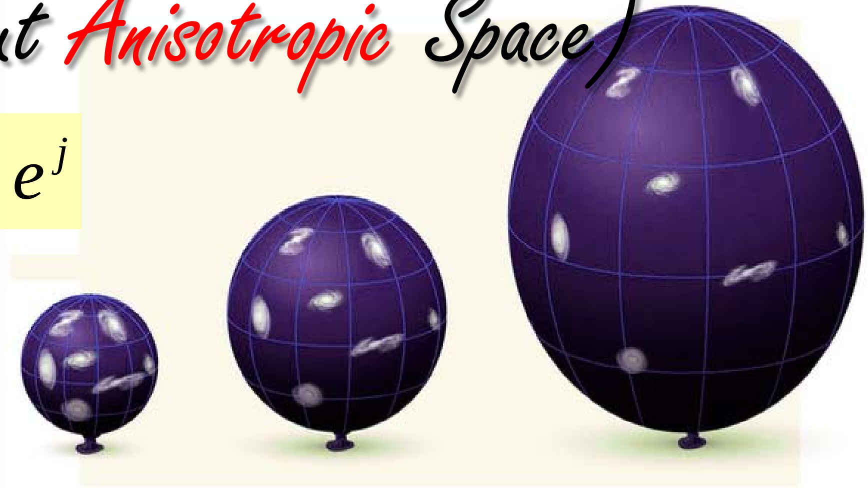

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- **Strong Energy Condition (SEC):**

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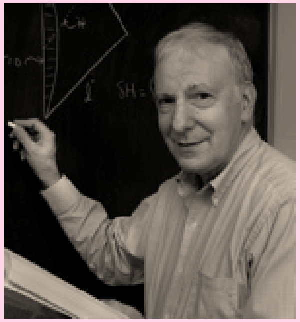
Wald's Cosmic No-Hair Theorem



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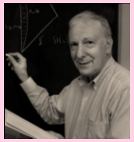

If the conditions of the Wald's theorem hold:

- Regardless of the **enlargement** of the **initial anisotropy**, system exponentially approaches the **isotropic** solution.
- **Inflation never ends** in these systems ⇓

  Inflationary models do **not** satisfy in Wald's theorem!

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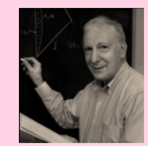

We need to extend Wald's theorem for inflationary settings.

Cosmic No-Hair **Theorem**

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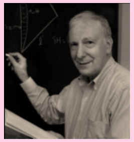

Extended Cosmic No-Hair **Theorem** for Inflation

(by A. Malek-Nejad & M. M. Sheikh-Jabbari 2012)

Extended Cosmic No-Hair Theorem for Inflation

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Theorem: In the **GR**, assume **Inflation** in **Bianchi-type** models with the total energy-momentum tensor as

$$T_{\mu\nu} = -\Lambda(t)\, g_{\mu\nu} + \mp_{\mu\nu}$$ where $\Lambda(t) \geq 0$ & $\dot{\Lambda}(t) \leq 0$

$\mp_{\mu\nu}$ satisfies **Strong** & **Weak energy conditions**.

1) In principle, **anisotropies** can **grow (in contrast** to the **cosmic no-hair conjecture)** *!!!*

2) however, there is an **upper-bound** on the **growth of anisotropies** of the order of the slow-roll parameter.

Extended Cosmic No-Hair Theorem for Inflation

@ It is always possible to: describe the **energy-momentum tensor** of any **inflationary system** as

$$T_{\mu\nu} = -\Lambda(t)\, g_{\mu\nu} + \mathrm{Ŧ}_{\mu\nu}$$

where $\Lambda(t) \geq 0$ &

$\mathrm{Ŧ}_{\mu\nu}$ satisfies **Strong** & **Weak Energy Conditions**.

A. M-N and M. M. S-J arXiv:1203.0219v2

Extended Cosmic No-Hair Theorem for Inflation

@ General form for the energy-momentum tensor:

$$T_{\mu\nu} = (\rho(t) + P(t))u_\mu u_\nu + P(t)g_{\mu\nu} + \Pi_{\mu\nu}(t)$$

$\Pi_{\mu\nu}(t)$ anisotropic stress tensor → $T_{ij}(t) \quad i \neq j$

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is the source of anisotropy dynamics!

A. M-N and M. M. S-J arXiv:1203.0219v2

Anisotropy Dynamics in 2 classes of inflationary models

- Scalar driven inflationary models
  - Ordinary multi-scalar filed models
  - K-inflation
  - DBI inflation
- Models of inflation involving vector gauge fields
  - Gauge-flation
  - Inflationary universe with anisotropic hair

Anisotropy Dynamics in 2 classes of inflationary models

- Scalar driven inflationary models
  - Ordinary multi-scalar filed models
  - K-inflation
  - DBI inflation

In all of the above cases T_{ij} , $i \neq j$ is identically zero. So, anisotropy damps out exponentially fast in few Hubble times.

Anisotropy Dynamics in 2 classes of inflationary models

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Gauge-flation:

Anisotropy Dynamics in 2 classes of inflationary models

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Gauge-flation:

(non-Abelian gauge field inflation) is a **novel inflationary scenario** in which inflation is driven by **su(2) non-Abelian gauge field minimally** coupled to **Einstein gravity**.

A. Maleknejad & M.M. Sheikh-Jabbari Phys. Rev. D 84 (2011)
A. Maleknejad & M.M. Sheikh-Jabbari arXiv:1102.1513

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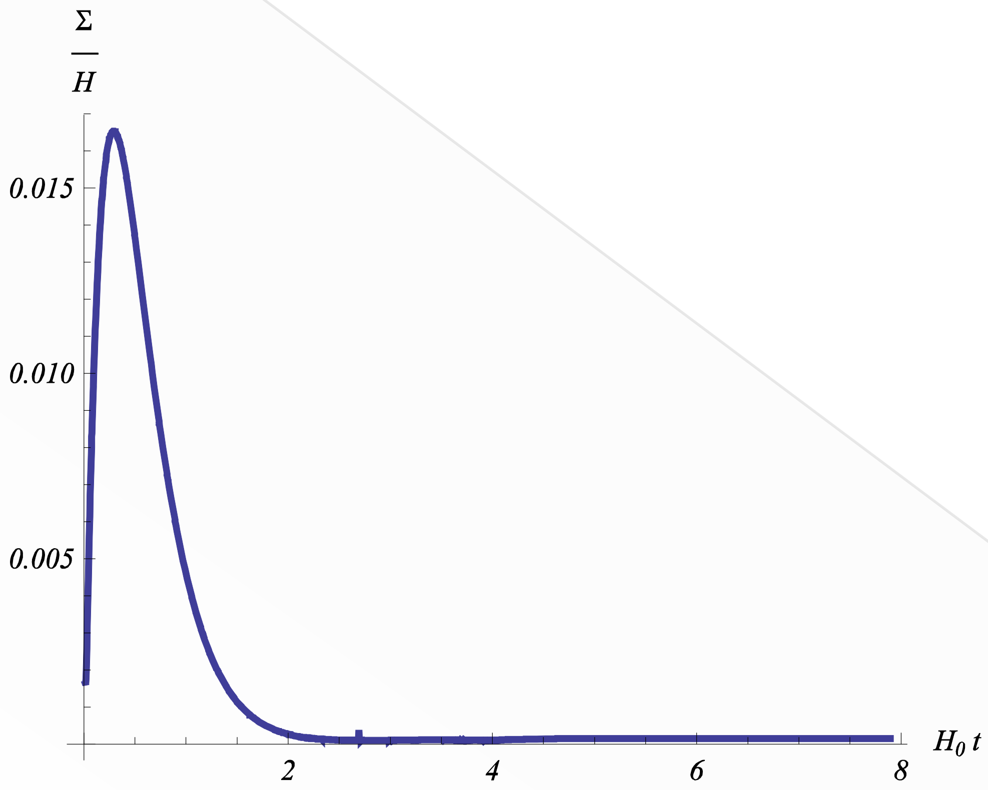
Due to its **vector nature**

⇓

Stability of Isotropic background **?**

A. Maleknejad , M.M. Sheikh-Jabbari and Jiro Soda, JCAP 1201,016 (2012)

Anisotropy in Gauge-flation



$\frac{\Sigma(t)}{H^2} := \frac{\dot\sigma(t)}{H^2}$ for a system with $\kappa = 3.77 \times 10^{15}$, $g = 10^{-1}$, $\psi_0 = 0.6 \times 10^{-3}$, $\dot\psi_0 = 10^{-10}$,

A. Maleknejad , M.M. Sheikh-Jabbari and Jiro Soda, JCAP 1201,016 (2012)

Anisotropy Dynamics in 2 classes of inflationary models

- Models of inflation involving vector gauge fields

Inflationary universe with anisotropic hair

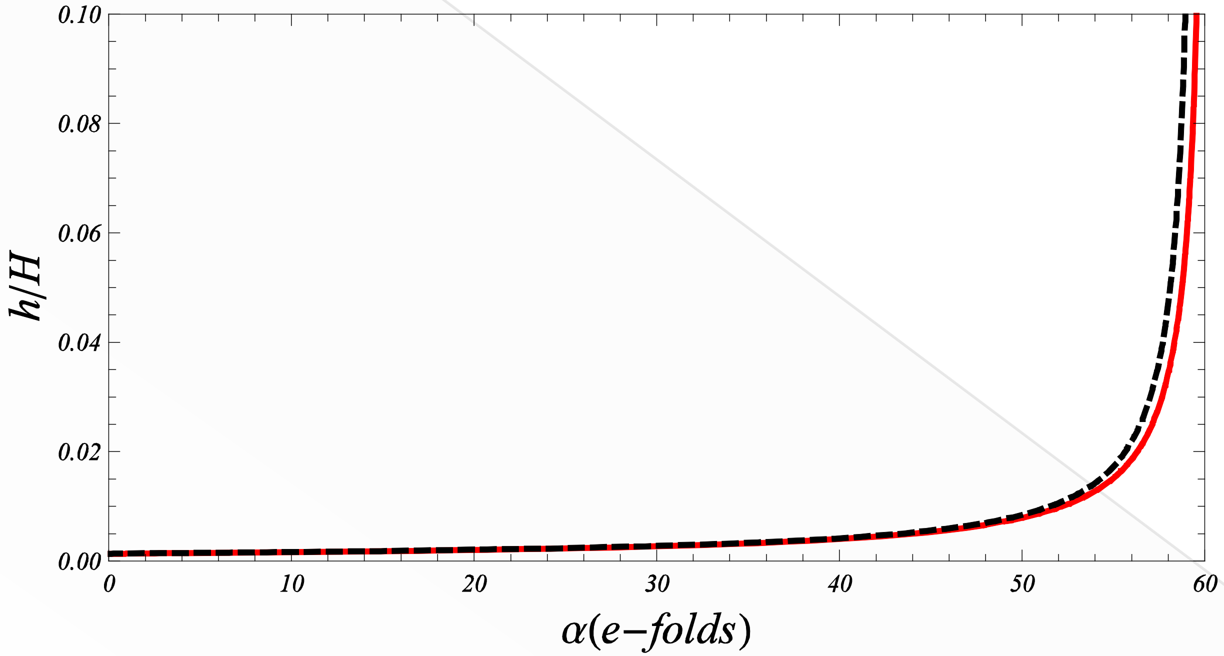
M. Watanabe, S. kanno and J. Soda

has introduced an inflationary model with **anisotropic hair**. Their model includes **a scalar field** as **inflation** coupled to a mass-less **U(1) gauge-field**.

M. Watanabe, S. kanno and J. Soda, Phys. Rev. Lett. 102, 191302 (2009)

Anisotropic Inflation in this Model

M. Watanabe, S. kanno and J. Soda, Phys. Rev. Lett. 102, 191302 (2009)



Hubble-normalized shear $\frac{h}{H} := \frac{\dot{\sigma}}{H}$ during inflation when $c = 2$ and $= \varphi_i = 11 M_{Pl}$.

B. Himmetoglu, JCAP 1003, 023 (2010)

Concluding Remarks

- we **Extended cosmic no-hair theorem** for general **inflationary setups.**
- We find, the behavior of **anisotropies** are governed by the **anisotropic stress tensor** (anisotropic part of $T_{\mu\nu}$)
- It is shown that **Anisotropies** can **grow** during inflation, but
- There is **an upper-bound value** on **their enlargement** (assuming slow-roll, its equal to $\frac{8}{3}(\varepsilon_0 - \eta_0)$)


Thank you