Entanglement and discord of pseudo-entangled state in noninertial frame

Hossein Mehri-Dehnavi

Research Center for Quantum Computing, Kinki University, Japan,
and
Institute for Advanced Studies in Basic Sciences, Gava Zang, Zanjan Iran

In collaboration with:

Robabe Rahimi (Institute for quantum computing, waterloo, Canada),
Hosein Mohammadzadeh, and Behrouz Mirza
(Isfahan University of technology, Isfahan, Iran)
Plan of Talk

• Introduction
• Entanglement, superdense coding protocol, and quantum discord
• Quantum information in accelerated frames
• Discussion and conclusion

Introduction

Moore’s Law

Source: Intel
Introduction

Digital Computer or Abacus?

Initialization

Input

Computation

Output/Readout
ENIAC (1946)

100,000,000,000,000,000,000 Bits

NMR QC
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Entanglement, Superdense coding, ...

Entanglement and its evaluation.

\[ |\Phi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \]

\[ |\psi_1\rangle|\psi_2\rangle = (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle) \]
\[ = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle \]

A simpler way is using entanglement measures, like as Negativity, logarithmic negativity, entanglement of formation, concurrence and ....

\[ N(\rho) := \log_2 \sum |\lambda_i(\rho^{pt})| \]

Where \( \rho^{pt} \) is the partial transpose of \( \rho \).
Entanglement, Superdense coding, ...

Superdense coding

\[ |\beta_{z,x}\rangle \equiv \frac{|0, x\rangle + (-1)^z|1, \bar{x}\rangle}{\sqrt{2}} \]

where \( z, x = 0, 1 \) and \( \bar{x} = 1 - x \)

- \( I \)
  - \( H \)
  - \( U_{zx} \)
  - \( H \)
  - \( Z_I \)

- \( S \)
  - \( \psi_0 \rangle \) (\( \rho_0 \))
  - \( \psi_1 \rangle \) (\( \rho_1 \))
  - \( \psi_2 \rangle \) (\( \rho_2 \))
  - \( \psi_3 \rangle \) (\( \rho_3 \))

\[ |\psi_1\rangle = |\beta_{00}\rangle \]
\[ |\psi_2\rangle = U_{zx}|\beta_{00}\rangle = |\beta_{zx}\rangle \]

\[ U_{zx} = Z^z X^x \]
\[ |\psi_3\rangle = U_{\text{disent}}|\psi_2\rangle = |z.x\rangle \]
Entanglement, Superdense coding, ...

Is entanglement necessary for SDC? No:
They have done the SDC experiment with NMR in room temperature.

\[ \rho = \frac{1 - p}{4} I + p |\Phi^+\rangle \langle \Phi^+ | \quad |\Phi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \]

The above state is entangled for \( p > 1/3 \), but in NMR experiment we have \( p \approx 10^{-5} \).

\[ \rho_{\text{sep}} = \sum_i w_i \rho_A^i \otimes \sigma_B^i \]
\[ \rho_{\text{pcc}} = \sum_{i=1}^{d^A} \sum_{j=1}^{d^B} e_{ij} |v_A^i\rangle \langle v_A^i | \otimes |v_B^j\rangle \langle v_B^j | \]

Therefore a better measure of quantumness is quantum discord.
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Study of the Werner (*Pseudo-Entanglement*) state in non-inertial frame

\[ \rho = \frac{1 - p}{4} I + p |\Phi^+\rangle \langle \Phi^+| \quad \quad |\Phi^+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \quad \quad p > 1/3 \]

*The Unruh-Davis Effect:*
When Rob accelerates uniformly through the Minkowski vacuum, his detector registers a number of particles.

For Spinor field we have

\[ |0_k\rangle_M = \sum_{n=0}^{1} A_n |n_k\rangle_I^+ |n_{-k}\rangle_{II} \]

\[ |0\rangle_M = \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II} \]

\[ |1\rangle_M = |1\rangle_I |0\rangle_{II} \]

\[ \cos r = \frac{1}{\sqrt{1 + e^{-2\pi \omega c/a}}} \quad \omega = \sqrt{\frac{k^2}{a} + m^2} \]
\[
\rho_{A, I, II} = \frac{1}{4} \begin{pmatrix}
(1 + p) \cos^2 r & 0 & 0 & \frac{1+p}{2} \sin 2r & 0 & 0 & 2p \cos r & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 - p & 0 & 0 & 0 & 0 & 0 \\
\frac{1+p}{2} \sin 2r & 0 & 0 & (1 + p) \sin^2 r & 0 & 0 & 2p \sin r & 0 \\
0 & 0 & 0 & 0 & (1 - p) \cos^2 r & 0 & 0 & \frac{1-p}{2} \sin 2r \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2p \cos r & 0 & 0 & 2p \sin r & 0 & 0 & 1 + p & 0 \\
0 & 0 & 0 & 0 & \frac{1-p}{2} \sin 2r & 0 & 0 & (1 - p) \sin^2 r
\end{pmatrix}.
\]

where, we have used the basis \(|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, and |111\rangle\), with \(|lmn\rangle := |l\rangle_A |m\rangle_I |n\rangle_{II}\).
Quantum discord for Alice Rob bipartite, which is nonzero for all $p>0$.

logarithmic negativity for Alice Rob bipartite. We have entanglement death for $1/3 > p > 3/7$. 

logarithmic negativity for Alice antiRob bipartite. It tends to $N(\rho_{A,1})$, in the limit of infinite acceleration.

Quantum discord for Alice antiRob bipartite, which is nonzero for all $r$ and $p$. 
Discussion and Conclusion:

• We have entanglement (not discord) death for some finite acceleration.
• It is harder to doing QC in accelerated frames.
Thank you for your attention
\[
\rho_{1,\Pi} = \frac{1}{2} \begin{pmatrix}
\cos^2 r & 0 & 0 & \sin r \cos r \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\sin r \cos r & 0 & 0 & \sin^2 r
\end{pmatrix}
\]
Collaborators:

Prof. Ali Mostafazadeh, Koc university, Istanbul, Turkey

Prof. Mikio Nakahara. Kinki University, Osaka, Japan

Prof. Behrouz Mirza, Isfahan University of Technology, Isfahan, Iran
Dr. Robabeh Rahimi, Osaka City University, Osaka, Japan
Hosein Mohammadzadeh, Isfahan University of Technology, Isfahan, Iran

Dr. Shapour Moradi, Razi University, Kermanshah, Iran.
Dr. Ivette Fuentes, University of Nottingham, Nottingham, United Kingdom
Entanglement & ENDOR Quantum Computing

Pseudo-pure state entanglement*:

\[
\begin{align*}
\ket{e_n^\downarrow} \xrightarrow{P_i^{(\pi/2)}} \frac{1}{\sqrt{2}} \left( \ket{e_n^\uparrow} + \ket{e_n^\downarrow} \right) \xrightarrow{P_s^{(\pi)}} \frac{1}{\sqrt{2}} \left( \ket{e_n^\uparrow} - \ket{e_n^\downarrow} \right)
\end{align*}
\]

Mehring et al., *PRL.* 90, 153001(2003)
“When we get to the very, very small world - say circuits of seven atoms - we have a lot of new things that would happen that represent completely new opportunities for design.

Atoms on a small scale behave like nothing on a large scale, for they satisfy the laws of quantum mechanics...”
1. Quantum **objects** are waves and can be in states of **superposition**......

   “quantum bit”:\n   \[
   \alpha |0\rangle + \beta |1\rangle
   \]

2. ...... as long as you **don't look**!

   \[
   \alpha |0\rangle + \beta |1\rangle \quad \text{or} \quad |0\rangle \leftrightarrow |1\rangle
   \]

   quantum measurement \(\Leftrightarrow\) flipping a coin