Double-Slit Interference Pattern for an Open Macroscopic Quantum System

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Open Macroscopic Quantum Systems

- ✓ The realistic picture of a quantum systems is an open one which continuously monitored by its environment.
- ✓ The key idea promoted by decoherence is the insight that realistic quantum systems are never isolated, but are immersed in the surrounding environment and interact continuously with it.
- ✓ Environmental entanglement and the resulting decoherence plays a crucial role in quantum to classical transition.
- ✓ The environment surrounding a quantum system can monitor some of the system observables. As a result, the eigenstates of those observables continuously decohere and can behave like classical states.

E. Joos, H. D. Zeh, *The emergence of classical properties through interaction with the environment*, Z.Phys. B: Condense. Matter, **59**, 223-243, (1985).

M. Schlosshauer, *Decoherence and the Quantum to Classical Transition*, Springer, (2007).

- ✓ Quantum mechanical interference effects may be washed out by the influence of the environment. This can happen regardless of the system in question being macroscopic or not.

- ✓ The macroscopic quantum systems are considered as a bridge between quantum and classical systems.

- ✓ Macroscopic quantum phenomena refer to quantum features in objects of ‘large’ sizes, systems with many components or degrees of freedom, organized in some ways where they can be identified as macroscopic objects.

- ➢ Superfluidity and Superconductivity (SQUIDs)
- ➢ Electro-mechanical and opto-mechanichal systems
- ➢ Bose- Einstein condensates

S. Takagi, *Macroscopic Quantum Tunneling*, Cambridge University Press, New York (2005).
A. O. Caldeira, *An introduction to macroscopic quantum phenomena and quantum dissipation,* Cambridge University Press, New York (2014).

- ✓ Many works have been done to explain double-slit experiment under different conditions such as interference for the macro-system in experimental and theoretical contexts.

- ✓ Nairz and Zeilinger studied the interference pattern for the fullerene molecule as a large object similar to a classical one.

- ✓ Hornberger investigated the effect of environment on interference pattern of fullerene as a macromolecule.

- ✓ Hornberger and others studied quantum interference of the clusters in experiment.

- ✓ Gerlich and others showed the quantum diffraction of large organic molecules.

O. Nairz, M. Arndt and Anton Zeilinger, Am. J. Phys. **71**, 319 (2003).
K. Hornberger, S. Uttenthaler, B. Brezger, L. Hackermuller, M. Arndt, and A. Zeilinger, Phys. Rev. Lett. **90**, 160401 (2003).
K. Hornberger, S. Gerlich, P. Haslinger, S. Nimmrichter and M. Arndt, Mod. Phys **84**, 157 (2012).
S. Gerlich, S. Eibenberger, M. Tomandl, S. Nimmrichter, K. Hornberger, P. J. Fagan, J.Tuxen, M. Mayor and M. Arndt, Nature Comm. **2**, 263 (2011).

Bilinear- Harmonic model of the environment

- ✓ We assume that the central system is a quantum harmonic oscillator. The entire system is composed of the system and the environment, in which the latter is supposed to be a collection of micro-oscillators.

- ✓ In dimensionless form of the Schrodinger equation, we define the characteristic parameter of length R_0 and energy U_0 as constant units of length and energy, respectively. Subsequently, for a particle of mass M, one can define the characteristic time as $\tau_0 = R_0/(U_0\, M)^{\frac{1}{2}}$. Since, U_0 acts like the kinetic energy of a given system, the unit of momentum could be define as $P_0 = (U_0\, M)^{\frac{1}{2}}$.

$$i\tilde{h}\frac{d\psi(t)}{dt} = H\psi(t)$$

$$[q,p] = i\tilde{h} \qquad [x_\alpha, p_\beta] = i\tilde{h}\delta_{\alpha\beta}$$

$$\tilde{h} = \hbar/R_0P_0$$
$$\tilde{h} = \lambda/2\pi\, R_0$$

S. Takagi, *Macroscopic Quantum Tunneling*, Cambridge University Press, New York (2005).

- The situation in which one gets $\tilde{h} \ll 1$, the system behaves quasi-classically.

- The values of $\tilde{h}$ between 0.01 to 0.1 are fair enough to show the macroscopic disposition of the proposed system.

$$H = H_s + H_e + H_{se}$$

$$H_s = \frac{1}{2} p^2 + \mathrm{V}(q)$$

$$H_e = \sum_{\alpha} (\frac{1}{2} p_\alpha{}^2 + \frac{1}{2} \omega_\alpha{}^2 x_\alpha{}^2 - \frac{1}{2} h\omega_\alpha)$$

$$H_{se} = -\sum_{\alpha} \omega_\alpha{}^2 f_\alpha(q)\ x_\alpha + \frac{1}{2} \sum_{\alpha} \omega_\alpha{}^2 [f_\alpha(q)]^2$$

α: environmental particles varies from 1 to N.

Separable model 1) $f_\alpha(q) = \gamma_\alpha\, f(q)$ $0 < \gamma_\alpha < 1$

2) $f(q) = \gamma_\alpha\, q$

Bilinear model $f_\alpha(q) = \gamma_\alpha\, q$

$$H = \frac{1}{2}\, p^2 + \bar{V}(q) + \sum_\alpha \left(\frac{1}{2} {p_\alpha}^2 + \frac{1}{2}(1-\gamma_\alpha){\omega_\alpha}^2{x_\alpha}^2 - \frac{1}{2} h\omega_\alpha\right) + \sum_\alpha \gamma_\alpha {\omega_\alpha}^2 (x_\alpha - q)^2$$

- For decoupling the Hamiltonian, we define plus-minus position and momentum coordinates:

$$\begin{pmatrix} x_+ \\ x_- \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} q' \\ x_\alpha \end{pmatrix}$$

$$\begin{pmatrix} p_+ \\ p_- \end{pmatrix} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} p' \\ p_\alpha \end{pmatrix}$$

R. M. McDermott and I. H. Redmount, arXive:0403184 (2004).

$$\theta = \frac{1}{2}\arctan\left(\frac{{\omega_\alpha}'^2}{\omega^2 - {\omega_\alpha}^2}\right)$$

$$\omega = \left({\omega_0}^2 + \sum_\alpha {\omega_\alpha}^2 {\gamma_\alpha}^2\right)^{\frac{1}{2}}$$

$$\omega_\alpha' = i\left(2{\omega_\alpha}^2\gamma_\alpha\right)^{\frac{1}{2}}$$

Assumption: For all particles of the environment, ω_α and γ_α are nearly the same.

$tan2\theta < 0$ ⇒ $\tilde{h} = \lambda_0/R_0 < \lambda_\alpha/R_0$

$$H = \sum_\alpha H_\alpha$$

$$H_\alpha = H_{+\alpha} + H_{-\alpha}$$

$$H_\alpha = \frac{{p_{+\alpha}}^2}{2} + \frac{1}{2}{\omega_{+\alpha}}^2{x_{+\alpha}}^2 + \frac{{p_{-\alpha}}^2}{2} + \frac{1}{2}{\omega_{-\alpha}}^2{x_{-\alpha}}^2$$

$$\omega_{+\alpha} = (\omega^2\cos^2\theta + {\omega_\alpha}^2 sin^2\theta + {\omega_\alpha}'^2 sin\theta\ cos\theta)^{\frac{1}{2}}$$

$$\omega_{-\alpha} = (\omega^2\cos^2\theta + {\omega_\alpha}^2 sin^2\theta - {\omega_\alpha}'^2 sin\theta\ cos\theta)^{\frac{1}{2}}$$

✓ Wave function of the ground state for the central system coupled to two environmental particles:

$$\psi_0(x_{+1}, x_{-1}, x_{+2}, x_{-2}) = \left(\frac{\omega_{+1}\,\omega_{-1}\omega_{+2}\,\omega_{-2}}{\pi^4\tilde{h}^4}\right)^{\frac{1}{4}}$$

$$\times \exp\left(\frac{-\omega_{+1}}{2\tilde{h}}{x_{+1}}^2\right)\exp\left(\frac{-\omega_{-1}}{2\tilde{h}}{x_{-1}}^2\right)\exp\left(\frac{-\omega_{+2}}{2\tilde{h}}{x_{+2}}^2\right)\exp\left(\frac{-\omega_{-2}}{2\tilde{h}}{x_{-2}}^2\right)$$

Formulation of double-slit diffraction pattern

- ✓ We assume that the system has been in interaction with the environment only in y −direction, so that regarding the x − direction, the state of the system behaves like a Gaussian wave packet independent of any environmental effect.

$b\ , b'$ ⟹ widths of the slits

$2a$ ⟹ depth

d, d' ⟹ distances of the slits from the origin

- ✓ The wave packet describing the incoming particle is factorized in its x and y dependences. It is assumed that x and y dependences of the wave function remain factorized during and after passing the slits.

$$\psi(x, y, t) = \chi(x, t)\ \phi(y, t)$$

A. Zecca, J. Theor. Phys. **38**, 911 (1999).

A. Zecca, Adv. Studies Theor. Phys. **7**, 287 (2013).

$$\chi(x,t) = [\frac{\xi}{\pi^{\frac{1}{2}}(1+i\tilde{h}\xi^2 t)}]^{\frac{1}{2}}\{exp\,[-\frac{\xi^2}{2}\frac{(x-x_0-k_{0x}t)^2}{1+i\tilde{h}\xi^2 t}+\frac{ik_{0x}t}{\tilde{h}}(x-x_0)-ik_{0x}{}^2 t/2\tilde{h}]\}$$

$$\phi(y,t) = [\frac{\beta}{\pi^{\frac{1}{2}}\left(1+i\tilde{h}\beta^2 t\right)}]^{\frac{1}{2}}\,exp[\frac{-\beta^2}{2}\frac{(y-y_0)^2}{1+i\tilde{h}\beta^2 t}]$$

Where

$$\beta = [\frac{f(\omega_{+1},\omega_{-1},\omega_{+2},\omega_{-2})}{\tilde{h}}]^{\frac{1}{2}}$$

- ✓ After the slit, the wave function in the y −direction evolve as:

$$\phi_I(y,t) = [\frac{\beta}{2\,\pi^{\frac{3}{2}}\,i\tilde{h}t}]^{\frac{1}{2}}\,exp[y^2\frac{i}{2\tilde{h}t}-y_0{}^2\frac{\beta^2}{2}]\int_I exp\left[-\xi^2\left(\frac{\beta^2}{2}-\frac{i}{2\tilde{h}t}\right)+\xi\left(y_0\beta^2-\frac{iy}{2\tilde{h}t}\right)\right]d\xi$$

Limiting cases of the problem

1) We suppose that the wave packet reaching the slits is narrower than both the slits:

$\Delta y = \frac{1}{\beta\sqrt{2}} \ll b, b'$

$$\varphi_I(y,t)\varphi_I{}^*(y,t)$$
$$\cong \frac{\beta\pi^{-\frac{3}{2}}}{\left(1+\tilde{h}^2\beta^4t^2\right)^{\frac{1}{2}}} \exp[-\beta^2 \frac{(y-y_0)^2}{1+\tilde{h}^2\beta^4t^2}][\int_{(y_0+d')\beta/\sqrt{2}}^{(y_0+b'+d')\beta/\sqrt{2}} \exp(-t^2)\,dt + \int_{(y_0-d-b)\beta/\sqrt{2}}^{(y_0-d)\beta/\sqrt{2}} \exp(-t^2)\,dt]$$

- This shows that $\varphi_I\ \varphi_I{}^*$ is a Gaussian-like distribution

- ✓ In this case, the diffraction pattern, for any value of $0.01 < \tilde{h} < 0.1$ has a Gaussian form.
- ✓ No interference pattern is seen here, since the particle passes through one slit.

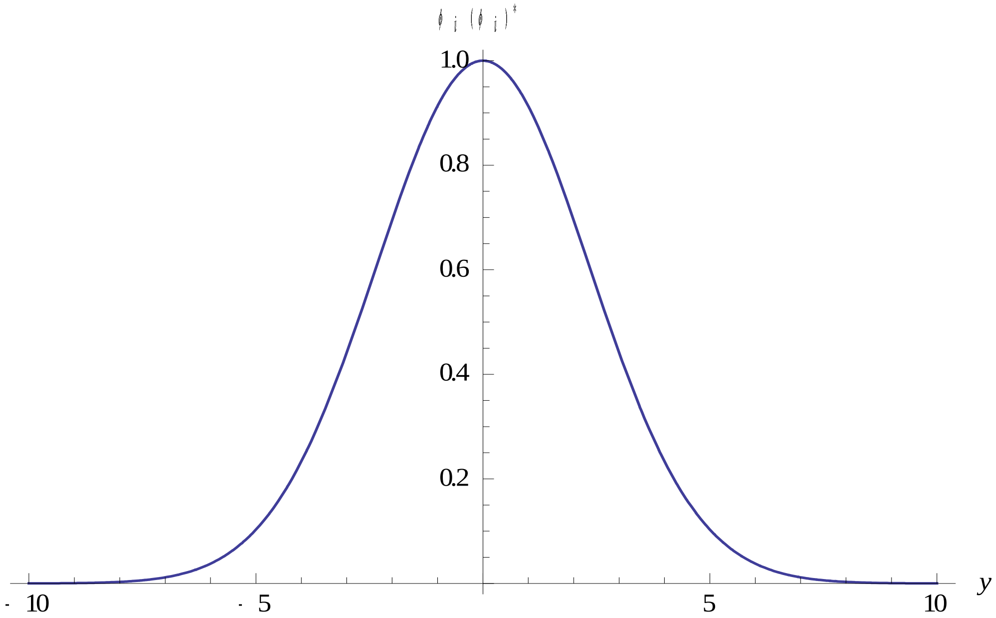

FIG. 1: Interference pattern for the case in which the incoming wave packet is too narrow with respect to both slits.

2) We assume that the wave packet reaching the slits has very large uncertainty, depicted by the $y-$ position probability distribution:

$$\Delta y = \frac{1}{\beta\sqrt{2}} \gg b, b'$$

- ✓ $d = d'$, $b = b'$

$$\varphi_I(y,t)\varphi_I{}^*(y,t) \cong \frac{2\beta b^2}{\pi^{\frac{3}{2}}\tilde{\hbar}t}\, exp(-\beta^2 {y_0}^2)\,\frac{sin^2\left(\frac{by}{2\tilde{\hbar}t}\right)}{\left(\frac{by}{2\tilde{\hbar}t}\right)^2}\, cos^2\left[\frac{y}{\tilde{\hbar}t}(d+b/2)\right]$$

- ✓ As expected, this probability has a maximum at $y_0 = 0$.

- ✓ If the separation of the slits is of order of the slit width ($d \cong$b), the factor containing the cosine will be practically negligible and the above expression gives the elementary diffraction pattern of a plane wave passing through a single slit.

- ✓ If the separation of the slit is much greater than their width ($d \gg$ b), the probability relation represents a high- frequency pattern.

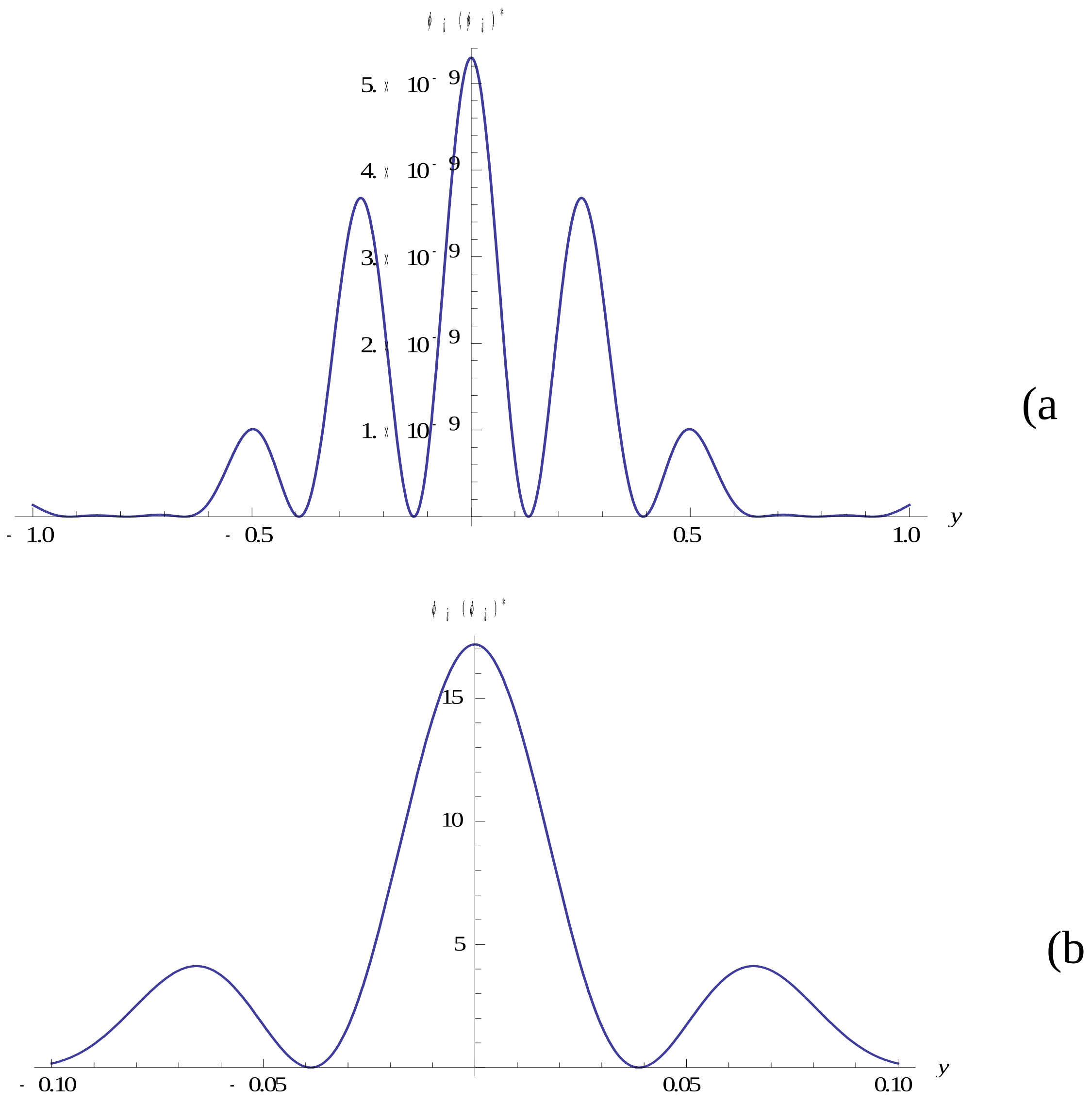

FIG. 2: Interference pattern for the case in which the incoming wave packet has a great uncertainty along the $y-$ direction with a) $\tilde{h} = 0.1$, b) $\tilde{h} = 0.01$

Conclusion

- ✓ Interference patterns for different quantum systems have been considered for many decades. In recent years, however, the experts have encountered with how the quantum-to- classical transition occurs, when the system shows classical trait.

- ✓ Taking into account the effects of an interacting environment on a quantum harmonic system via a simple oscillating model, we have shown that when the quantumness of the system is evanesced (measured by the parameter $\tilde{h}$), interference fringes are diminished in accordance with known patterns observed for macro-molecules (Fig. 2b).

- ✓ The environmental effects are not important when the incoming wave packet somehow describes the position state of the system in a given direction (Fig.1).

- ✓ We have now a controllable parameter $\tilde{h}$ by which we can follow and demonstrate the effects of the environment on quantum behavior of the system. This may open new door to the way one can better understand the emergence of classical appearance of the physical world in an interactive manner.

- “Life is like riding a bicycle.
To keep your balance, you must keep moving.”

- Albert Einstein (1879-1955)