Cluster Operator Approach

An extension of spin wave theory

Abdollah Langari

Sharif University of Technology

http://sina.sharif.edu/~langari

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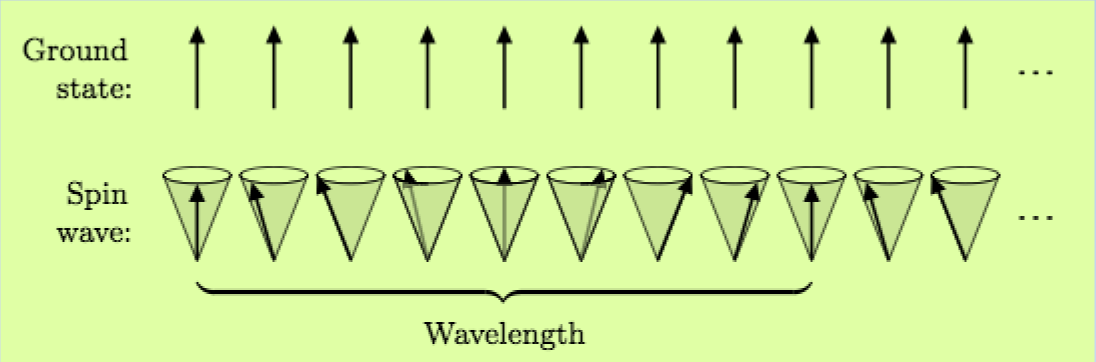
Spin wave theory

Ferromagnetic Heisenberg model on the spin-1/2 chain

$$\hat{H} = -J \sum_{<i,j>} \hat{S}_i . \hat{S}_j$$

$$S \to \infty$$



$$\hat{S}_i^{\pm} = S_i^x \pm i S_i^y$$

$$\hat{H} = -J \sum_i \hat{S}_i^z \hat{S}_{i+1}^z + \frac{1}{2} (\hat{S}_i^+ \hat{S}_{i+1}^- + \hat{S}_i^- \hat{S}_{i+1}^+)$$

Spin wave theory

Ferromagnetic Heisenberg model on the spin-1/2 chain

$$\hat{H} = -J \sum_{<i,j>} \hat{S}_i . \hat{S}_j$$

Holstein-Primakoff transformation:

$$\hat{S}_i^z = S - a_i^+ a_i$$

$$\hat{S}_i^+ = (2S - a_i^+ a_i)^{1/2} a_i$$

$$\hat{S}_i^- = a_i^+ (2S - a_i^+ a_i)^{1/2}$$

Linear SWT

$$< a_i^+ a_i > \prec\prec 2S$$

$$\hat{S}_i^z = S - a_i^+ a_i$$

$$\hat{S}_i^+ \cong (2S)^{1/2} a_i$$

$$\hat{S}_i^- \cong (2S)^{1/2} a_i^+$$

Spin wave theory

Ferromagnetic Heisenberg model on the spin-1/2 chain

$$\hat{H} = -J \sum_{<i,j>} \hat{S}_i . \hat{S}_j$$

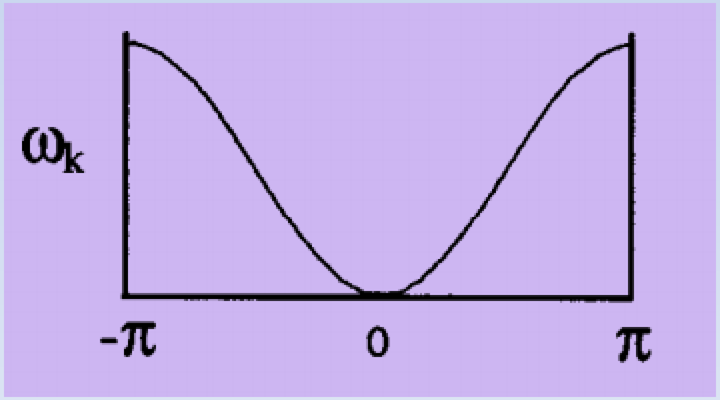
$$\hat{H}_{eff} = -JNS^2 + JS \sum_i (a_{i+1}^+ - a_i^+)(a_{i+1} - a_i)$$

Fourier Transformation: $a_i = \frac{1}{\sqrt{N}} \sum_k^{B.Z.} e^{-ik.r_i} a_k, \quad a_i^+ = \frac{1}{\sqrt{N}} \sum_k^{B.Z.} e^{ik.r_i} a_k^+$

$$\hat{H}_{eff} = -JNS^2 + \sum_k^{B.Z.} \hbar\omega_k a_k^+ a_k$$

$\hbar\omega_k = 2JS(1 - \cos k) = 4JS \sin^2(k/2)$

$k \to 0 \quad , \hbar\omega_k \to JSk^2$



Multi-Spin flip excitations

Frustrated antiferromagnetic J_1-J_2 Heisenberg model
on the Honeycomb lattice

$$H = J_1 \sum_{<i,j>} \vec{S}_i.\vec{S}_j + J_2 \sum_{<<i,j>>} \vec{S}_i.\vec{S}_j$$

$Classical\ S \to \infty\ Limit:$

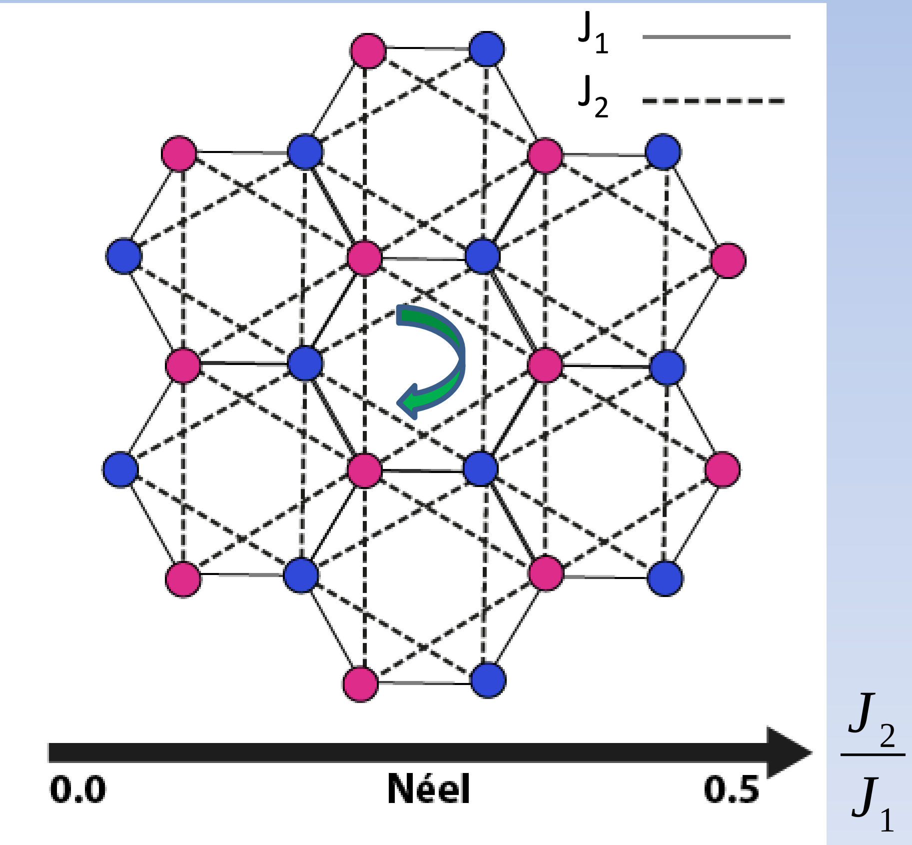
$Single\ spin\ flip\ :\ \Delta E_s = 6J_1 - 12J_2$

$Plaquette\ flip\ :\ \Delta E_h = 12J_1 - 48J_2$

$$\Delta E_h < \Delta E_s \Rightarrow \frac{1}{6} < \frac{J_2}{J_1} < \frac{1}{2}$$

$Zero-Energy\ Plaquette\ Flip:$

$$\Delta E_h = 0 \Rightarrow \frac{J_2}{J_1} = \frac{1}{4}$$



Classical Phase diagram

Multi-Spin flip excitations

Frustrated antiferromagnetic J_1-J_2 Heisenberg model
on the Checkerboard lattice

$$Classical\ S \to \infty\ Limit:$$

$$Single\ spin\ flip\ :\ \Delta E_s^{Néel} = 2J_1 - J_2\ ,$$

$$\Delta E_s^{Collinear} = J_2$$

$$Plaquette\ flip\ :\ \Delta E_p^{Néel} = 4(J_1 - J_2)\ ,$$

$$\Delta E_p^{Collinear} = 4(J_2 - J_1)$$

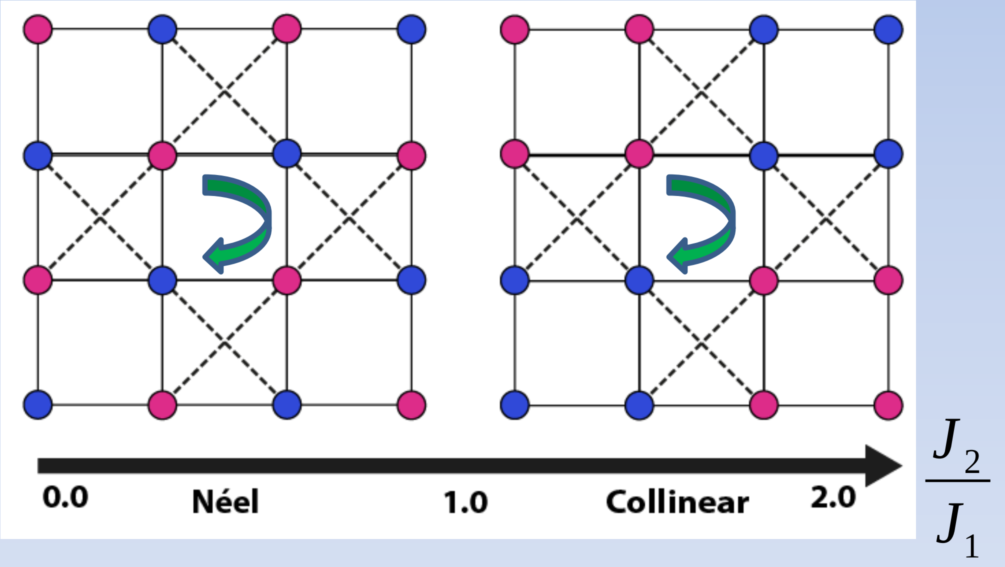
$$\Delta E_p < \Delta E_s \Rightarrow \frac{2}{3} < \frac{J_2}{J_1} < \frac{4}{3}$$

$$Zero-Energy\ Plaquette\ Flip:$$

$$\Delta E_p = 0 \Rightarrow \frac{J_2}{J_1} = 1$$

$$H = J_1 \sum_{<i,j>} \vec{S}_i . \vec{S}_j + J_2 \sum_{<<i,j>>} \vec{S}_i . \vec{S}_j$$

J_1 ———
J_2 - - - -

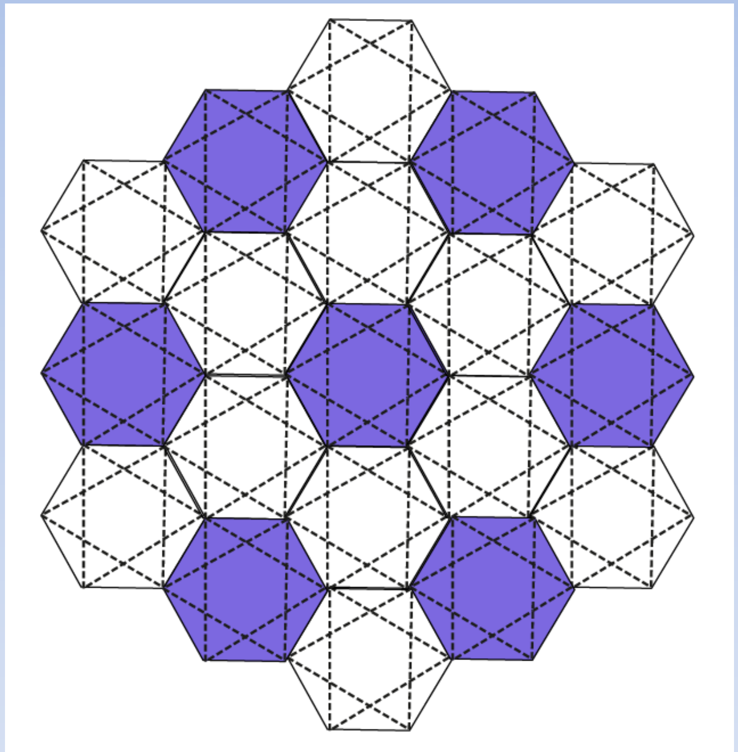


Classical Phase diagram

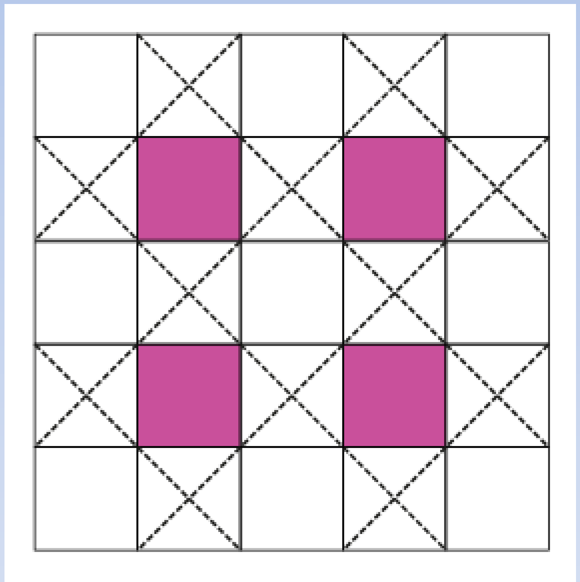
L.P. Henry, et al, PRB (2012) 85,134427

Cluster Operator Theory

Based on a plaquette ordered background
with plaquette-type excitations



Honeycomb Lattice



Checkerboard Lattice

Sachdev and Bhatt, PRB (1990) 43,9323

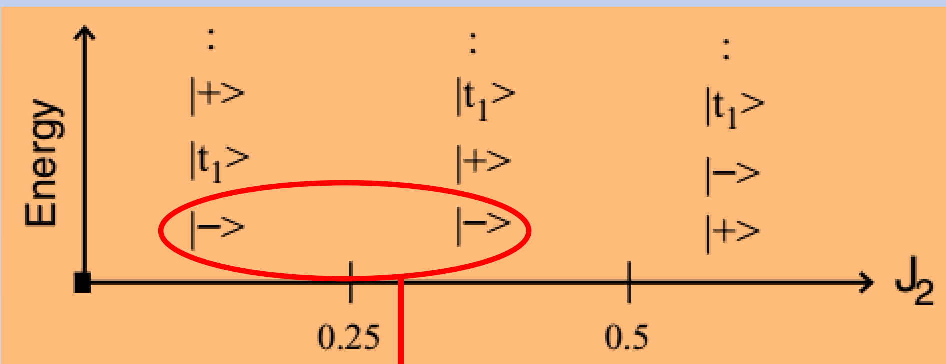
Zhitomirsky and Ueda, PRB (1996) 54,139007

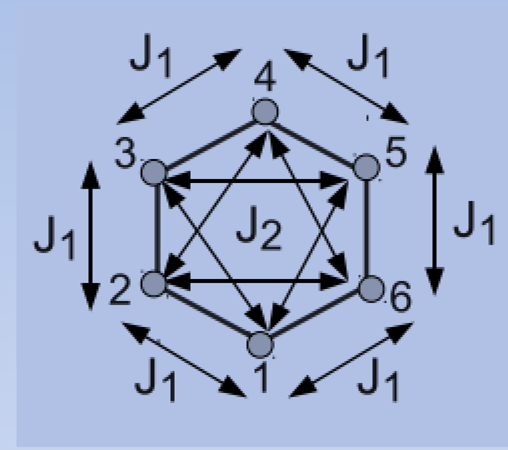
Cluster Operator Theory

$J_1 - J_2$ Heisenberg model on the honeycomb Lattice

Single Hexagon Problem: (we set $J_1 = 1$)

$$H_{Single-Hex} = \sum_{<i,j>} \vec{S}_i . \vec{S}_j + J_2 \sum_{<<i,j>>} \vec{S}_i . \vec{S}_j$$





$64 = 2^6 \, states$

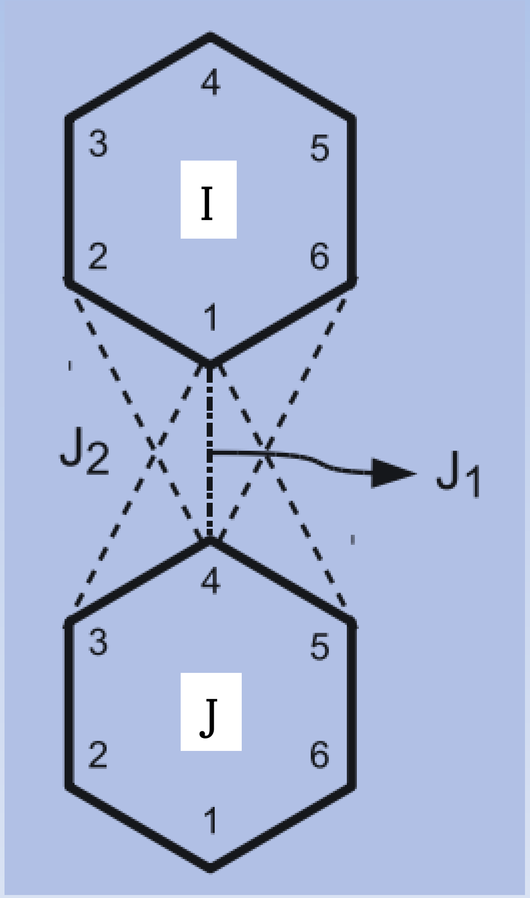
R.Ganesh, et al, PRB (2013) 87,054413

Unique ground state of the single hexagon for $0.0 < J_2 < 0.5$,
the range that we predict existence of a plaquette ordered ground state.

Cluster Operator Theory

$J_1 - J_2$ Heisenberg model on the honeycomb Lattice

In the absence of interaction between plaquettes,
all shaded plaquettes are in their unique grundstates.



Interaction between plaquettes:

$$H_{IJ} = J_1 S_1^I . S_4^J + J_2 S_2^I . S_4^J + J_2 S_6^I . S_4^J + J_2 S_1^I . S_3^J + J_2 S_1^I . S_5^J$$

The interaction term hybridizes the ground state of each plaquette with its other excited states:

Amplitudes of possible interplaquette interactions:

$$\langle uv | H_{IJ} | -- \rangle$$
$$\langle u- | H_{IJ} | -v \rangle$$
$$\langle -v | H_{IJ} | u- \rangle$$
$$\langle -- | H_{IJ} | uv \rangle$$

R.Ganesh, et al, PRB (2013) 87,054413

Cluster Operator Theory

$J_1 - J_2$ Heisenberg model on the honeycomb Lattice

Effective Hamiltonian: A boson scheme

$$\left|u\right\rangle_I = b^+_{I,u}\left|0\right\rangle, \qquad u = 1,\ldots.,64$$

Bose-condensation assumption of groundstate bosons:

$$b_{I,-} \equiv b^+_{I,-} \equiv \overline{p}$$

To preserve the Hilbert space,
the total occupancy of bosons per plaquette should be unity:

$$N\overline{p}^2 + \sum_{I,u=2,..,64} b^+_{I,u} b_{I,u} = N$$

R.Ganesh, et al, PRB (2013) 87,054413

Cluster Operator Theory

$J_1 - J_2$ Heisenberg model on the honeycomb Lattice

Effective Hamiltonian: A boson scheme

$$H_{eff} = \sum_{I} \in_1 \bar{p}^2 + \sum_{I,u=2}^{64} \in_u b_{I,u}^{+} b_{I,u} - \mu[N\bar{p}^2 + \sum_{I,u=2}^{64} b_{I,u}^{+} b_{I,u} - N]$$

$$+ \bar{p}^2 \sum_{<I,J>} \sum_{u,v=2}^{64} [\langle uv | H_{IJ} | --\rangle b_{I,u}^{+} b_{J,v}^{+} + \langle u - | H_{IJ} | - v \rangle b_{I,u}^{+} b_{J,v} + H.C.]$$

After going to momentum space and doing a Bogoliubov transformation:

$$H_{eff} = N\mu + N\bar{p}^2(\in_{-} - \mu) - \frac{1}{2} N \sum_{u} (\in_u - \mu) + \sum_{k,v} (\frac{1}{2} + \gamma_{k,v}^{+} \gamma_{k,v}) \omega_{k,v}(\mu, \bar{p})$$

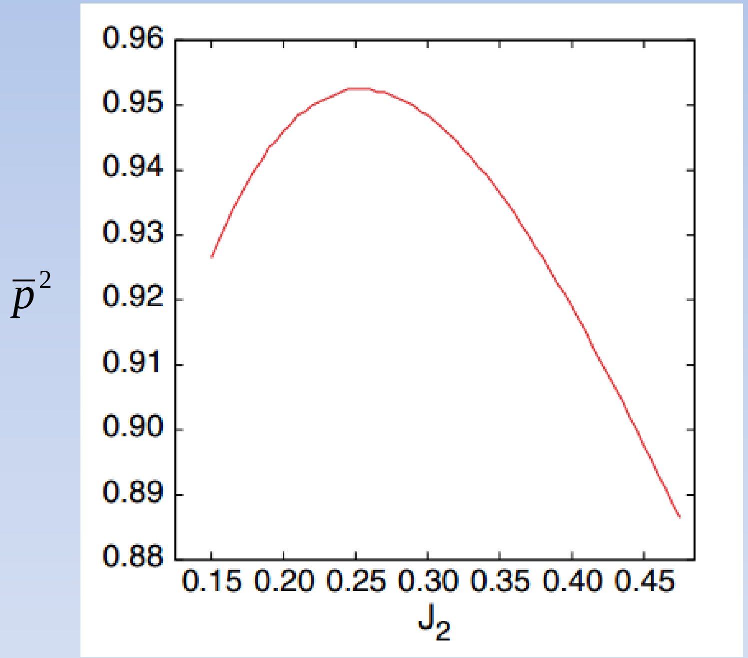
Self-consistent equations: $\frac{\partial \langle H \rangle}{\partial \mu} = 0, \quad \frac{\partial \langle H \rangle}{\partial \bar{p}} = 0$

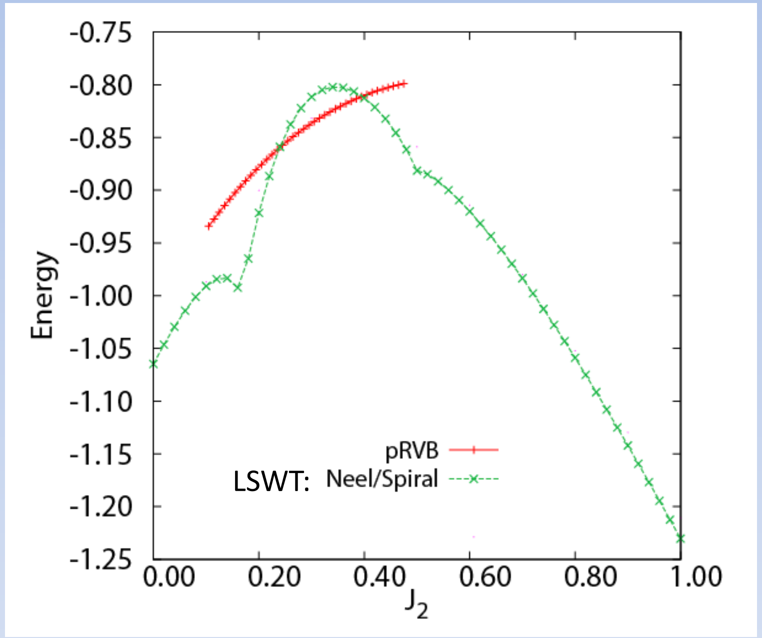
R.Ganesh, et al, PRB (2013) 87,054413

Cluster Operator Theory

$J_1 - J_2$ Heisenberg model on the honeycomb Lattice

Results





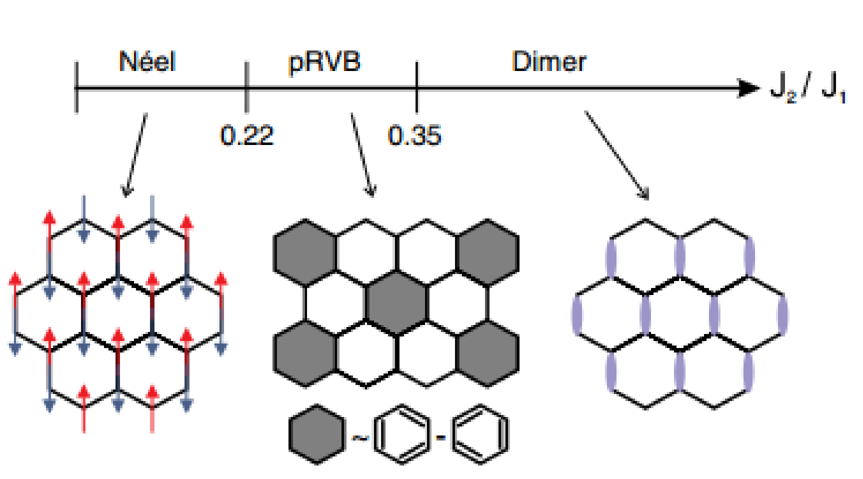
Plaquette order is strongest around $J_2/J_1=0.25$, where we showed that there is a zero-energy plaquette flip excitation in classical limit.

R.Ganesh, et al, PRB (2013) 87,054413

Cluster Operator Theory

$J_1 - J_2$ Heisenberg model on the honeycomb Lattice

Results



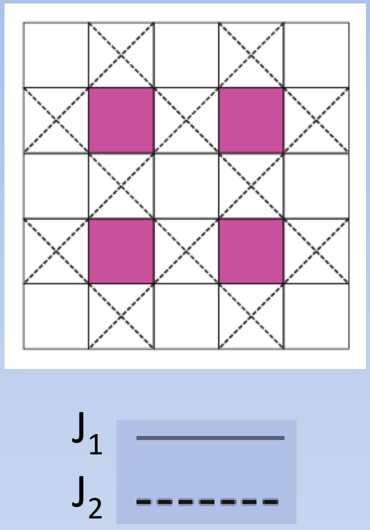
Phase diagram of the model obtained from DMRG calculation

R.Ganesh, et al , PRL (2013) 110,127203

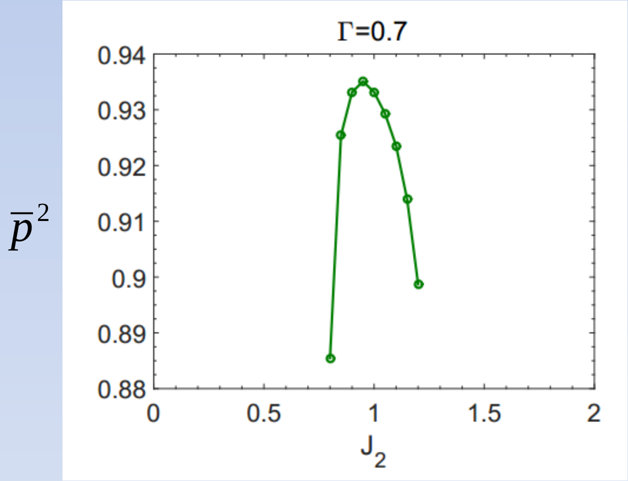
Cluster Operator Theory

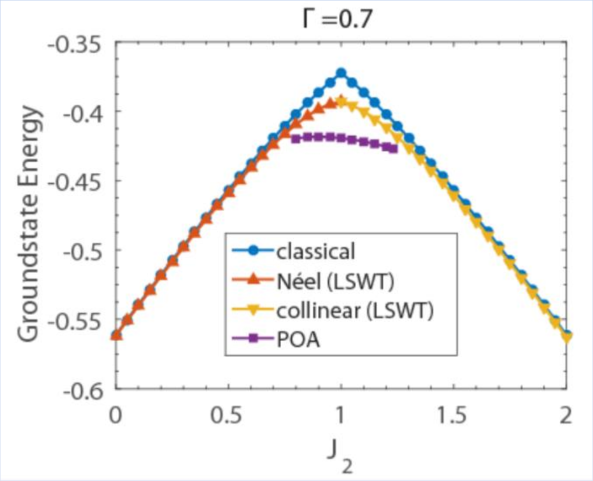
Transverse field J_1-J_2 Ising model on the checkerboard lattice

$$H = J_1 \sum_{<i,j>} S_i^z S_j^z + J_2 \sum_{<<i,j>>} S_i^z S_j^z - \Gamma \sum_i S_i^x$$



Results





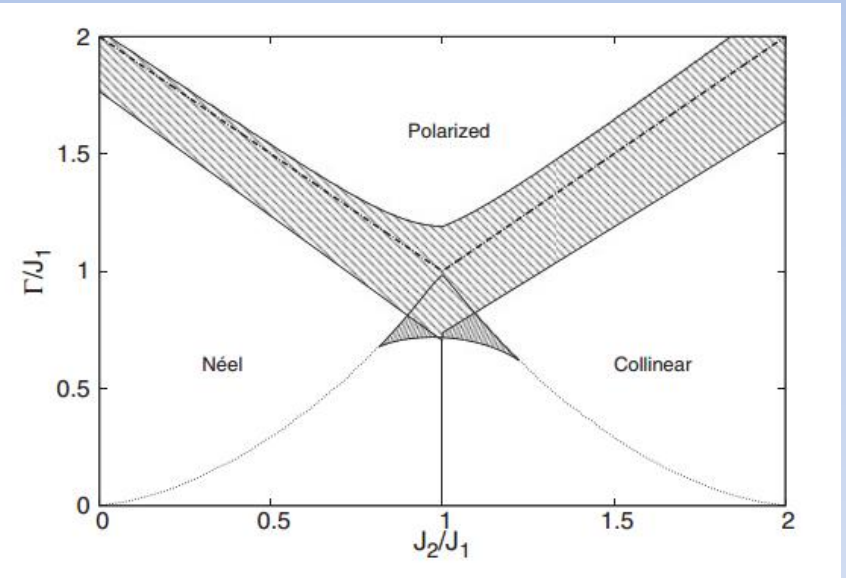
Plaquette order is strongest around J_2/J_1=1.0, where we showed that there is a zero-energy plaquette flip excitation in classical limit.

Sadrzadeh, et.al. Eur. Phys. J. B (2015) 88:259

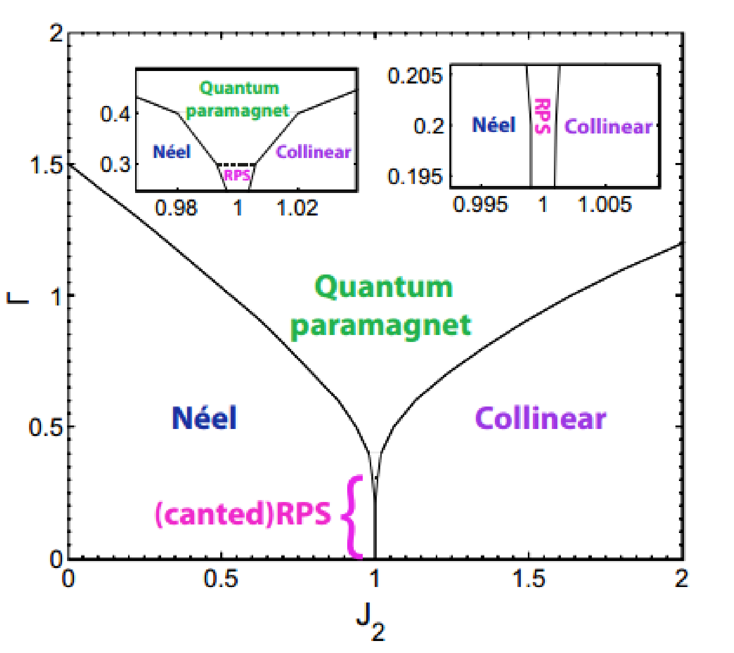
Cluster Operator Theory

Transverse field J_1-J_2 Ising model on the checkerboard lattice

Results



Phase diagram of the model obtained from LSWT,
L.P. Henry, et al,
PRB (2012) 85,134427



Phase diagram of the model obtained from COA,
Sadrzadeh, et.al.
Eur. Phys. J. B (2015) 88:259

Cluster Operator Approach

an analogy to spin wave theory

SWT

An initial classical ordered ground state (m=1)
+
Single spin flip Quantum fluctuations

→

Reduction of the order parameter (m<1)
+
Correction on the ground state energy
+
Harmonic spectrum of low-energy excitations

COA

An initial cluster ordered ground state ($\bar{p}=1$)
+
Multi-spin Quantum fluctuations

→

Reduction of the Bose condensation density ($\bar{p}<1$)
+
Correction on the ground state energy
+
An-harmonic spectrum of low-energy excitations

Thanks for your attention

and

Special thanks to M. Sadrzadeh who helped me in preparing this lecture.