The Uses of Bubbles

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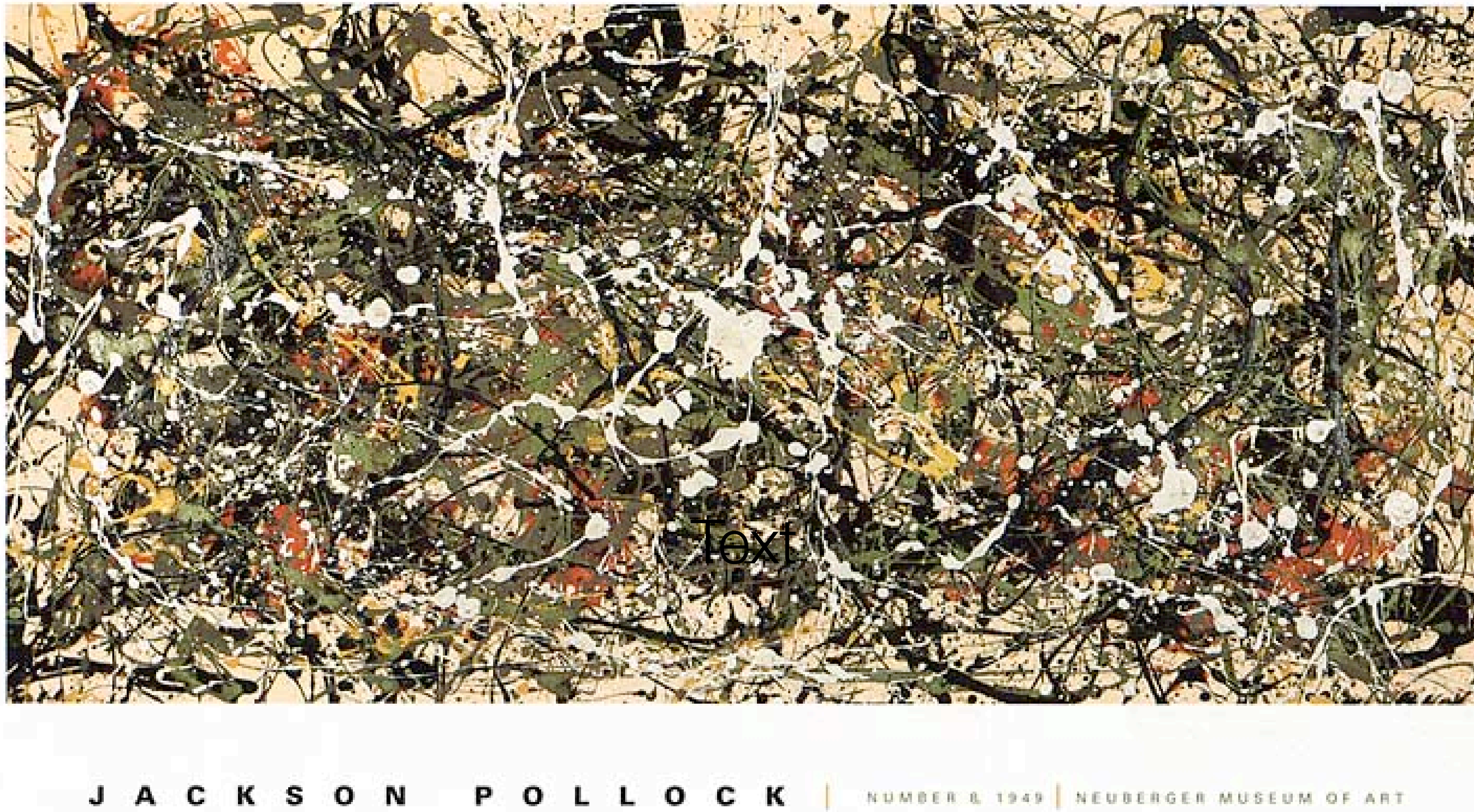
Second Year Talk
DAMTP



the vacuum landscape

a generic feature of string theory?

Introduction: Why study vacua?

- The cosmological constant problem
- Inflation
- Beginning of the Universe

The Cosmological Constant

- The cosmological constant problem: various experiments (supernovae, WMAP) observe acceleration.
- However, the density of this cosmological constant is nowhere near the vacuum energy of the standard model constituents: $\rho_\Lambda^{\text{observed}} \simeq 10^{120} \rho_\Lambda^{\text{theoretical}}$
- Any ultimate theory must explain why this density is small, positive, and very nearly constant.
- Two main approaches: dynamical and anthropic.

Dynamical relaxation

- Bousso-Polchinski: neutralize cosmological constant via four-form field strength:

$$F_7 = dA_6 = *_{11} F_4$$

$$\lambda = \lambda_{bare} + \sum_{i=1}^{N} c_i n_i^2$$

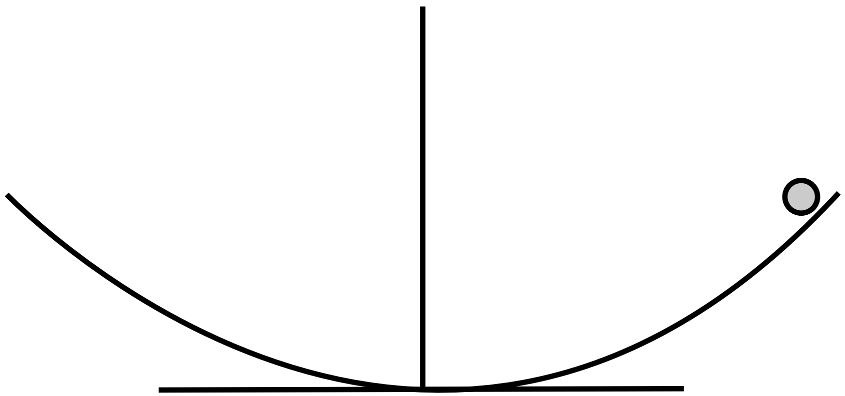
- Turok-Steinhardt: field moves through potential of the form

$$V(\phi) = -M^4 \cos\left(\frac{\phi}{f}\right) + \epsilon \frac{\phi}{2\pi f}$$

- In a cyclic universe there is no “empty Universe” problem.

Initial conditions

- Canonical inflationary model:



- Various attempts (Freese et al, Watson et al) to start the Universe in some top vacuum state (perhaps the Planck scale) and tunnel down to the current state.

- This sort of model does not rely on a scalar field inflaton.

Starting the Universe

- Vilenkin, Linde: Universe spontaneously nucleates in a de Sitter space.
- Hartle-Hawking: wave function of the Universe is a path integral over all compact Euclidean histories terminating at a certain 3D configuration
- Both proposals must deal with “nothing”.
  - HH: Compact four-geometries interpolate between “nothing” and a 3-geometry.
  - Vilenkin: Minisuperspace model with non-singular boundary a=0, finite field.
  - Can bubble methods describe “nothing”?

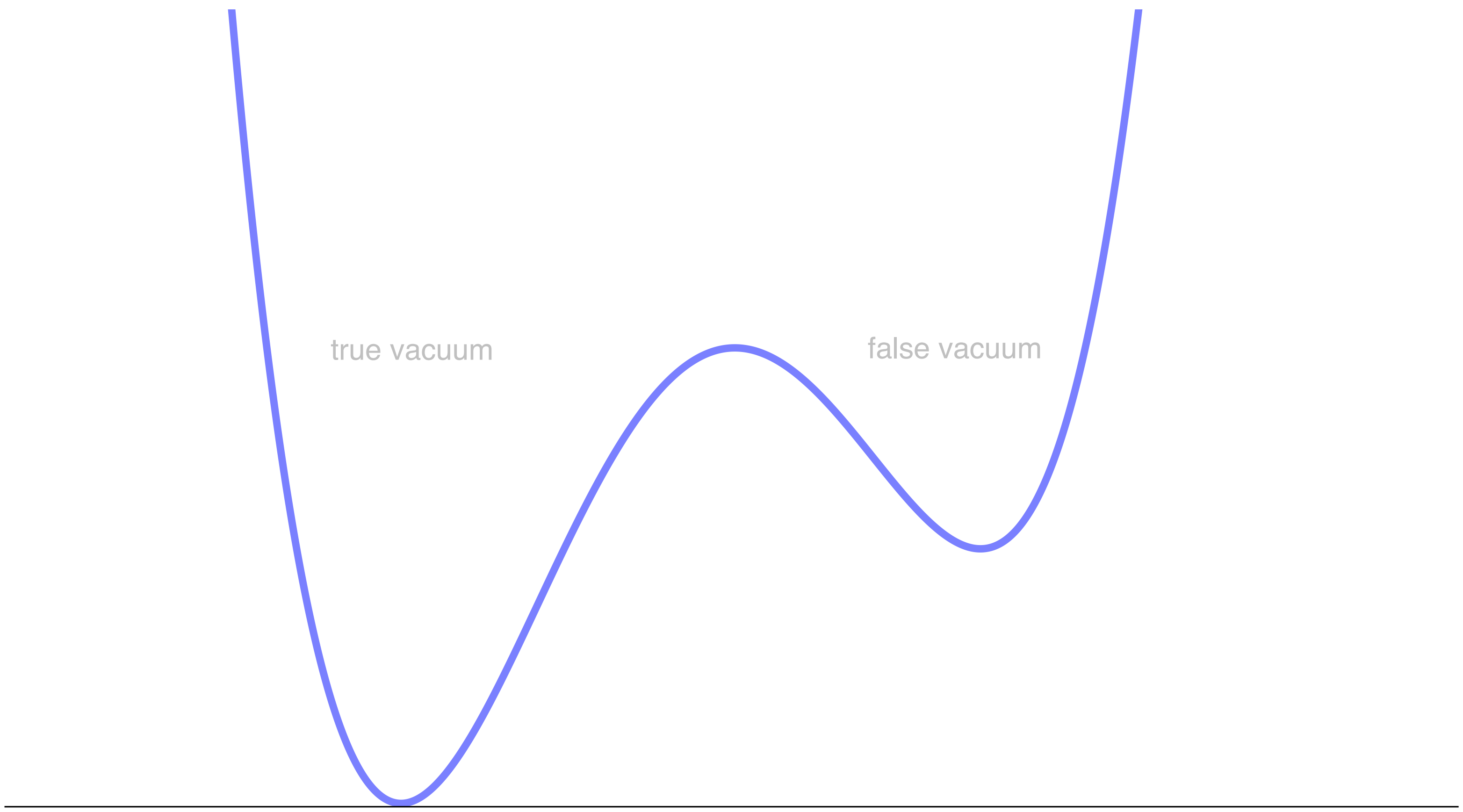
Outline

- Review of the Coleman-DeLuccia mechanism.
- Reformulation using GR formalism.
- Conformal diagrams for bubble nucleation.
- Implications for chain/bubble-driven inflation.
- Some strange results
- Conclusions

Coleman & DeLuccia

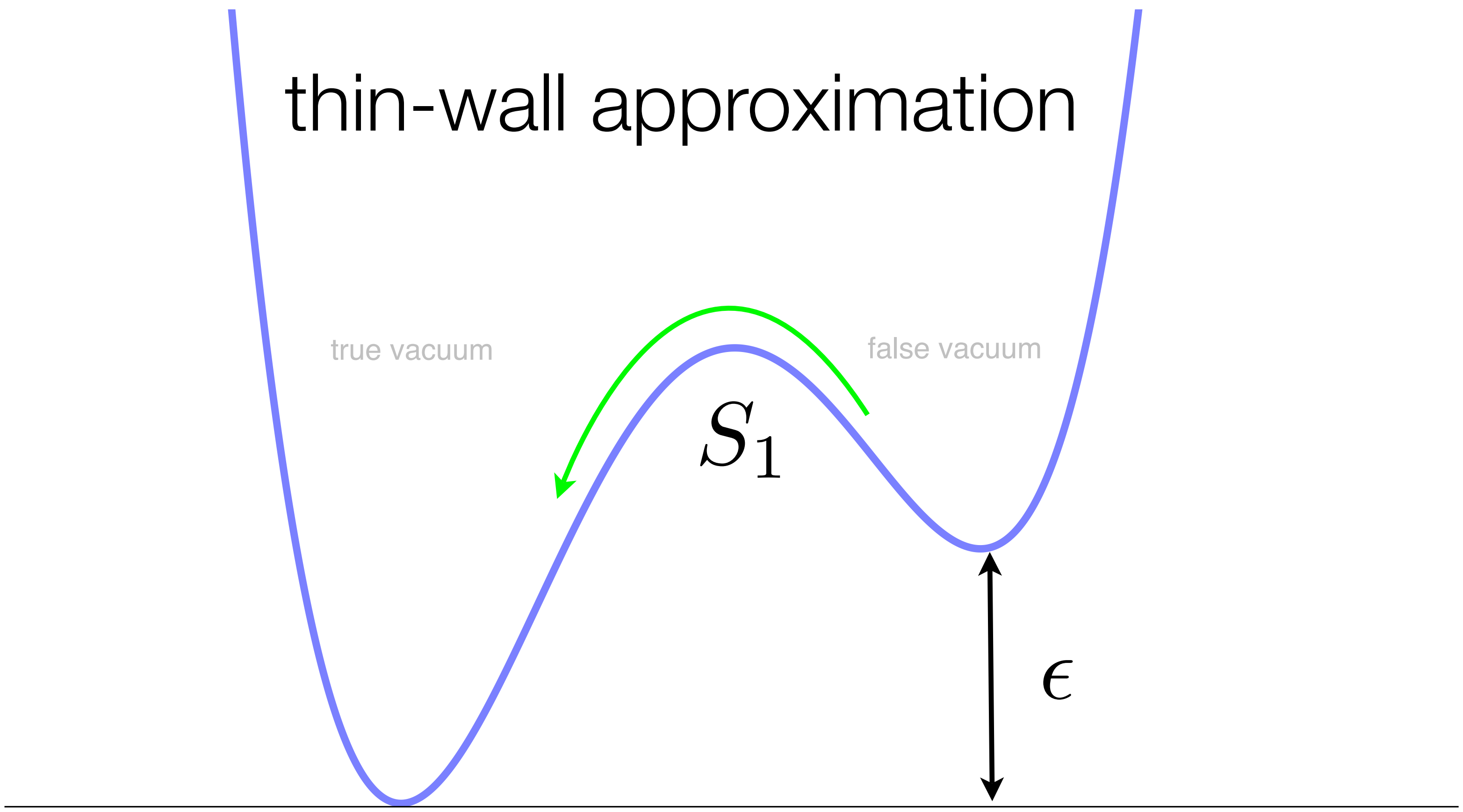
bubble of true vacuum forms
inside false vacuum

Review: scalar field with gravity



$$S = \int d^4x \, \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \partial \nu \phi - V(\phi) - (16\pi G)^{-1} R \right)$$

Review: scalar field with gravity



$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \partial \nu \phi - V(\phi) - (16\pi G)^{-1} R \right)$$

Review: scalar field with gravity

- Find "bounce": solution to Euclidean equations of motion obeying appropriate boundary conditions.

- Evaluate Euclidean action of the bounce solution.

- Tunnel rate per unit volume is $\frac{\Gamma}{V} = \left|\frac{\delta^2 S}{\delta\phi^2}\right| e^{-S_E(\phi_{bounce})}$

- CD find Euclidean action $B = \frac{27\pi^2 S_1^4}{2\epsilon^3 \left(1 + 3r_b^2/(8\pi G)\right)^2}$

- Bubble radius $r_b = \frac{12 S_1}{4\epsilon + 24\pi G S_1^2}$

- Can we reformulate this in a purely GR context?

an example: de Sitter decays to Minkowski

- Metric on either side (static coordinates)

$$ds^2 = -A(r)dt_M^2 + \frac{1}{A(r)}dr^2 + r^2 d\Omega^2.$$

$$A_M(r) = 1$$

$$A_{dS}(r) = (1 - \chi^2 r^2)$$

- Boundary: timelike brane in r-direction (assuming spherical symmetry)

$$n_\mu = (-\dot{r}, \dot{t}, 0, 0) \qquad \left(\cdot = \frac{d}{d\tau}\right)$$

- Israel matching condition gives

$$K^i_j(\text{Minkowski}) - K^i_j(\text{dS}) = -4\pi G T \delta^i_j.$$

Equation of motion

- θ-θ component of Israel matching gives a simple equation of motion:

$$\dot{r}^2 = c^2 r^2 - 1$$

$$c^2 = \left(\frac{\chi_2^2 + \kappa^2 - \chi_1^2}{2\kappa} \right)^2$$

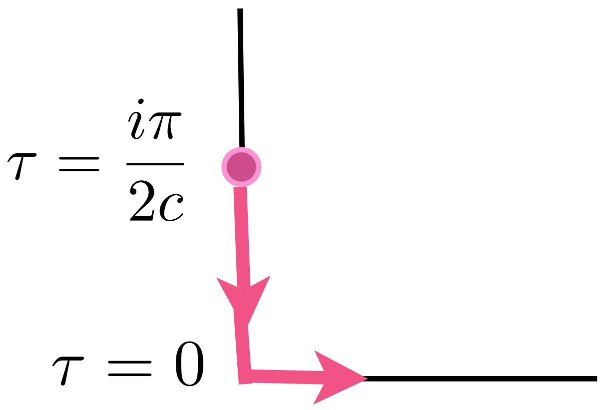
- Classical turning point: $\dot{r} = 0$

- No way to shrink the bubble back to zero size in real time.

- Interpretation: bubble of true vacuum nucleates at finite size 1/c.

imaginary time

- evaluate action along complex time contour.
- imaginary part of the action corresponds to Euclidean action.
- bubble nucleates at $\tau = 0$
- spacetimes become compact: de Sitter has topology of a sphere.
- “instanton”= matching solution.

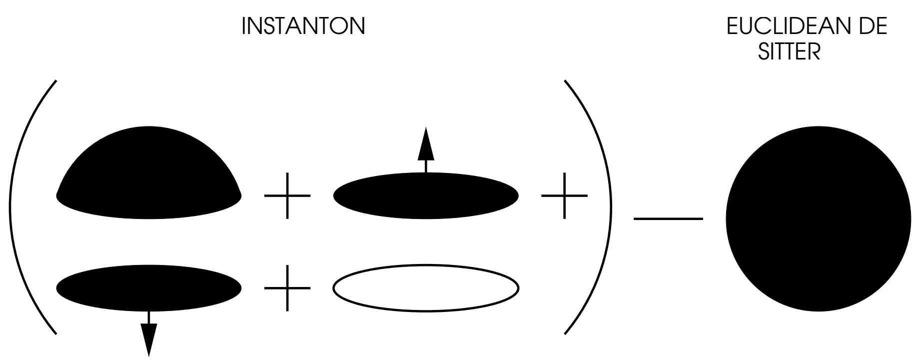


instanton action

- the action for this cut and glued spacetime is

$$S_{grav} = \frac{1}{16\pi G}\int_M d^4x\,\sqrt{-g}(R_{dS} - 2\Lambda_{dS}) + \frac{1}{16\pi G}\int_M d^4x\,\sqrt{-g}(R_M - 2\Lambda_M) + \frac{1}{8\pi G}\int_{\partial M} d^3x\,\sqrt{-h}K_{dS} + \frac{1}{8\pi G}\int_{\partial M} d^3x\,\sqrt{-h}K_M - T\int_{wall} d^3x\,\sqrt{-h}$$

- schematically, we depict it as:



- the imaginary portion is

$$B = \frac{\kappa\pi}{2Gc^3} - \frac{3\pi}{4Gc^2} + \frac{\pi}{2G\chi^2} - \frac{\pi\sqrt{c^2-\chi^2}}{2G\chi^2 c} - \frac{\pi\sqrt{c^2-x^2}}{2Gc^3}.$$

Tension and horizons

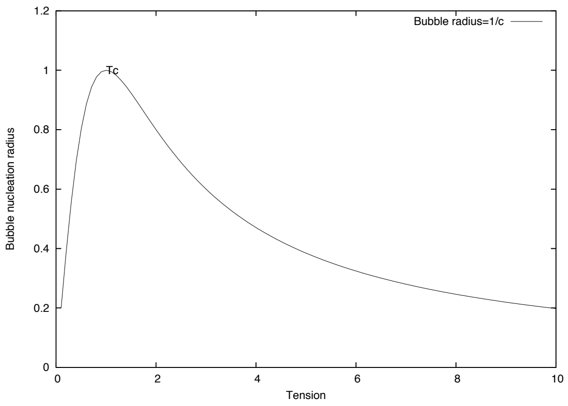
- As tension is increased, the bubble radius increases up to $\kappa_{crit} = \chi$

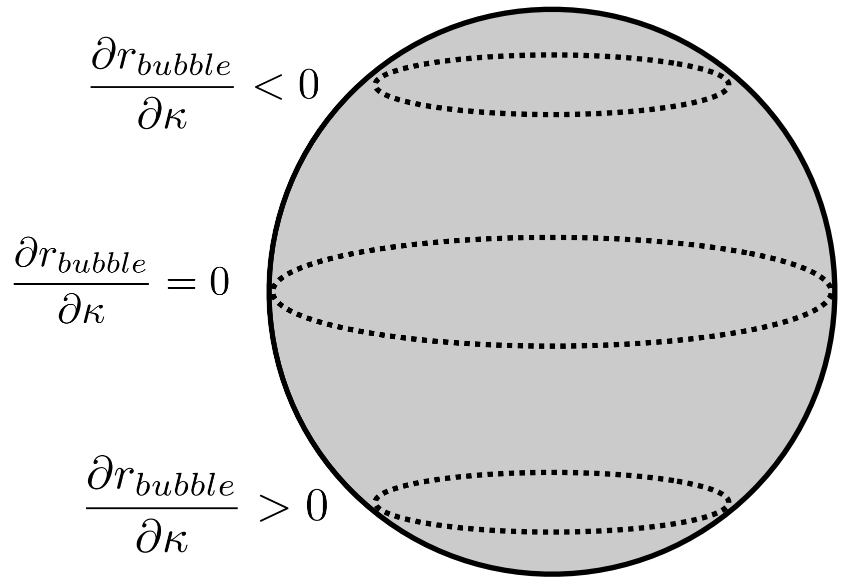
- Each value of wall tension corresponds to two nucleation radii.

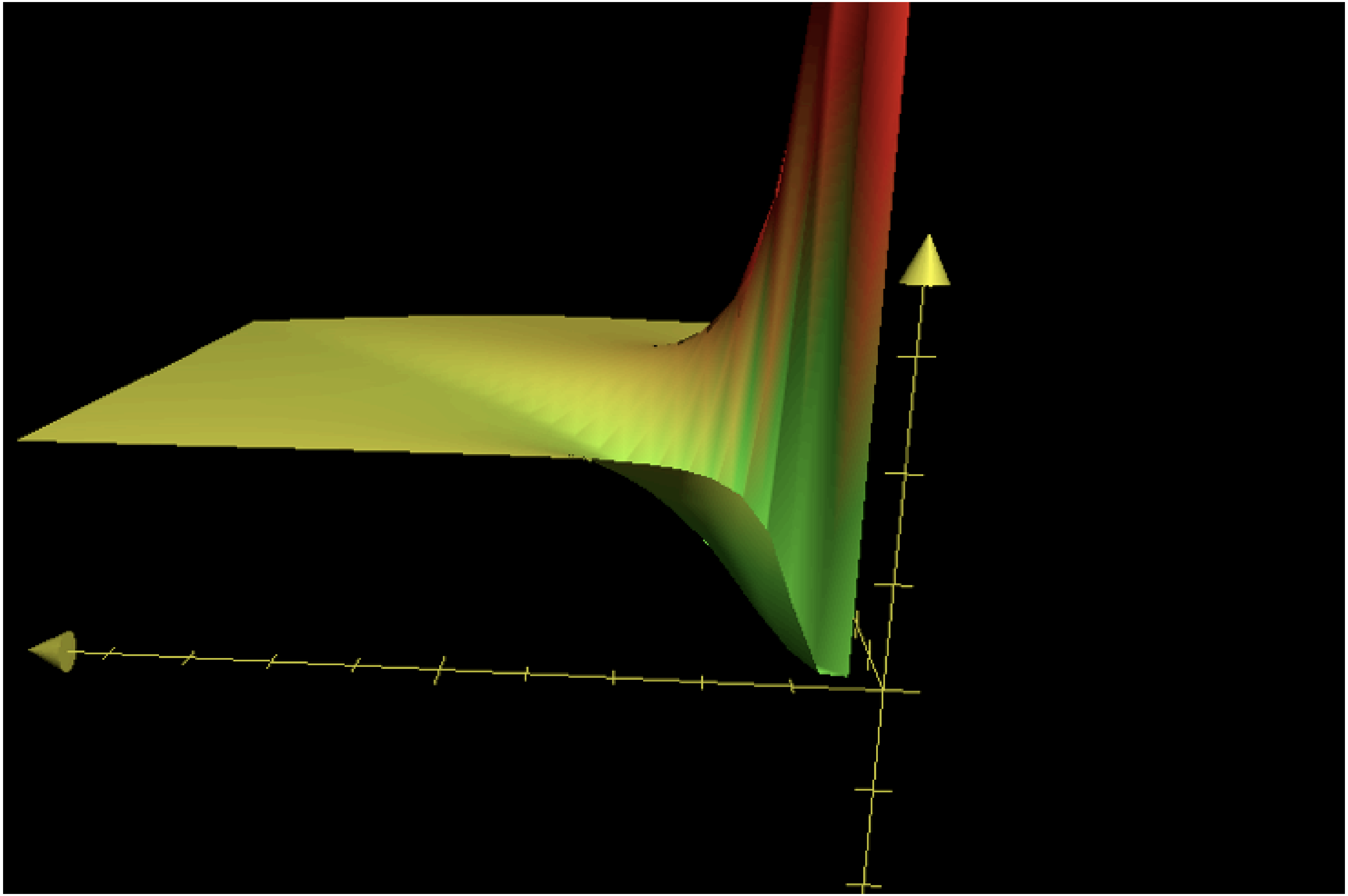
- Flaw in the coordinate system: only covers part of the manifold.

- $r = \chi$ is the cosmological horizon.

- So does the theory predict superhorizon bubbles for large wall tensions?







plot of tunnel rate vs. membrane tension and top vacuum de sitter radius.
normalized and sensible only for tensions less than the critical tension.

Conformal diagram

- Change to coordinates (T, ψ, θ, ϕ) in which metric is

$$ds^2 = \frac{\sec^2(T)}{\chi^2}(-dT^2 + d\Omega^2_{(3)})$$

$$0 < \psi < \pi$$
$$-\frac{\pi}{2} < T < \frac{\pi}{2}$$

- Wall trajectory is then

$$\frac{c^2}{\chi^2}\sin^2\psi\cos^2\psi = (\frac{c^2}{\chi^2} - 1)\sin^2 T\cos^2 T + \cos^2 T\cos^2\psi$$

- For all subhorizon bubbles, $\mathcal{I}_+$ is reached at $\psi = \dfrac{\pi}{2}$
- For horizon-size bubbles it is reached at $\psi = \pi$
- What about superhorizon size bubbles?

Implications

- Two solutions for $\sin^2 \psi$
- Solution in blue corresponds to a subhorizon bubble reaching conformal infinity at $\psi = \frac{\pi}{2}$
- Solution in red corresponds to a horizon-size bubble reaching conformal infinity at $\psi = \pi$
- This is the only decreasing solution!
- Superhorizon bubbles are not tunneling solutions.

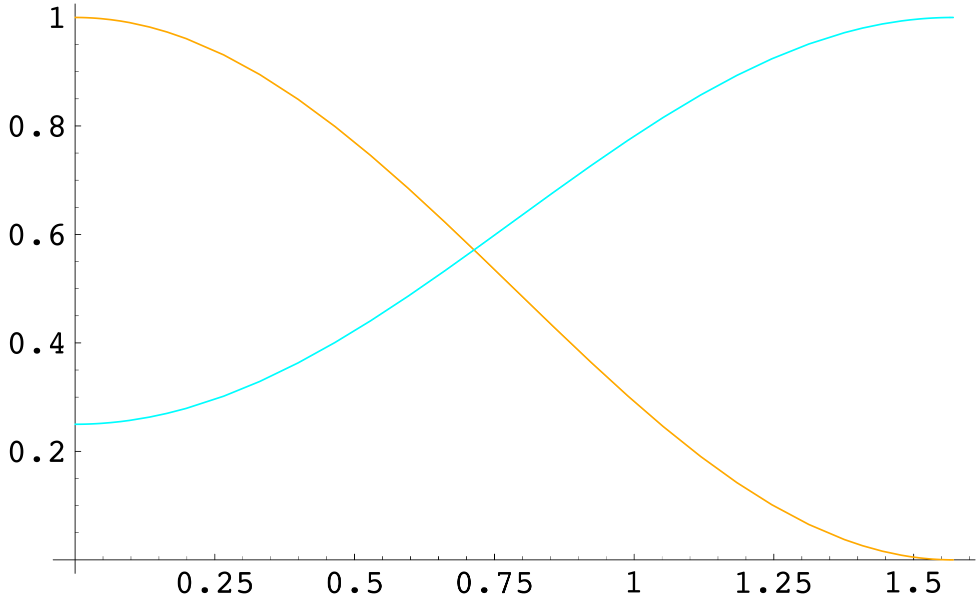
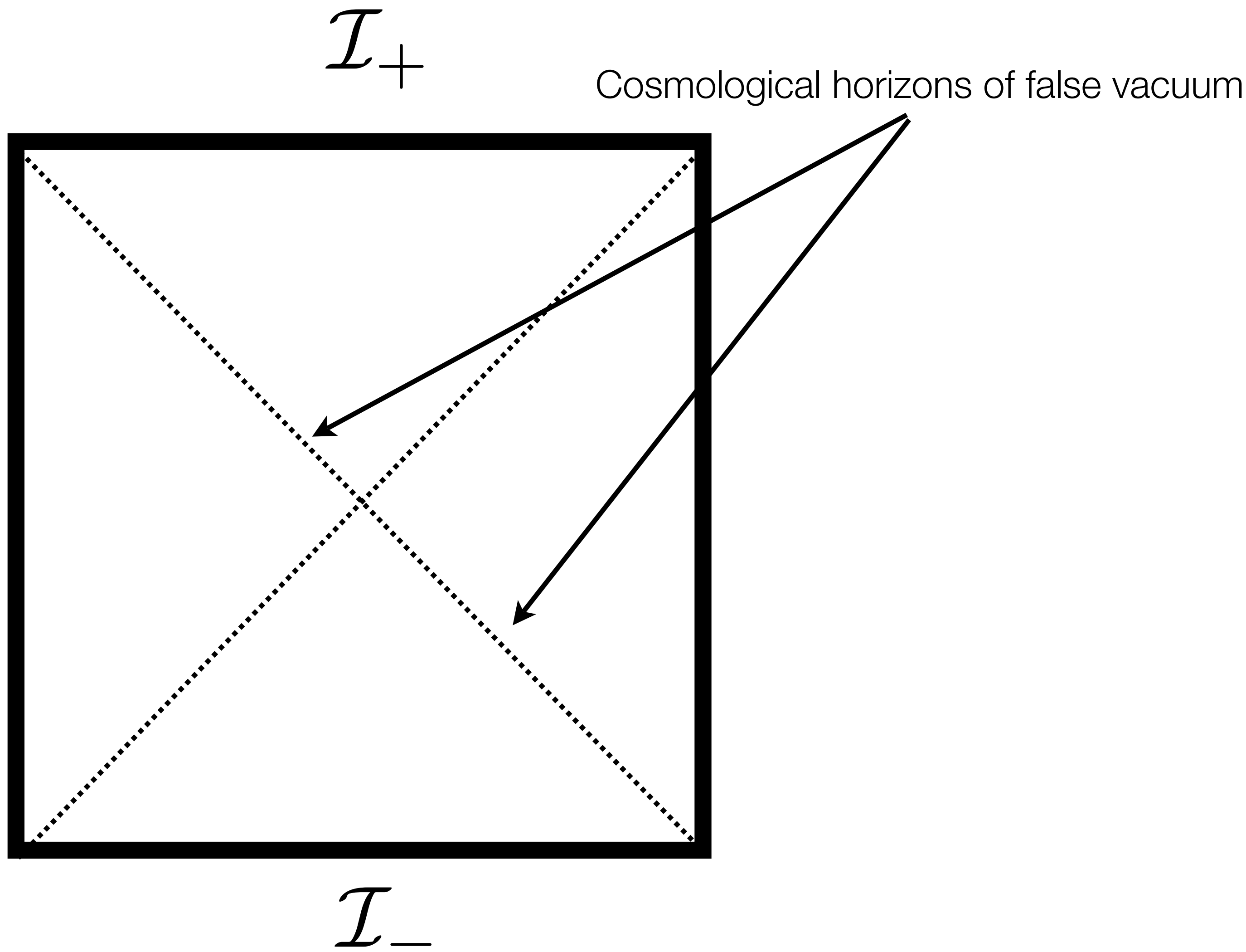
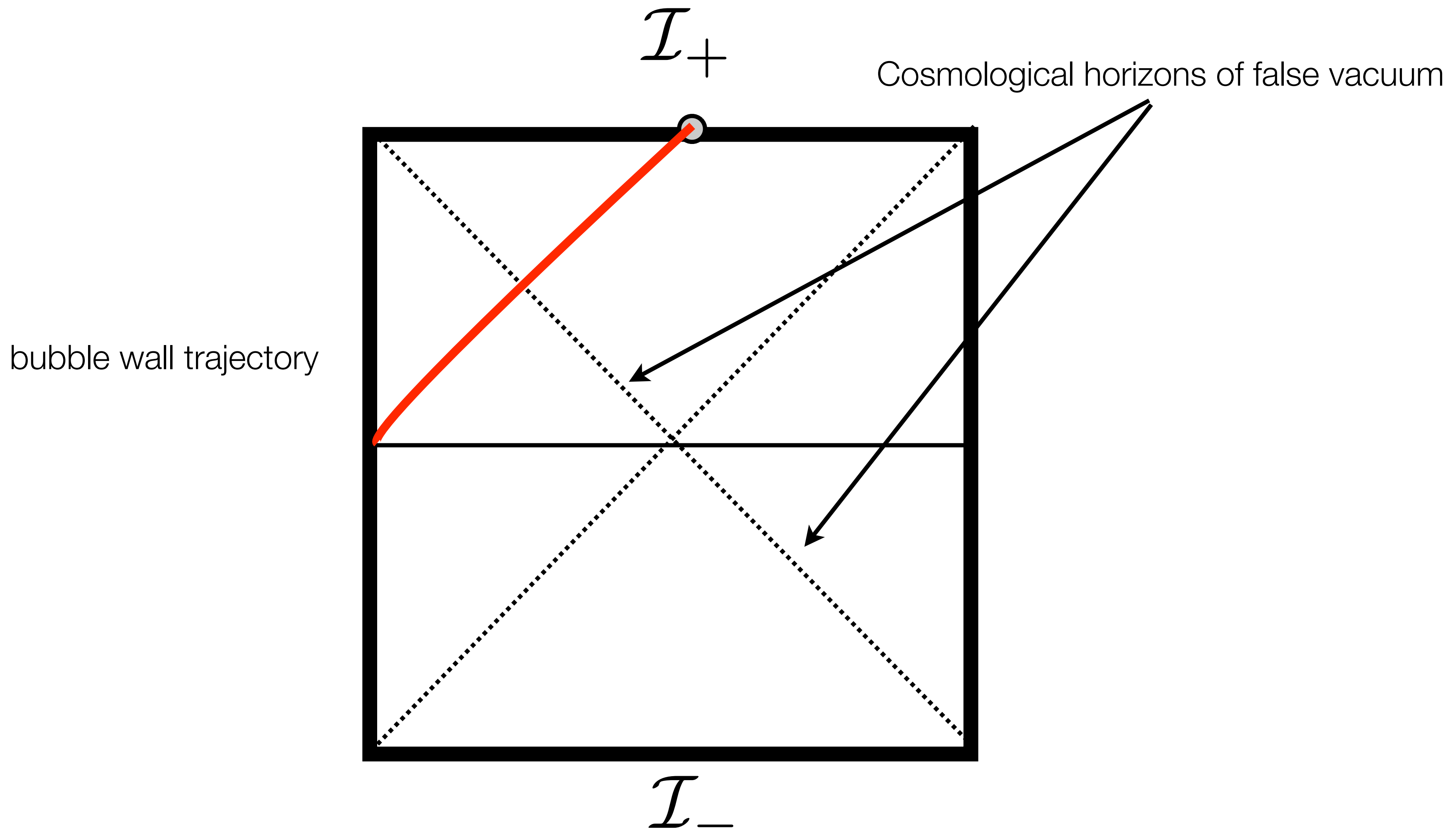


FIG. 6: Two allowed trajectories for the the bubble wall.

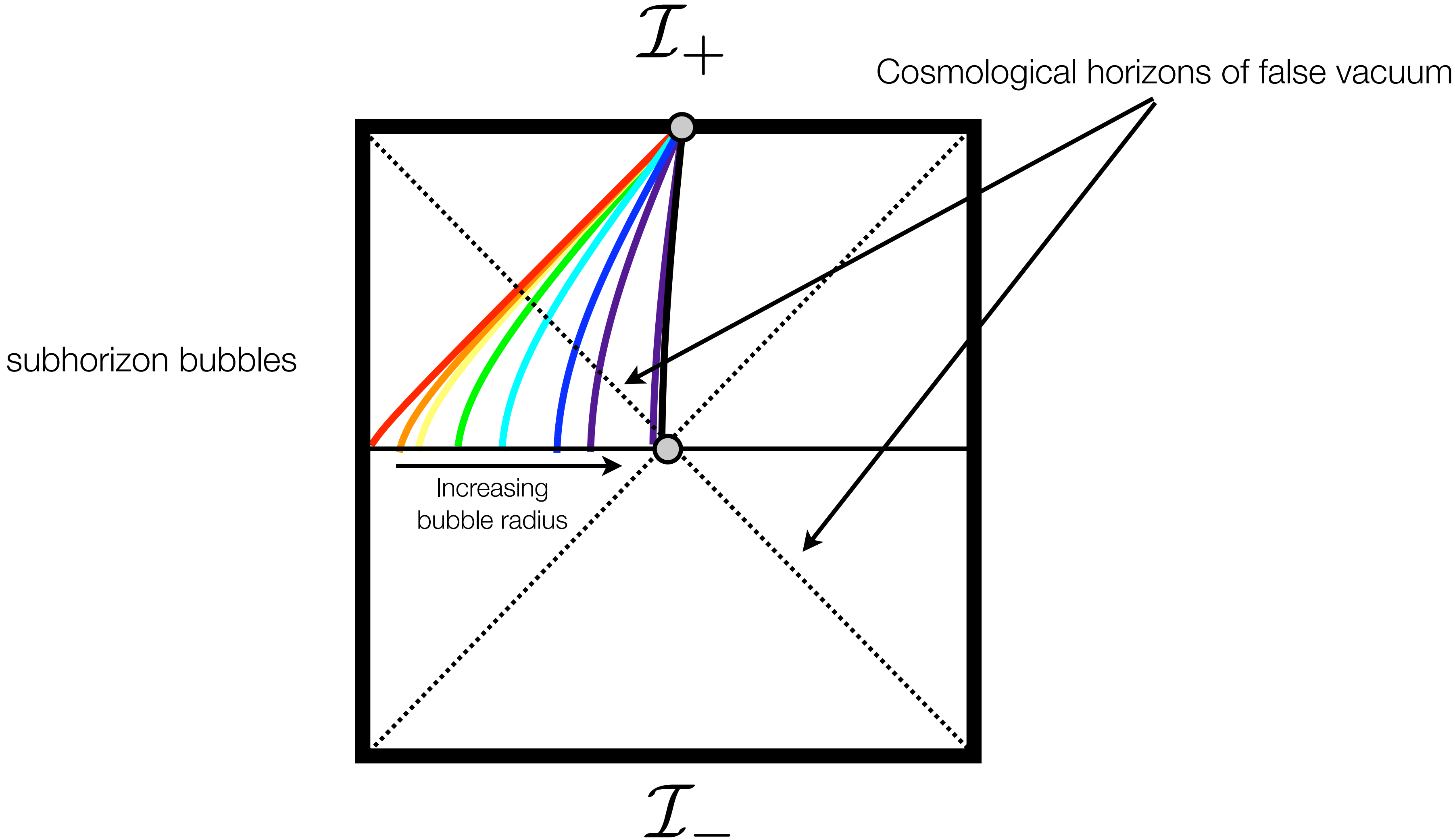
Bubble nucleation in de Sitter



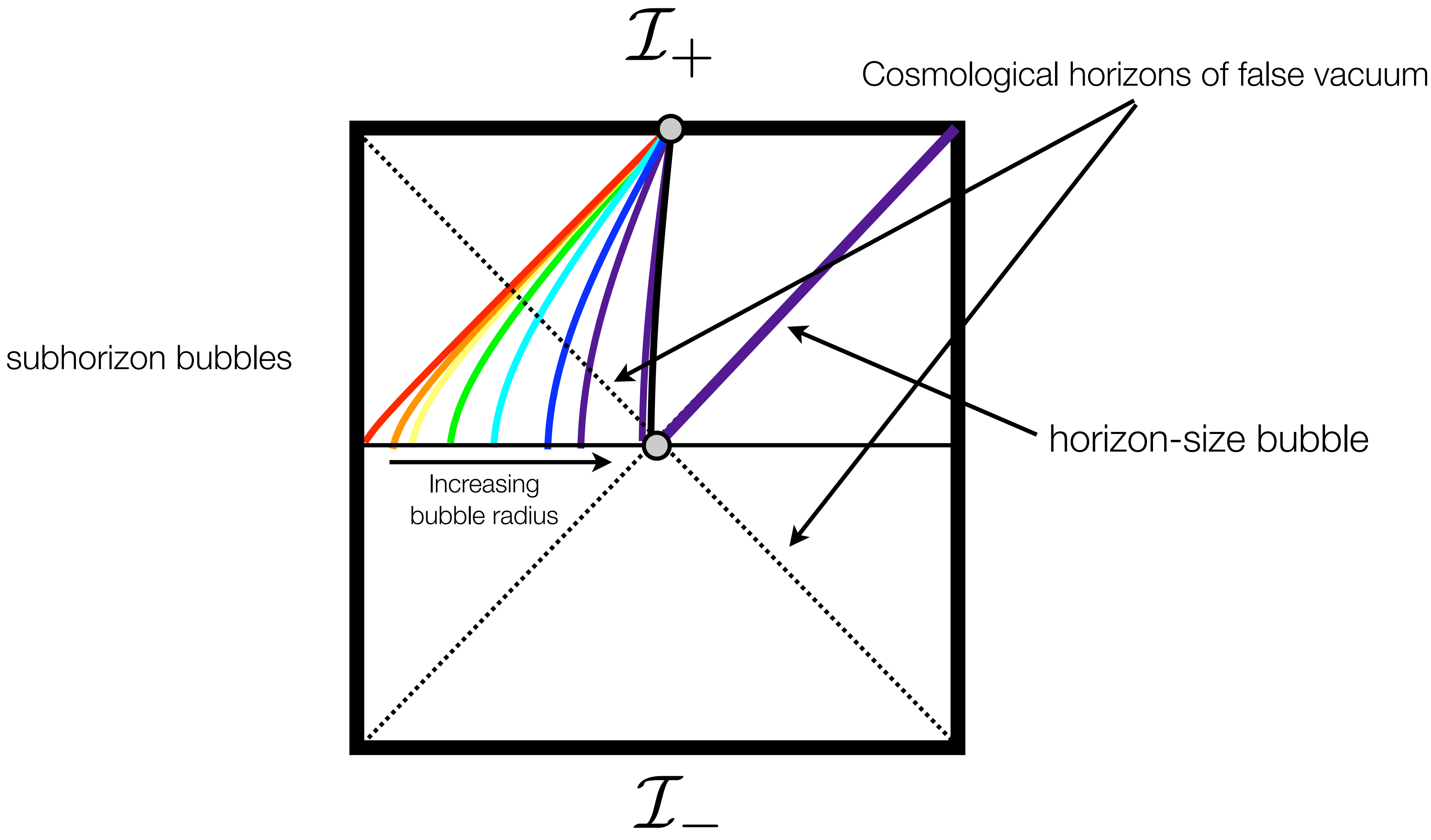
Bubble nucleation in de Sitter



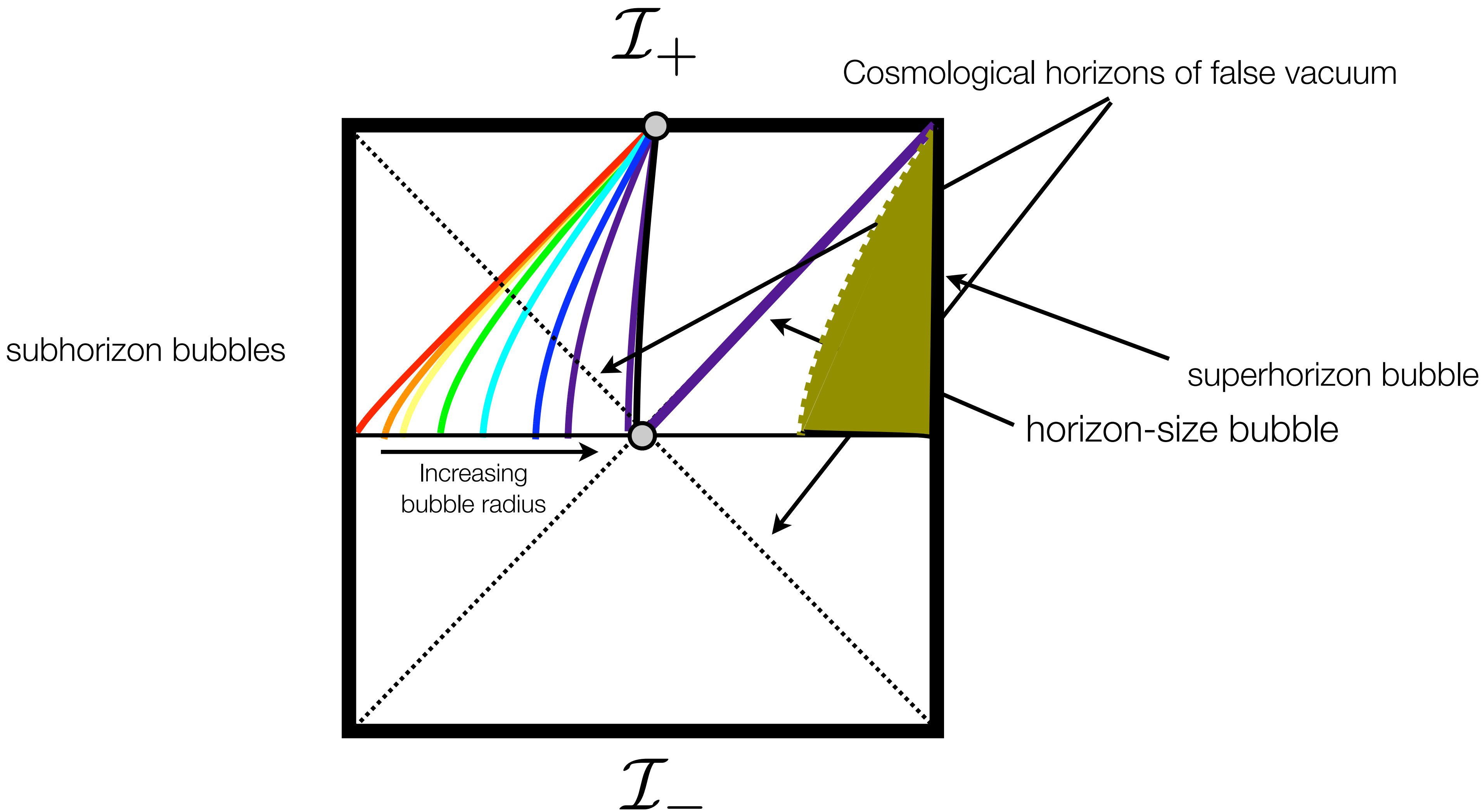
Bubble nucleation in de Sitter



Bubble nucleation in de Sitter



Bubble nucleation in de Sitter

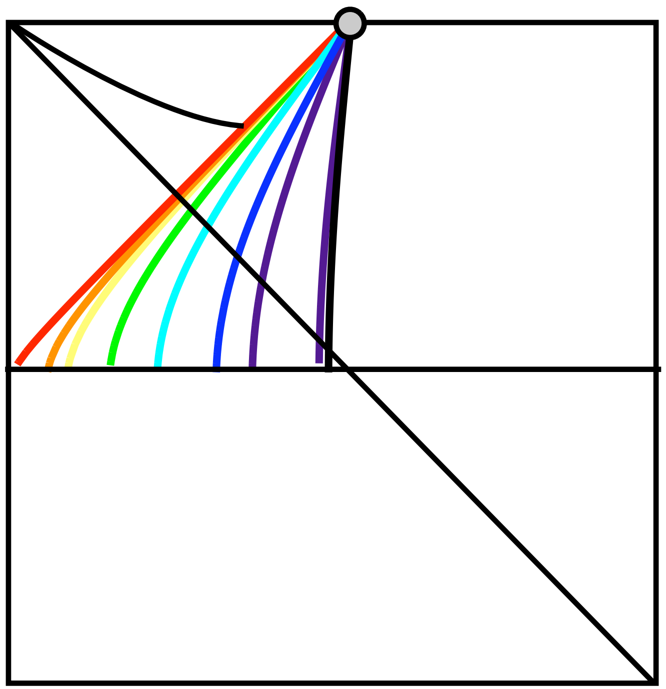


de Sitter - de Sitter decay

- True vacuum has lower cosmological constant than false vacuum.
- So will have different horizon structure:

$$\cos \psi_2 = \left(\frac{\chi_2}{\chi_1} \right)^2 \sin T_2$$

- As the Hubble parameter of the false vacuum space goes to zero the horizon approaches $\mathcal{I}_+$: flat space has no horizons!



chain inflation: freese, liu, spolyar, watson, perry, kane, etc.

- Inflation happens via tunneling in some vacuum landscape
- no need for artificial initial conditions: start at Planck scale
- tunnel rate per unit volume is

$$\frac{\Gamma}{V} = Ae^{-B}$$

bubble-driven inflation: a simple model

- tunneling rate between two vacua is

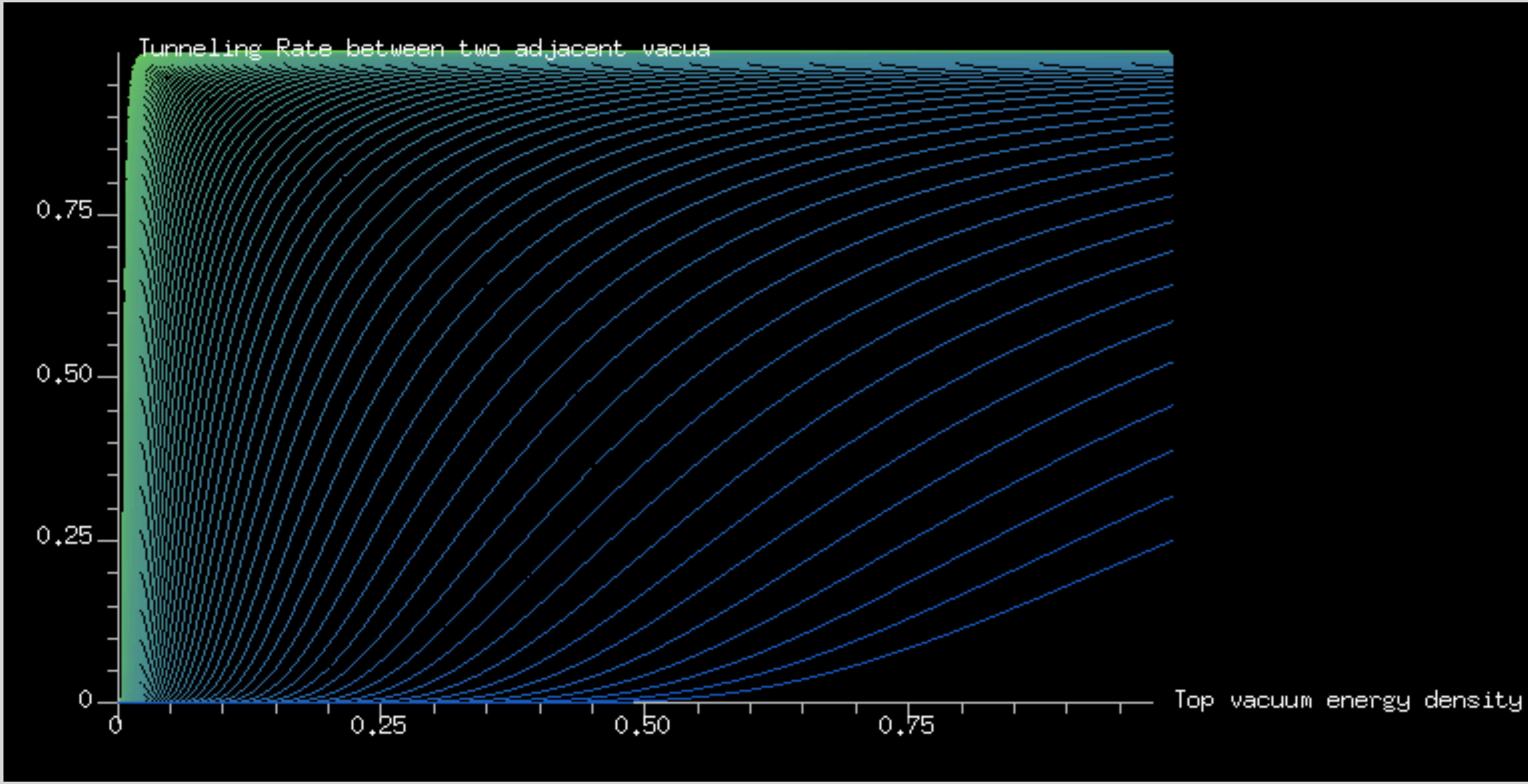
$$\frac{\Gamma}{V} = M\exp\left\{\frac{\pi}{2\mathrm{G}}\left(\frac{(1-\chi 1^2\mathrm{r}_\mathrm{b}^2)^{3/2}-1}{\chi 1^2}-\frac{(1-\chi 2^2\mathrm{r}_\mathrm{b}^2)^{3/2}-1}{\chi 2^2}+\mathrm{r}_\mathrm{b}^2\right)\right\}$$

- M is a one-loop determinant factor which we'll take to be $\simeq \dfrac{1}{r_b^4}$

- V is the Hubble volume.

- We assume the Universe starts in some top vacuum = 1 and that the vacua are evenly spaced.

- Also assume uniform tension on the membranes separating vacuum regions.



Initially, tunneling is simple.

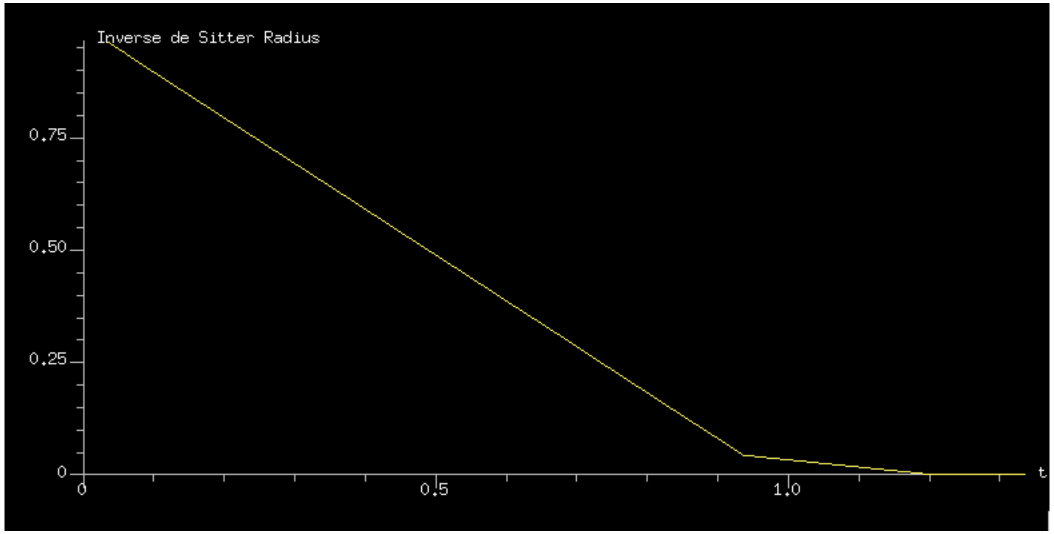
this depends on the tension.

bounds on inflation

- "slow-roll" period ends when tunneling becomes prohibitively slow:

$$\frac{\kappa}{\chi_{end}} = \frac{d\chi}{\chi_{max}} \to \chi end = N_{vac}\kappa.$$

- Number of e-folds is then $N_{efolds} \sim N_{vac}(1 - \kappa N_{vac})$



causality again

- note that if the vacuum spacing is sufficiently large($d\chi > \kappa$) we never have to worry about causality issues.

- this means that “slow-roll” ends when the lifetime of a vacuum exceeds the Hubble time of that vacuum: $\Gamma \simeq H$

- however, for small vacuum spacings, there is a point where semiclassical tunneling is FORBIDDEN by causality:

$$r_{bubble} > r_{dS}$$

- This final state is semiclassically stable!

Is nothing unstable?

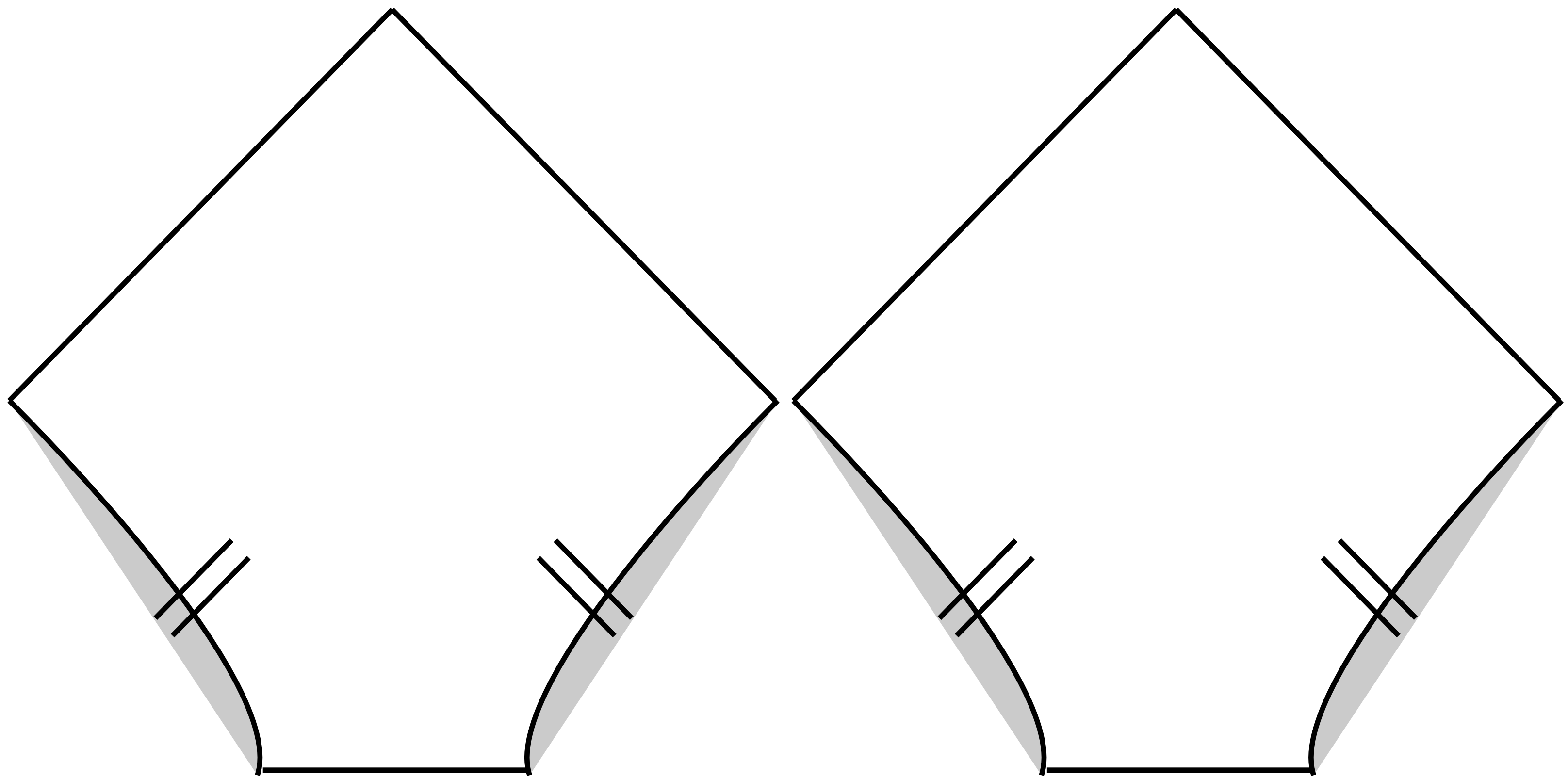
- Recall de Sitter - Minkowski decay example:

$$B = \frac{\kappa\pi}{2Gc^3} - \frac{3\pi}{4Gc^2} + \frac{\pi}{2G\chi^2} - \frac{\pi\sqrt{c^2-\chi^2}}{2G\chi^2 c} - \frac{\pi\sqrt{c^2-x^2}}{2Gc^3}.$$

$$\boxed{\lim_{\chi\to 0} B = \frac{4\pi}{G\kappa^2}}$$

- What does this correspond to physically?

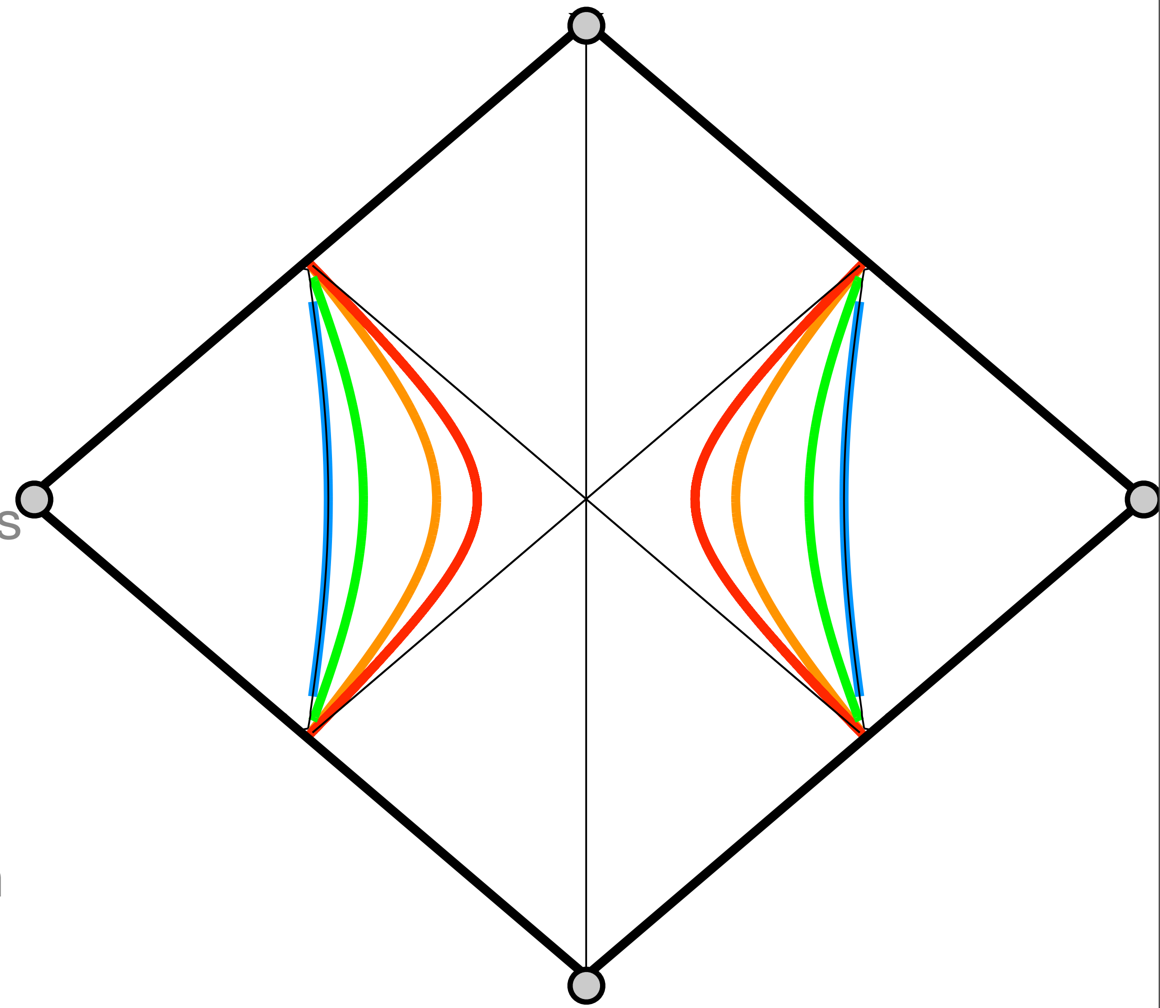
A very silly instanton



Two “wedges” of flat space appear from nothing.

More bounds on tension

- In this limit, the bubble radius is trivially related to the wall tension:
$$r = \frac{\kappa}{2}$$
- In the usual conformal coordinates for flat space, the trajectory is
$$\tan(T+R)\tan(T-R) = -r_b^2$$
- As before, this forces all trajectories to reach null infinity at the same spatial coordinate.



- So even though flat space has no horizons, we can place a bound on tension, allowing a sensible interpretation of the solution.

Conclusions

- Causality issues and horizons dictate semiclassical stability.
- There is a maximum allowable wall tension to create tunneling instantons.
- if, as predicted, $N_{vac} \sim 10^{500}$, difficult to get chain inflation without large, fine-tuned degeneracies.
- Instanton methods predict that “nothing” (g=0) is unstable to the formation of bounded wedges of spacetime.
- Lots of interesting further work to be done.