Challenges with Averaging in GR

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Standard Model of Cosmology

- Universe (as a whole) homogeneous and isotropic,
  - Expansion
  - Spherical Symmetry
- A global geometry in which inhomogeneities are smoothed out but yet observable at different scales.
- Compatibility with the cosmological principle in large scale.

Inhomogeneities- Motivations

- Visual Universe is Inhomogeneous in different scales.
- Proper understanding of the dark energy requires the study of inhomogeneities and their effect on the observational parameters as well as the averaging process in GR.

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Inhomogeneities- Approaches

- As linear perturbations to the FRW background,
  - -Divergence due to the growth of perturbation at the epoch of structure formation
  - - …
- The non-perturbative method including volume averaging of inhomogeneities.

Volume averaging in GR

- Covariant Averaging(Zalaletdinov's Method)
  - *Aseem Paranjape, arxiv:0705.2380*

- Spatial Averaging(Buchert's Method)
  - *Buchert T, gr-qc/9906015, gr-qc/0102049*

Volume averaging in GR

- Perfect and irrotational cosmic fluid
- Spatial Average:

$$\langle f \rangle \equiv \frac{1}{V_D} \int_D f dV$$

- Spatial averaging does not commute with time derivative:

$$\langle f \rangle^{\bullet} - \langle \dot{f} \rangle = \langle f\theta \rangle - \langle f \rangle \langle \theta \rangle$$

$$\theta_D \equiv \langle \theta \rangle \equiv \frac{\dot{V}}{V} = 3\frac{\dot{a}_D}{a_D} = 3H_D$$

- Average of Einstein equations using Hamiltonian constraint and Raychaudhuri equation

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}(\rho_D + \rho_Q + \rho_D)$$

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3}(\rho_D + 4\rho_Q)$$

- Backreaction term *Q*

$$Q \equiv -2\left\langle \sigma^2 \right\rangle + \frac{2}{3}\left\langle \left(\theta - \langle\theta\rangle\right)^2 \right\rangle = -2A - 6H_D^2$$

$$A \equiv \left[\left\langle \sigma^2 \right\rangle - \frac{1}{3}\left\langle \theta^2 \right\rangle\right]$$

Spatial Averaging: Deficiencies

- All of our observations are along the lightcone, but the averaging is on time slices,
- Non-causal effects due to the constant volume for averaging,
- Gauge Dependence:

  Does the results of averaging depend on the specific choice of gauge? Yes or No,

  Aseem Paranjape, astro-ph/0605195, 0609481

- Singularities, Formation of caustics.

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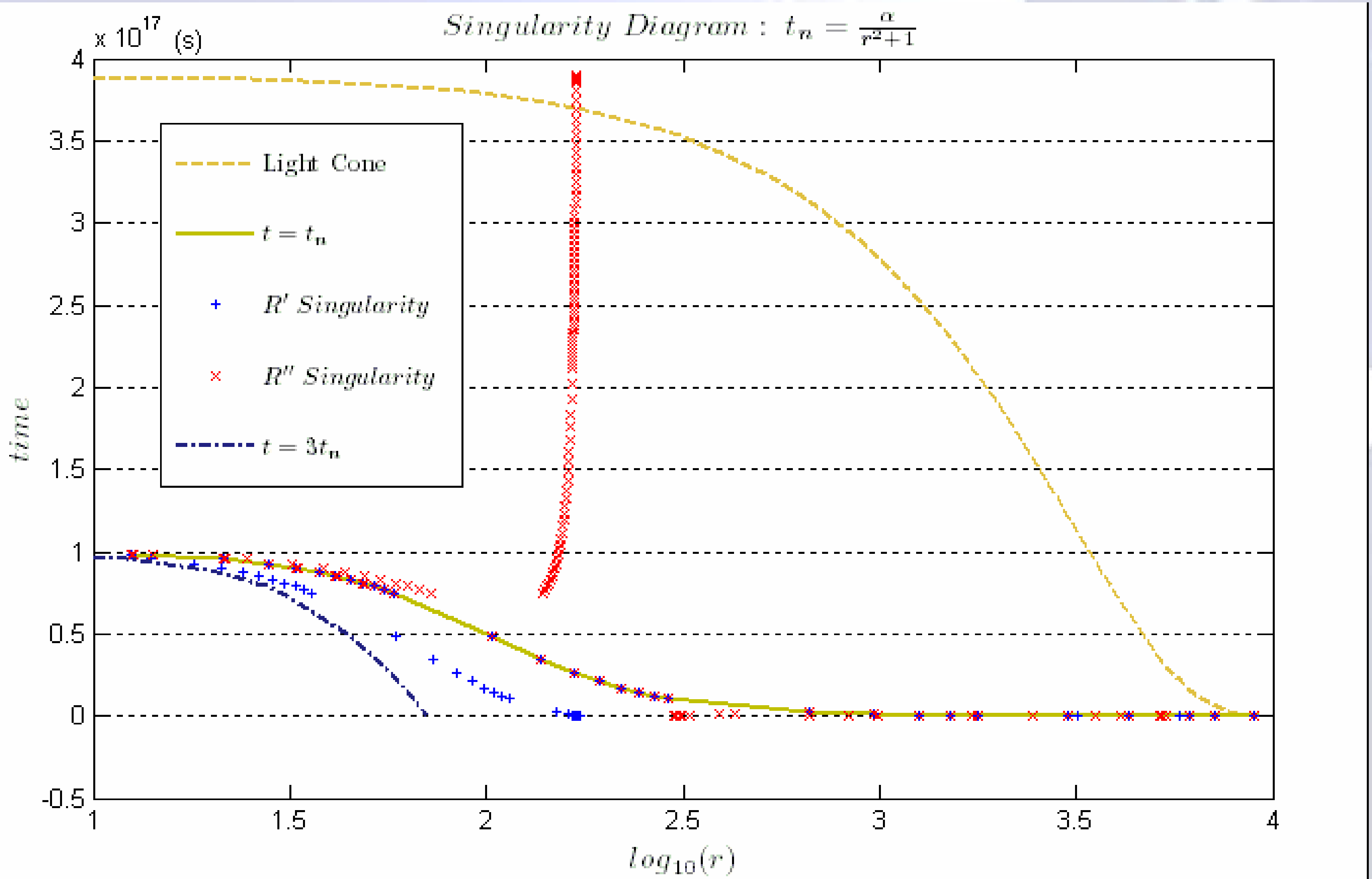
Singularities



LTB Solution

- A marginally bound LTB metric with *E(r)=0*

$$ds^2 = -c^2 dt^2 + R'^2 dr^2 + R^2(r,t)\left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

- Corresponding Einstein equations

$$\dot{R}^2(r,t) = \frac{2GM(r)}{R}$$

$$4\pi\rho(r,t) = \frac{M'(r)}{R^2 R'}$$

- Integration of density function:

$$M(r) \equiv 4\pi \int_0^{R(r,t)} \rho(r,t) R^2 \, dR$$

- Scale factor:

$$a(t,r) \equiv \frac{R(t,r)}{r}$$

- Comoving density

$$\rho_c(r) \equiv \frac{6M(r)}{r^3}$$

- Solution of field equations:

$$R(r,t) = \left[\frac{9GM(r)}{2}\right]^{\frac{1}{3}} \left[t - t_n(r)\right]^{\frac{2}{3}}$$

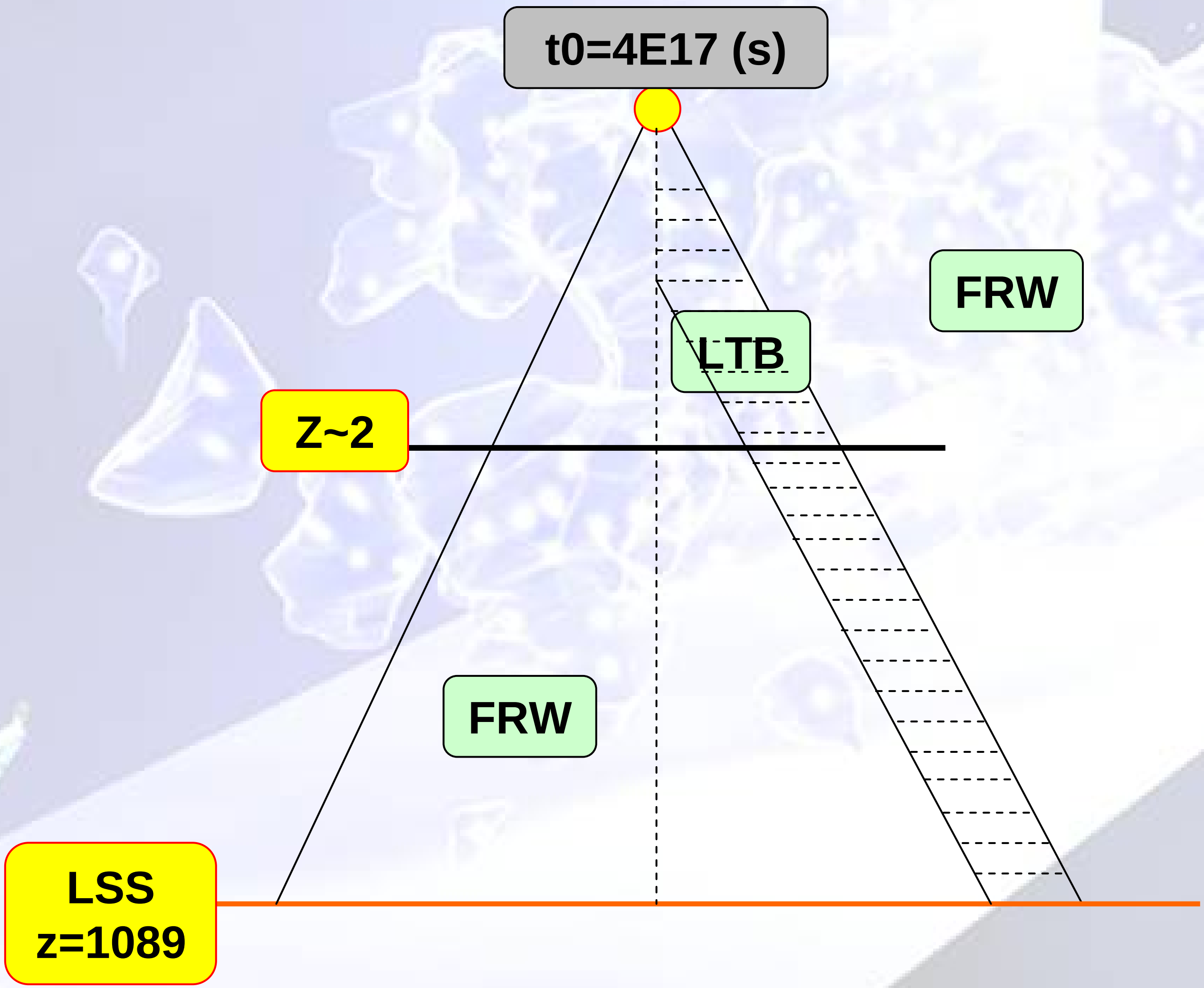
$$a(r,t) = \left[\frac{3}{4} G \rho_c(r)\right]^{\frac{1}{3}} \left[t - t_n(r)\right]^{\frac{2}{3}}$$

- Bang time selection
  - – Avoiding singularities
  - – Asymptotic behaviour for large r to be FRW metric

$$t_n = \frac{\alpha}{r^4 + r^2 + 1}$$

in which $\alpha = 10^{-17}$ and r is scaled to 100 Mpc.

Modifying the averaging process

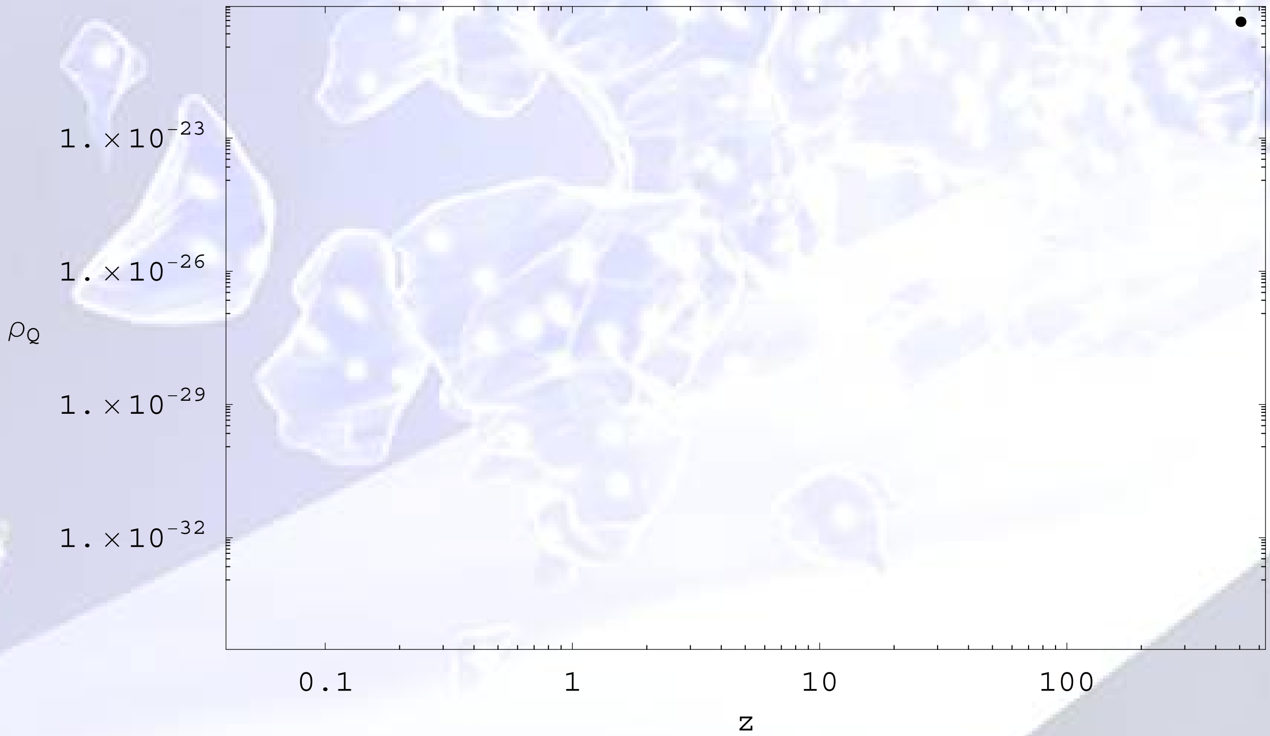


Modifying the Averaging Process

- In-lightcone averaging,
- On- lightcone averaging,
  - Domain of integration for each value of redshift:
    - From the vicinity of lightcone up to the ***r*** value corresponding to the size of horizon in the same redshift.

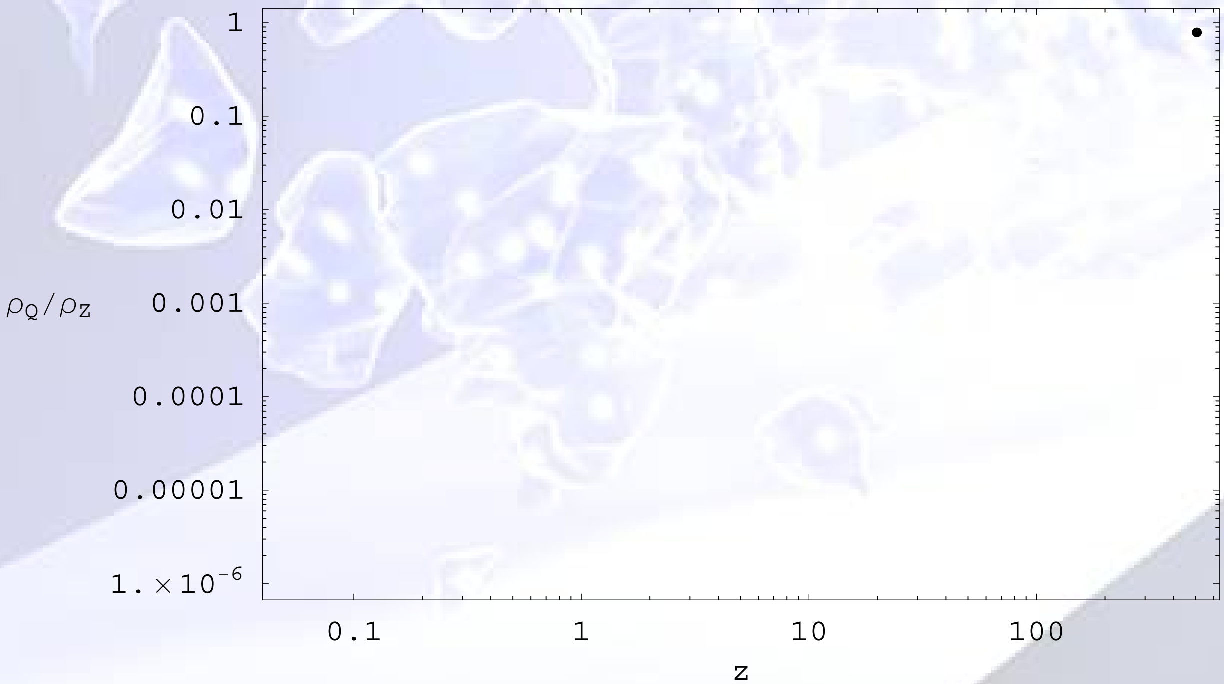
Results

- Backreaction density versus redshift



Results

- Backreaction to matter density ratio versus redshift



Results ?

- Numerical results show:

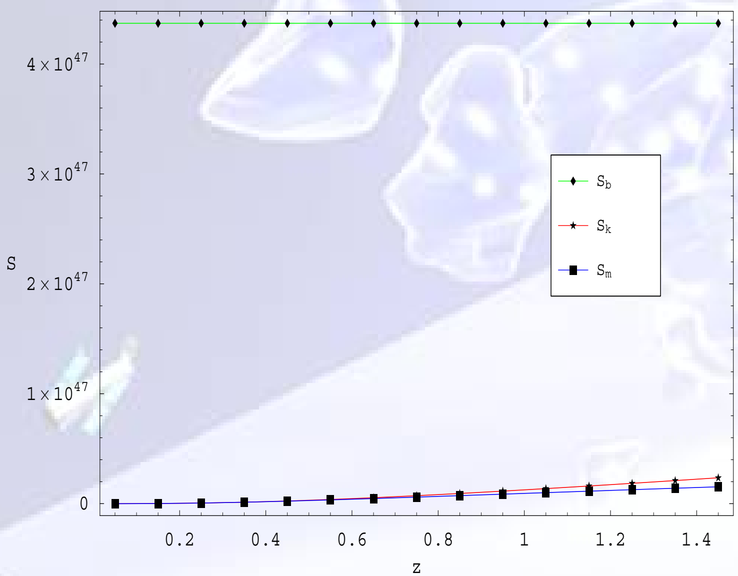
$\rho_D \neq 0$ guage dependence ?

ρ_D evolves through time ?

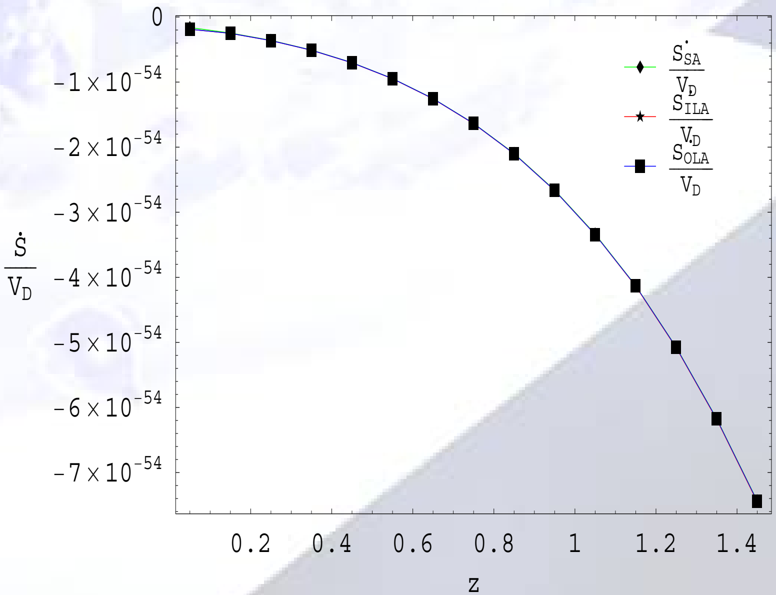
$\rho_D = p_D$ Backreaction is positive so we get positive presure ?

Kullback-Leibler Relative Information Entropy as a measure of inhomogeneity

- Total Entropy versus redshift



- Entropy production rate per unit volume versus redshift



Future work

- Different bang times,
- LTB solutions with $E(r) \neq 0$,
- Interpretation of KL entropy as structure formation

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