Are these results surprising?

- We can make contact with usual Pert. theory around FLRW

$$ds^2 = a^2(\tau)\left[-(1+2\Phi)\,d\tau^2 + (1-2\Phi)\,dx^2\right]$$

Assuming $u \ll 1$ and $E \ll 1$ the LTB metric becomes

$$ds^2 = \frac{9\tau^4}{\bar{H}^2}\Big\{-d\tau^2 + \Big[1 + 2R_2\gamma^2\tau^2(r k)' - 20 R_2 \gamma^2 r^2 k(r)\Big]dr^2 + r^2 d\Omega\left[1 + 2R_2\gamma^2\tau^2 k(r)\right]\Big\} \qquad (1)$$

From here we can calculate the gauge invariant combinations Φ and Ψ (Bardeen potentials).

Given a generic metric (perturbed to linear order, only scalar fluctuations):

$$ds^2 = a^2(\tau)\Big\{-(1+2\Psi)\,d\tau^2 + 2\,\partial_i w\,d\tau\,dx^i + \left[(1-2\phi)\,\delta_{ij} + D_{ij}\chi\right] dx^i dx^j\Big\}$$

Which, in spherical spatial coordinates becomes

$$ds^2 = a^2(\tau)\Big[-(1+2\Psi)\,d\tau^2 + 2w'\,dr\,d\tau + \left(1-2\phi+\frac{2}{3}\mathcal{E}\right)dr^2 + \left(1-2\phi-\frac{1}{3}\mathcal{E}\right)r^2 d\Omega^2\Big\} \qquad (2)$$

where $\mathcal{E} = \chi'' + \frac{\chi'}{r}$

Then Ψ and Φ are given by

$$\Psi = \Psi + \frac{1}{a}\left[\left(-w + \frac{\chi_\tau}{2}\right)a\right]_\tau$$

$$\Phi = \phi + \frac{1}{6}\nabla^2\chi - \frac{a_\tau}{a}\left(w - \frac{\chi_\tau}{2}\right)$$

Just by comparing (1) with (2) we can calculate Ψ and Φ for the LTB metric

$$|\Psi| = |\Phi| = 6R_2\gamma^2\int^{r} d\hat{r}\;k(\tilde{r})\,\tilde{r} < \frac{3}{5}k_{MAX}\left(\frac{L}{r_H}\right)^2$$

So Φ is indeed small

Moreover we can understand the two kind of corrections

(1) On the redshift of the sources inside the patch

(2) on the net effect for a photon that travels through a patch.

In fact using the weak field Newtonian metric

$$\frac{\delta z}{1+z} \simeq \overbrace{\Phi_S - \Phi_O}^{\text{negligible}} + \overbrace{v_S^i \cdot e^i - \cancel{v_O^i \cdot e^i}}^{\text{large near observer}}$$

$$+ \underbrace{\int d\tau \frac{\partial \Phi}{\partial \tau}}_{\text{net effect through a patch}}$$

dipole of the observer (if observer at the center, it is zero)

(1)

Now $v_S{}^i = \frac{\partial^i \Phi}{3}$ ~~[illegible]~~

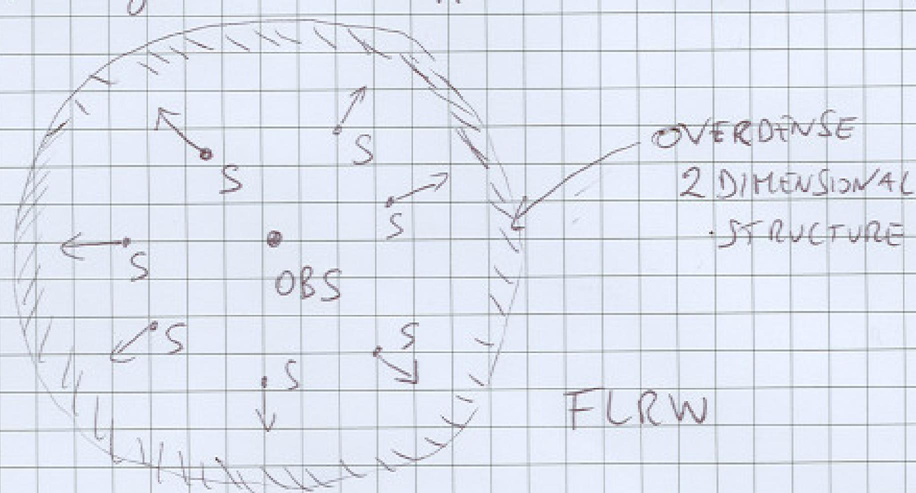
$$= \frac{d}{dz}\left[6 R_2 \gamma^2 \int k(z)\, z\, dz\right]$$

موضوع : تاریخ :

It gives exactly the large correction on $\bar{z}$ that we have computed in LTB!

$\Longrightarrow$ All the large effects that one can see in LTB (with $u \ll 1$ and $E \ll 1$) are just peculiar velocities of the sources (in the radial direction)

So, if we live in the center of a local void all the sources inside the void move towards the boundary and their redshift gets a Doppler correction



That gives a large correction to the local Hubble flow.

In order to mimick acceleration with a reasonable fit of the SN data we need a patch that contains all the low-redshift SN _

(See COMPUTER SLIDES)

- So we have a simple understanding on how a local void can mimick acceleration
- This can be compatible with CMB 1st peak measurements and BAO measurements, assuming low Hubble constant ($h \sim 0.5$) in the outside FLRW region

(See COMPUTER SLIDES)

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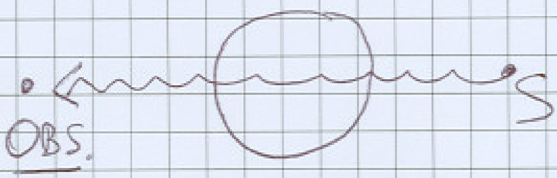
(2) Let's go back to study the net effect for a photon travelling through LTB



In the usual Newtonian gauge this is given by

$$\frac{\delta z}{1+z} \simeq \int \frac{\partial \Phi}{\partial \tau}\, d\tau$$

Φ at linear order is constant

But [Panek, Ap. J. 388, 225-233 (1992)]

a simple estimate gives at nonlinear level in δ:

$$\int d\tau \frac{\partial \Phi}{\partial \tau} \simeq \frac{\Phi}{t_c}\,\Delta t$$

travelling time inside the structure
$\Delta t \sim L$

typical time scale of change of gravitational potential
$t_c \simeq \frac{L}{v_c}$

typical velocity inside the structure

$$\simeq \boxed{\Phi\, v_c}$$

Using $\nabla^2 \Phi = 4\pi G \bar{\rho}\, \delta a^2$

We get:

$$\frac{\delta z}{1+z} \simeq \begin{cases} \delta^2 \left(\frac{L}{R_H}\right)^3 & \left(\text{for } v_c = |\vec{\nabla}\Phi| \right. \\ & \left. \text{linear perturbations}\right) \\ \delta^{3/2} \left(\frac{L}{R_H}\right)^3 & \left(\text{for } v_c^2 \simeq \Phi \right. \\ & \left. \text{virialized structures}\right) \end{cases}$$

- The same result $\delta z \sim \delta^2 \left(\frac{L}{R_H}\right)^3$
  comes out from a detailed second order
  treatment of light propagation
  [see astro-ph / 9702234 ~~and~~ (general formalism)
  and astro-ph / 0702555]

- SEE RESULTS IN PLOT FIG. 5 of
  astro-ph / 0702555
  The cubic scaling is confirmed also
  by numerics.

- OBS : For a patch of radius $\sim 200/h$ Mpc
  and $\delta \simeq 0.2$ we get roughly
  10^{-5} : VISIBLE in the CMB!
  (see Inoue and Silk '06)

P4PCO

• WHAT ABOUT corrections to D_L?

Numerics in astro-ph/0702555 give negligible corrections

(but Brouzakis, Tetradis Tzavara '07 claim larger correction)

FINAL QUESTIONS:

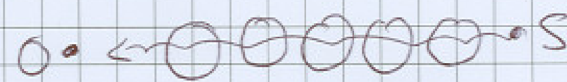
① What happens if $k(r)$ is large (large u)?

- It is possible to study it numerically: no significant difference
- Analytically we estimate also the effect to go ~~...~~ as $\left(L/R_H\right)^3$ to <u>all orders</u> in u

In fact (astro-ph/0702555) we find a correction on proper time $\tau(r)$

$$\frac{|\delta\tau|}{\tau_0} \leq 1.5 \left(L/R_H\right)^3$$

② Do corrections build up if going through several patches?



Yes, but still the effect is too small to affect Supernova Observations
[but what about CMB?]

③ What happens for large $E(r)$?

The backreaction Q_D (as in the general formalism) may be non-zero.

But numerics did not show any surprising results
[in the Onion model, at large distance from the center $E(r) \gg 1$)