



The Bargmann-Wigner Equation: Filed equation for arbitrary spin

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Motivation

- **De-Broglie fusion theory of light:**
 - 1) It was motivated by De Broglie to derive Maxwell's equation as emergence field equation of two spin $\frac{1}{2}$ massless particles (Neutrino in that time)
 - 2) Bargmann and Wigner in their seminal paper in 1948 group theoretically speaking, generalize this approach to the field equation for arbitrary spin

Review

- Dirac equation:

$$(\boldsymbol{\gamma} \cdot \hat{\partial} + \mathbf{m}) \psi = 0$$

- Set of Gamma matrices

$$I, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu}$$

Note: they form a complete set.

Charge conjugation operator

$$C\psi = \psi^*$$

$$C\gamma_5 C^{-1} = \gamma_5$$

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T$$

Procedure

- 1) We symmetries spinor of rank "n" , $\Psi_{\alpha\beta..}$
- 2) It satisfies Dirac equation for each spinor index:

$$(\gamma.\partial + m)_{\alpha\alpha'}\Psi_{\alpha'\beta\dots} = 0$$

Spin-1 Particle

- Group theoretically speaking, the symmetric state of two spin- $\frac{1}{2}$ particles gives spin-1 particle. B-W method helps us to derive the field equation corresponding to new field equation. In spin-1 case it means we can derive Maxwell's (Proca) equation from Dirac equation!

Spin-1

$$\Psi_{\alpha\beta} = A^\mu (\gamma_\mu C)_{\alpha\beta} + F^{\mu\nu} (\sigma_{\mu\nu} C)_{\alpha\beta}$$

- Now we contract above equation with;

$$(C^{-1} \gamma_5)_{\beta\delta}, (C^{-1} \gamma_5 \gamma_\nu)_{\beta\delta}, (C^{-1})_{\beta\delta}$$

Spin-3/2 (Rarita-Schwinger eq)

- We can continue to make a spin 3/2 particle as follow;

$$\Psi_{\rho\alpha\beta} = A^{\mu}{}_{\rho} (\gamma_{\mu} C)_{\alpha\beta} + F^{\mu\nu}{}_{\rho} (\sigma_{\mu\nu} C)_{\alpha\beta}$$

- Follow the same method of contraction.

Modified BW

- Q: We examine if fully anti-symmetrization spinor gives us what we expect.
- A: Yes.

Spin-0

- Here we try to (re)derive Klein-Gordon equation from second rank anti-symmetric spinor.
- The general method is as before, except we changed the expansion of the spinor with respect to anti-symmetric gamma matrices.

Spin-0

$$\Psi_{\alpha\beta} = A(\gamma_5 C)_{\alpha\beta} + B^\nu (\gamma_5 \gamma_\nu C)_{\alpha\beta} + F(C)_{\alpha\beta}$$

- Now, we contract above spinor with symmetric gamma matrices.

$$(C^{-1} \gamma_\mu)_{\beta\delta}, (C^{-1} \sigma_{\mu\nu})_{\beta\delta}$$



Summary

- Starting from Dirac equation one can derive higher spin field
- By modifying BW we examine validity of the Anti-Symmetric state



Thank You

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