Prospects for Inflation from String Theory

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Outline

- Motivation for Inflation from String Theory
- A quick Review on String Theory
- Brane Inflation
- Moduli Stabilization
- Warped Brane Inflation
- DBI Inflation
- Cosmic Strings

Motivation for String Cosmology

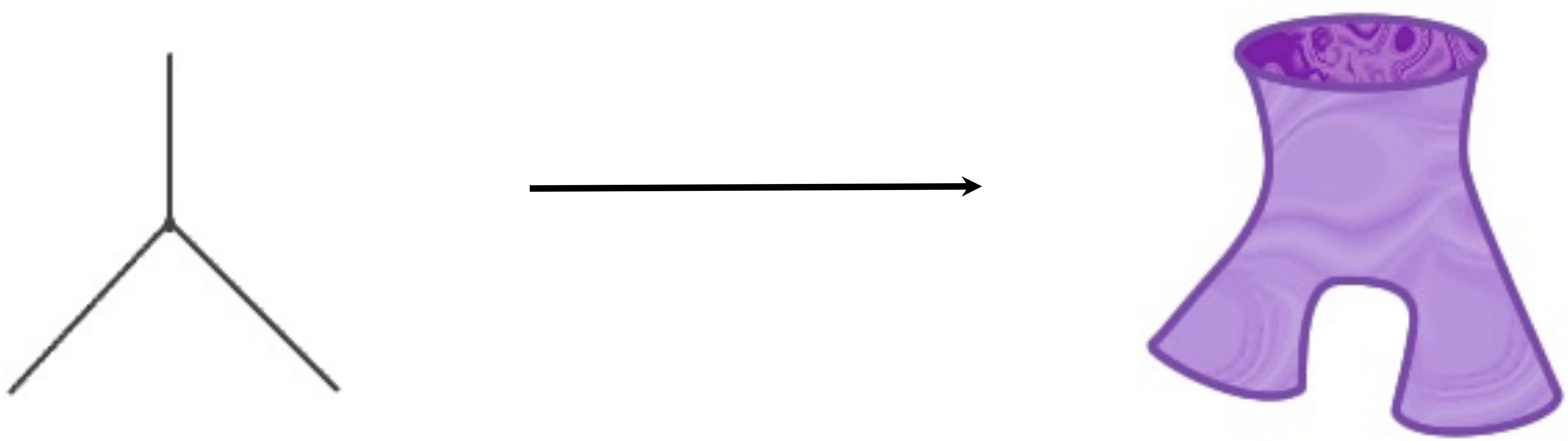
- Recent observations strongly support inflation as the origin of big bang and the structure formation in universe.
- However, the origin of the inflation is not known from a fundamental theory point of view.
- String theory, on the other hand, is a consistent theory of gravity. So far it has failed to make contact with the real world.
- Since the scale of inflation is very high, possibly GUT scale, it is natural to expect that string theory was relevant in driving inflation.
- This would provide a unique chance to test the relevance of string theory to the real world.

String Theory

- All known particles are low-lying string vibrations.

- String theory is a consistent theory of quantum gravity.
  It contains the graviton in its spectrum.

- All interactions consist of the splitting and joining of these elementary strings:



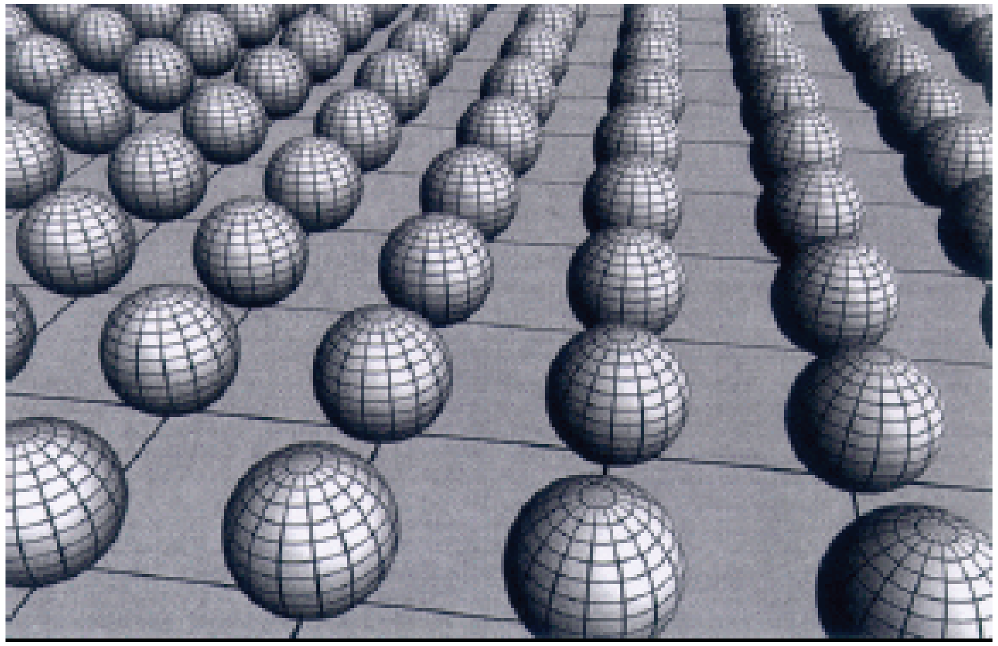
- In string theory, gauge fields(photons) are given by open strings while gravity is represented by closed strings.

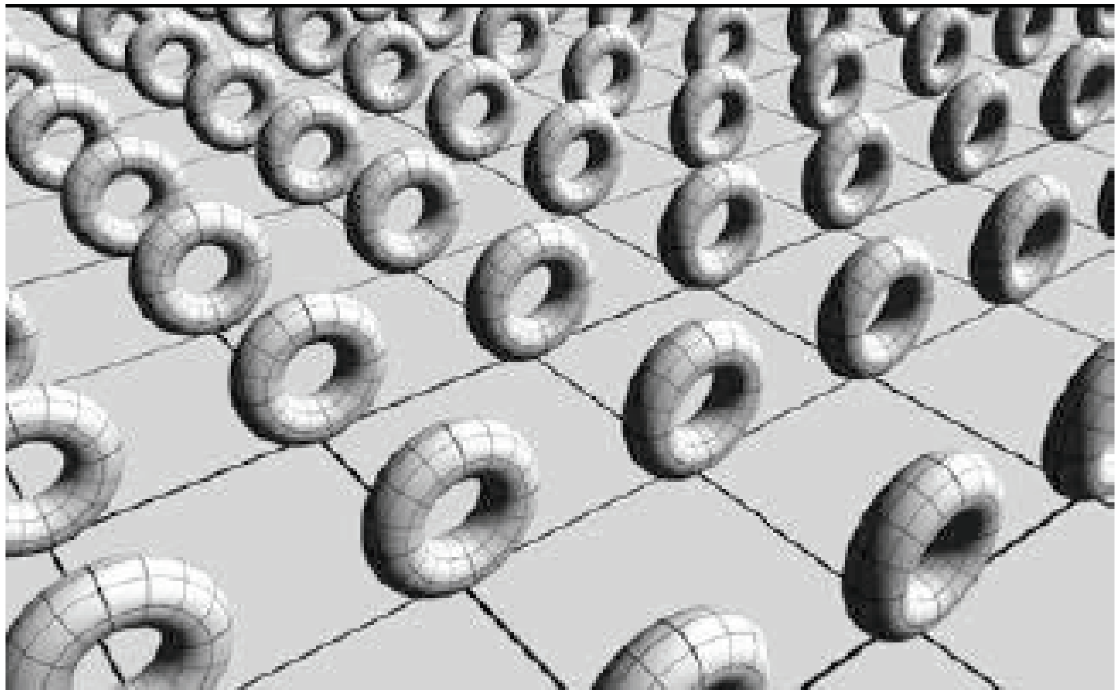
Some Novel Aspects of String Theory

- A consistent string theory requires 9 spatial dimensions.
- We see only 3 of them! To account this discrepancy, it is assumed that the 6 extra dimensions are curled up on a manifold, the Calabi-Yau manifold.
- Depending on how one compactify the extra dimensions, one can get many solutions: 10^{500}

$\downarrow$

String Theory Landscape

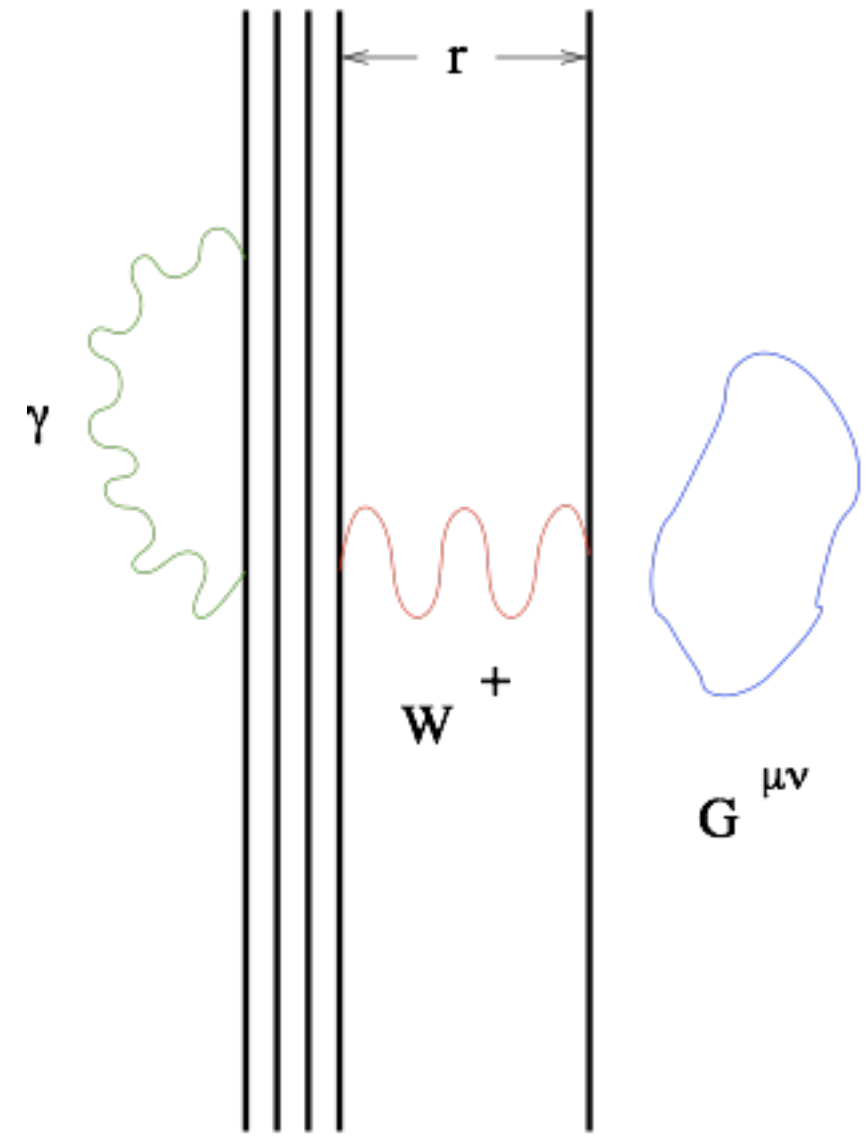




B. Greene, The Elegant Universe

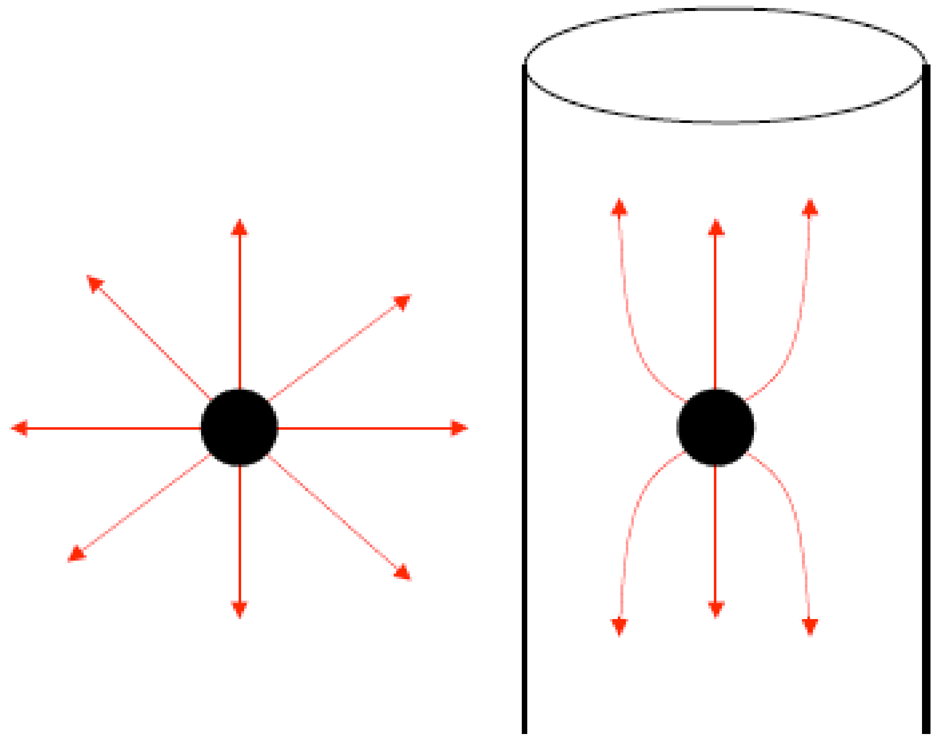
D-Branes

- Originally it was a theory of strings, but recent developments have shown that it also contains higher dimensional defects, D-branes.
- Usually open strings end-points satisfy Neumann boundary conditions and strings end-points can move with the speed of light.



- In the presence of D-branes, Dirichlet boundary conditions are also possible and the open strings end-points can live on these hyper-surfaces.
- Photons(open strings) are confined to the branes while closed strings modes, gravitons, can propagate to the extra dimensions, the bulk.

The Brane World Scenario



- In brane world picture, we (the Standard Model particle physics) are confined to a D3-brane, a three-dimensional hyper-surface in a higher dimensional space-time. The D3-brane spans our entire Universe.
- The gravity can propagate in the extra dimensions, bulk. This can explain why gravity is so weak compared to electro magnetic interactions. Force laws fall faster in higher dimensions because there are more rooms for gravity to escape. For example in 4+n space-time:

$$V \sim G_n \frac{M_1 M_2}{r^{n+1}} \longrightarrow G_N \sim \frac{G_n}{V}$$

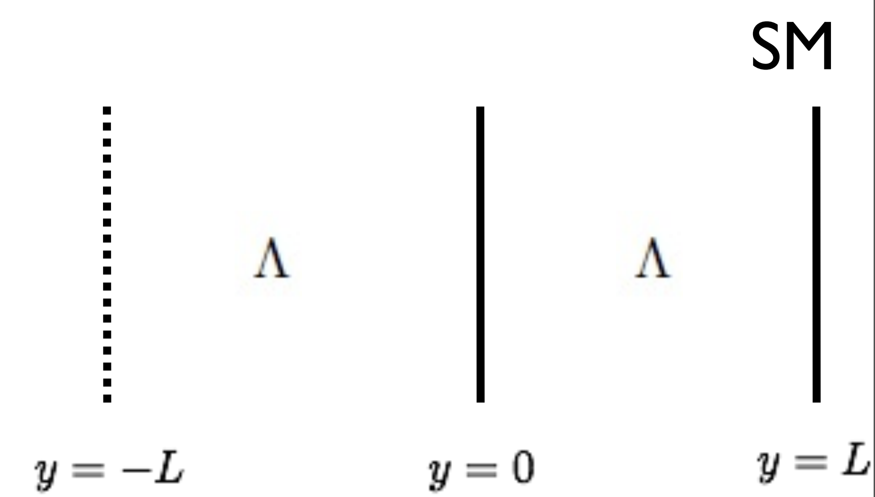
- If n=2, then the size of the extra dimensions can be as large as 0.01 mm! This is within the reach of the current table-top experiments.

Warped Brane World

Randall and Sundrum
hep-ph/9905221, hep-th/9906064

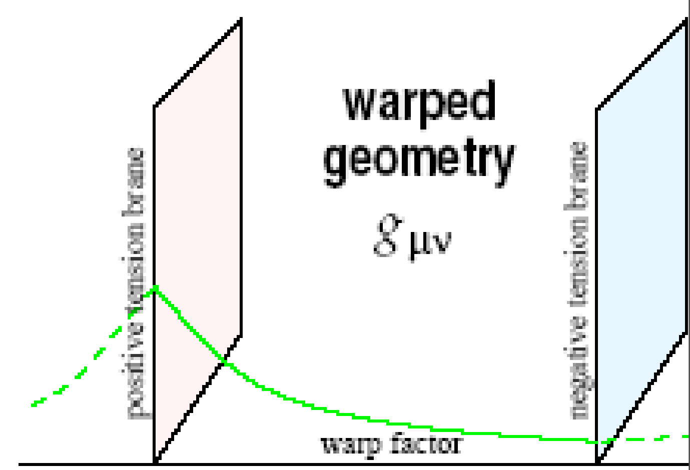
We start with a five-dimensional space with a negative bulk cosmological constant and branes with tensions

$$S = \int d^4x \int_{-L}^{L} dy \sqrt{-g}\left[-\Lambda + 2\, M^3 R\right] - \int d^4x\, V_{vis} - \int d^4x\, V_{hid}$$



The space-timer is five-dimensional warped geometry

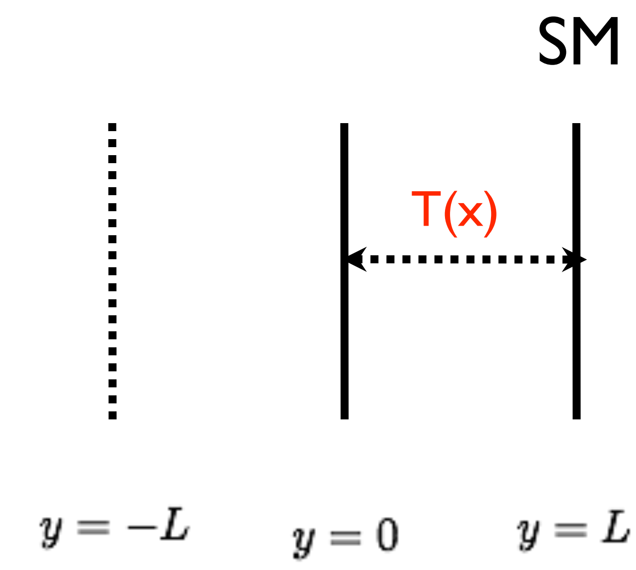
$$ds^2 = e^{-2k|y|}\, \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$



subject to

$$V_{hid} = -V_{vis} = 24 M^3 k, \quad \Lambda = -24 M^3 k^2$$

The 4-D physics



$$ds^2 = e^{-2kT(x)}\left[\eta_{\mu\nu} + \bar{h}_{\mu\nu}\right] dx^\mu dx^\nu + T(x)^2 dy^2 \qquad \bar{g}_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \bar{h}_{\mu\nu}(x).$$

The four-dimensional gravitational action is

$$S_{eff} = 2\int d^4x \int_{-L}^{L} dy\, M^3 e^{-2ky}\sqrt{-\bar{g}}\bar{R} + \ldots$$

which results in

$$M_P^2 = M^3 L \int_{-L}^{L} dy\, e^{-2ky} \simeq \frac{M^3}{k}$$

Now suppose one localize the standard model particle physics (SM) on the "visible" branes. Consider a scalar field with bare mass m_0:

$$m_{phys} = e^{-kL}\, m_0$$

To solve the hierarchy problem one only needs kL ~ 35

Problem with RS scenario: One has to stabilize the distance between branes, the field T(x).

Moduli In String Theory

- String theory is a higher dimensional theory. After compactification to four dimensions one gets many scalar fields, moduli, depending on details of compactifications.

- Good News: Due to large number of moduli, it is possible that some of them play the role of inflaton field. Examples are: Tachyon Inflation, Racetrack Inflation, DBI-Inflation, D3-D7 Inflation, Brane Inflation, Warped Brane Inflation. In general, the inflaton field is either an open string mode(brane position) or a closed string mode (complex Kahler moduli).

- Bad News: These moduli may couple to the inflaton field and interfere with the slow roll conditions. Furthermore, If they are not stabilized they will destroy the success of late time big bang cosmology, like big bang nucleosynthesis.

- The first task in string cosmology is the understanding of moduli and their stabilization mechanism.

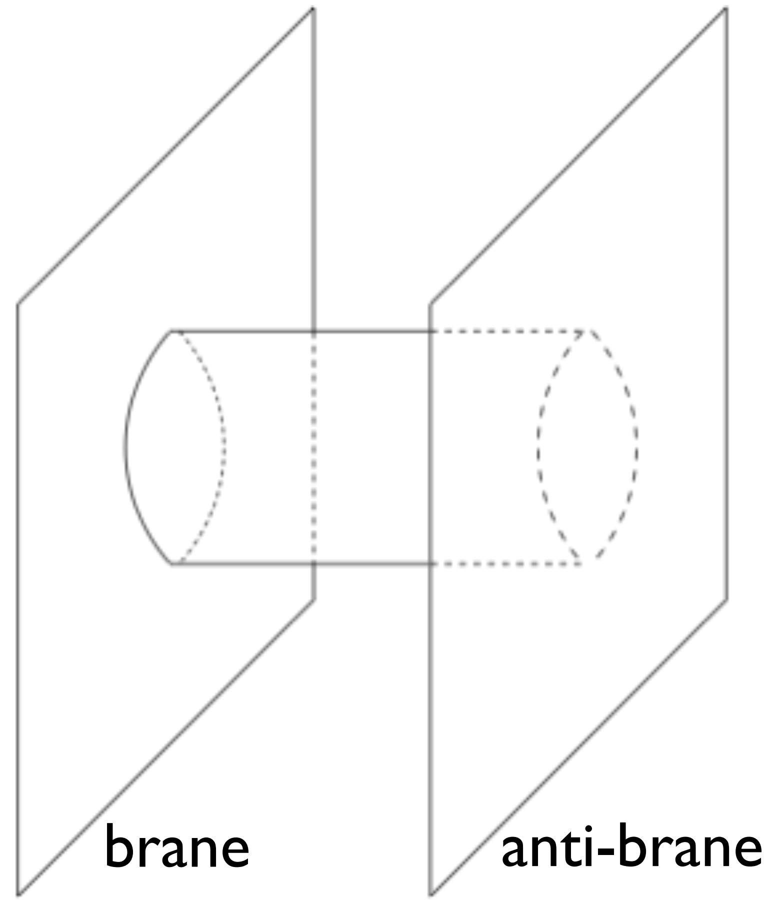
Different Types of Moduli

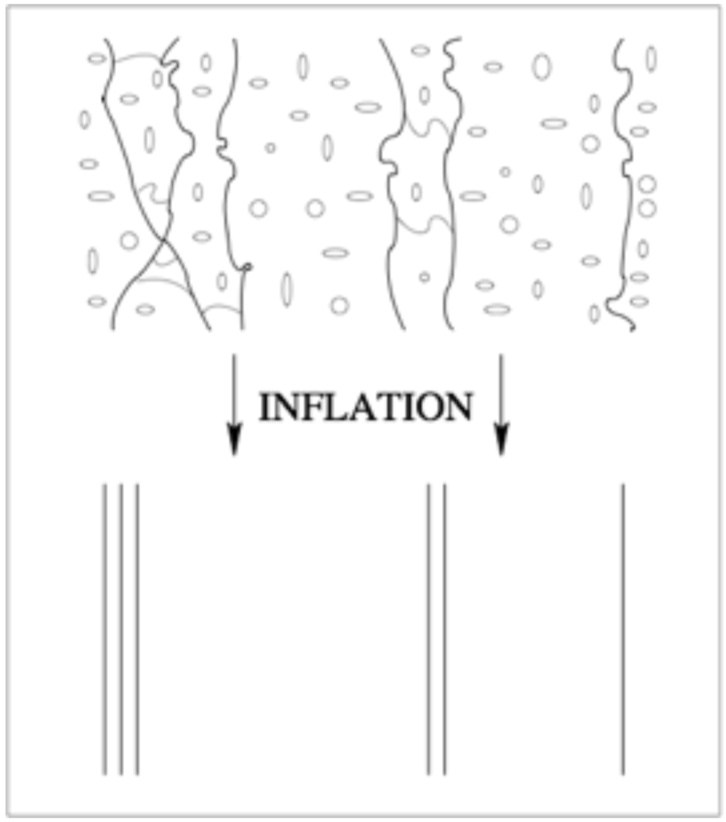
- Typically Moduli are divided in two classes:
- Complex structures, like dilaton and shape of internal cycles inside Calabi-Yau(CY) manifold.
- Kahler structures which determine the overall volume of the CY compactification.
- Usually complex structures can be fixed by turning internal fluxes, while the stabilization of Kahler moduli is non-trivial and needs some non-perturbative effects.

Brane Inflation

G. Dvali and H. Tye, hep-ph/9812483

- In brane inflation the inflaton field is the distance between brane and anti-brane.
- There is an attractive force between brane and anti-brane. If the potential is flat enough one can get enough inflation.
- When the distance between brane and anti-brane is at the order of string scale, a tachyon develops. Inflation ends when brane and anti-brane collide.
- Problem: In flat CY, the potential is too strong to achieve the slow-roll conditions for inflation.

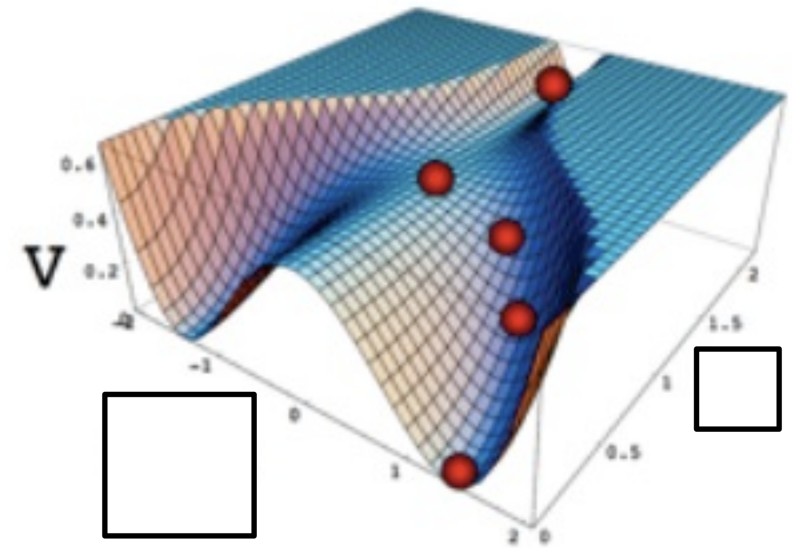




Basic properties of brane -anti brane inflation:

The potential between a p-brane and anti p-brane is

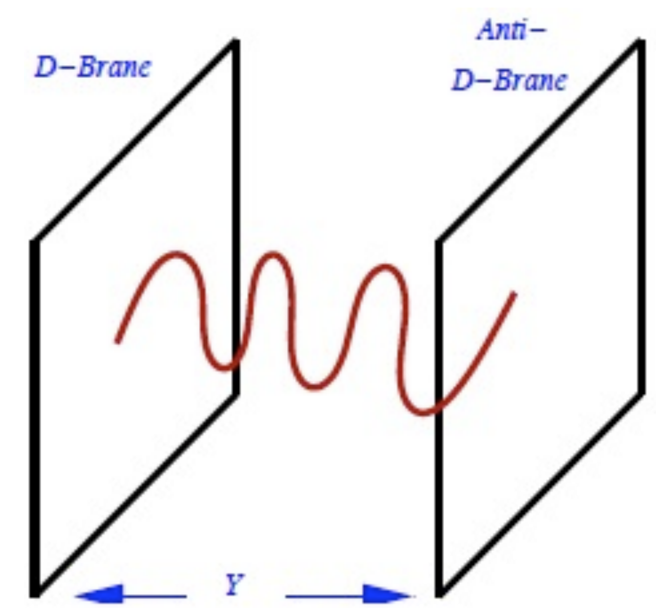
$$V(r) = A - \frac{B}{y^{7-p}}$$



where

$$A \sim m_s^4 \qquad B \sim \kappa_{10}^2 T_p^2$$

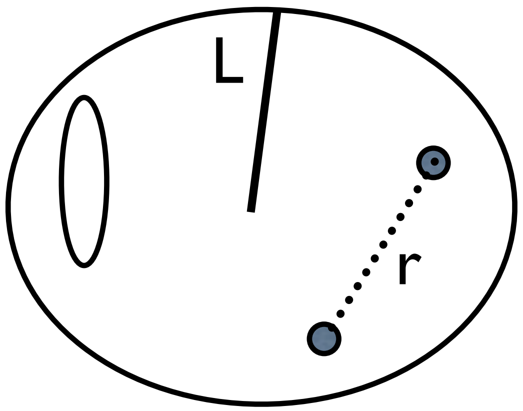
$$\alpha' = m_s^{-2} \qquad T_p = \frac{1}{g_s (2\pi\alpha')^{\frac{p+1}{2}}} \qquad \kappa_{10}^2 \equiv \tfrac{1}{2}(2\pi)^7 g_s^2 (\alpha')^4.$$



Brane inflation is a realization of hybrid inflation

$$\alpha' m_{tachyon}^2 = \frac{y^2}{4\pi^2\alpha'} - \frac{1}{2}$$

Problem with the Original Brane Inflation



The potential between D3 and anti-D3 branes is

$$V(r) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3}{M_{10,Pl}^8 r^4}\right) . \qquad T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2}.$$

The where r is the distance between them in extra dimensions. In terms of canonically normalized field $\phi = \sqrt{T_3} r$ the potential is

$$V(\phi) = 2T_3\left(1 - \frac{1}{2\pi^3}\frac{T_3^3}{M_{10,Pl}^8 \phi^4}\right) .$$

To have slow-roll inflation one requires that $\epsilon, \eta \ll 1$ where

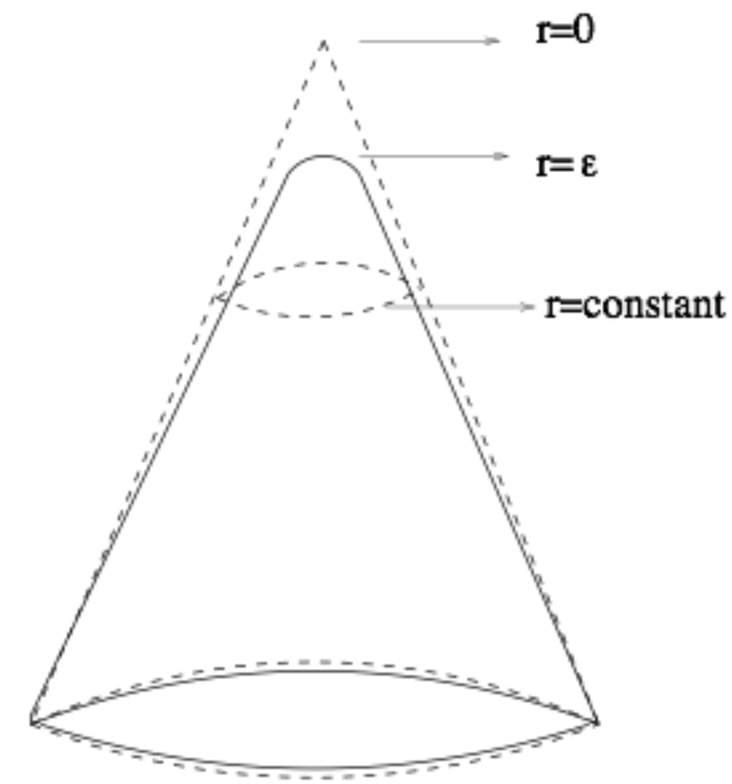
$$\epsilon \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta \equiv M_{Pl}^2 \frac{V''}{V} .$$

Noting that $M_{Pl}^2 = M_{10,Pl}^8 L^6$ where L is the typical size of extra dimension yields

$$\eta = -\frac{10}{\pi^3}(L/r)^6 \sim -0.3(L/r)^6$$

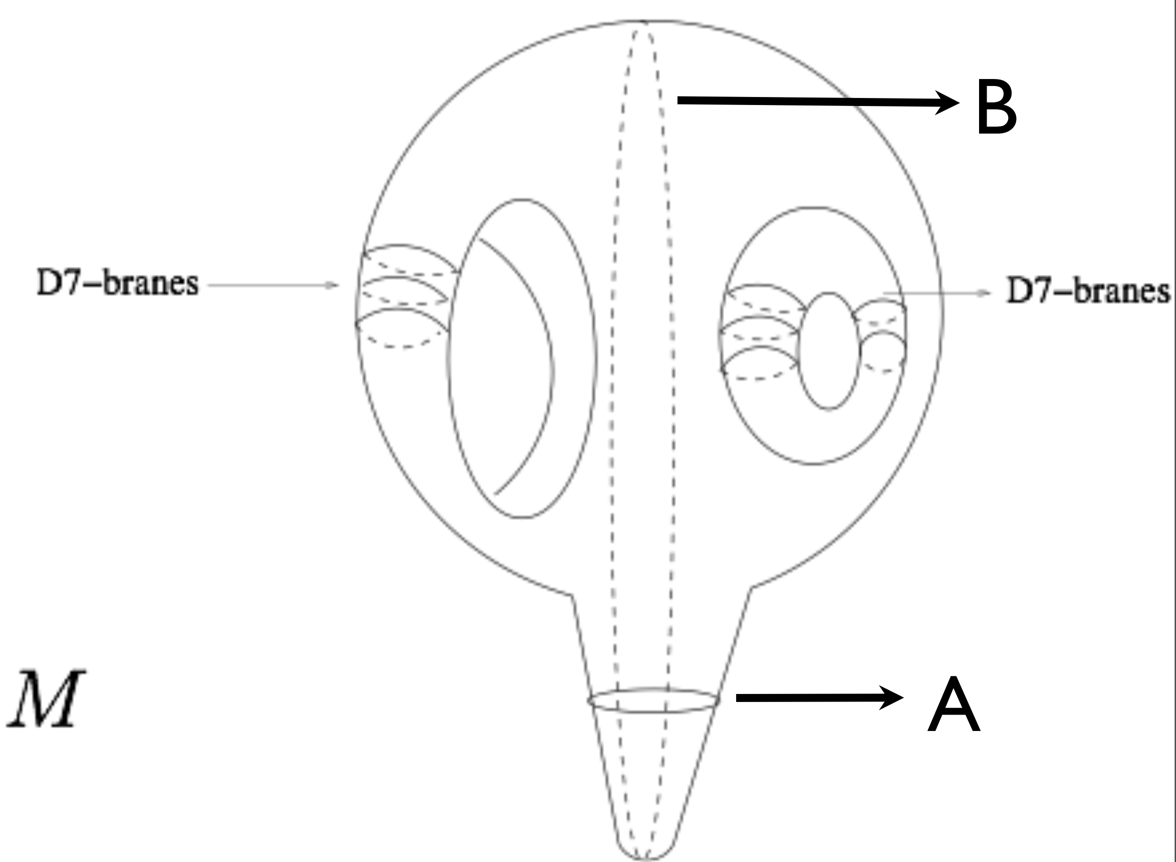
Hence $\eta \ll 1$ is possible only for r > L which is a contradiction!

Complex Structure Stabilization



- Particular example studied carefully is a deformed conifold in IIB string theory(Kelebanov and Sttrassler, hep-th/0007191).

- A deformed conifold is defined by $$\sum_{i=1}^{4} w_i^2 = \epsilon^2$$



- There are two 3-cycles, called A and B. One can turn on M units of RR 3 forms F3 on A cycle and K units of NSNS 3 forms H3 on B cycles:

$$\frac{1}{4\pi^2 l_s^2}\int_B H_3 \;=\; -K, \qquad \frac{1}{4\pi^2 l_s^2}\int_A F_3 = M$$

The Warped Deformed Conifold

- By turning these fluxes one can create a warped geometry like Randall-Sundrum(RS) scenario(Kachru, Giddings and Polchiski, hep-th/0105097).
- The metric inside the conifold is almost an AdS metric:

$$ds^2 = h(r)^2 \left(-dt^2 + a(t)^2 d\vec{x}^2\right) + h(r)^{-2} dr^2 \qquad h(r) = \frac{r}{R}:$$

- Where R is the characteristic length scale of the AdS geometry

$$R^4 = \frac{27}{4}\pi g_s N \alpha'^2 .$$

- N=MK is the effective background D3-brane charge. The warp factor at the end of the throat is given by

$$h_A = e^{-2\pi K/3g_s M}$$

Volume Modulus Stabilization

- In GKP, they could stabilize the complex structure like axion-dilaton field. However, the volume modulus can not be stabilized by flux compactifications.

- In supergravity limit the potential for moduli is

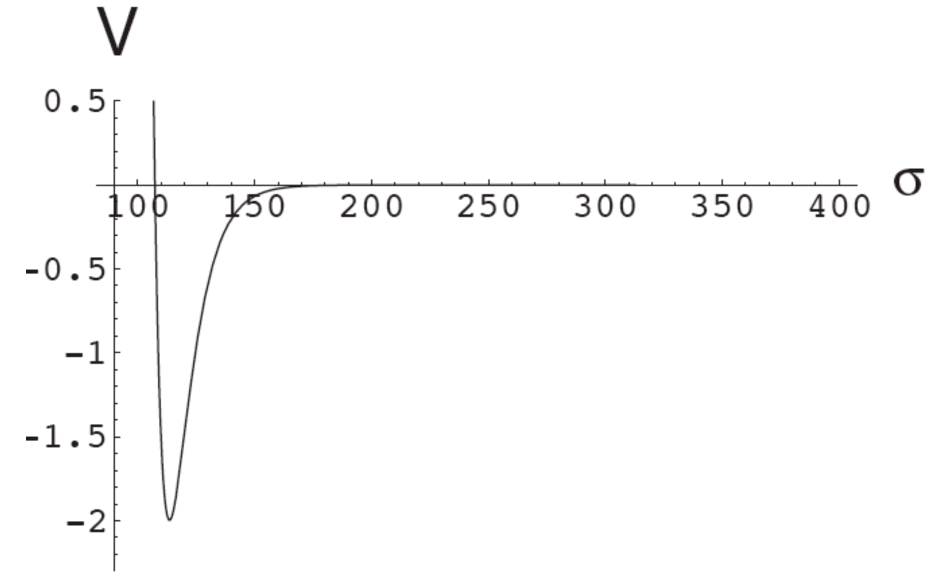
$$V_F = e^{\mathcal{K}} \sum_{a,b} \left(g^{\bar{a}b} \overline{D_a W} D_b W - 3|W|^2 \right) \to e^{\mathcal{K}} \sum_{i,j} \left(g^{\bar{i}j} \overline{D_i W} D_j W \right)$$

- Where the Kahler potential and the superpotential are

$$\mathcal{K} = -\ln[-i(\rho - \bar{\rho})] - \ln[-i(\tau - \bar{\tau})] \qquad W = W(\tau)$$

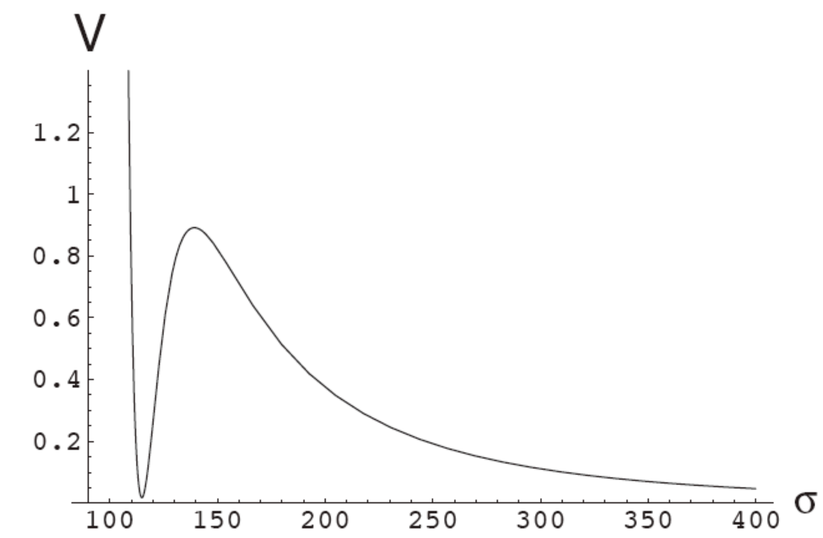
- Because W is independent of the volume modulus(Kahler modulus), the potential is independent of the volume modulus and it can not be stabilized in GKP.

KKLT hep-th/0301240

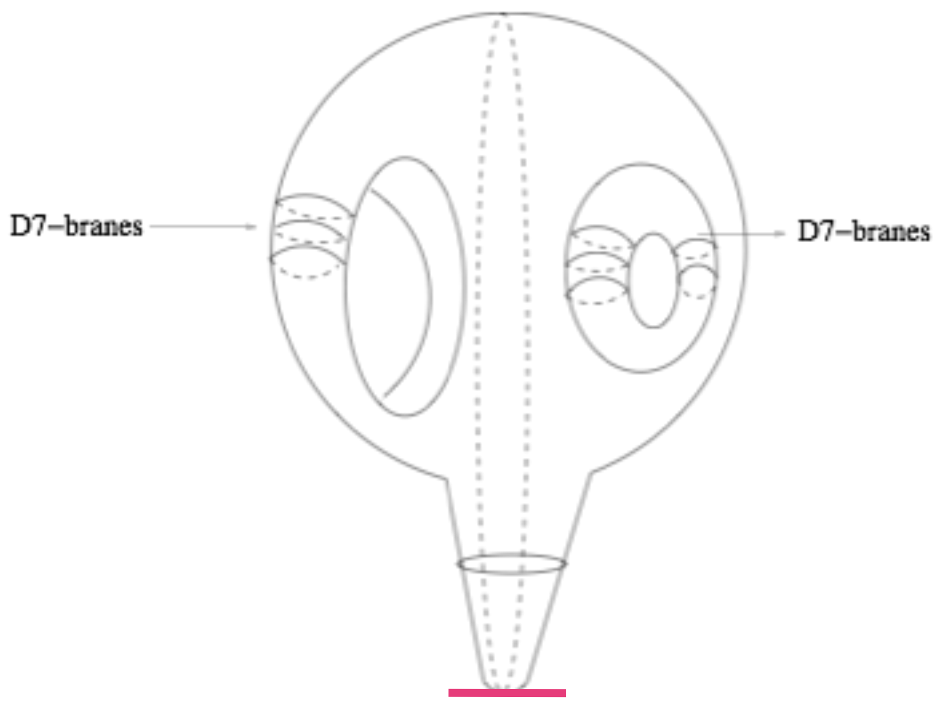


- Kachru and collaborators realized that non-perturbative effects on superpotential can create ρ dependence in W.

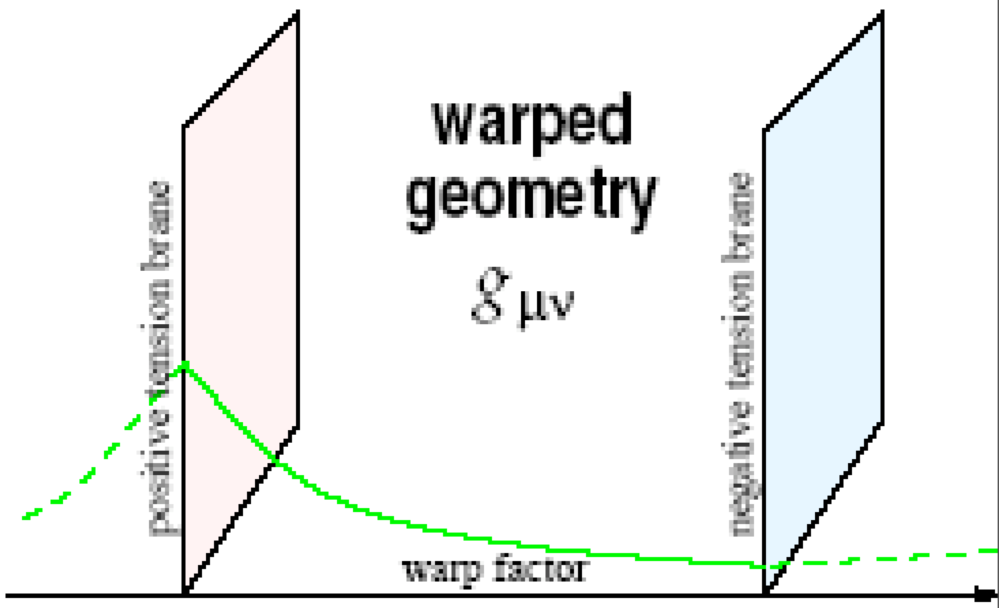
$$W = W_0 + Ae^{ia\rho}$$



- One can stabilize the volume modulus while the vacuum is a supersymmetric AdS minimum.
- To obtain a 4D theory with positive cosmological constant, they uplift the AdS vacuum by adding some anti-D3 brane at the bottom of the conifold.

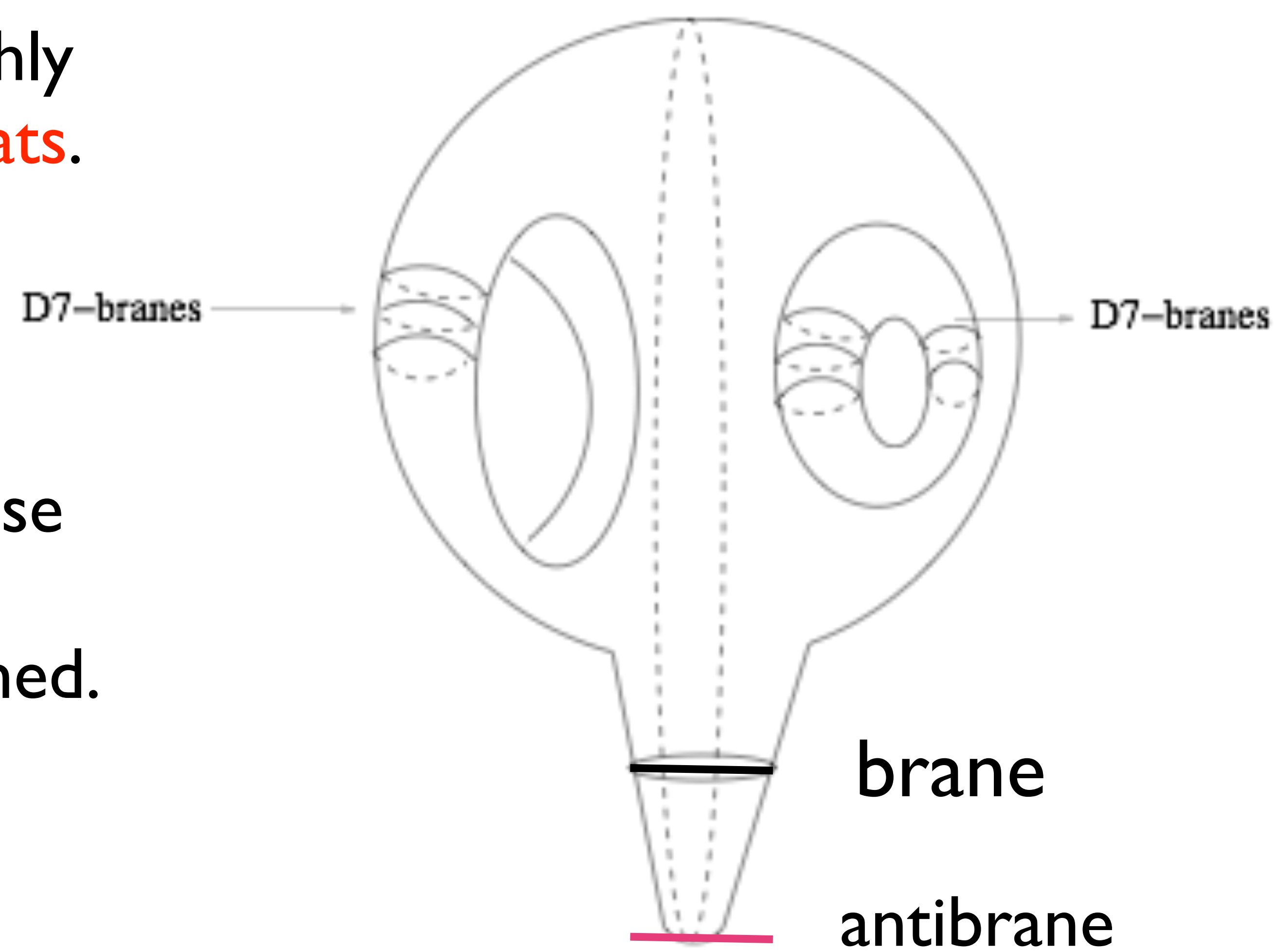


Warped Brane Inflation: Overview



- Warped Geometry is a method to flatten the potential between brane and anti-brane.

- There are localized regions in the bulk of the Calabi-Yau compactification which are highly warped. These regions are called the throats. Usually there are many of them.



- By putting the brane and anti-brane in these throats the force between them becomes weaker and enough inflation can be obtained. (KKLMMT: hep-th/0308055)

Warped Brane Inflation

KKLMMT: hep-th/0308055)

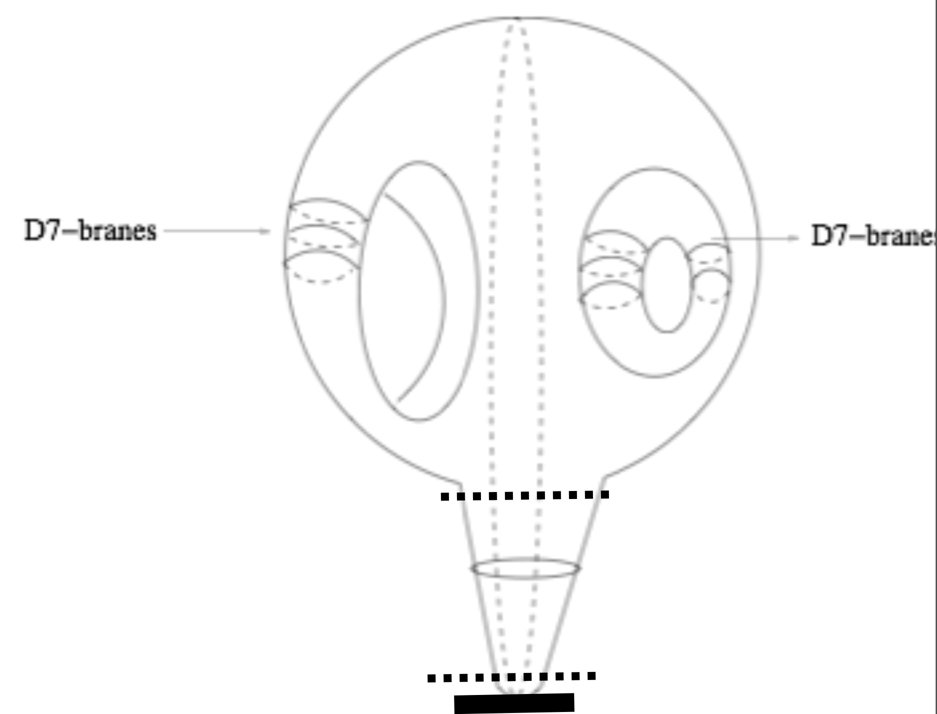
The background geometry is locally an AdS

$$ds^2 = h(r)^2\left(-dt^2 + a(t)^2 d\mathbf{x}^2\right) + h(r)^{-2}dr^2 + R^2 ds_5^2 \qquad h(r) = \frac{r}{R}$$

where N is the background charges(branes). The action is

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R + S_{local}$$

where the local action is



$$S_{local} = T_3\int d^3x\sqrt{-|g_{ab}|} \pm \frac{T_3}{g_s}\int d^4x C^{(4)}_{01234}$$

with $$T_3 = \frac{1}{(2\pi)^3 g_s \alpha'^2}. \qquad C^{(4)}_{01234} = \frac{R^4}{r^4}$$

The local action becomes

$$S_{local} = -T_3\int d^4x\, a(t)^3\left[h^4\sqrt{1 - h^{-4}\dot{\phi}^2 T_3^{-1}} \mp h^4\right]$$

In the slow-roll limit the gravitational attraction and the RR repulsion of D3-branes forces cancel out and and

$$S_{D3} = \frac{1}{2}\int d^4x\, \dot{\phi}^2$$

On the other hand for the anti-D3 branes, both forces are attractive and the anti brane is attracted towards the bottom of the throat with potential

$$V_0 = 2\, T_3\, h_A^4(r_0) = 2T_3 \left(\frac{r_0}{R}\right)^4$$

Once the anti-brane is at the bottom of the throat it attracts the D3-branes with the Coulombic potential

$$V_{D\bar{D}} \propto \frac{\phi_A^8}{N\phi^4} \qquad \phi_A = \sqrt{T_3} r_0$$

The total potential of the system is

$$V = V_A + V_{D\bar{D}} = 2\, T_3 h_A^4 \left(1 - \frac{1}{N}\frac{\phi_A^4}{\phi^4}\right)$$

So far we have considered only local effect. There are back-reactions of volume modulus and fluxes on the dynamics of the mobile branes. One expect the total inflationary potential to be

$$V = V_K + V_A + V_{D\bar{D}} = \frac{1}{2}\beta H^2\phi^2 + 2T_3 h_A^4 \left(1 - \frac{1}{N_A}\frac{\phi_A^4}{\phi^4}\right)$$

H. F. and H. Tye, hep-th/0501099

where β indicates the unknown corrections from the bulk.

The slow-roll parameter is

$$\eta = M_P^2 \frac{V''}{V} = \frac{\beta}{3} - \frac{20}{N_A}\frac{{M_P}^2\phi_A^4}{\phi^6}$$

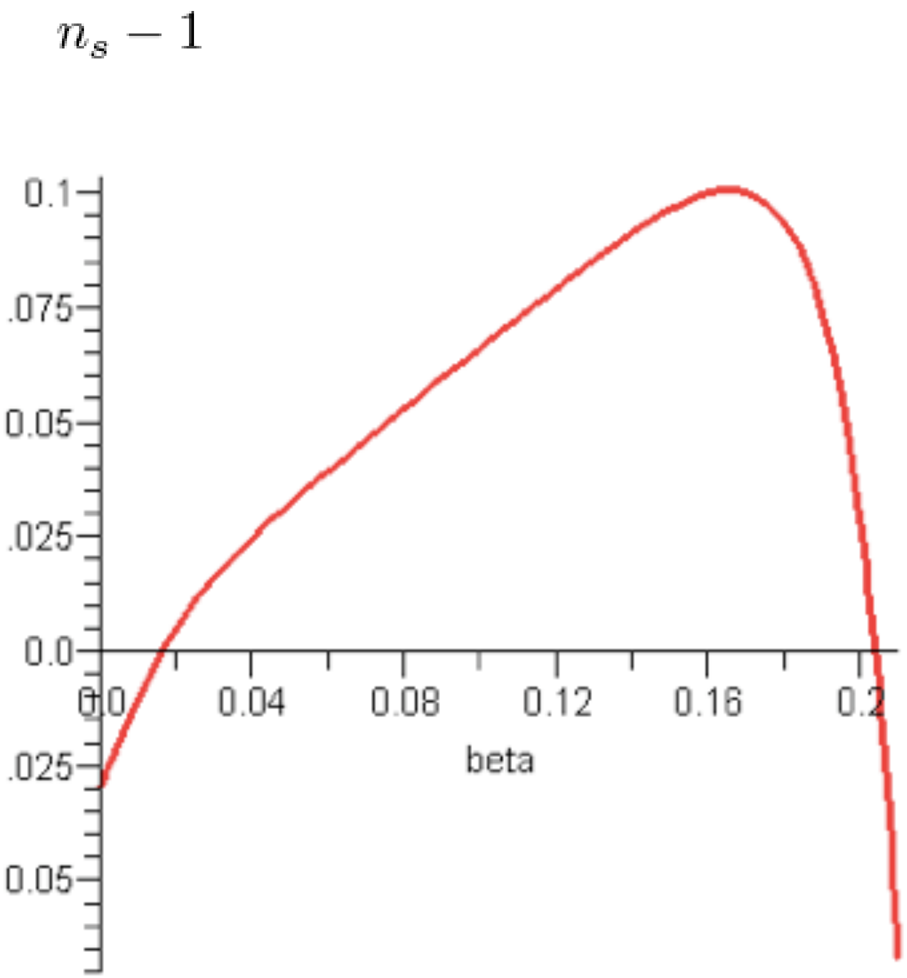
Inflation ends when $\eta \sim -1$. This can be used to determine the end value of inflaton field

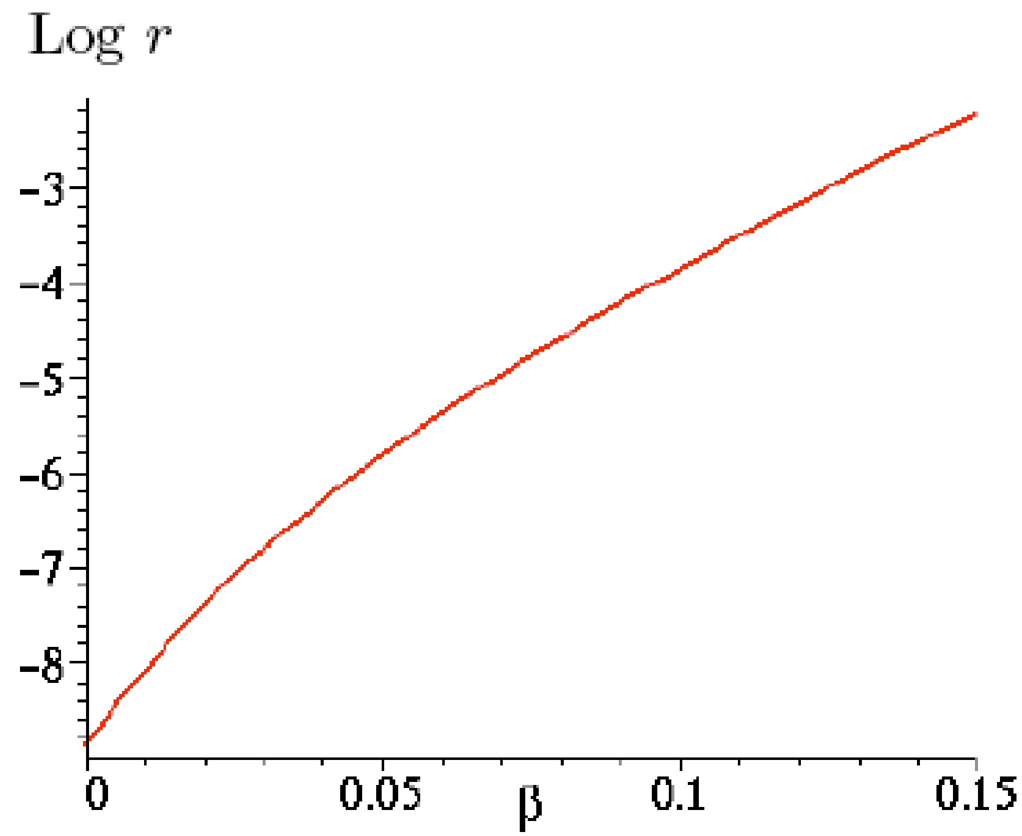
$$\phi_{\mathrm{f}}^6 = \frac{1}{(1+\beta/3)}\left(\frac{20}{N_A} M_{\mathrm{P}}^2 \phi_A^4\right).$$

Calculating the physical observables quantities, one has

$$\begin{aligned} \delta_H &\equiv \frac{1}{\sqrt{75}\pi}\frac{1}{M_{\mathrm{P}}^3}\frac{V^{3/2}}{V'} \\ &= \left(\frac{2^{11}}{3\times 5^6 \times \pi^4}\right)^{1/6} N_{\mathrm{e}}^{5/6}\left(\frac{T_3}{M_{\mathrm{P}}^4} h_A^4\right)^{1/3} f(\beta)^{-2/3} \end{aligned}$$

$$f(\beta) \simeq \left[\frac{2\beta N_{\mathrm{e}}}{\mathrm{e}^{2\beta N_{\mathrm{e}}} - 1}\right]^{5/4} \mathrm{e}^{3\beta N_{\mathrm{e}}}$$





$$n_s \sim 0.98 + \beta \qquad\qquad \log r \sim -8.8 + 60\beta$$

Basic predictions: Usually brane inflation is a small field inflation scenario:

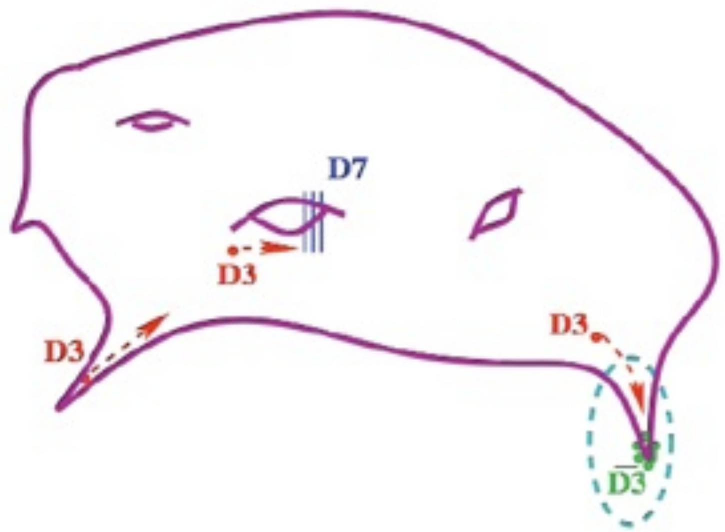
$$\frac{\Delta\phi}{M_P} < 1 \qquad \to r << 1.$$

DBI brane inflation

Alishahiha, Silverstein, Tong , hep-th/0404084
Shandera, Tye, hep-th/0601099

The action of mobile D3-brane(inflaton field) is

$$S = -T_3 \int d^4x\, a^3(t) \left[h^4 \sqrt{1 - h^{-4}\dot{\phi}^2 T_3^{-1}} + T_3^{-1} V(\phi) - h^4 \right]$$



where the inflaton potential is

$$V = \frac{m^2}{2}\phi^2 + V_0 \left(1 - \frac{vV_0}{4\pi^2\phi^4} \frac{(\gamma+1)^2}{4\gamma} \right)$$

Here $\gamma(\phi)$ is the Lorentz factor indicating the ultra-relativistic motion of the brane:

$$\gamma(\phi) = \frac{1}{\sqrt{1 - h^{-4}\dot{\phi}^2 T_3^{-1}}}$$

The energy density and the pressure is

$$\rho = T_3 h^4 (\gamma - 1) + V$$
$$p = T_3 h^4 (\gamma - 1)/\gamma - V$$

Now the cosmological evolutions are given by

$$3H^2 = \frac{1}{M_p^2}\rho \qquad\qquad 2\frac{\ddot{a}}{a} + H^2 = -\frac{1}{M_p^2}p$$

$$\ddot{\phi} - \frac{6h'}{h}\dot{\phi}^2 + 4T_3h^3h' + \frac{3H}{\gamma^2}\dot{\phi} + \left(V' - 4T_3h^3h'\right)\frac{1}{\gamma^3} = 0$$

$$\frac{\ddot{a}}{a} = \frac{V}{3M_p^2} - \frac{T(\gamma + 2 - 3/\gamma)}{6M_p^2}$$ ← The kinetic energy contributes negatively

It is more convenient to use ϕ as the clock:

$$V(\phi) = 3M_p^2H(\phi)^2 - T_3h(\phi)^4(\gamma(\phi) - 1)$$
$$\gamma(\phi) = \sqrt{1 + 4M_p^4T_3^{-1}h(\phi)^{-4}H'(\phi)^2}$$
$$\dot{\phi}(\phi) = \frac{-2M_p^2H'(\phi)}{\gamma(\phi)}$$

Cosmological Perturbation Theory

Now we allow for the perturbations in metric and inflaton field

$$ds^2 = -(1+2\Phi)dt^2 + a(t)^2(1-2\Phi)dx^2 \qquad \phi = \phi(t) + \delta(x,t)$$

In terms of gauge invariant curvature perturbation $\zeta = \frac{H}{\dot{\phi}}\delta + \Phi$

the dynamics of perturbation in momentum space is

$$\frac{d^2 u_k}{d\tau^2} + \left(\frac{k^2}{\gamma^2} - \frac{1}{z}\frac{d^2 z}{d\tau^2}\right) u_k = 0$$

where $z = \frac{a\dot{\phi}\gamma^{3/2}}{H}$ $\qquad u = \zeta z$

Now the fluctuations propagate with the speed of sound

$$c_s^2 = \frac{\partial p}{\partial \dot{\phi}} / \frac{\partial \rho}{\partial \dot{\phi}} = \frac{1}{\gamma^2}$$

The power spectrum of the curvature perturbation is

$$\mathcal{P}_R^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}}|\frac{u_k}{z}| = 2^{\nu-3/2}\frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})}(1-\epsilon_D)^{\nu-1/2}\frac{H^2}{2\pi|\dot{\phi}|}|_{k=aH\gamma}$$

where the modified slow-roll parameters are defined by

$$\epsilon_D \equiv \frac{2M_p^2}{\gamma}\left(\frac{H'(\phi)}{H(\phi)}\right)^2$$
$$\eta_D \equiv \frac{2M_p^2}{\gamma}\left(\frac{H''(\phi)}{H(\phi)}\right)$$
$$\kappa_D \equiv \frac{2M_p^2}{\gamma}\left(\frac{H'}{H}\frac{\gamma'}{\gamma}\right)$$

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon_D)$$

the spectral index is

$$n_s - 1 = \frac{d\ \ln \mathcal{P}_R}{d\ \ln\ k} \sim (1+\epsilon_D+\kappa_D)(-4\epsilon_D+2\eta_D-2\kappa_D)\cdot$$

and the standard consistency relation is modified into

$$r = \frac{16\epsilon_D}{\gamma}$$

Different Limits:

1- Slow-roll $\quad \gamma_{60} \simeq \gamma_1 \simeq 1 \quad\quad 0 \leq \beta \leq 0.02$

negligible gravity wave, no non-gaussianity

2-Intermediate $\quad \gamma_{60} \gg 1 \quad , \quad \gamma_1 \simeq 1$

there can be appreciable gravity wave, no significant non-gaussianity

3-Ultra-relativistic $\quad \gamma_{60} \gg 1 \quad , \quad \gamma_1 \gg 1$

significant non-gaussianity $\quad f_{NL} = 0.32\gamma^2 < 300$

f_{NL} measures 3-point function

P. Creminelli, A. Nicolis, I. Senatore, M. Tegmark, M. Zaldarriaga astro-ph/0509029

Ultra-relativistic limit

In this limit one can check that $H \propto \phi$ is a solution and

$$H(\phi) = \frac{\hat{m}}{M_P}\frac{\phi}{6}$$
$$\gamma(\phi) = \sqrt{\frac{2\lambda}{3}}\frac{\hat{m} M_P}{\phi^2}$$

$$\lambda = T_3 R^4 \propto N v$$

where

$$\hat{m}^2 = m^2 + 2M_P m\sqrt{\frac{2}{3\lambda}}\left(1+\frac{2M_P^2}{3m^2\lambda}\right)^{1/2} + \frac{4M_P^2}{3\lambda}.$$

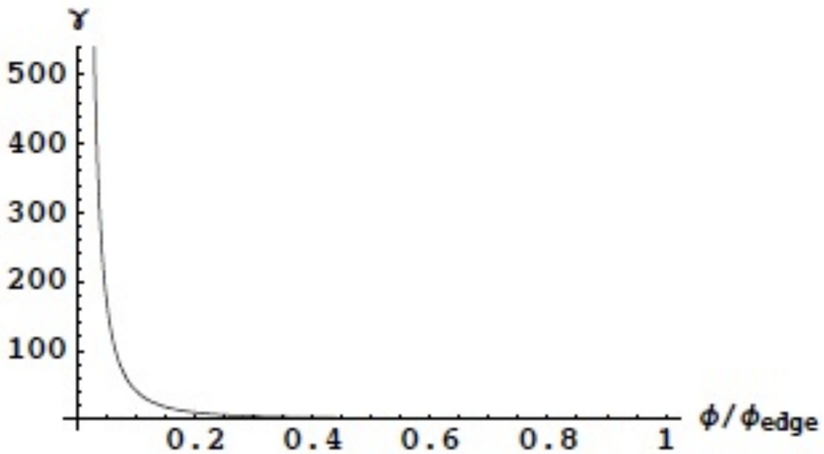
One can check that

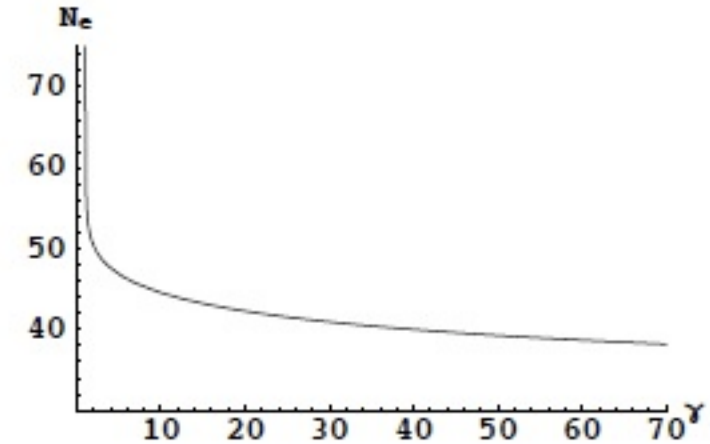
$$N_e \simeq \frac{\sqrt{\lambda}\, m}{\sqrt{6}\, M_P}\ln\left(\frac{\phi_i}{\phi_f}\right) \qquad \delta_H \simeq \frac{N_e^2}{5\pi\sqrt{\lambda}}.$$

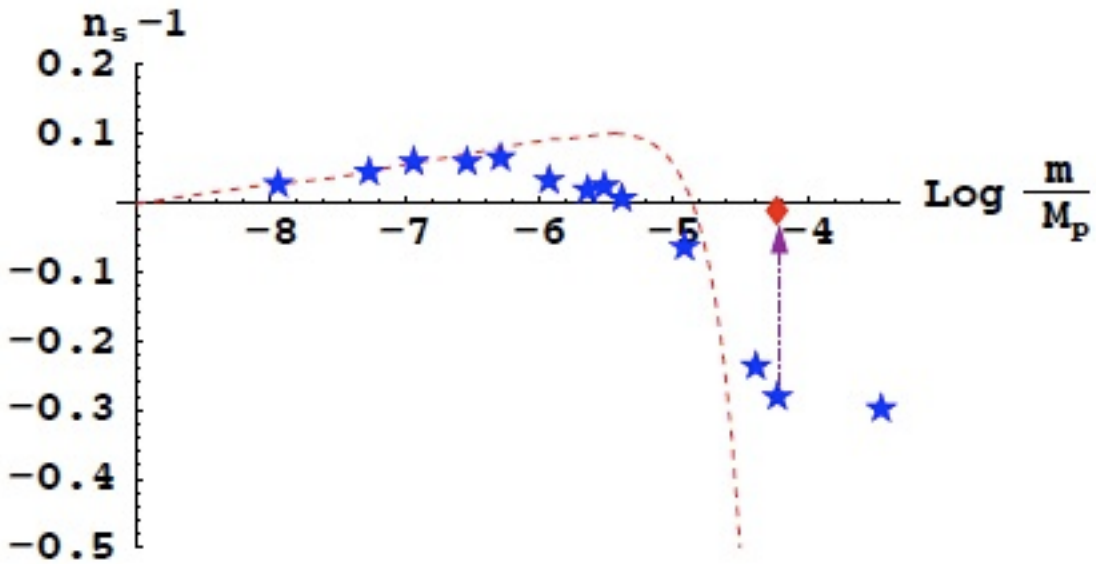
With $\delta_H \simeq 2\times 10^{-5}$ and $N_e \simeq 60$ to solve the horizon and the flatness problem

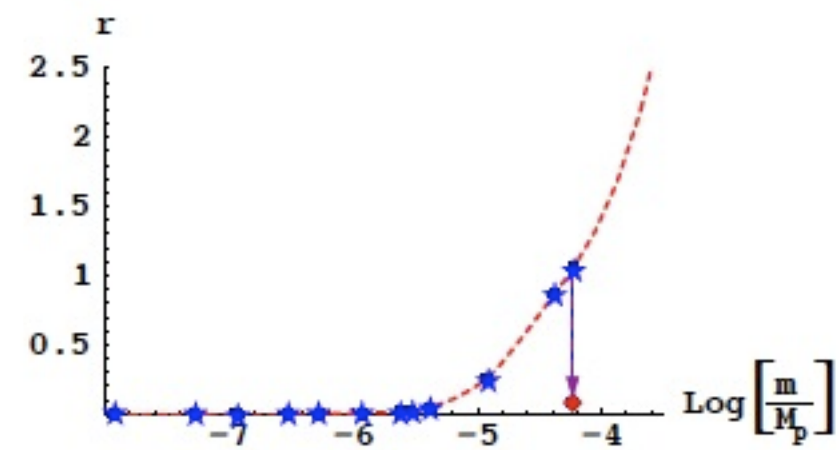
$$\lambda \sim N v \sim 10^{14}$$

Shandera, Tye,   hep-th/0601099









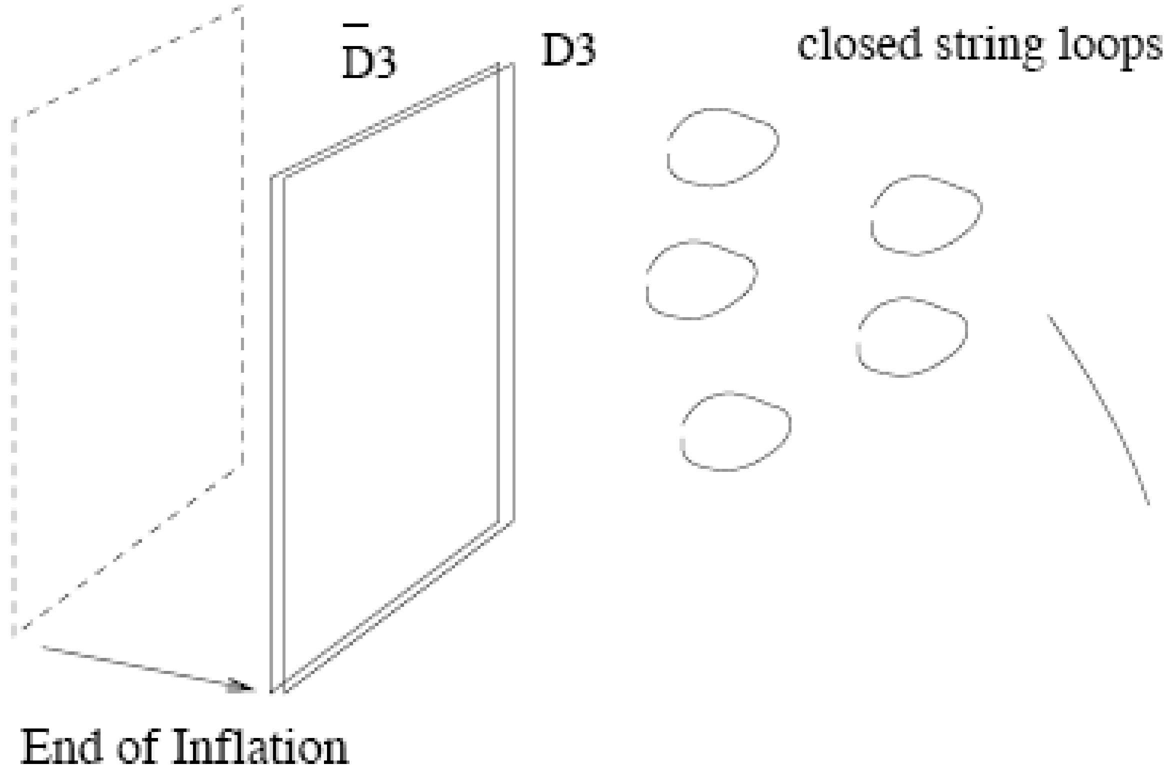
$$f_{NL} \simeq \left(\frac{m}{M_p}\right)^2 \left(\frac{M_p}{m_s h_A}\right)^4$$

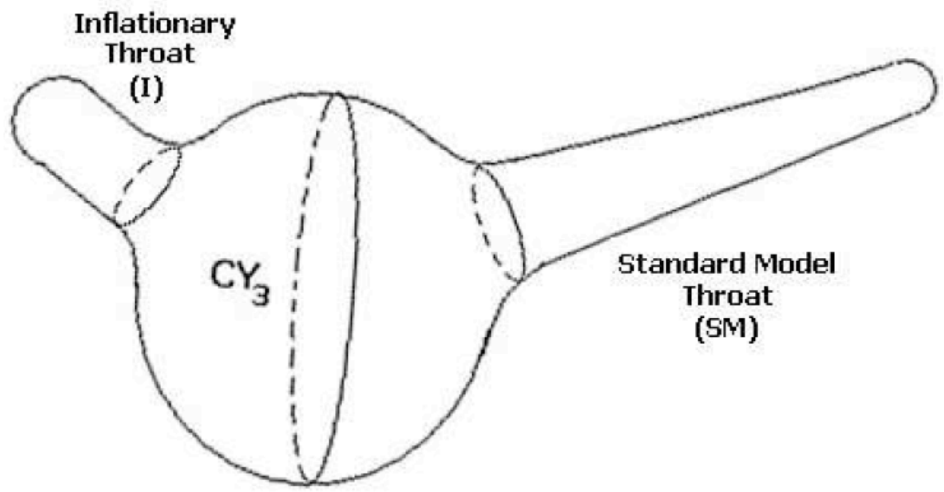
$$\lambda \sim Nv \sim 10^{14}$$

$$\frac{m}{M_p} \simeq 10^{-6}$$

Reheating in Brane Inflation

- At the end of brane inflation all energy stored in brane&anti-brane system is transferred to closed string modes.
- In the brane world picture, it is assumed tha the Standard Model of particle physics is confined on some D3-branes.
- Reheating Problem: There should exist a mechanism to channel the energy from the closed strings modes to the SM fields.
- Multi-throat compactification: Usually one can create many warped throats in CY. Different throats can have different energy scales. In one throat the inflation can take place while in another throat the SM fields are located.





Kofman and Yi

String model Parameters

H. Tye

- string scale $M_s^2 = 1/\alpha'$
- string coupling g_s
- warp factor h_A
- Number of D3 flux N_A
- inflaton mass m
- When r saturates the observational bound, non-Gaussianity is small, and vice versa.

Cosmological parameters :

$$\delta_H, n_s, dn_s/d\ln k, r, n_t, f_{NL}, G\mu$$

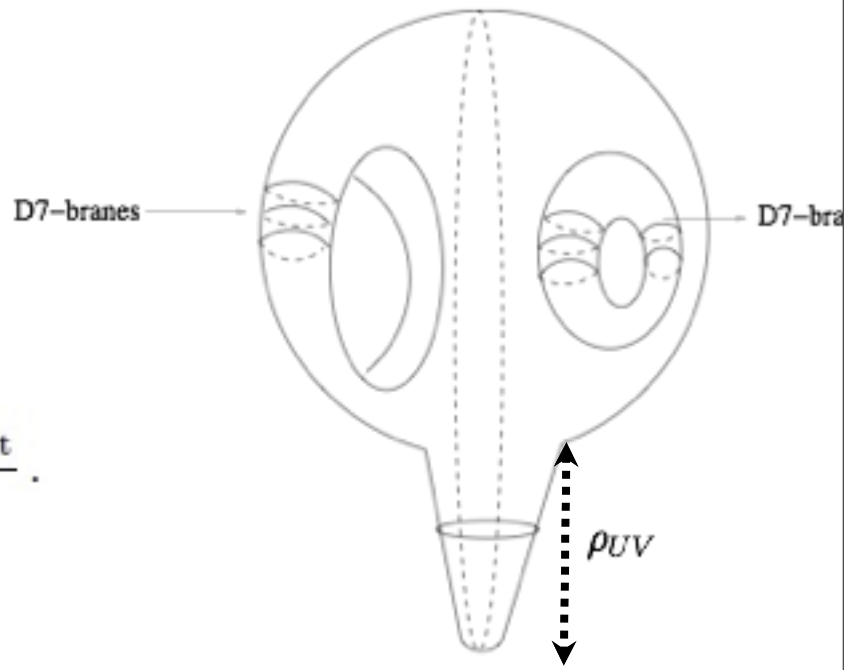
Baumann-McAllister Bound

Starting with the ten-dimensional action

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}R + \ldots \qquad \kappa_{10}^2 \equiv \tfrac{1}{2}(2\pi)^7 g_s^2(\alpha')^4.$$

The effective four dimensional Plank mass is given by

$$M_P^2 \equiv \frac{V_6^w}{\kappa_{10}^2} \qquad V_6^w \equiv (V_6^w)_{\rm bulk} + (V_6^w)_{\rm throat}\,.$$



This gives an upper bound on the value of M_P

$$M_P^2 > \frac{(V_6^w)_{\rm throat}}{\kappa_{10}^2}\,.$$

For the warped model studied so far

$$ds^2 = h(r)^2\left(-dt^2 + a(t)^2 d\mathbf{x}^2\right) + h(r)^{-2}dr^2 + R^2 ds_5^2 \qquad h(r) = \frac{r}{R}$$

one has

$$(V_6^w)_{\rm throat} = 2\pi^4 g_s N(\alpha')^2\,\rho_{UV}^2\,.$$

Noting that $\phi = \sqrt{T_3} r$ and $T_3 \equiv \frac{1}{(2\pi)^3} \frac{1}{g_s (\alpha')^2}$

this leads to

$$\left(\frac{\Delta\varphi}{M_P}\right)^2 < \frac{T_3 \rho_{UV}^2}{M_P^2} < \frac{T_3 \kappa_{10}^2 \rho_{UV}^2}{(V_6^w)_{\mathrm{throat}}}$$

which implies $$\left(\frac{\Delta\varphi}{M_P}\right)^2 < \frac{4}{N}.$$

Combined with the Lyth bound $$r_{\mathrm{CMB}} = \frac{8}{(\mathcal{N}_{\mathrm{eff}})^2} \left(\frac{\Delta\varphi}{M_P}\right)^2.$$

One obtains

$$\frac{r_{\mathrm{CMB}}}{0.009} \lesssim \frac{4}{N}.$$

For DBI case this changes into

$$N < r f_{NL}$$

With r < 0.2 from WMAP and $f_{NL} < 100$ leads to N < 40.

An overview of brane inflation

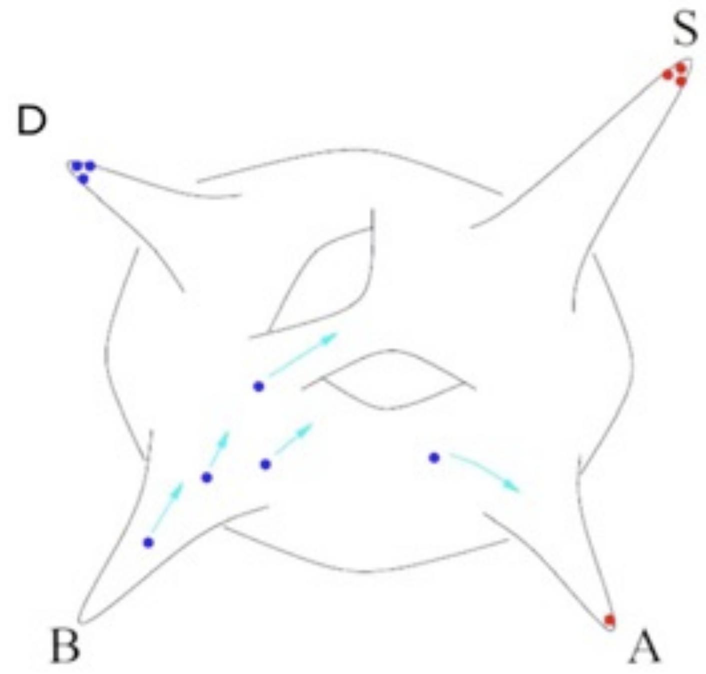
Combining the ideas studied so far, one can obtain different variation of brane inflation:

1- Single throat slow-roll brane inflation

2- multiple throat slow-roll brane inflation

3- Slow-roll + DBI brane inflation

4- D3-D7 brane inflation in the bulk or in the throat



How to embed SM in models of brane inflation? It multiple throats scenario one can consider that in some throat brane inflation takes place while in other throats SM is located.

The question of reheating in brane inflation? This seems to be challenging, specially in multi-throat brane inflation.

Cosmic Strings in Brane Inflation

- At the end of brane inflation cosmic strings are copiously produced (S. Sarangi and H. Tye, hep-th/0204074).
- Both fundamental strings (F-strings) and D1-brane (D-strings) are formed. A (p,q) string is the bound state of p F-strings and q D-strings.
- They can be detected via lensing, their effects on CMB or by gravitational wave bursts.
- The most important quantity in cosmic strings observations is their tension in term of Planck mass, $G\,\mu$.
- Constraints on $G\,\mu$

CMB :	$G\mu \leq 3 \times 10^{-7}$	Wyman, Pogosian, Wassarman
Pulsar timing	$G\mu \leq 1.5 \times 10^{-8}$	Jenet et al, astro-ph/0609013

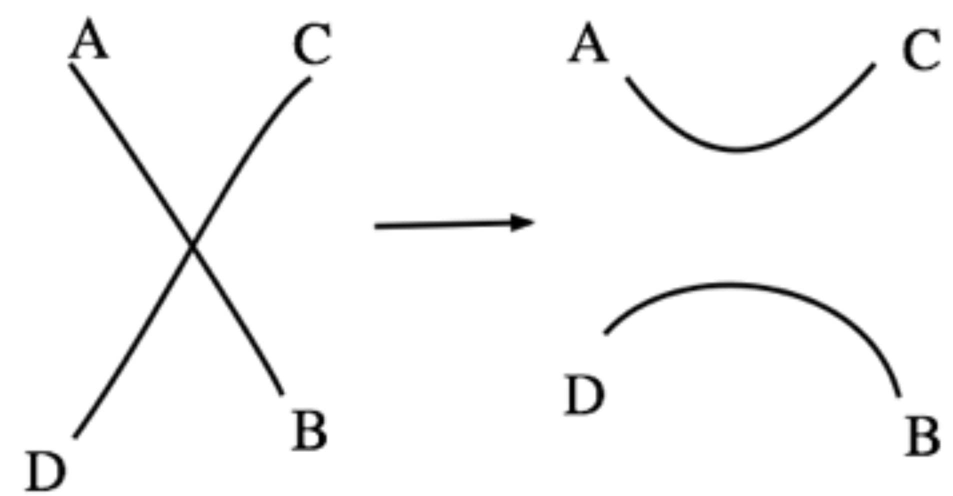
- In KKLMMT, in the absence of conformal coupling, one obtains

$$G\mu = 4 \times 10^{-10}\,.$$

H.F. and H. Tye , hep-th/0501099

Cosmic superstrings evolution

The evolution of the cosmic strings network crucially depends on the intercommutaion probability, P . For ordinary gauge strings, P is equal to unity: the strings exchange partners.



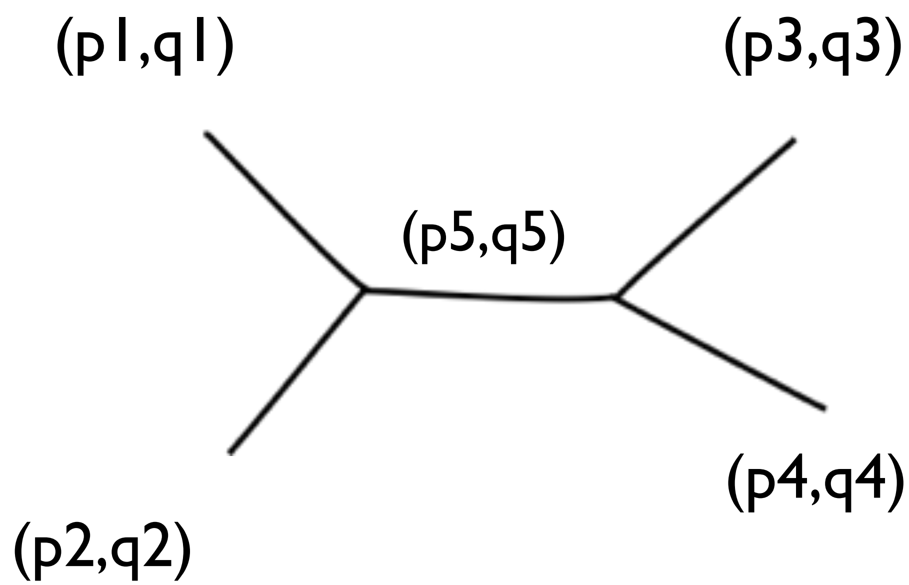
For cosmic superstrings in flat background, P is calculated by Jackson, Jones & Polchinski, hep-th/ 0405229 :

F-F : $P \sim g_s^2 \quad \rightarrow \quad 10^{-3} < P < 1$

D-D : The result depends on the details of the compactification. One can obtain

$$10^{-3} < P < 1$$

For cosmic superstrings of different types, the situation is quite different. Instead of exchanging the partners, they form (p,q) strings at junction(Copeland, Kibble and Steer, hep-th/0611243).



$$\sum_i p_i = 0 \qquad \sum_i q_i = 0$$

The formation of junctions may put strong bounds on network evolution. For example, the strings may combine to form a web of strings and freeze. So it may dominates the energy density and overclose the universe.

It may have novel effects in lensing (Shlaer&Wyman, hep-th/0509177) . One may find three or more identical images, or identical images along with images where only parts of the objects are present.

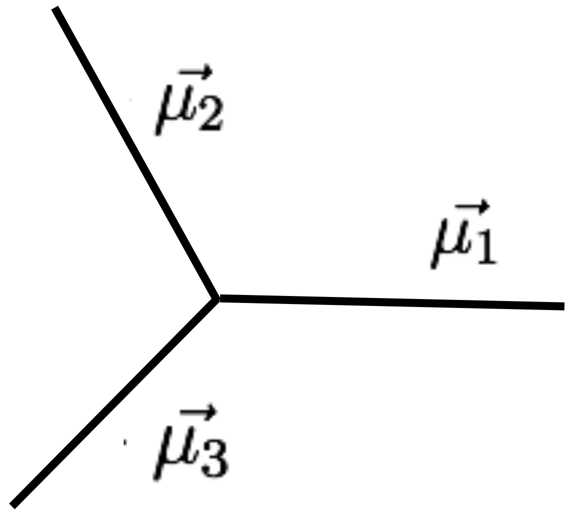
Strings at a stationary junction

R. Brandenberger, H.F. , J. Karouby,
arXiv:0710.1636

Consider N semi-infinite strings at an stationary junction.

$$S = -\sum_i \mu_i \int dt\, d\sigma \sqrt{-|\gamma_i|}\,,$$

$$\gamma_{i\,mn} = g_{\mu\nu}\partial_m X_i^{\mu}\,\partial_n X_i^{\nu}\,.$$



The condition for the junction to be stationary
is the force balance condition:

$$\vec{\mu_1} + \vec{\mu_2} + \vec{\mu_3} = 0$$

We would like to find the geometry around this configuration:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,,$$

Solving The Einstein equation, one obtains

$$h_{ab} = 4\,G \sum_i \mu_i \left(\delta_{ab} - n_{i\,a}\, n_{i\,b}\right) \ln\left(\frac{r - \vec{r}\,.\,\vec{n}_i}{r_0}\right)$$

Example: Consider an infinite strings, extended along the z-direction. In our formalism, this corresponds to:

$$\vec{n}_1 = -\vec{n}_2.$$

The non-zero components of the metric are:

$$\begin{aligned} h_{xx} = h_{yy} &= 4\,G\,\mu\,\ln\left(\frac{r^2 - (r.\vec{n}_1)^2}{r_0^2}\right) \\ &= 8\,G\,\mu\,\ln\left(\frac{r_\perp}{r_0}\right)\,. \end{aligned}$$

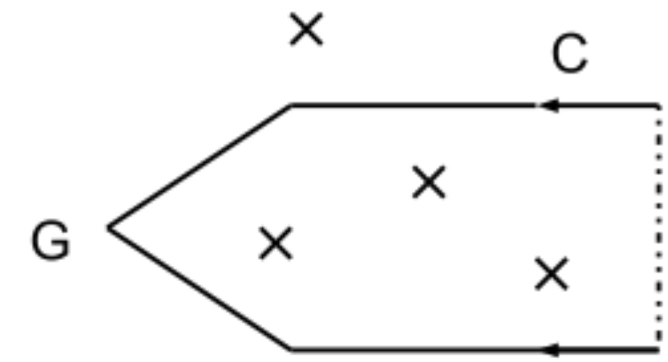
The flatness of the geometry

There are six non-zero components of the Riemann tensor. In 3-dimensions, they can be expressed in terms of Ricci tensor. For example:

$$R_{xyxy} = \frac{1}{2}(R_{zz} - R_{xx} - R_{yy})$$

This explicitly shows that the geometry is flat. One can check that this is due to the force balance condition.

CMB anisotropies and lensing



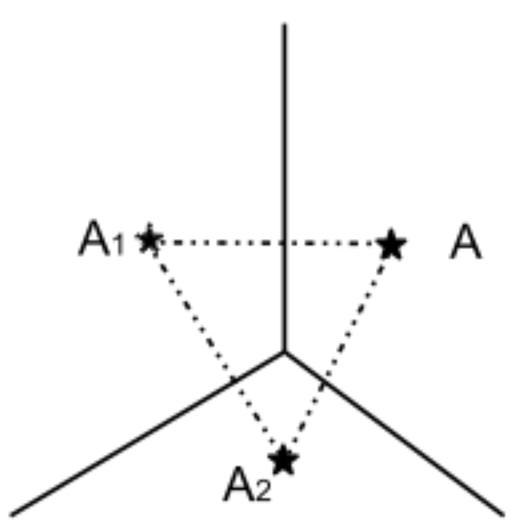
Consider two parallel light arrays are emitted from infinity. The relative change in their velocities is

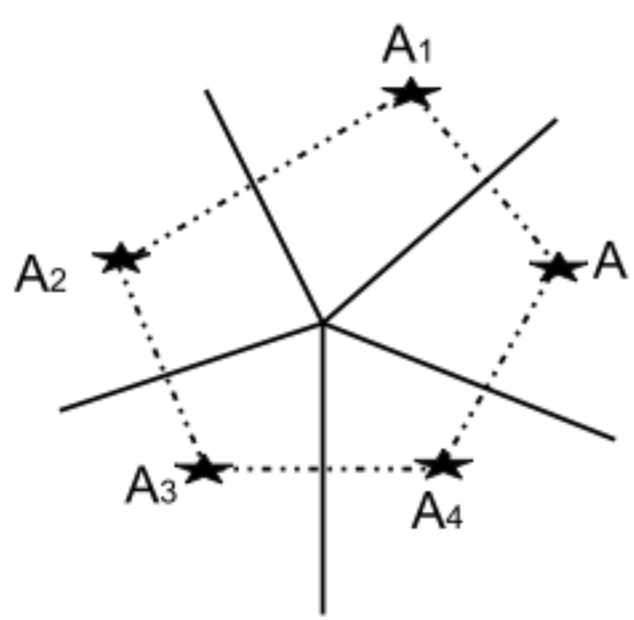
$$\delta v^{\alpha} = -\frac{1}{2}\int_S R^{\alpha}_{\beta\gamma\lambda}\, v^{\beta}\, dx^{\gamma}\wedge dx^{\lambda}$$

This gives $$\delta\vec{v} = -8\pi\, G\, \vec{k}\times\sum_i \vec{\mu}_i$$

The angle between the light rays is $$\Delta = |\delta\vec{v}| = 8\pi\, G\, \mu_{eff}\,, \qquad \mu_{eff} = |\vec{k}\times\sum_i \vec{\mu}_i|\,.$$

Lensing





CMB anisotropies:

Relativistically moving strings produce CMB anisotropies on CMB

$$\frac{\delta T}{T} = G\mu\gamma(v)v$$

For anisotropies induced by a moving junction we have

$$\delta\omega = \omega \int R_{0zzy}\, dy\, dz\,.$$

This gives

$$\frac{\delta\omega}{\omega} = 8\pi G\,\gamma \left| \vec{v}\,.\,(\vec{k} \times \sum_i \vec{\mu}_i) \right|$$

The details of the picture depends on the direction of line of sight and the direction of light arrays. The observer may see a junction with legs of different temperature anisotropies.

Matrix Inflation(M-Flation)

Suppose inflation is driven by non-commutative matrices:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\sum_i \mathrm{Tr}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i\right) - V(\Phi_i,[\Phi_i,\Phi_j])\right)$$

Example :

$$V = \mathrm{Tr}\left(-\frac{\lambda}{4}[\Phi_i,\Phi_j][\Phi_i,\Phi_j] + \frac{i\kappa}{3}\epsilon_{jkl}[\Phi_k,\Phi_l]\Phi_j + \frac{m^2}{2}\Phi_i^2\right)$$

where Φ_i are $N \times N$ matrices.

The equations of motion are

$$H^2 = \frac{1}{3M_P^2}\left(-\frac{1}{2}\mathrm{Tr}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i\right) + V(\Phi_i,[\Phi_i,\Phi_j])\right)$$

$$\ddot{\Phi}_l + 3H\dot{\Phi}_l + \lambda\,[\Phi_j,\,[\Phi_l,\Phi_j]\,] - i\,\kappa\,\epsilon_{ljk}[\Phi_j,\Phi_k] + m^2\Phi_l = 0$$

Truncation to SU(2)sector

$$\Phi_i = \hat{\phi}(t) J_i \ , \qquad i = 1, 2, 3.$$

where J_i are the N-dimensional irreducible representation of SU(2) algebra.

$$[J_i, J_j] = i\,\epsilon_{ijk} J_k \ , \qquad \mathrm{Tr}(J_i\, J_j) = \frac{N}{12}(N^2 - 1)\,\delta_{ij} \, .$$

Plugging this ansatz into the action, we obtain

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \mathrm{Tr} J^2 \left(-\frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{\lambda}{2} \hat{\phi}^4 + \frac{2\kappa}{3} \hat{\phi}^3 - \frac{m^2}{2} \hat{\phi}^2 \right) \right]$$

where $\mathrm{Tr} J^2 = \mathrm{Tr}(J_i^2) = N(N^2 - 1)/4.$

Upon field re-definition

$$\hat{\phi} = \left(\mathrm{Tr} J^2\right)^{-1/2} \phi = \left[\frac{N}{4}(N^2-1)\right]^{-1/2} \phi$$

The effective potential is

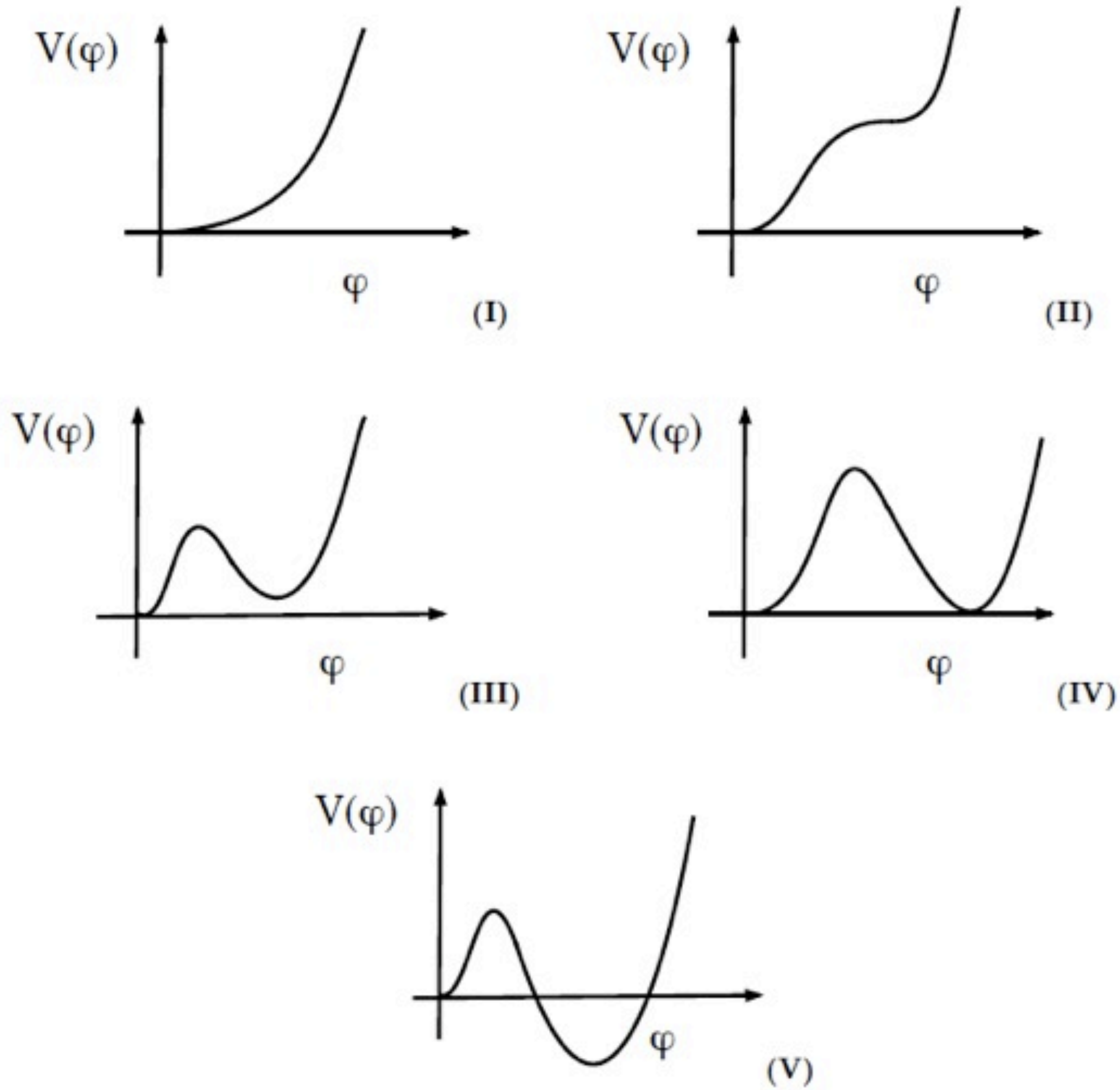
$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$

Where

$$\lambda_{eff} = \frac{2\lambda}{\mathrm{Tr} J^2} = \frac{8\lambda}{N(N^2-1)}\,, \quad \kappa_{eff} = \frac{\kappa}{\sqrt{\mathrm{Tr} J^2}} = \frac{2\,\kappa}{\sqrt{N(N^2-1)}}$$

The Shape of the Potential

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$



- **Case I)** $\kappa^2 < 2m^2\lambda$.

$$\sin^2\Theta \equiv \frac{2m^2\lambda}{\kappa^2} \qquad \Theta \in [0, \pi/2].$$

- **Case II)** $\Theta = \frac{\pi}{2}$.
- **Case III)** $\sin^{-1}(\frac{8}{9}) < \Theta < \frac{\pi}{2}$.
- **Case IV)** $\Theta = \sin^{-1}(\frac{8}{9})$.
- **Case V)** $0 \leq \Theta < \sin^{-1}(\frac{8}{9})$.

Examples:

1- Chaotic Inflation $V = \frac{1}{4}\lambda_{eff}\,\phi^4$: $m = \kappa = 0$

To fit the CMB observation, we need

$$\lambda_{eff} \sim 10^{-14} \qquad \Delta\phi \sim 10 M_P.$$

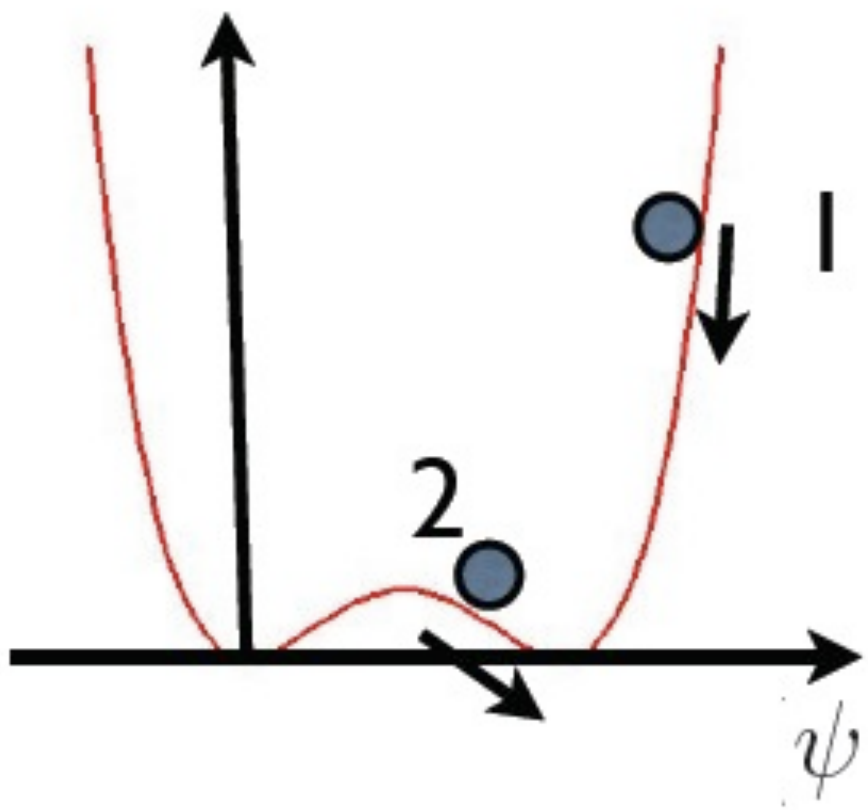
On the other hand $\lambda_{eff} \sim \lambda N^{-3}$ $\Delta\hat{\phi} \sim N^{-3/2}\Delta\phi$

One obtains $N \sim 10^5$ $\Delta\hat{\phi} \sim 10^{-7} M_P$

Due to large running of field values, a considerable amount of gravity waves can be produced. $r \simeq 0.26$

The scalar spectral index is $n_{\mathcal{R}} \simeq 0.949$

2- Symmetry breaking potential:



$$V_0 = \frac{\lambda_{eff}}{4}\phi^2(\phi-\mu)^2 \qquad \mu \equiv \sqrt{2}m/\sqrt{\lambda_{eff}}.$$

I: $\phi_i > \mu$

To fit the observational constraints

$$\phi_i \simeq 43.57 M_P \quad , \quad \phi_f \simeq 27.07 M_P \quad , \quad \mu \simeq 26 M_P.$$

$$\lambda_{eff} \simeq 4.91 \times 10^{-14}, \quad m \simeq 4.07 \times 10^{-6} M_P, \quad \kappa_{eff} \simeq 9.57 \times 10^{-13} M_P.$$

II: $\mu/2 < \phi_i < \mu$

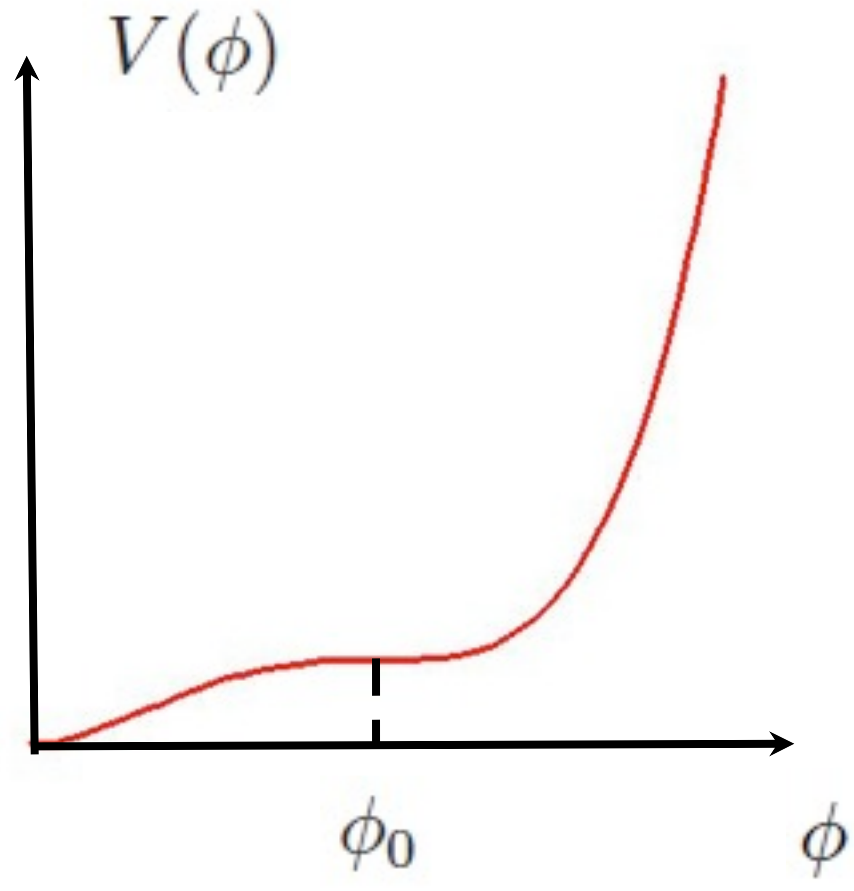
$$\phi_i \simeq 23.5 M_P \quad , \quad \phi_f \simeq 35.03 M_P \quad , \quad \mu \simeq 36 M_P.$$

$$\lambda_{eff} \simeq 7.18 \times 10^{-14}, \quad m \simeq 6.82 \times 10^{-6} M_P, \quad \kappa_{eff} \simeq 1.94 \times 10^{-12} M_P.$$

One finds $N \sim 10^5.$ $\Delta\hat{\phi} \sim 10^{-7} M_P$

3-Saddle-point Inflation

$\kappa = \sqrt{2\lambda}\, m$



$$V(\phi) \simeq V(\phi_0) + \frac{1}{3!} V'''(\phi_0)(\phi - \phi_0)^3$$

$$V(\phi_0) = \frac{m^2}{12}\phi_0^2 \quad , \quad V'''(\phi_0) = \frac{2m^2}{\phi_0} .$$

The CMB observables are given by

$$n_s \simeq 1 - \frac{4}{N_e} \quad , \quad \delta_H \simeq \frac{2}{5\pi} \frac{\lambda_{eff} M_P}{m} N_e^2 .$$

$$\lambda_{eff} = (\frac{9\, r}{32})^{1/3} \left(\frac{5\pi}{8} \delta_H \right)^2 (1 - n_s)^{8/3} .$$

The upper bound r < 0.2 from WMAP5, and ns=0.96, gives

$$\lambda_{eff} \lesssim 10^{-13} \text{ and } N \gtrsim 10^5$$

Reheating ?

We have not provided a mechanism of reheating where the energy from the ψ_{mn} particles are transferred into SM particles.

One scenario in M-flation in string theory: We may imagine that SM fields are localized on branes as open strings gauge fields $A_\mu^{(a)}$

This can naturally be embedded in model noting that $D_a\Phi^i = \partial_a\Phi^i + i[A_a, \Phi^i]$

As an estimate of Preheat temperature, suppose we have an instant preheating

$$N^2T^4 \sim 3H^2M_P^2$$

for large N one can get sufficiently small reheat temperature.

Non-Gaussianity?

Due to multiple-field nature of the model, there would be plenty of NG produced. It would be interesting to calculate primordial NGs and compare it with observation.

Motivation from string theory

- When N D-branes are located on top of each other the gauge symmetry enhances to U(N)

$$A_a = A_a^{(n)} T_n \qquad , \qquad F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$$

$$D_a \Phi^i = \partial_a \Phi^i + i[A_a, \Phi^i]$$

The action for N coincident branes is

$$S = -T_3 \int d^4x \, \mathrm{STr}\left(\sqrt{-|g_{ab}|}\sqrt{|Q^i_j|}\right) + \frac{\mu_3}{2}\int d^4x \, \mathrm{STr}\left([\Phi_i, \Phi_j] C^{(6)}_{ij0123}\right)$$

Where

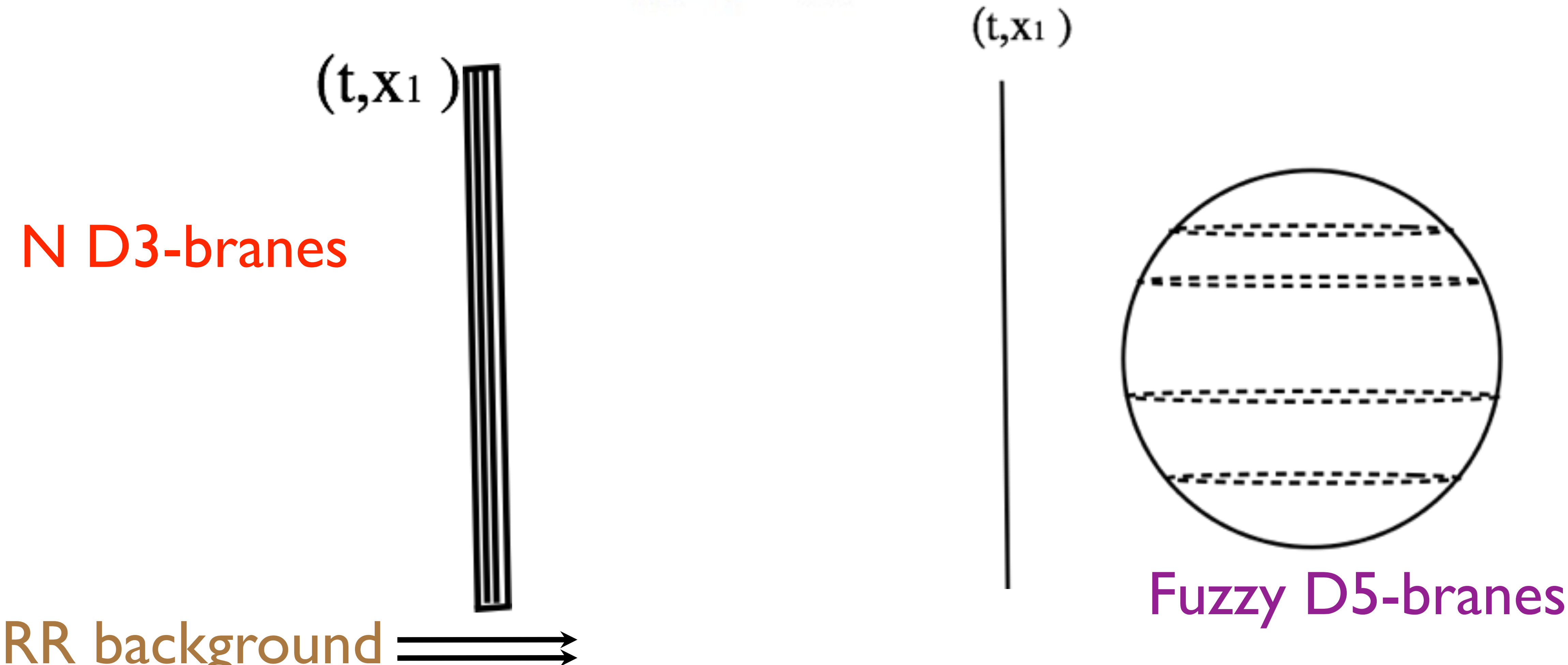
$$Q^j_k = \delta^i_j + 2\pi i \, \alpha' \, [\Phi_j, \Phi_k]$$

Consider the RR background $C^{(6)}_{jk0123} = -\frac{2i}{3}\kappa\,\epsilon_{jkl}\,\Phi_l$

Expanding the action up to leading terms, one obtains

$$S= -\frac{1}{2}\sum_i \mathrm{Tr}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i\right) -\frac{\lambda}{4}[\Phi_i,\Phi_j][\Phi_i,\Phi_j] + \frac{i\kappa}{3}\epsilon_{jkl}[\Phi_k,\Phi_l]\Phi_j$$

with $\lambda = 2\pi g_s\ ,\qquad \hat{\kappa} = \frac{\kappa}{g_s\cdot\sqrt{2\pi g_s}}$

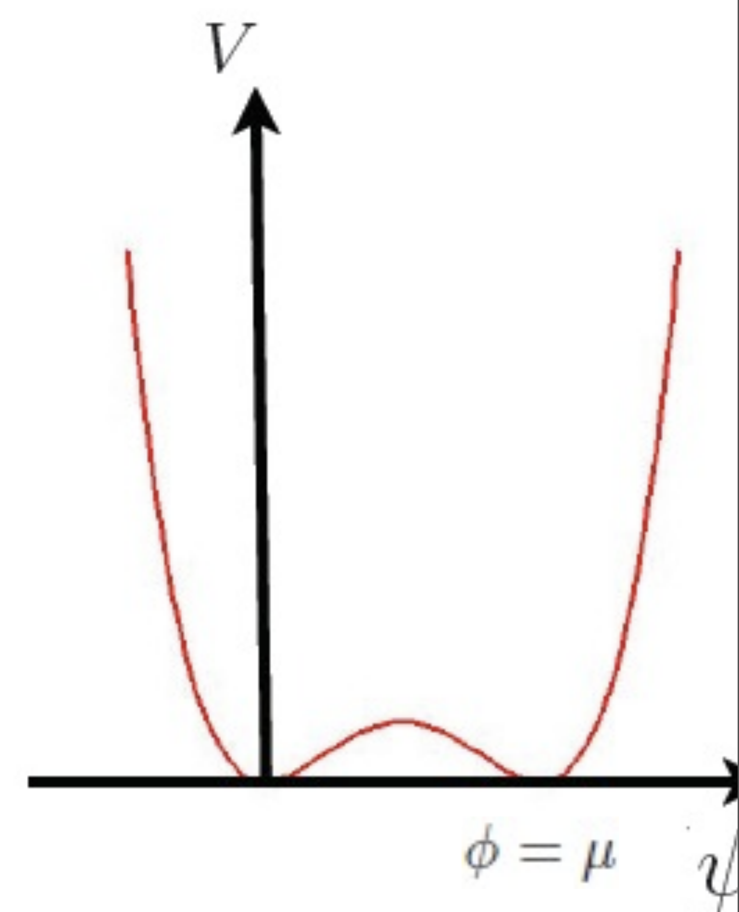


As mentioned the potential is

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$

The condition $\lambda m^2 = 4\kappa^2/9$ is required for background to be susy.

$$V_0 = \frac{\lambda_{eff}}{4}\phi^2\left(\phi - \mu\right)^2$$



The minimum $\phi = \mu$ is the susy vacuum.

This corresponds to the solution where N D-3 branes blow up into a fuzzy D5-branes.

Geometrically, ϕ is the radius of the fuzzy two-sphere.

The Landscape of Matrix Inflation

n-block Matrix Inflation: $$\Phi_i = \sum_\alpha \hat{\phi}_\alpha J_i^\alpha \quad , \quad i = 1,2,3$$

Upon rescaling the potential becomes

$$V(\phi_\alpha) = \sum_\alpha \frac{\lambda_\alpha}{4}\phi_\alpha^4 - \frac{2\kappa_\alpha}{3}\phi_\alpha^3 + \frac{m^2}{2}\phi_\alpha^2$$

Where

$$\hat{\phi}_\alpha = \left[\frac{1}{4}N_\alpha(N_\alpha^2 - 1)\right]^{-1/2} \phi_\alpha \ , \qquad \lambda_\alpha = \frac{8\lambda}{N_\alpha(N_\alpha^2-1)} \ , \qquad \kappa_\alpha = \frac{2\,\kappa}{\sqrt{N_\alpha(N_\alpha^2-1)}} .$$

The cosmological evolutions are given by

$$H^2 = \frac{1}{3M_P^2}\sum_{\alpha=1}^{n}\left(\frac{1}{2}\dot{\phi}_\alpha^2 + V_\alpha(\phi_\alpha)\right)$$

$$\ddot{\phi}_\alpha + 3H\dot{\phi}_\alpha + \partial_{\phi_\alpha} V_\alpha = 0 \ ,$$

Counting the Number of Vacua in the Landscape:

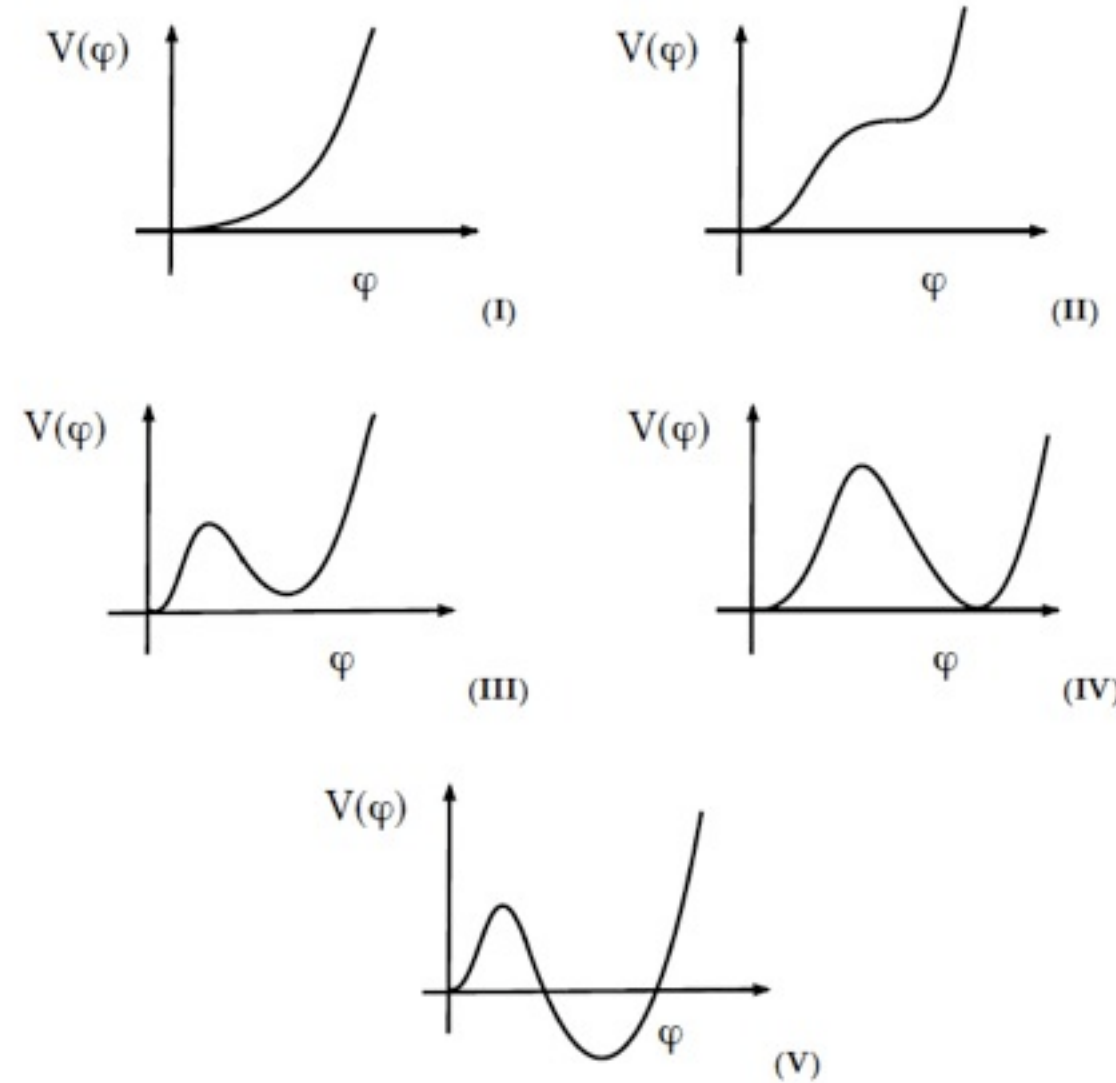
$$\sum_{i=1}^{n} N_i = N,\ 1 \leq N_i \leq N_{i+1}.$$

Single block: 1 Vacuum

Two blocks: $[\frac{1}{2}N]$ Vacuum

......

n-blocks: $\sim \frac{2^N}{N}$ Vacuum



The shape is controlled by the ratio $\kappa^2/(m^2\lambda)$
and is independent of N_α

String Theory Landscape

Linde

Conclusion

- All observations strongly support inflation as a theory of early Universe and structure formation. But there is no deep theoretical understanding of its origin.
- String theory has many light fields, the moduli. It is hoped that some of these moduli may be the inflaton field. For this to work, one must stablize the other unwanted moduli.
- Brane Inflation is an interesting realization of inflation in string theory.It predicts cosmic (super)strings which can have importnat cosmological consequences. Also in models of DBI brane inflation, significant NG is produced which can be used to constraint the model.
- Matrix Inflation is another realization of inflation in string theory which may help to solve the conventional fine-tuning problem associated with chaotic inflation.