Electron Electric Dipole Moment from Lepton Flavor Violation

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A neutral non-relativistic particle of spin S can be described by following Hamiltonian :

$$H = -\mu \mathbf{B} \cdot \frac{\mathbf{S}}{S} - d\mathbf{E} \cdot \frac{\mathbf{S}}{S}$$

P(B.S) = B.S **P(E.S) = -E.S**

T(B.S) = B.S **T(E.S) = -E.S**

In QFT, spin-1/2 particle interact with the electromagnetic tensor $F_{\mu\nu}$ through:

$$L = \frac{-i}{2} d_f \overline{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}$$

Which in the non-relativistic limit reads:

$$L = d_f \psi_A^{+} \vec{\sigma}.\vec{E} \psi_A$$

Elementary particles can possess **EDMs**, only if the CP-symmetry is violated.

For this reason, studying EDMs of the elementary particles is of prime importance as it can teach us about CP-violation which is closely related to the creation of the baryon asymmetry of the universe.

SM of elementary particles allows for CP-violation:

1. θ-term in QCD
2. CKM matrix in quark sector

In fact in the Kaon and B-meson sector, CP symmetry has been observed to be violated in accordance With the Standard Model

The maximum possible values of EDMs in the context of SM are extremely small

So far electric dipole moment for the electron or neutron has not been detected.

$$d_e \sim 10^{-38}\ e\ \mathrm{cm}$$
$$d_n \sim 10^{-31}\ e\ \mathrm{cm\ to}\ 10^{-33}$$

$\ll$

$$|d_e| < 1.4 \times 10^{-27}\ e\ \mathrm{cm}$$
$$|d_n| < 3.0 \times 10^{-26}\ e\ \mathrm{cm}.$$

Measurement of EDMs much larger than the SM prediction would indicate new sources of CP-violation with origin in physics beyond the SM.

While the Standard Model contains just one CP phase, more phases can appear in extensions of SM. For example, Higgs doublet model, little Higgs Model or Extra dimension Models and

MSSM

The Minimal Supersymmetric Standard Model (MSSM) is the most popular model beyond the SM. The general MSSM introduces 44 sources of CP-violation. Mainly for the sake of simplicity, studies in the literature are concentrated on the Constraint models for instance mSUGRA model.

Taking the values of MSSM parameters which are phenomenologically favorable (M_{SUSY} =100 GeV and phases~1), one finds that the EDMs of the electron, neutron and mercury exceed the experimental bounds by several orders of magnitude. In principle, to suppress the EDMs to below their experimental bounds, three possibilities exist:

1. The first generation slepton and the first two generation squarks are very heavy. However

- the production and study of these particles at LHC and ILC will be difficult.

- with large sfermion masses the annihilation rate of the Lightest Supersymmetric Particle (LSP) will be too low and the relic density of the LSP be larger than expected from the cosmological observations.

For these reasons, this assumption are not phenomenologically favorable .

2. The phases are zero or very small which means that there will not be any interesting display of CP-violation in colliders. Moreover, electroweak baryogenesis cannot happen in this case.

3. The contribution of the phases cancel each other. This assumption have been extensively studied in the literatures.

Recent neutrino data proves that Lepton Flavor (LF) has been violated in nature.

In the context of MSSM there are several sources for CP-violation as well as for LFV which can lead to effects exceeding the present experimental bounds.

In this paper, we will focus on the possible effects of the phase of A_τ (trilinear coupling in soft Supersymmetric term) in present LFV sources on The electric dipole moment of the electron. One of the goals of the proposed state-of -the-art ILC project is studying CP-violation. Also it is possible detecting of decay and production of stau which can be affected by phase of A_τ .

Model Building

We consider the minimal Supersymmetric Standard Model with superpotential:

$$W_{MSSM} = Y_u \widehat{u^c}\ \hat{Q}\cdot\widehat{H_u} - Y_d \widehat{d^c}\ \hat{Q}\cdot\widehat{H_d} - Y_e \widehat{e^c}\ \hat{L}\cdot\widehat{H_d} - \mu\ \widehat{H_u}\cdot\widehat{H_d}$$

The soft supersymmetry breaking at the electroweak scale

$$\begin{aligned} \mathcal{L}_{\rm soft}^{\rm MSSM} &= -\,1/2\left(M_1\widetilde{B}\widetilde{B} + M_2\widetilde{W}\widetilde{W} + \text{H.c.}\right) \\ &- (A_i Y_i \delta_{ij} + A_{ij})\widetilde{e^c_{Ri}}\ \widetilde{L_j}\cdot H_d + \text{H.c.}) - \widetilde{L_i}^\dagger\ (m^2_{\tilde{e}_L})_{ij}\widetilde{L_j} - \widetilde{e^c_{Ri}}^\dagger\ (m^2_{\tilde{e}_R})_{ij}\widetilde{e^c_{Rj}} \\ &- m^2_{H_u}\ H_u^\dagger\ H_u - m^2_{H_d}\ H_d^\dagger\ H_d - (\ B_H\ H_u\cdot H_d + \text{H.c.}), \qquad (2) \end{aligned}$$

Notice that we have relaxed universality assumption and divided the trilinear coupling to a flavor diagonal part and a LFV part.

The Hermitian mass matrix of sleptin can in general be written in this form:

$$L_{\text{slepton}} = -\begin{pmatrix} \tilde{e}_L^\dagger & \tilde{e}_R^\dagger \end{pmatrix}_i \begin{pmatrix} m_L^2 & m_{LR}^{2\dagger} \\ m_{LR}^2 & m_R^2 \end{pmatrix}_{ij} \begin{pmatrix} \tilde{e}_L \\ \tilde{e}_R \end{pmatrix}_j$$

Where i and j indices determine the flavor

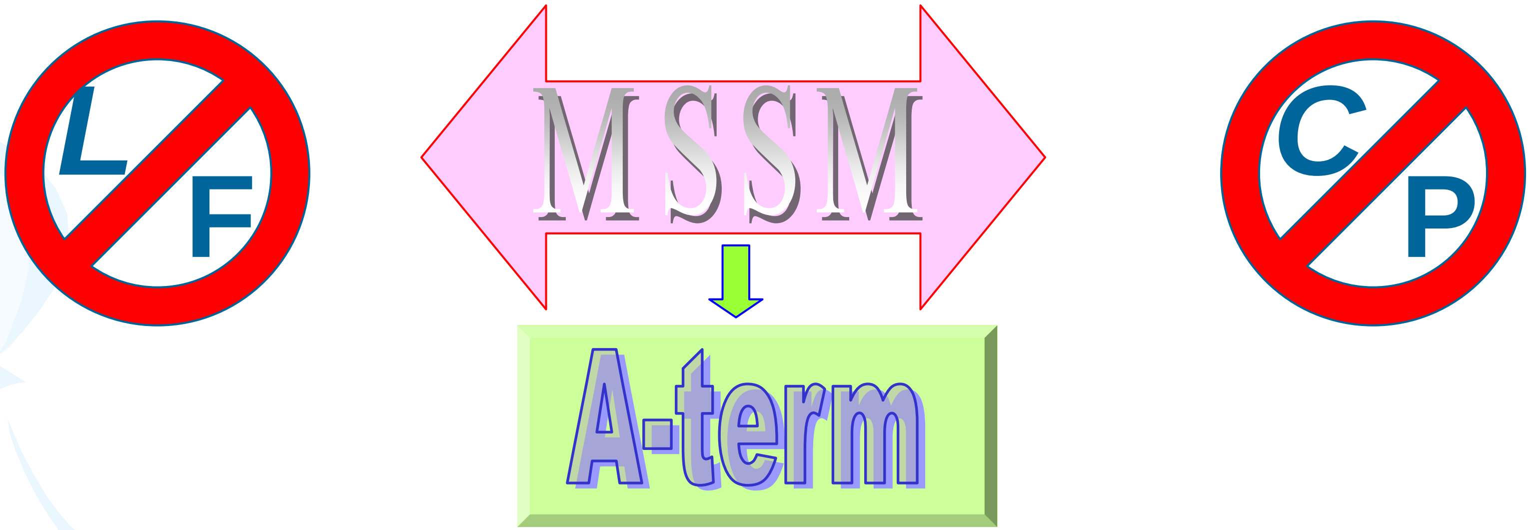
$$(m_L^2)_{ij} = (m_{\tilde{e}_L}^2)_{ij} + (m_e^2)_i \delta_{ij} + m_Z^2 \cos 2\beta \left(-\frac{1}{2} + \sin^2\theta_W\right)\delta_{ij}$$

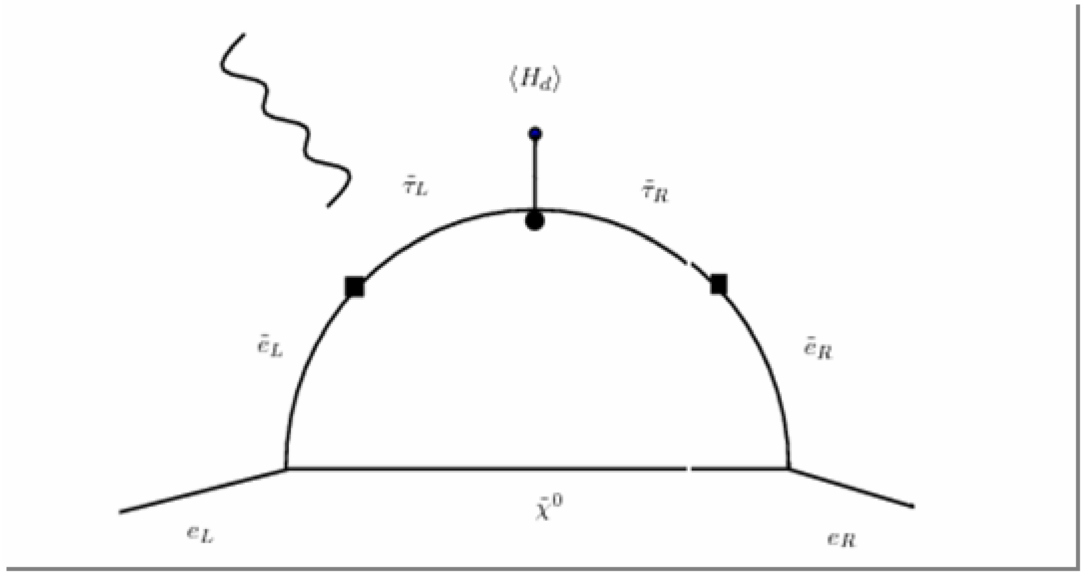
$$(m_R^2)_{ij} = (m_{\tilde{e}_R}^2)_{ij} + (m_e^2)_i \delta_{ij} - m_Z^2 \cos 2\beta \sin^2\theta_W \delta_{ij}$$

$$(m_{LR}^2)_{ij} = m_i (A_i - \mu^* \tan\beta)\delta_{ij} + A_{ij}\langle H_d \rangle$$

Notice that we consider 6 * 6 matrices for mass matrices of slepton

For LFV case, the A-term associated with a definite lepton flavor can in principle affect the EDM of a lepton of another flavor.





In the above figure, for illustrative purposes, we have displayed the mass insertion approximation to show that A_τ can contribute to d_e and Br($\tau \rightarrow e\,\gamma$). For calculating of d_e, we will use the exact formulae (without the mass insertion approximation).

In order to study the EDM of electron, we have to first consider the bounds on the LFV masses and A-terms from the bounds on the LFV decay modes of the charged leptons.

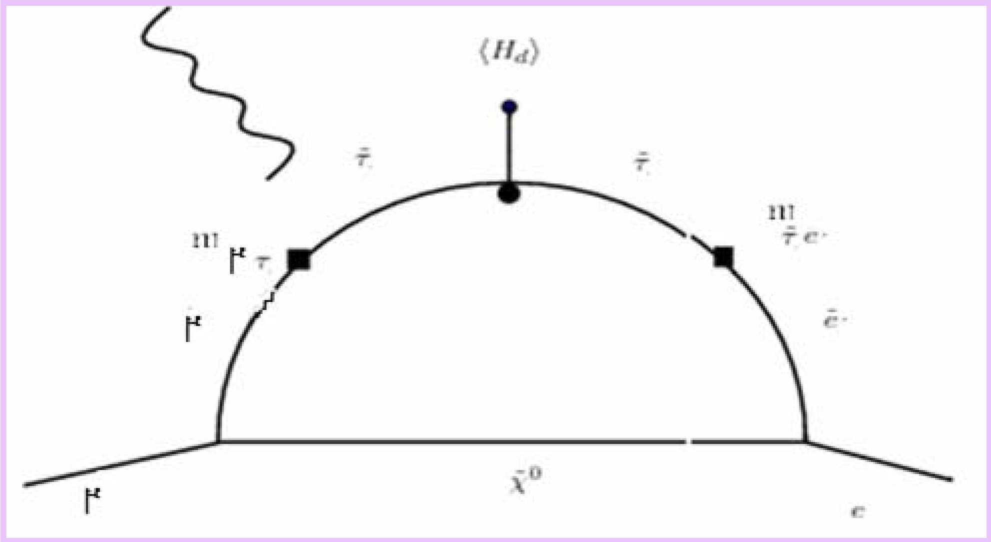
$$\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$$

Throughout this paper we will set the following LFV elements equal to zero:

$$(m_L^2)_{e\mu} = (m_R^2)_{e\mu} = 0 \quad \text{and} \quad A_{e\mu} = A_{\mu e} = 0.$$

$$\mathrm{Br}(\tau \to e\gamma) < 1.1 \times 10^{-7}$$

$$\mathrm{Br}(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$$



$$(m_L^2)_{\mu\tau} = (m_R^2)_{\mu\tau} = 0 \quad \text{and} \quad A_{\mu\tau} = A_{\tau\mu}.$$

As shown in the literature, integrating out the heavy supersymmetric states, $\tau \to e\gamma$ can be described by the following effective Lagrangian

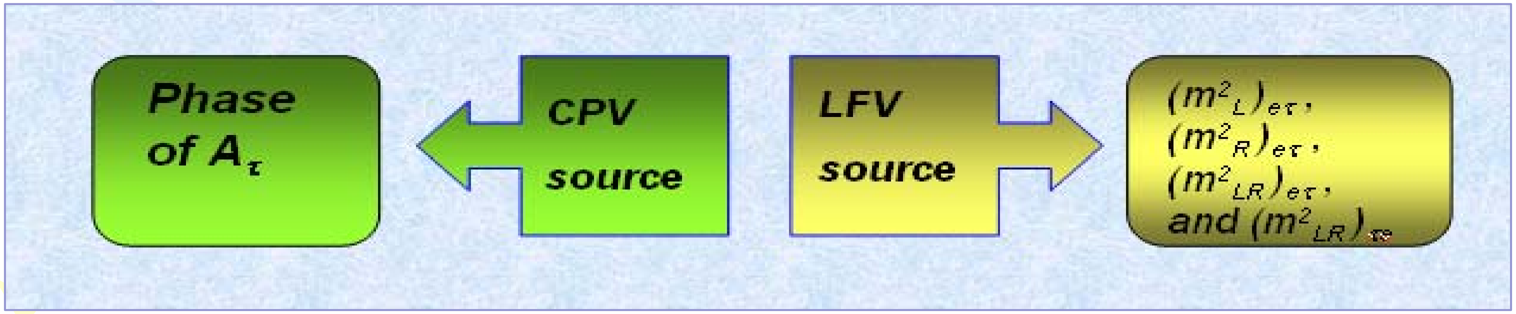
$$e\epsilon_\alpha^\dagger m_\tau q_\beta \left[\bar{e}_R \sigma^{\alpha\beta} (A_L)_{e\tau} \tau_L + \bar{e}_L \sigma^{\alpha\beta} (A_R)_{e\tau} \tau_R\right] + \text{H.c.}$$

$$\Gamma(\tau \to e\gamma) = \frac{e^2}{16\pi} m_\tau^5 (|(A_L)_{e\tau}|^2 + |(A_R)_{e\tau}|^2)$$

$$\frac{d\Gamma(\tau \to e\gamma)}{d\cos\theta} = \frac{e^2}{32\pi} m_\tau^5 \left[(|(A_L)_{e\tau}|^2 (1 + \cos\theta) + |(A_R)_{e\tau}|^2 (1 - \cos\theta) \right]$$

$$A_P = 4 \times \frac{\int_0^1 \frac{d\Gamma(\tau\to e\gamma)}{d\cos\theta} d\cos\theta - \int_{-1}^0 \frac{d\Gamma(\tau\to e\gamma)}{d\cos\theta} d\cos\theta}{\Gamma(\tau \to e\gamma)} = \frac{|(A_L)_{e\tau}|^2 - |(A_R)_{e\tau}|^2}{|(A_L)_{e\tau}|^2 + |(A_R)_{e\tau}|^2}$$

Studying LFV signals at a e^+e^- colliders can helps us in this direction.



New Contribution to d_e in the presence of LFV

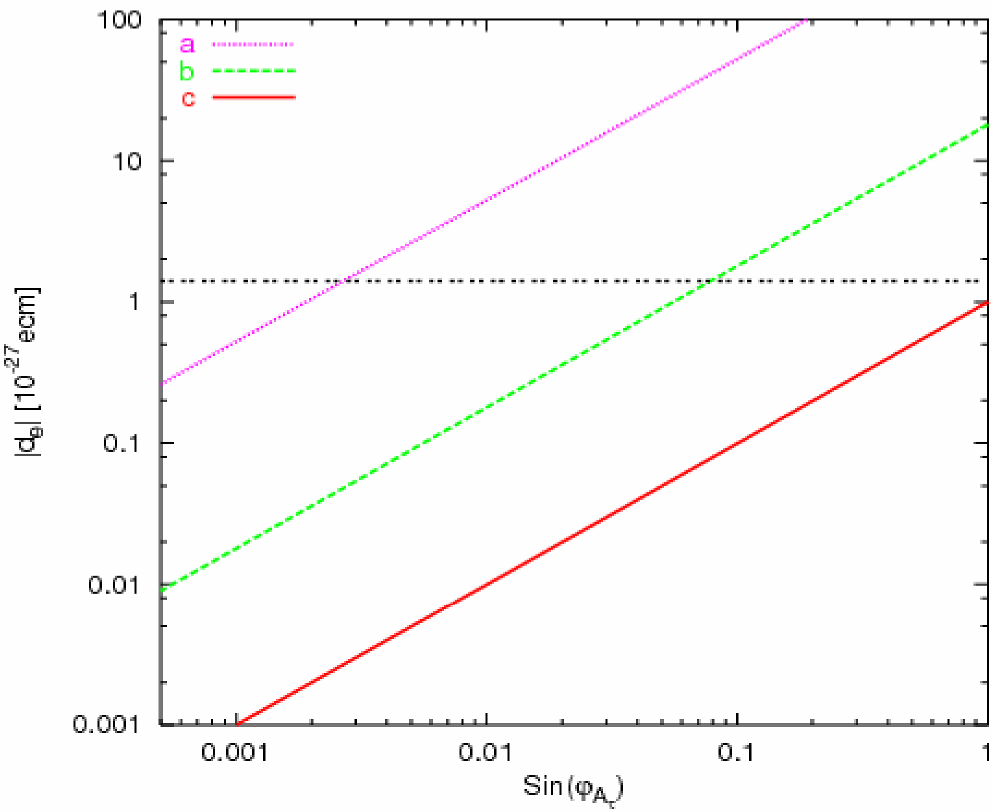
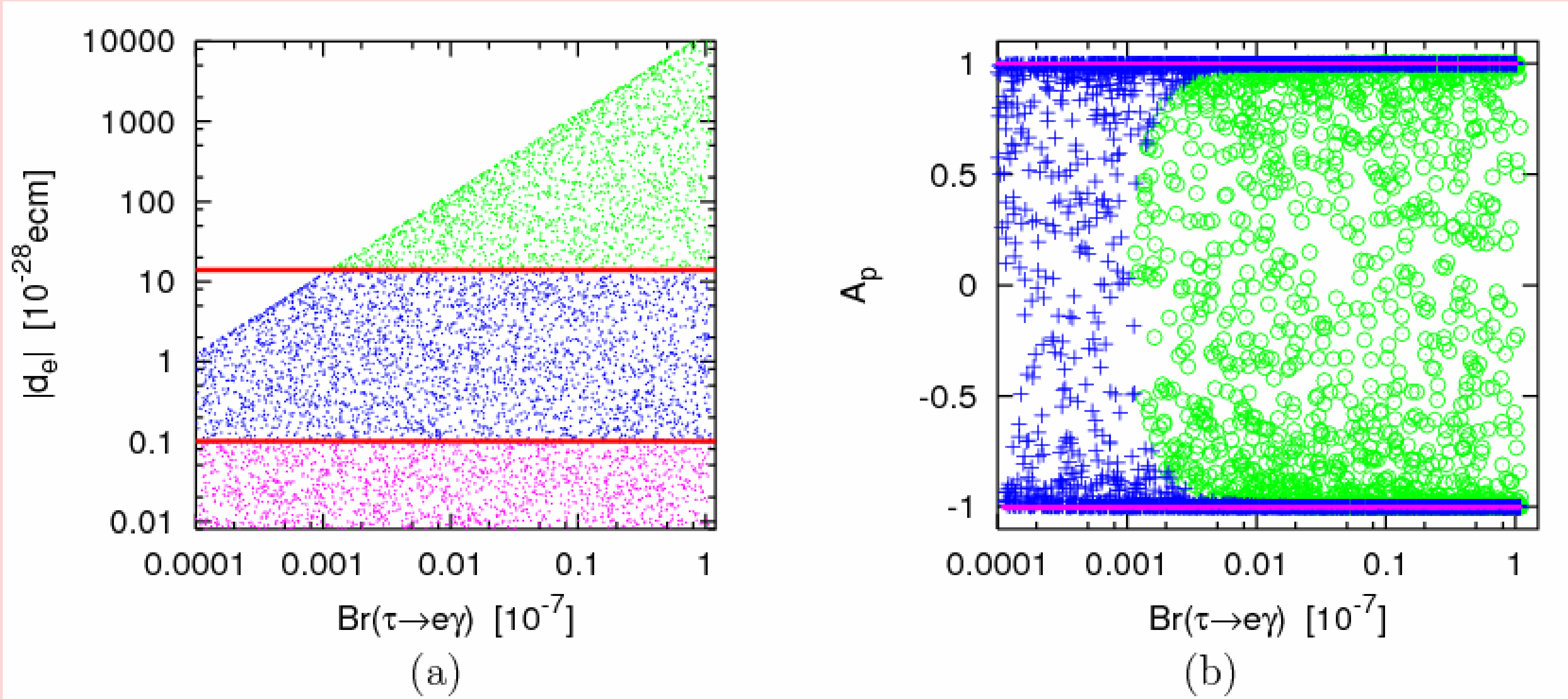


Figure 2: d_e versus $\sin\phi_{A_\tau}$. The input parameters correspond to the α benchmark proposed in [17]: $|\mu| = 375$ GeV, $m_0 = 210$ GeV, $M_{1/2} = 285$ GeV and $\tan\beta = 10$ and we have set A_τ=500 GeV. All the LFV elements of the slepton mass matrix are set to zero except $(m_L^2)_{e\tau}$ and $(m_R^2)_{e\tau}$. The dotted (pink) line labeled (a) corresponds to $(m_L^2)_{e\tau}$=3500 GeV^2 and $(m_R^2)_{e\tau}$=15000 GeV^2. The dashed (green) line labeled (b) corresponds to $(m_L^2)_{e\tau}$=50 GeV^2 and $(m_R^2)_{e\tau}$=37000 GeV^2. The solid (red) line labeled (c) corresponds to $(m_L^2)_{e\tau}$=3500 GeV^2 and $(m_R^2)_{e\tau}$=30 GeV^2. The horizontal doted line at 1.4×10^{-27} e cm depicts the present experimental limit [12] on d_e.

- ***This diagram demonstrates that for $(m^2_L)_{e\tau}$ and $(m_R{}^2)_{e\tau}$ close to their bounds from Br($\tau \to e\,\gamma$), a very strong bound on phase of A_τ can be derived.***

- ***For non-zero $(m^2_L)_{e\tau}$, and $(m^2_R)_{e\tau}$ the phase of A_τ can induce a contribution to the d_e.***
- ***For definite values of the off-diagonal mass elements, the bound on d_e can be interpreted as a bound on $Im(A_\tau)$.***



re 3: a) Scatter plot of d_e versus Br($\tau \to e\gamma$). The input parameters espond to the α benchmark proposed in [18]: $|\mu| = 375$ GeV, $m_0 =$ 0 GeV, $M_{1/2} = 285$ GeV and $\tan\beta = 10$. We have however set $\phi_{A_\tau} = \pi/2$ and $|A_\tau| = 500$ GeV. The values of $(m_L^2)_{e\tau}$ and $(m_R^2)_{e\tau}$ are randomly chosen respectively from $(0.59\ \mathrm{GeV}^2, 5.9 \times 10^3\ \mathrm{GeV}^2)$ and $(3.7\ \mathrm{GeV}^2, 3.7 \times 10^4\ \mathrm{GeV}^2)$ at a logarithmic scale. $(m_{LR}^2)_{e\tau}$ and $(m_{LR}^2)_{\tau e}$ pick up random values at a logarithmic scale from the interval $(0.12\ \mathrm{GeV}^2, 1.2 \times 10^3\ \mathrm{GeV}^2)$. The horizontal line at 1.4×10^{-27} e cm depicts the present experimental limit [13] and the one at 10^{-29} e cm shows the limit that can be probed in the near future [3]. b) Scatter plot of A_P versus Br($\tau \to e\gamma$). For each scatter point in Fig. 3-a there is a counterpart in Fig. 3-b corresponding to the same input values for the $e\tau$ elements which is shown with the same color and symbol.

- This figure demonstrates the correlation Between A_p and d_e.
- To illustrate the correlation between A_p and d_e, we have shown the corresponding scatter points in Fig-a and Fig-b with the same color.
- We conclude that for $A_{e\tau} = A_{\tau e} = 0$, The bound on d_e can be satisfied If BR is very small or A_p is close to ± 1

If $Br(\tau \to e\gamma)$ is close to its present bound and A_P takes a value in the interval (-1,1), we expect the bound on $Im(A_\tau)$ to be more stringent than the bound on $Im(A_e)$.

Within this scenario, if future search find $5\times 10^{-10} < Br(\tau \to e\gamma)$ and $-0.9 < A_p < 0.9$ the bound on d_e should be interpreted as a bound on phase A_τ

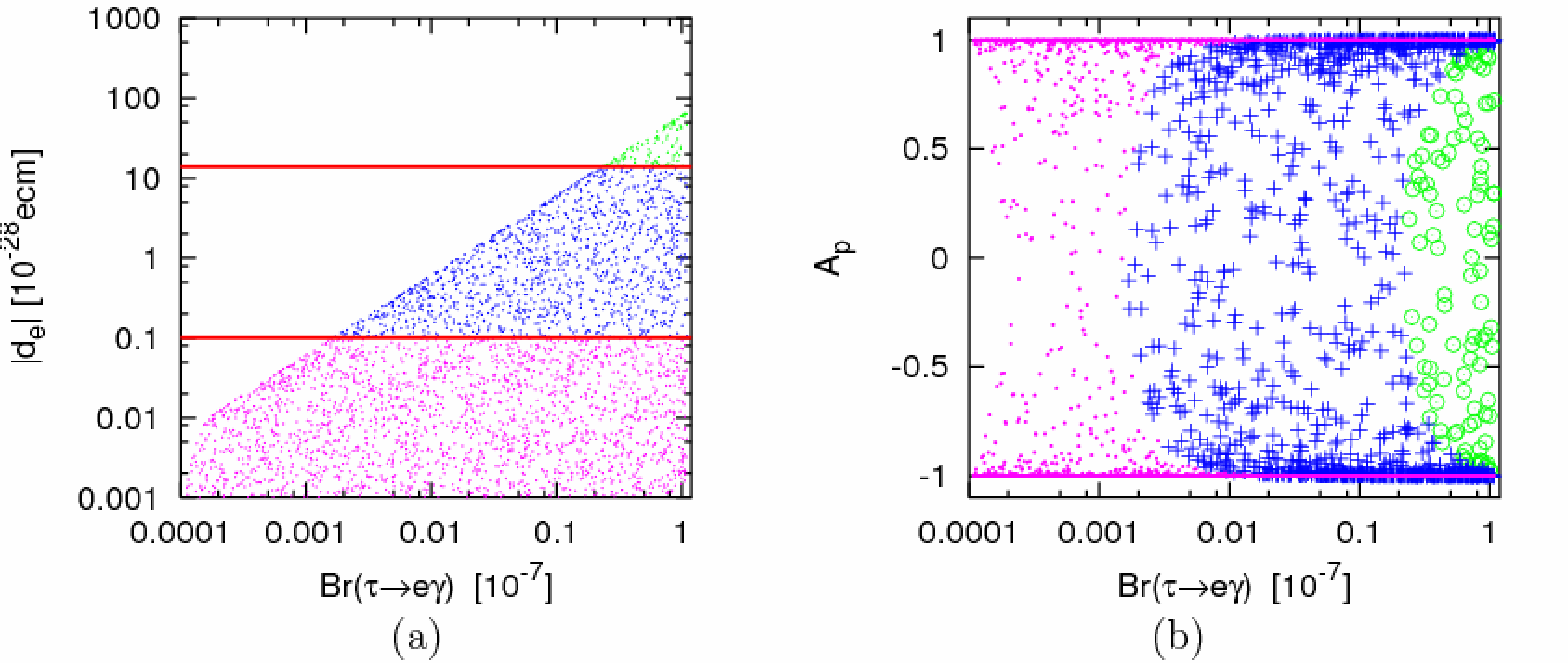
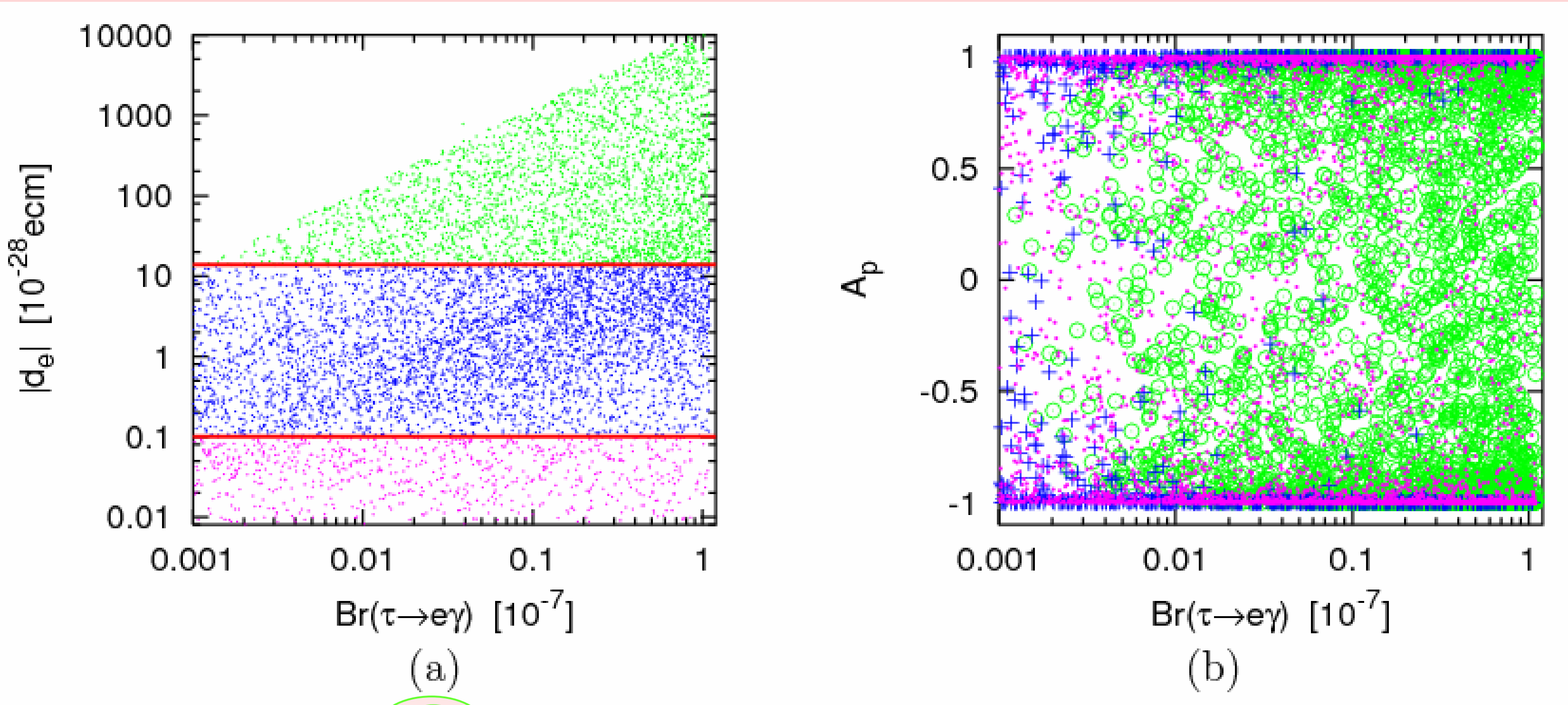


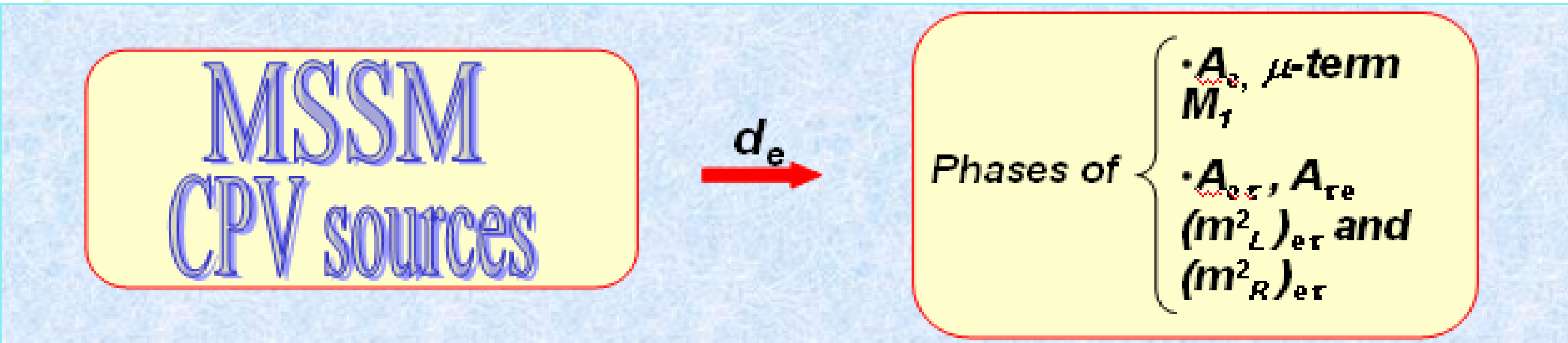
Figure 4: Similar to Fig. 3 except that $(m_L^2)_{e\tau} = (m_R^2)_{e\tau} = 0$ and instead $(m_{LR}^2)_{e\tau}(= A_{e\tau}\langle H_d\rangle)$ and $(m_{LR}^2)_{\tau e}(= A_{\tau e}\langle H_d\rangle)$ pick up random values at a logarithmic scale from $(1.2 \times 10^{-3}\ \mathrm{GeV}^2, 1.2 \times 10^3\ \mathrm{GeV}^2)$.



Input parameters are similar to previous figure expect that here in addition to $(m^2_L)_{e\tau}$ and $(m^2_R)_{e\tau}$, $(m^2_{LR})_{e\tau}$ and ,$(m^2_{RL})_{e\tau}$ are allowed to be nonzero.

- The significant point is that setting All the eτ mass elements nonzero, The correlation among A_p ,d_e and Br(τ→eγ) is lost.
- As result, Without independent Knowledge of the ratios of LFV elements, we can not derive any conclusive bound on phase of A_τ

Similar Effects (Degeneracy) by different sources of CP-violation



The following figures display the degeneracies between possible CP-violating phases.

Fountain Diagram

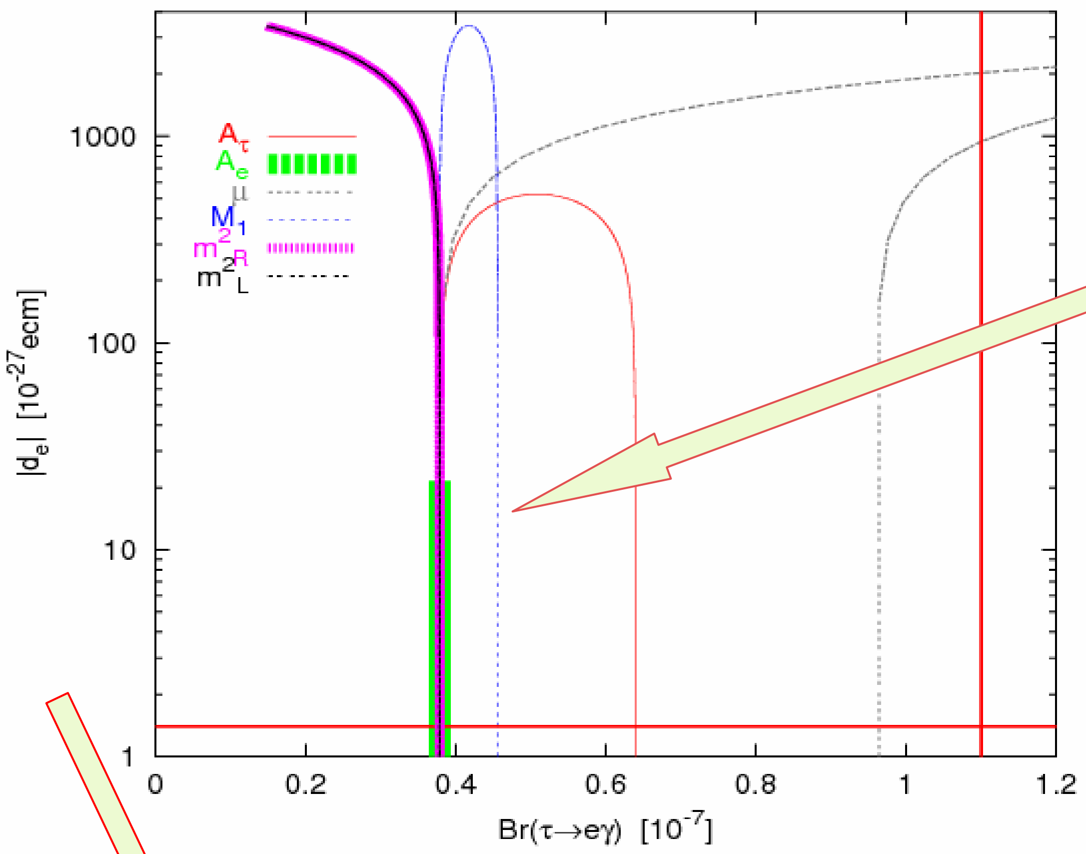


Figure 7: d_e versus Br($\tau \to e\gamma$) as the CP-violating phases vary between zero and π. The input parameters correspond to the α benchmark proposed in [17]:$|\mu|$ = 375 GeV, m_0 = 210 GeV, $M_{1/2}$ = 285 GeV and $\tan\beta$ = 10 and we have set A_τ=A_e=500 GeV. All the LFV elements of the slepton mass matrix are set zero except that $(m^2_L)_{e\tau}$=3500 GeV^2 and $(m^2_R)_{e\tau}$=15000 GeV^2. To draw the curves all phases are set zero except one that varies between 0 and π.

This line shows that Br($\tau \to e\,\gamma$) does not significantly change as phase A_e varies.

Notice that the effects of The rest of phases can exceed the maximal contribution from Phase of A_e by more than one order of magnitude.

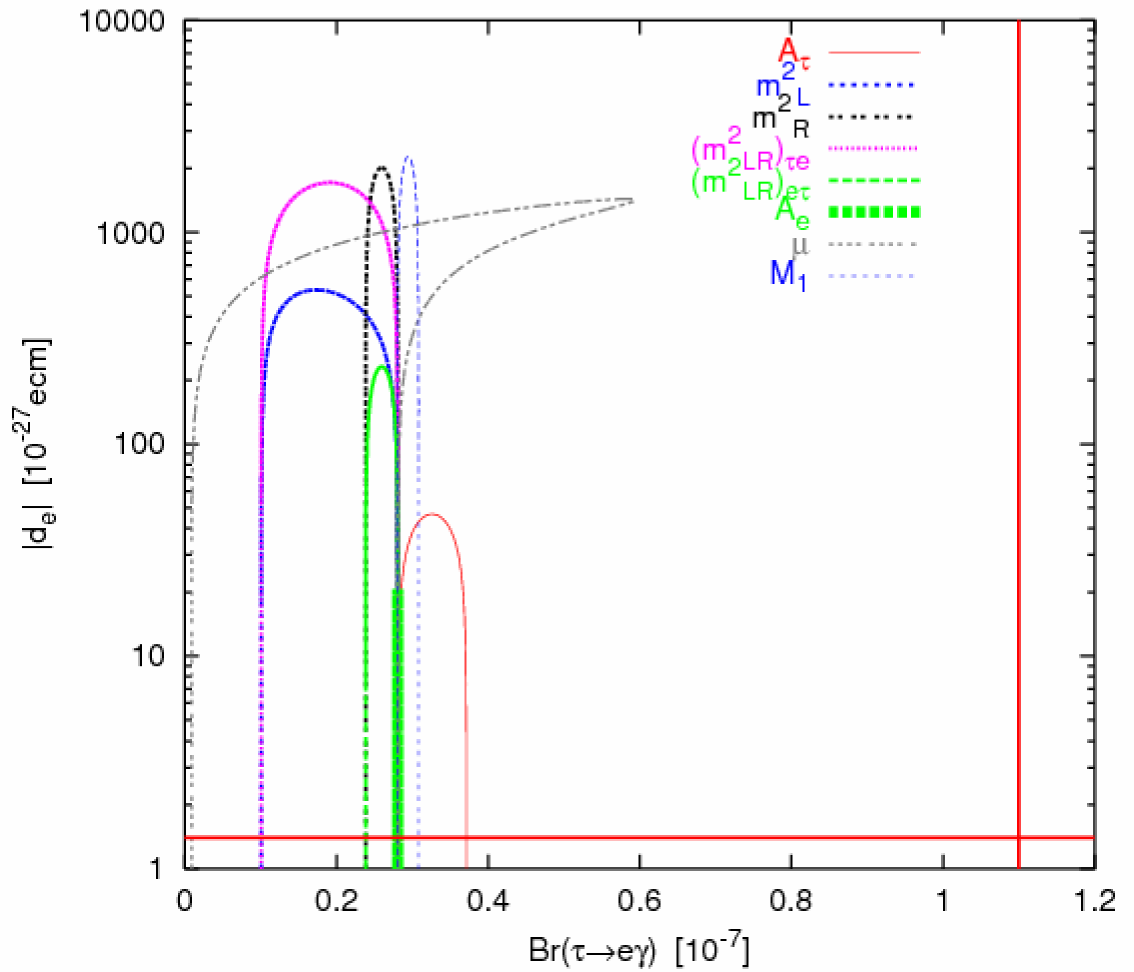


Figure 8: Similarly to Fig. 7 except that here we have set $(m_L^2)_{e\tau}$ = 1000 GeV^2, $(m_R^2)_{e\tau}$=5000 GeV^2 and $(m_{LR}^2)_{e\tau} = (m_{LR}^2)_{\tau e} = 300$ GeV^2. The thin solid red curve, light dash-dotted grey curve, light blue dashed curve, solid dashed dark blue curve and thick black dotted curve respectively correspond to the varying phase of A_τ, μ, M_1, $(m_L^2)_{e\tau}$ and $(m_R^2)_{e\tau}$.

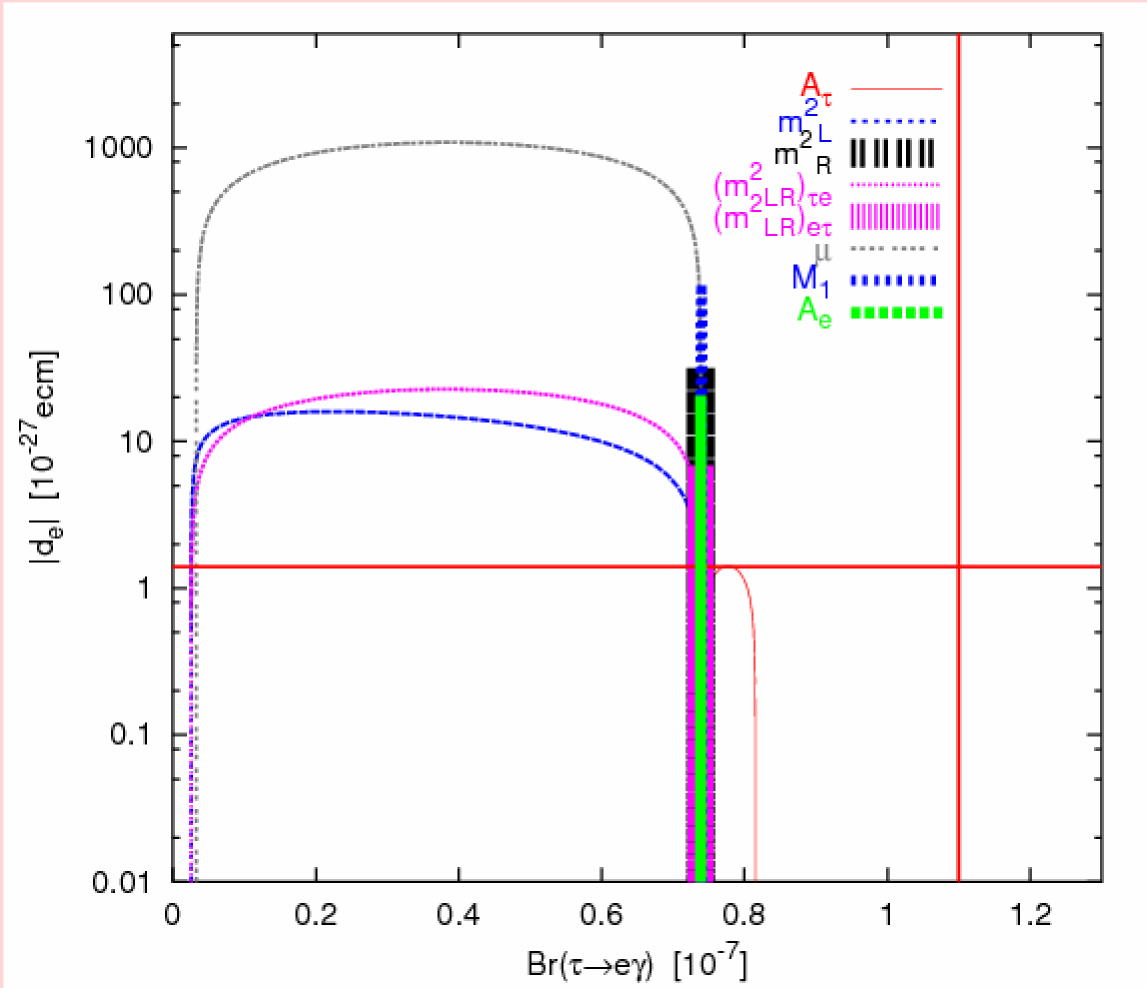


Figure 9: Similarly to Fig. 7 except that here $(m_L^2)_{e\tau} = 3000$ GeV^2, $(m_R^2)_{e\tau} = 50$ GeV^2, $(m_{LR}^2)_{e\tau}(= A_{e\tau}\langle H_d\rangle)$=3 GeV^2 and $(m_{LR}^2)_{\tau e}(= A_{\tau e}\langle H_d\rangle)$=400 GeV^2. The thin solid red curve, light dotted grey curve, thin dotted blue curve and thin dotted pink curve respectively correspond to varying phase of A_τ, μ, $(m_L^2)_{e\tau}$ and $(m_{LR}^2)_{\tau e}$. The pink, green, black and dark blue thick vertical lines at Br$(\tau \to e\gamma) = 7.5 \times 10^{-8}$ (which reach up $d_e = 6.8 \times 10^{-27}, 2 \times 10^{-26}, 3.2 \times 10^{-26}, 1.1 \times 10^{-25}$ e cm) depict d_e versus Br$(\tau \to e\gamma)$ as the phases of respectively $(m_{LR}^2)_{e\tau}$, A_e, $(m_R^2)_{e\tau}$ and M_1 vary between 0 and π.

Conclusions

We have studied the effects of the phase of trilinear A-coupling of the staus on d_e in the presence of nonzero LFV $e\tau$ elements of the slepton mass matrix. We have shown :

- *For a large portion of the parameter space consistent with the present bound on Br($\tau \to e\gamma$) the contribution of phase of A_τ to d_e can exceed the present bound by several orders of magnitude.*

- *The effect of A_τ on d_e strongly depends on the ratios of the LFV slepton masses $(m^2_L)_{e\tau}/(m^2_R)_{e\tau}$ and $(m^2_{LR})_{e\tau}/(m^2_{RL})_{e\tau}$. We have shown that for specific case that $(m^2_{LR})_{e\tau}=(m^2_{RL})_{e\tau}=0$* by measuring the asymmetry A_p we can solve this ambiguity.

- *When phase of A_τ is the only source of CP-violation which contribute to d_e, we have derived bounds on phase of A_τ for various values of the LFV elements giving rise to $Br(\tau \rightarrow e\gamma)$ close to the present bound.*

- *Contributions from phases of* $\boldsymbol{(m^2_L)_{e\tau}}$, $\boldsymbol{(m^2_R)_{e\tau}}$, $\boldsymbol{(m^2_{RL})_{e\tau}}$, *and* $\boldsymbol{(m^2_{LR})_{e\tau}}$, *can cancel the effect of phase of A_τ on d_e.*