A New Class of Invariants in Lepton Sector

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A. E. and Yasaman Farzan; hep-ph\0711.0448

OUTLINE:

- Introduction, a brief review of the invariants in Quark and lepton sectors.
- Invariants, introducing a new class of invariants in Lepton sector.
- CP Violation, formulating conditions for CP conservation in terms of invariants.
- Symmetries of mass matrices, $\mu - \tau$ exchange and reflection symmetries.
- Texture zero matrices, formulating the two zeros texture conditions.
- Conclusion.

INTRODUCTION

★ Quark sector:

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}}\, J^{\mu}_{\rm CC}\, W^{\dagger}_{\mu} + {\rm h.c.}$$

Charged Current Lagrangian

$$J^{\mu}_{\rm CC} = (\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L})\, \gamma^{\mu}\, V_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix}$$

Charged Current

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM Mixing Matrix

INTRODUCTION

$$V_{\rm CKM} = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta} & c_{23}\, c_{13} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$

Presence of phase results in CP-violation

Phase of each element of CKM Matrix can be changed by rephasing of Quark fields

Standard Parameterization

$$J_{\rm CP} = |\,{\rm Im}\,(V_{ij} V_{kl} V_{il}^* V_{kj}^*)\,|\,; \quad i \neq k\,,\ j \neq l$$

Invariant under basis transformation

C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); Z. Phys. C **29**, 491 (1985)

$$J = (3.08^{+0.16}_{-0.18}) \times 10^{-5} \ \text{ at } \ 95\% \ \text{ C.L.}$$

Summary of Quark sector

- Any CP-violating observable is proportional to **Jarlskog** invariant. In Quark sector there is only ***one*** invariant.
-

INTRODUCTION

★ Lepton sector:

$$-\mathcal{L} = (m_\ell)_{\alpha\beta}\overline{\ell_{R\alpha}}\ell_{L\beta} + \frac{1}{2}(m_\nu)_{\alpha\beta}\overline{(\nu_{L\alpha})^c}\nu_{L\beta} + \text{H.c.}$$

Mass terms in Lepton sector (Majorana Neutrinos)

$$-\mathcal{L}_{CC} = \frac{g}{\sqrt{2}}(\overline{e}_L, \overline{\mu}_L, \overline{\tau}_L)\gamma^\mu U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} W_\mu^+ + h.c.$$

Charged Current Interaction for Massive Neutrinos

✦ Hereafter we assume only three active Majorana Neutrinos (Low energy)

INTRODUCTION

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}e^{i\delta} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}e^{i\delta} \end{pmatrix} \mathrm{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})$$

$$c_{ij} = \cos\theta_{ij},\ s_{ij} = \sin\theta_{ij}$$

Two additional CP-violating phases because of the Majorana nature of Neutrinos

Oscillation parameter		central value	99% CL range
solar mass splitting	Δm^2_{12}	$= (8.0 \pm 0.3)\,10^{-5}\mathrm{eV}^2$	$(7.2 \div 8.9)\,10^{-5}\mathrm{eV}^2$
atmospheric mass splitting	$\lvert\Delta m^2_{23}\rvert$	$= (2.5 \pm 0.3)\,10^{-3}\mathrm{eV}^2$	$(1.7 \div 3.3)\,10^{-3}\mathrm{eV}^2$
solar mixing angle	$\tan^2\theta_{12}$	$= 0.45 \pm 0.05$	$30^\circ < \theta_{12} < 38^\circ$
atmospheric mixing angle	$\sin^2 2\theta_{23}$	$= 1.02 \pm 0.04$	$36^\circ < \theta_{23} < 54^\circ$
'CHOOZ' mixing angle	$\sin^2 2\theta_{13}$	$= 0 \pm 0.05$	$\theta_{13} < 10^\circ$

non-oscillation parameter	probed by	experimental limit at 99% CL	99% CL range normal hierarchy	99% CL range inverted hierarchy
ee-entry of m	$0\nu2\beta$	$m_{ee} < 0.38\,h\,\mathrm{eV}$	$(1.1 \div 4.5)\,\mathrm{meV}$	$(12 \div 57)\,\mathrm{meV}$
$(m^\dagger m)^{1/2}_{ee}$	β-decay	$m_{\nu_e} < 2.0\,\mathrm{eV}$	$(4.6 \div 10)\,\mathrm{meV}$	$(42 \div 57)\,\mathrm{meV}$
$m_1 + m_2 + m_3$	cosmology	$m_{\rm cosmo} < 0.94\,\mathrm{eV}$	$(51 \div 66)\,\mathrm{meV}$	$(83 \div 114)\,\mathrm{meV}$

A. Strumia, F. Vissani; [hep-ph/0503246]

INVARIANTS

Consider the following transformation on charged leptons and neutrinos :

$$\begin{cases} \ell_{L\alpha} \rightarrow U_{\alpha\beta}\,\ell_{L\beta} \\ \nu_{L\alpha} \rightarrow U_{\alpha\beta}\,\nu_{L\beta} \\ \ell_{R\alpha} \rightarrow V_{\alpha\beta}\,\ell_{R\beta} \end{cases}$$

Invariance of the Lepton sector Lagrangian implies

$$\begin{cases} m_\ell \rightarrow V\, m_\ell\, U^\dagger\,, \\ m_\nu \rightarrow U^*\, m_\nu\, U^\dagger \end{cases}$$

$$\sum_i [a_i m_\nu (m_\ell^\dagger m_\ell)^{n_i} + b_i (m_\ell^T m_\ell^*)^{m_i} m_\nu] \rightarrow U^* \sum_i [a_i m_\nu (m_\ell^\dagger m_\ell)^{n_i} + b_i (m_\ell^T m_\ell^*)^{m_i} m_\nu]\, U^\dagger$$

INVARIANTS

The following quantities are invariant under basis transformation :

$$\frac{\mathrm{Det}[\sum_i (a_i m_\nu (m_\ell^\dagger m_\ell)^{n_i} + b_i (m_\ell^T m_\ell^*)^{m_i} m_\nu)]}{\mathrm{Det}[\sum_i (a_i' m_\nu (m_\ell^\dagger m_\ell)^{n_i'} + b_i' (m_\ell^T m_\ell^*)^{m_i'} m_\nu)]}$$

AND

$$\mathrm{Det}[\sum_i (a_i m_\nu (m_\ell^\dagger m_\ell)^{n_i} + b_i (m_\ell^T m_\ell^*)^{m_i} m_\nu)] \left(\mathrm{Det}[\sum_i (a_i' m_\nu (m_\ell^\dagger m_\ell)^{n_i'} + b_i' (m_\ell^T m_\ell^*)^{m_i'} m_\nu)]\right)^*$$

For our work we considered the following combination :

$$M(A,B) = m_\nu + A[m_\nu (m_\ell^\dagger m_\ell) + (m_\ell^T m_\ell^*) m_\nu] + B[(m_\ell^T m_\ell^*)^2 m_\nu - m_\nu (m_\ell^\dagger m_\ell)^2]$$

INVARIANTS

By choosing the following values for A and B, we obtained :

$$\mathcal{E}_1 \equiv \mathrm{Det}\left[M\left(A=\frac{-(m_\mu^2+m_\tau^2)}{(m_e^2+m_\mu^2)(m_e^2+m_\tau^2)}, B=\frac{-1}{(m_e^2+m_\mu^2)(m_e^2+m_\tau^2)}\right)\right]\Big/ S_1$$

$$\mathcal{E}_2 \equiv \mathrm{Det}\left[M\left(A=\frac{-(m_\tau^2+m_e^2)}{(m_\mu^2+m_\tau^2)(m_\mu^2+m_e^2)}, B=\frac{-1}{(m_\mu^2+m_\tau^2)(m_\mu^2+m_e^2)}\right)\right]\Big/ S_2$$

$$\mathcal{E}_3 \equiv \mathrm{Det}\left[M\left(A=\frac{-(m_e^2+m_\mu^2)}{(m_\tau^2+m_e^2)(m_\tau^2+m_\mu^2)}, B=\frac{-1}{(m_\tau^2+m_e^2)(m_\tau^2+m_\mu^2)}\right)\right]\Big/ S_3$$

AND

$$\mathcal{F}_1 \equiv \mathrm{Det}\left[M\left(A=\frac{-1}{2m_\tau^2}, B=\frac{-1}{2m_\tau^2(m_\tau^2+m_e^2)}\right)\right]\Big/ T_1$$

$$\mathcal{F}_2 \equiv \mathrm{Det}\left[M\left(A=\frac{-1}{2m_e^2}, B=\frac{-1}{2m_e^2(m_e^2+m_\mu^2)}\right)\right]\Big/ T_2$$

$$\mathcal{F}_3 \equiv \mathrm{Det}\left[M\left(A=\frac{-1}{2m_\mu^2}, B=\frac{-1}{2m_\mu^2(m_\mu^2+m_\tau^2)}\right)\right]\Big/ T_3$$

Where :

$$S_1 = \frac{-(m_e^4+m_e^2m_\mu^2+m_e^2m_\tau^2-m_\mu^2m_\tau^2-2m_\mu^4)(m_e^4+m_e^2m_\mu^2+m_e^2m_\tau^2-m_\mu^2m_\tau^2-2m_\tau^4)}{(m_e^2+m_\mu^2)^2(m_e^2+m_\tau^2)^2}$$

$$T_1 = \frac{(m_\mu^2-m_e^2)(m_\mu^2-m_\tau^2)(m_\tau^2-m_e^2)(2m_\tau^4+m_\tau^2m_e^2-m_\tau^2m_\mu^2-m_e^2m_\mu^2-m_\mu^4)}{4m_\tau^6(m_\tau^2+m_e^2)^2}$$

INVARIANTS

If we choose the following basis (charged lepton basis) :

$$m_\ell^\dagger m_\ell = m_\ell^T m_\ell^* = \begin{pmatrix} {m_e}^2 & 0 & 0 \\ 0 & {m_\mu}^2 & 0 \\ 0 & 0 & {m_\tau}^2 \end{pmatrix} \text{ and } m_\nu = \begin{pmatrix} m_{ee} & m_{e\mu} & m_{e\tau} \\ m_{\mu e} & m_{\mu\mu} & m_{\mu\tau} \\ m_{\tau e} & m_{\tau\mu} & m_{\tau\tau} \end{pmatrix}$$

$$\mathcal{E}_1 = m_{ee}(m_{\mu\tau}^2 - m_{\mu\mu}m_{\tau\tau})$$
$$\mathcal{E}_2 = m_{\mu\mu}(m_{e\tau}^2 - m_{ee}m_{\tau\tau})$$
$$\mathcal{E}_3 = m_{\tau\tau}(m_{e\mu}^2 - m_{ee}m_{\mu\mu})$$

$$\mathcal{F}_1 = m_{\mu\tau}(m_{e\mu}m_{e\tau} - m_{ee}m_{\mu\tau})$$
$$\mathcal{F}_2 = m_{e\tau}(m_{e\mu}m_{\mu\tau} - m_{\mu\mu}m_{e\tau})$$
$$\mathcal{F}_3 = m_{e\mu}(m_{e\tau}m_{\mu\tau} - m_{\tau\tau}m_{e\mu})$$

Notice that :

$$\begin{cases} \mathcal{E}_i \rightarrow \mathrm{Det}[U^*]\,\mathcal{E}_i\,\mathrm{Det}[U^\dagger] \\ \mathcal{F}_i \rightarrow \mathrm{Det}[U^*]\,\mathcal{F}_i\,\mathrm{Det}[U^\dagger] \end{cases}$$

We can construct invariants by taking their ratios or multiplying one of them by the other one

INVARIANTS

In lepton sector with massive Majorana neutrinos
there are 9 independent parameters

$$\left\{ \theta_{12}\,,\theta_{23}\,,\theta_{13}\,,\Delta m^2_{21}\,,\Delta m^2_{31}\,,\delta\ ,\ mass\ scale\,,\alpha_{21}\,,\alpha_{31} \right\}$$

There are more invariants in Lepton sector than Quark sector

- It can be shown that only four out of the six combinations $\mathcal{E}_i$ and $\mathcal{F}_i$ are independent → Three independent invariants

CP VIOLATION

- In Lepton sector there are more than one source of CP-violation. So, unlike the Quark sector, we need more than one invariant to formulate CP-invariance.

- These invariants can be used to formulating CP-violation in a basis independent way.

- As an example les us consider models that in them : $\mathrm{Det}[m_\nu] = 0$

G. C. Branco [arXiv:hep-ph/0311348] ; T. Yanagida [arXiv:hep-ph/0212341]; I. Aizawa [arXiv:hep-ph/0502135]; B. Chauhan [arXiv:hep-ph/0602084]; .....

- It can be shown that in these scenarios, vanishing of the Jarlskog invariant and realness of $\mathcal{E}_1/\mathcal{E}_2$, are the sufficient conditions for CP-invariance.

For details see the next slide!

CP VIOLATION

- In models with $\mathrm{Det}[m_\nu] = 0$ there exists two possibility

$$\begin{cases} m_1 = 0 & \text{Normal Hierarchy} \\ m_3 = 0 & \text{Inverted Hierarchy} \end{cases}$$

- Normal Hierarchy: In this case $\mathcal{J} = \mathrm{Im}[\mathcal{E}_1/\mathcal{E}_2] = 0$, imply (other than CP-conservation) :

$$\frac{s_{13}^2 m_3 \sin\alpha}{s_{23}^2 c_{13}^2 m_3 \sin\alpha} = \frac{m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2 \cos\alpha}{m_2 (c_{12}c_{23} - s_{12}s_{13}s_{23})^2 + m_3 s_{23}^2 c_{13}^2 \cos\alpha}$$

$$\Downarrow$$

$$\boxed{m_2 \left[s_{12}^2 s_{23}^2 c_{13}^4 - s_{13}^2 (c_{12}c_{23} - s_{12}s_{13}s_{23})^2 \right] = 0}$$

$$\Uparrow$$

This equation is not compatible with the present data!

CP VIOLATION

★ Inverted Hierarchy: In this case $\mathcal{J} = \mathrm{Im}[\mathcal{E}_1/\mathcal{E}_2] = 0$, imply (other than CP-conservation) :

$$\frac{s_{12}^2 m_2 \sin\beta}{(c_{12}c_{23} - s_{12}s_{13}s_{23})^2 m_2 \sin\beta} = \frac{m_1 c_{12}^2 + m_2 s_{12}^2 \cos\beta}{m_1(s_{12}c_{23} + c_{12}s_{13}s_{23})^2 + m_2(c_{12}c_{23} - s_{12}s_{23}s_{13})^2 \cos\beta}$$

$$s_{13} = \frac{c_{23}}{s_{23}} \frac{c_{12}^2 - s_{12}^2}{\sin 2\theta_{12}}$$

This equation is not compatible with the present data!

With the present data (at 99% C.L.)

- Right hand side $>$ 0.18
- Left hand side $<$ 0.16

SYMMETRIES OF MASS MATRIX

$\mu - \tau$ exchange symmetry

OR

symmetry under $\nu_\mu \leftrightarrow \nu_\tau$

R. N. Mohapatra et.al. [arXiv:hep-ph/0507312];
Z. Xing et.al. [arXiv;hep-ph/0607091];
K. Matsuda et.al. [arXiv;hep-ph/0606142];
S. Kaneko et.al. [arXiv;hep-ph/0504074].

★ In the charged lepton basis this symmetry means :

$$m_{e\mu} = m_{e\tau} \quad , \quad m_{\mu\mu} = m_{\tau\tau}$$

$$\mathcal{E}_2 = \mathcal{E}_3 \quad , \quad \mathcal{F}_2 = \mathcal{F}_3$$

Necessary condition for exchange symmetry

SYMMETRIES OF MASS MATRIX

$\mu - \tau$ reflection symmetry

OR

symmetry under $\nu_\mu \leftrightarrow \nu_\tau^*$

P. Harrison et.al. [arXiv:hep-ph/0210197];
W. Grimus et.al. [arXiv:hep-ph/0305309];
A. Ghosal [arXiv:hep-ph/0304090];
N. Nimai Singh et.al. [arXiv:hep-ph/0707.2713].

★ In the charged lepton basis this symmetry means :

$$m_{e\mu} = m_{e\tau}^* \quad , \quad m_{\mu\mu} = m_{\tau\tau}^* \quad , \quad m_{ee} = m_{ee}^* \quad , \quad m_{\mu\tau} = m_{\mu\tau}^*$$

$$\frac{\mathcal{E}_2 - \mathcal{E}_3}{\mathcal{F}_1} = -\left(\frac{\mathcal{E}_2 - \mathcal{E}_3}{\mathcal{F}_1}\right)^* \quad , \quad \frac{\mathcal{F}_2 - \mathcal{F}_3}{\mathcal{E}_1} = -\left(\frac{\mathcal{F}_2 - \mathcal{F}_3}{\mathcal{E}_1}\right)^*$$

$$\frac{\mathcal{E}_2 - \mathcal{E}_3}{\mathcal{E}_1} = -\left(\frac{\mathcal{E}_2 - \mathcal{E}_3}{\mathcal{E}_1}\right)^* .$$

Necessary condition for reflection symmetry

TEXTURE ZERO MATRICES

Label	Neutrino mass matrix m_ν	Pattern of $\mathcal{E}_i$ and $\mathcal{F}_i$
A_1	$\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\mathcal{E}_1 = \mathcal{E}_3 = \mathcal{F}_1 = \mathcal{F}_3 = 0\,,\ \mathcal{E}_2 = -\mathcal{F}_2$
A_2	$\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{F}_1 = \mathcal{F}_2 = 0\,,\ \mathcal{E}_3 = -\mathcal{F}_3$
B_1	$\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	$\mathcal{E}_2 = \mathcal{F}_2 = 0\,,\ \mathcal{E}_1 = -\mathcal{F}_1\,,\ \mathcal{E}_3 = -\mathcal{F}_3$
B_2	$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$\mathcal{E}_3 = \mathcal{F}_3 = 0\,,\ \mathcal{E}_1 = -\mathcal{F}_1\,,\ \mathcal{E}_2 = -\mathcal{F}_2$
B_3	$\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$\mathcal{E}_2 = \mathcal{E}_3 = \mathcal{F}_2 = \mathcal{F}_3 = 0\,,\ \mathcal{E}_1 = -\mathcal{F}_1$
B_4	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$	$\mathcal{E}_2 = \mathcal{E}_3 = \mathcal{F}_2 = \mathcal{F}_3 = 0\,,\ \mathcal{E}_1 = -\mathcal{F}_1$
C	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	$\mathcal{E}_2 = \mathcal{E}_3 = 0\,,\ \mathcal{F}_2 = \mathcal{F}_3 = \mathcal{F}_1 + \mathcal{E}_1 \neq 0$

No three zeros texture are compatible with present data

These two zeros textures was introduced by:

P. Frampton, S. Glashow, D. Marfatia; [hep-ph /0201008]

And investigated by many authors:

S. Dev et.al. [arXiv:hep-ph/0708.3321];

S. Rajpoot [arXiv:hep-ph/0703185];

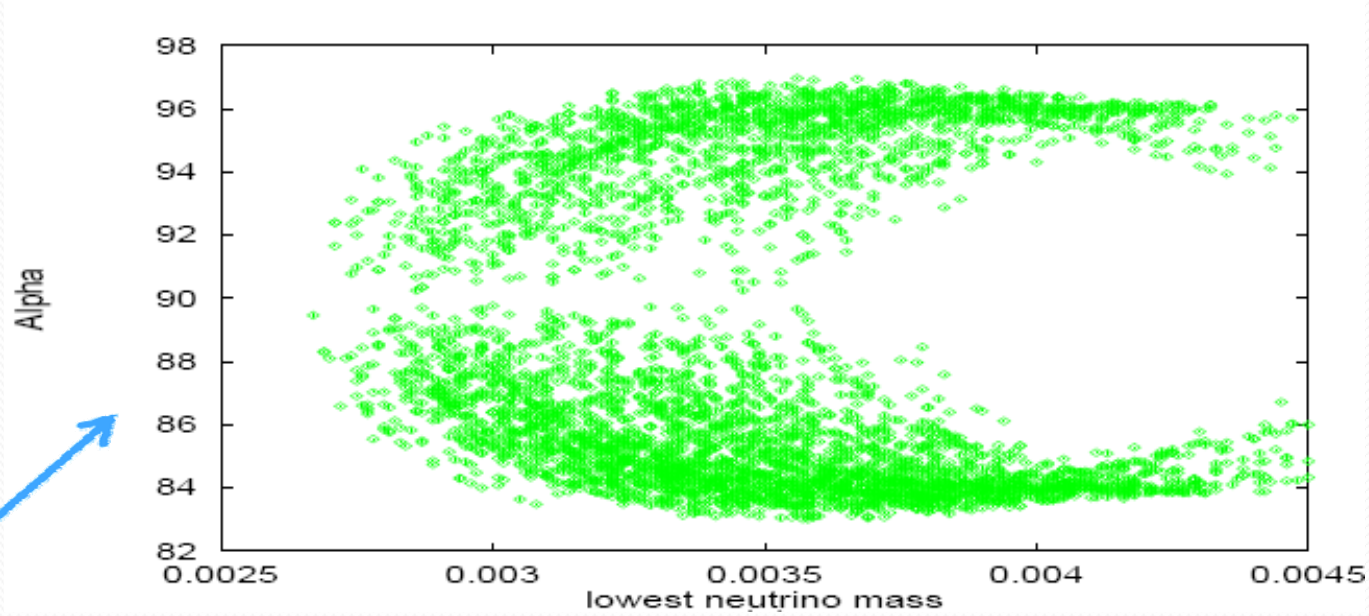
T. Fukuyama et.al. [arXiv:hep-ph/0702284];

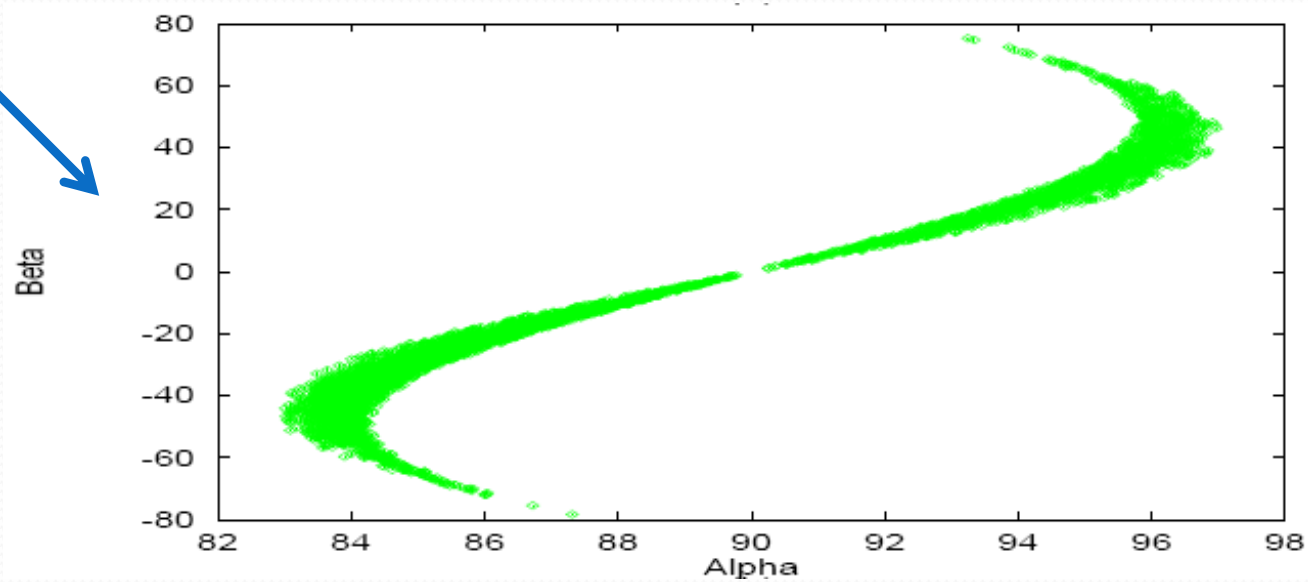
Z. Xing et.al. [arXiv:hep-ph/0307007];

……..

Some Phenomenological consequences of two zero texture matrices

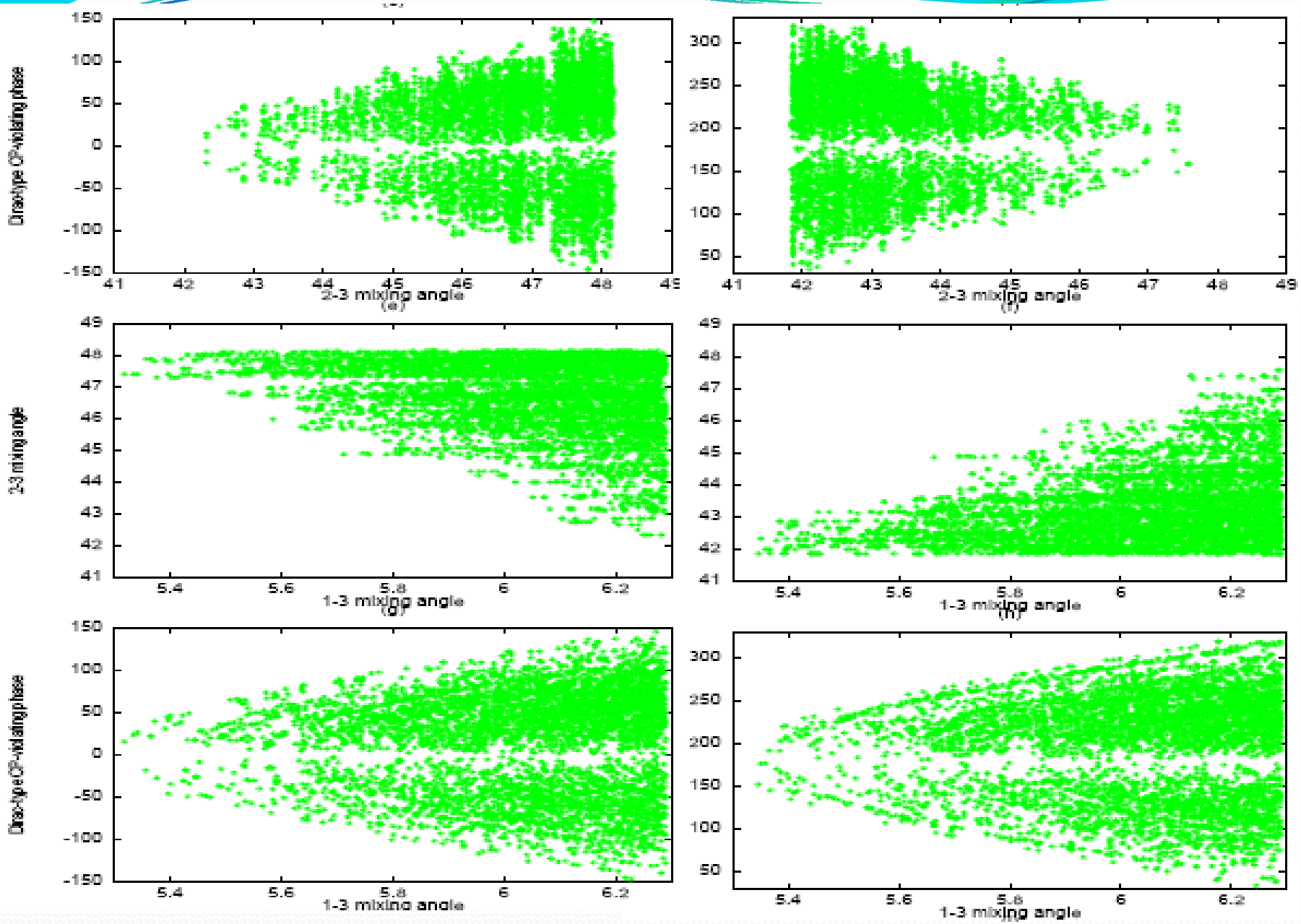
A





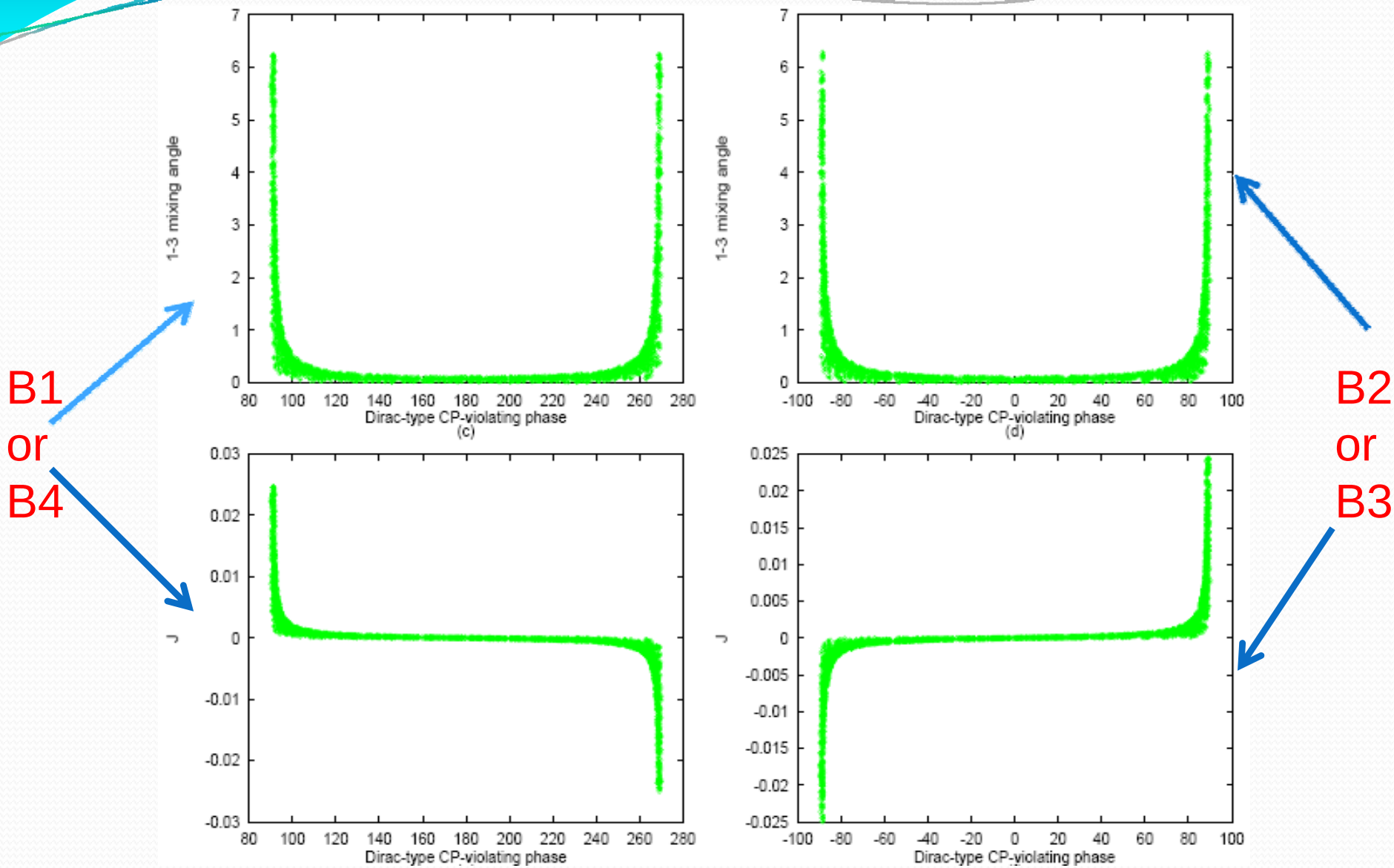
S. Dev et.al. [arXiv:hep-ph/0612102]

Some Phenomenological consequences of two zero texture matrices



S. Dev et.al. [arXiv:hep-ph/0612102]

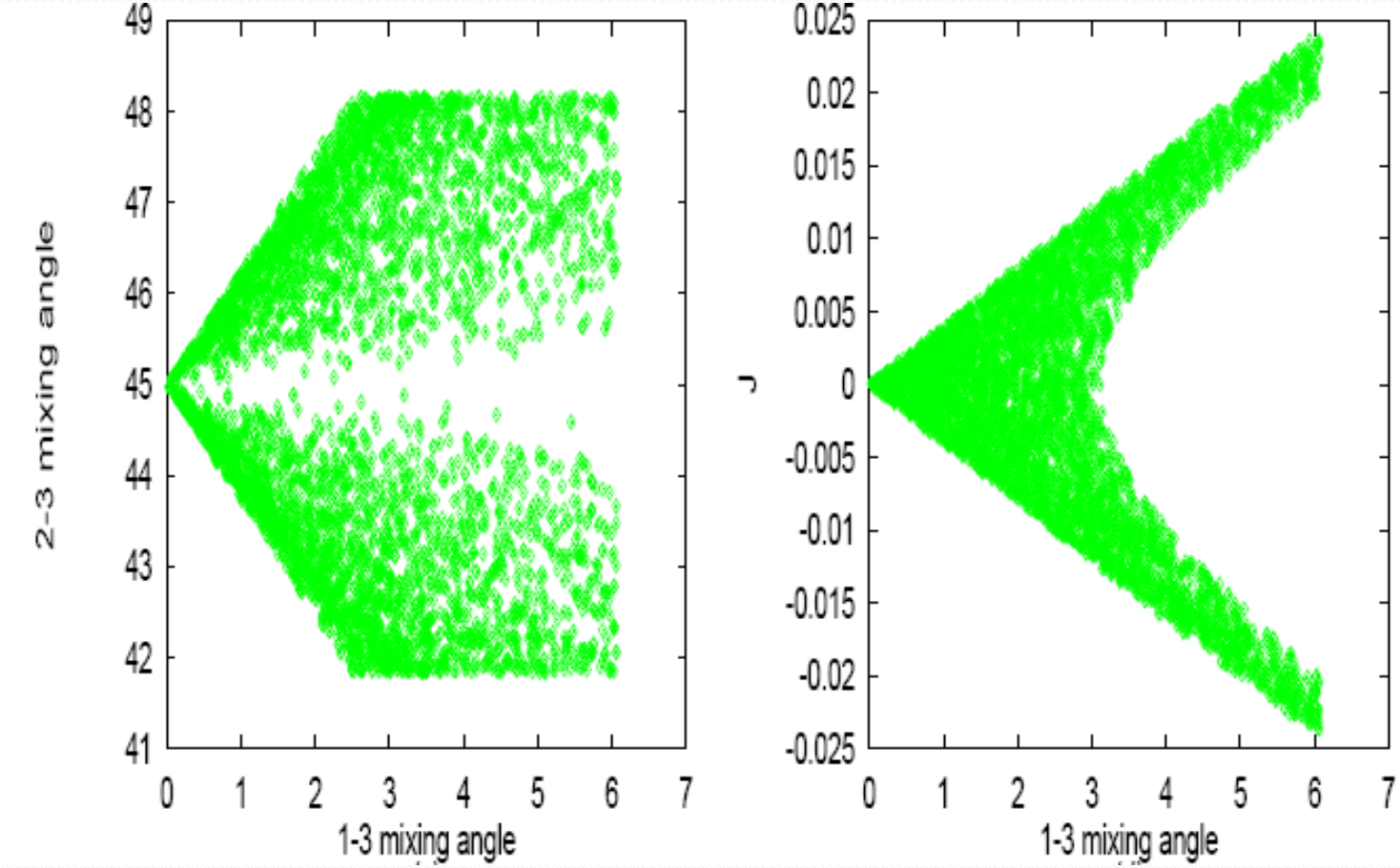
Some Phenomenological consequences of two zero texture matrices



S. Dev et.al. [arXiv:hep-ph/0612102]

Some Phenomenological consequences of two zero texture matrices

C



S. Dev et.al. [arXiv:hep-ph/0612102]

TEXTURE ZERO MATRICES

★ Let us consider the textures A_1 and A_2 . For both of these textures :

$$\mathcal{F}_1 = \mathcal{E}_1 = 0$$

Remember:

$$\mathcal{E}_1 = m_{ee}(m_{\mu\tau}^2 - m_{\mu\mu}m_{\tau\tau})$$

$$\mathcal{F}_1 = m_{\mu\tau}(m_{e\mu}m_{e\tau} - m_{ee}m_{\mu\tau})$$

If $m_{ee}, m_{\mu\tau} \neq 0$,

then $\mathcal{F}_1 = \mathcal{E}_1 = 0$ ➡

$$m_1 = -m_2 \frac{c_{12}^2 - c_{12}s_{12}s_{13}\tan\theta_{23}e^{i\delta}}{s_{12}^2 + s_{12}c_{12}s_{13}\tan\theta_{23}e^{i\delta}}$$ ⬅ Incompatible with data

$m_3 = 0, s_{13} = 0$ ➡ $\mathcal{E}_2, \mathcal{E}_3 \neq 0$

At least one of the $m_{ee}, m_{\mu\tau}$ must be zero

TEXTURE ZERO MATRICES

$$\mathcal{E}_1 = \mathcal{F}_1 = 0$$
$$m_{ee} \neq 0$$
$$m_{\mu\tau} = 0$$

$$\boxed{m_{\mu\mu} m_{\tau\tau} = 0}$$

Incompatible with data

$$\mathcal{E}_1 = \mathcal{F}_1 = 0$$
$$m_{ee} = 0$$
$$m_{\mu\tau} \neq 0$$

$$\boxed{m_{e\mu} m_{e\tau} = 0}$$

The desired result!

$$\mathcal{E}_1 = \mathcal{E}_3 = \mathcal{F}_1 = \mathcal{F}_3 = 0 \quad \Longleftrightarrow \quad A_1$$

$$\mathcal{E}_1 = \mathcal{E}_2 = \mathcal{F}_1 = \mathcal{F}_2 = 0 \quad \Longleftrightarrow \quad A_2$$

TEXTURE ZERO MATRICES

Consider texture B_1 . In this case we have $\mathcal{E}_2 = \mathcal{F}_2 = 0$.

This condition, in addition to $m_{\mu\mu} = m_{e\tau} = 0$, have the solution:

$$\begin{cases} |m_2|^2 - |m_1|^2 &= |m_1|^2 \cos\delta(4s_{13})/(s_{12}c_{12}) \\ |m_3|^2 - |m_1|^2 &= |m_1|^2(s_{23}^2 - c_{23}^2)/c_{23}^4 \end{cases}$$

$$\Downarrow$$

$$\boxed{s_{13}\cos\delta/(s_{23}^2 - c_{23}^2) \sim (\Delta m_{sol}^2)/(\Delta m_{atm}^2) \ll 1}$$

This relation can be tested by future experiments T2K , NOvA, KATRIN and T2KK. With the present data this solution is still acceptable.

TEXTURE ZERO MATRICES

- For the case B_2, we have $\mathcal{E}_3 = \mathcal{F}_3 = 0$.

  This condition, in addition to $m_{e\mu} = m_{\tau\tau} = 0$, have the solution:

$$\begin{cases} |m_2|^2 - |m_1|^2 &= -|m_1|^2 \cos\delta(4s_{13})/(s_{12}c_{12}) \\ |m_3|^2 - |m_1|^2 &= -|m_1|^2(s_{23}^2 - c_{23}^2)/c_{23}^4 \end{cases}$$

- Like the texture B_1 , this relation also can be tested in future.

- For the textures B_3 and B_4 :

$$\mathcal{E}_2 = \mathcal{F}_2 = \mathcal{E}_3 = \mathcal{F}_3 = 0 \quad \Longleftrightarrow \quad B_3 \text{ or } B_4.$$

TEXTURE ZERO MATRICES

For the case C , we have $\mathcal{E}_2 = \mathcal{E}_3 = 0$.

This condition, in addition to $m_{\mu\mu} = m_{\tau\tau} = 0$, have the solution:

$$\frac{|m_2|^2 - |m_1|^2}{|m_1|^2} = \frac{\cos 2\theta_{12}}{s_{12}^4},$$
$$\frac{|m_3|^2 - |m_1|^2}{|m_1|^2} \simeq \frac{\cos^2(2\theta_{23}) c_{12}^2}{4 s_{23}^4 c_{23}^2 s_{13}^2 s_{12}^2}$$

$$\Delta m^2_{atm}/\Delta m^2_{sol} = (s_{12}^2 c_{12}^2/\cos 2\theta_{12})(1/4 s_{23}^4 c_{23}^2)[\cos^2(2\theta_{23})/s_{13}^2] \gg 1$$

CONCLUSION:

- We have defined quantities called $\mathcal{E}_i$ and $\mathcal{F}_i$ as determinants of the combinations of m_ℓ^2 and m_ν that are linear in m_ν and second degree polynomials in m_ℓ^2. Then we constructed from them invariants under basis transformations.

- We have formulated the conditions for CP conservation and $\mu - \tau$ exchange and reflection symmetries in terms of the introduced invariants.

- We formulated the two zeros texture matrices in terms of these invariants, so a basis independent formulation of these matrices.

THE END.