Minimal supersymmetric standard model

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Standard model

Building blocks of nature

Bosons

Fermions

Elementary particles

Main forces in Standard Model (SM):

Gravitation

Electrodynamics γ

Weak interaction $W^{\pm} \quad Z^0$

Strong interaction

Higgs ?!

Fermions of the SM

Leptons

Quarks

1st generation

$$\begin{bmatrix} \nu_{eL} \\ e^-_L \end{bmatrix} \qquad e^-_R$$

$$\begin{bmatrix} u_L \\ d_L \end{bmatrix} \qquad u_R \quad d_R$$

2nd generation

$$\begin{bmatrix} \nu_{\mu L} \\ \mu^-_L \end{bmatrix} \qquad \mu^-_R$$

$$\begin{bmatrix} c_L \\ s_L \end{bmatrix} \qquad c_R \quad s_R$$

3rd generation

$$\begin{bmatrix} \nu_{\tau L} \\ \tau^-_L \end{bmatrix} \qquad \tau^-_R$$

$$\begin{bmatrix} t_L \\ b_L \end{bmatrix} \qquad t_R \quad b_R$$

Notation:

$$L_\alpha \equiv \begin{bmatrix} \nu_{\alpha L} \\ \ell^-_{\alpha L} \end{bmatrix} \qquad \ell^-_{\alpha R}$$

Lagrangian of the leptons

EM $$-eA_\mu \sum_\alpha (\bar{\ell}_{\alpha L}\gamma^\mu \ell_{\alpha L} + \bar{\ell}_{\alpha R}\gamma^\mu \ell_{\alpha R})$$

NC: $$\frac{eZ_\mu}{\sin\theta_w \cos\theta_w}\left[\sum_\alpha (\frac{\bar{\nu}_{\alpha L}\gamma^\mu \nu_{\alpha L}}{2} + \frac{\bar{\ell}_{\alpha L}(2\sin^2\theta_w - 1)\gamma^\mu \ell_{\alpha L}}{2} + \sin^2\theta_w \bar{\ell}_{\alpha R}\gamma^\mu \ell_{\alpha R})\right]$$

CC: $$\frac{e}{\sqrt{2}\sin\theta_w \cos\theta_w}\left[\sum_\alpha (\bar{\nu}_{\alpha L}\gamma^\mu \ell_{\alpha L} W_\mu^+ + \bar{\ell}_{\alpha L}\gamma^\mu \nu_{\alpha L} W_\mu^-)\right]$$

Mass: $$m_{\alpha\beta}\bar{\ell}_{\alpha R}\ell_{\beta L} + \text{H.c.}$$

In the general basis

In Old SM: $$m_\nu = 0$$

Reminder

- Quantum mechanics (schrodinger equation)

+

Special relativity

Quantum field theory (Dirac Equation for fermions)

- Quantum Field Theory

$\Downarrow$

- Matter $\Longleftrightarrow$ Antimatter

What is the anti-particle of the electron?

What is the anti-particle of the muon?

What is the anti-particle of the tau?

What is the anti-particle of the up-quark?

What is the anti-particle of photon?

What is the anti-particle of the Z boson?

What is the anti-particle of the neutrino?

$$\nu_e + X \to e^- + Y \qquad \bar{\nu}_e + X \to e^+ + Y$$

$$\nu_\mu + X \to \mu^- + Y \qquad \bar{\nu}_\mu + X \to \mu^+ + Y$$

$$\nu_\tau + X \to \tau^- + Y \qquad \bar{\nu}_\tau + X \to \tau^+ + Y$$

Modern question: Are the neutrino and antineutrino the same particles?

More technically: Are neutrinos of Majorana nature ?

Supersymmetry

Each particle has superpartner with exactly the same quantum numbers but with different spin.

Fermion	→	sfermion
Boson	→	name+ino

Naming the super-partners of the superpartners

Electron	Selectron •
Muon	Smuon •
Tau	Stau •
Top	Stop •
Neutrino	Sneutrino •

$$f \leftrightarrow \tilde{f}$$

$$q \leftrightarrow \tilde{q}$$

$$l \leftrightarrow \tilde{l}$$

squark

slepton

How many degrees of freedom does a Dirac fermion such as electron have?

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$P_L = \frac{1 - \gamma^5}{2}$$

$$P_R = \frac{1 + \gamma^5}{2}$$

$$f_L^- \quad f_R^- \quad f_L^+ \quad f_R^+$$

$$\downarrow$$

$$\tilde{f}_L^- \quad \tilde{f}_R^- \quad \tilde{f}_L^+ \quad \tilde{f}_R^+$$

Naming the superpartners of the bosons

- Higgs Higgsino
- Gluon Gluino
- W Wino
- Photon ???
- Z ???

- Neutralino $\neq$ sneutrino
- Chargino

Why beyond Standard Model

- Theoretical motivation

- Hierarchy problem
- Quest for Unification

Observational Motivation

- Dark matter candidate
- Neutrino mass

References

Martin, A supersymmetric primer,
Hep-ph/9709356 (*last update*)

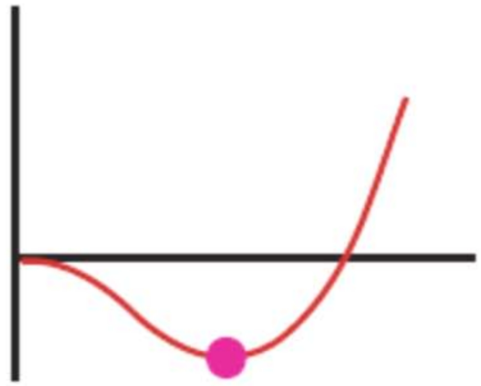
Peskin's lecture notes
http://www.slac.stanford.edu/~mpeskin

With his permission, I shall Adwidely use his material in the following but I have ded some other material of my own, too.

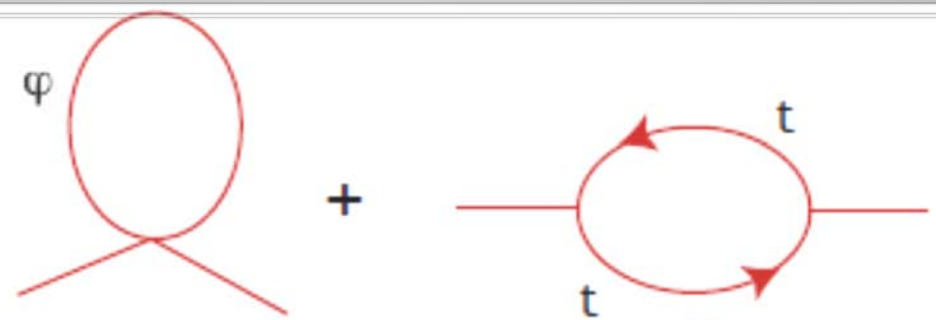
Higgs mechanism

$$V = \mu^2|\varphi|^2 + \lambda|\varphi|^4$$

with $\mu^2 < 0$.



Hierarchy problem



$$\mu^2 = \mu^2_{\mathrm{bare}} + \frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \cdots$$

Cutoff

$$\Lambda \sim m_{\mathrm{Pl}} \sim 10^{19}\ \mathrm{GeV}\,,\quad |\mu| \sim 100\ \mathrm{GeV}$$

Solutions

Technicolor ~~Electroweak precision data~~

Cancelation because of a new symmetry

shift symmetry ➤ little Higgs models

gauge symmetry ➤ extra dimension models

chiral symmetry ➤ supersymmetry models

Unification

- Electric force +
- Magnetic force

- Electromagnetism (prediction??)

Unification

- Electromagnetism

$$+$$

- Weak interaction

- Electroweak interactions

$$SU(2) \times U(1)$$

Prediction??

Grand unification

- Electroweak

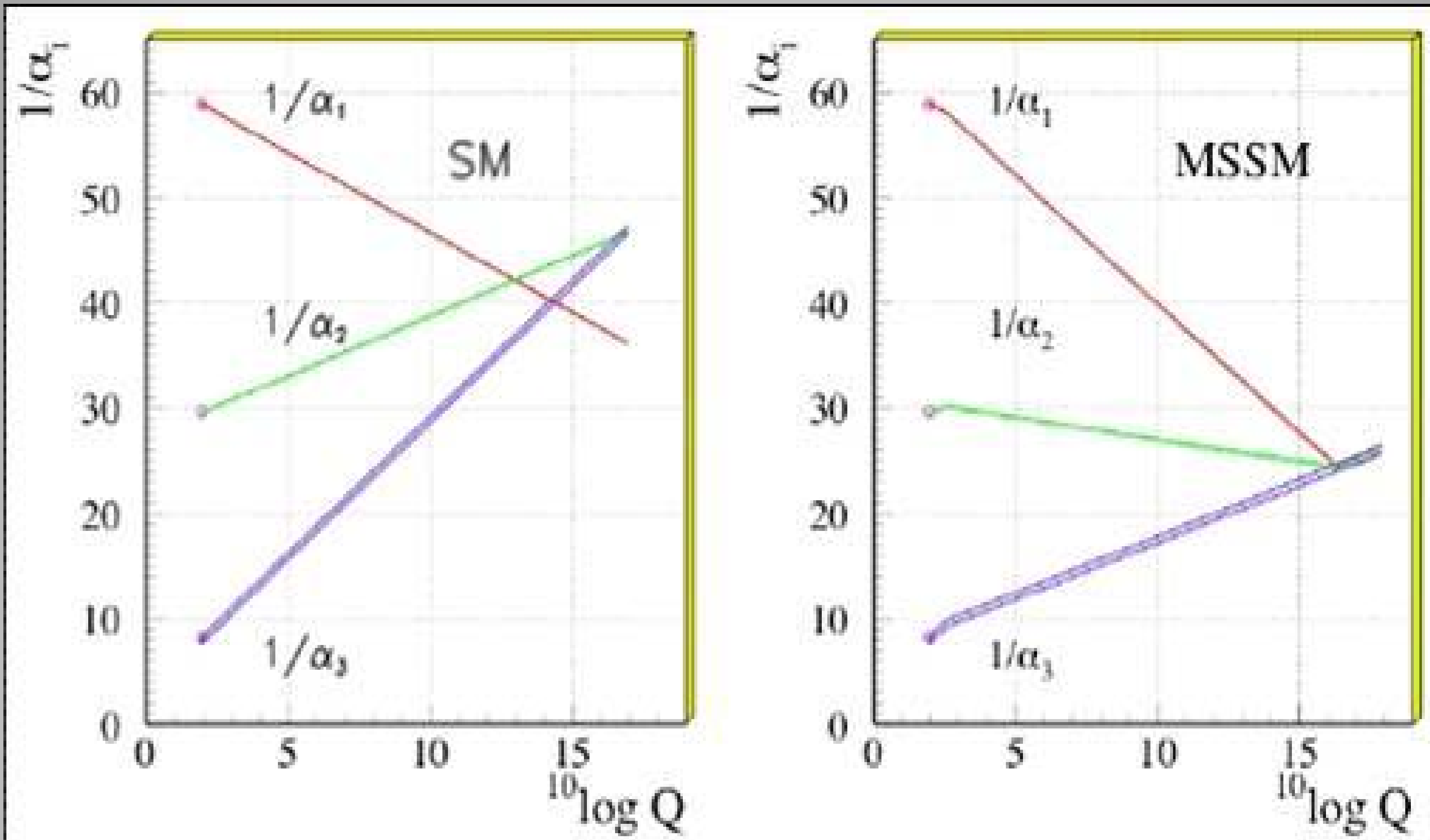
$$SU(2) \times U(1)$$

- Strong interaction

$$SU(3)$$

Grand Unification

$$SO(10) \quad SU(5)$$



SM
MSSM
1/α1
1/α2
1/α3
1/αi
0
10
20
30
40
50
60
0
5
10
15
10log Q

Susy

$$\boxed{\delta_\epsilon \phi = \epsilon \cdot \psi}$$

$$\boxed{[\epsilon \cdot Q, \phi] = \epsilon \cdot \psi} \Rightarrow \boxed{[Q_\alpha, M^{ab}] = ??}$$

$$\boxed{[Q_\alpha, H] = 0} \Rightarrow \boxed{[Q_\alpha^\dagger, H] = 0}$$

- Already lots of implications!

$$\boxed{[Q_\alpha, H] = 0}$$

$$\boxed{[Q^\dagger_\alpha, H] = 0}$$

Generalized Jacobian Identity

$$\Downarrow$$

$$\boxed{[\{Q_\alpha, Q^\dagger_\beta\}, H] = 0}$$

Implication of Lorentz symmetry

$$\boxed{\{Q_\alpha, Q^\dagger_\beta\} = 2\sigma^\mu_{\alpha\beta} R_\mu}$$

$$\boxed{\sigma^\mu_{\alpha\beta} = (1_{2\times 2}, \vec{\sigma})^\mu_{\alpha\beta}}$$

Can R_μ vanish?

- Consider an arbitrary state

$$\langle\psi|\{Q_\alpha,Q_\alpha^\dagger\}|\psi\rangle = \langle\psi|Q_\alpha Q_\alpha^\dagger|\psi\rangle + \langle\psi|Q_\alpha^\dagger Q_\alpha|\psi\rangle$$
$$= \|Q_\alpha^\dagger|\psi\rangle\|^2 + \|Q_\alpha|\psi\rangle\|^2$$

- Either charges nullify all states or R_μ is a non-zero conserved vector field.

- For a one-particle system:

$$R_\mu = r P_\mu$$

$$A + B \rightarrow A + B$$

P_μ Conservation: $P_A + P_B = P'_A + P'_B$

R_μ Conservation : $r_A P_A + r_B P_B = r_A P'_A + r_B P'_B$

Coleman and Mandula:

If $r_A \neq r_B$, $S = 1$.

Thus, $R_\mu = r P_\mu$

$$\{Q_\alpha, Q^\dagger_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu$$

$$\{Q_1, Q^\dagger_1\} = 2(E - P^3) \qquad \{Q_2, Q^\dagger_2\} = 2(E + P^3)$$

$\forall$ single state $|\psi\rangle$,

$Q_1|\psi\rangle \neq 0$ and/or $Q_2|\psi\rangle \neq 0$.

Important lesson

- You cannot be selective when you supersymmetrize a theory.

More than one pair of Q

$$\{Q_\alpha, Q_\beta\} = 0 \qquad \mathcal{N} = 1$$

More than a pair:

$$\{Q^i_\alpha, Q^j_\beta\} = Z^{ij} c_{\alpha\beta} \qquad Z^{ij} = -Z^{ji}$$

Central charge: Z^{ij}

$$c = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Possible values of $\mathcal{N}$

- Chiral,vector super-multiplet $\mathcal{N}=1$

- Other possibilities with closed representation $\mathcal{N}=2,4$

- $\mathcal{N}>8$ requires spin larger than 2

Spin half representation of Lorentz group

- Two spin half representation:

$$\boxed{\begin{aligned}\psi_L &\to (1 - i\vec{\alpha}\cdot\vec{\sigma}/2 - \vec{\beta}\cdot\vec{\sigma}/2)\psi_L \\ \psi_R &\to (1 - i\vec{\alpha}\cdot\vec{\sigma}/2 + \vec{\beta}\cdot\vec{\sigma}/2)\psi_R\end{aligned}}$$

A Lorentz invariant: $\psi_{1L}^T c\psi_{2L}$

$-c\psi_L^*$ transforms like right-handed

$$\Psi = \begin{pmatrix} \psi_{1L} \\ \psi_{2R} \end{pmatrix} = \begin{pmatrix} \psi_{1L} \\ -c\psi_{2L}^* \end{pmatrix}$$

$$\begin{aligned} \mathcal{L} &= \overline{\Psi} i\gamma \cdot \partial \Psi - M\overline{\Psi}\Psi \\ &= \psi_{1L}^\dagger i\overline{\sigma} \cdot \partial \psi_{1L} + \psi_{2L}^\dagger i\overline{\sigma} \cdot \partial \psi_{2L} \\ &\quad -(m\psi_{1L}^T c\psi_{2L} - m^*\psi_{1L}^\dagger c\psi_{2L}^*) \end{aligned}$$

with m = M.

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \overline{\sigma}^m & 0 \end{pmatrix}$$

$$\psi_{1L}^T c\psi_{2L} = \psi_{2L}^T c\psi_{1L}$$

$$(\psi_{1L}^T c\psi_{2L})^* = \psi_{2L}^\dagger(-c)\psi_{1L}^* = -\psi_{1L}^\dagger c\psi_{2L}^*$$

Mass term

$$\mathcal{L} = \psi_k^\dagger i\bar{\sigma}\cdot\partial\psi_k - (m_{jk}\psi_j^T c\psi_k - m_{jk}^*\psi_j^\dagger c\psi_k^*)$$

$$m_{jk} = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix}$$ gives a Dirac fermion

$m_{jk} = m\delta_{jk}$ gives a Majorana fermion

Algebra

$$\{Q_\alpha, Q^\dagger_\beta\} = 2\sigma^m_{\alpha\beta} P_m$$

$$[\delta_\xi, \delta_\eta] = 2i[\xi^\dagger \overline{\sigma}^m \eta - \eta^\dagger \overline{\sigma}^m \xi]\, \partial_m$$

Chiral supermultiplet

$$\delta_\xi \phi = \sqrt{2}\xi^T c\psi$$

$$\delta_\xi \psi = \sqrt{2} i\sigma^n c\xi^* \partial_n \phi + \sqrt{2}\xi F$$

$$\delta_\xi F = -\sqrt{2} i\xi^\dagger \overline{\sigma}^m \partial_m \psi$$

Consistent Lagrangian?? Consistent transformation??

$$
\begin{aligned}
[\delta_\xi, \delta_\eta]\phi = & - \delta_\xi(\sqrt{2}\eta^T c\psi) - (\xi \leftrightarrow \eta) \\
= & \ \ \sqrt{2}\eta^T \ \sqrt{2} i\sigma^n c\xi^* \partial_n \phi - (\xi \leftrightarrow \eta) \\
= & \ \ 2i\eta^T c\sigma^n c\xi^* \partial_n \phi - (\xi \leftrightarrow \eta) \\
= & \ \ 2i[\xi^\dagger \overline{\sigma}^n \eta - \eta^\dagger \overline{\sigma}^n \xi] \ \partial_n \phi
\end{aligned}
$$

- We must check if the following is invariant up to a total derivative:

$$\mathcal{L} = \partial^m \phi^* \partial_m \phi + \psi^\dagger i \overline{\sigma} \cdot \partial \psi + F^* F$$

Superpotential

- An analytical function $W(\phi)$

$$\cancel{W(\phi, \phi^\dagger)}$$

$$\mathcal{L}_W = F_k \frac{\partial W}{\partial \phi_k} - \frac{1}{2}\psi_j^T c \psi_k \frac{\partial W}{\partial \phi_j \partial \phi_k}$$

$$\mathcal{L} = \partial^m \phi_k^* \partial_m \phi_k + \psi_k^\dagger i \bar{\sigma}^m \partial_m \psi_k + F_k^* F_k + (\mathcal{L}_W + h.c.)$$

$$\boxed{\mathcal{L}_W = F_k \frac{\partial W}{\partial \phi_k} - \frac{1}{2} \psi_j^T c \psi_k \frac{\partial W}{\partial \phi_j \partial \phi_k}}$$

Eliminating Auxiliary F

$$\boxed{V_F = \sum_k \left| \frac{\partial W}{\partial \phi_k} \right|^2}$$

Notice the sign

$$\{Q_1, Q_1^\dagger\} = 2(E - P^3) \qquad \{Q_2, Q_2^\dagger\} = 2(E + P^3)$$

$$\Downarrow$$

$$\langle 0|(\{Q_1, Q_1^\dagger\} + \{Q_2, Q_2^\dagger\})|0\rangle = 4\langle 0|H|0\rangle \geq 0$$

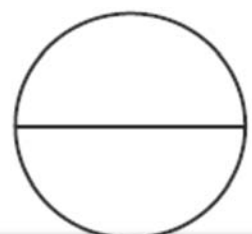
Vacuum energy vanishes if and only if

$$Q_\alpha |0\rangle = Q_\alpha^\dagger |0\rangle = 0$$

$$\langle 0| \left[\xi^T cQ, \psi_k\right] |0\rangle = \langle 0| \sqrt{2} i\sigma^n \xi^* \partial_n \phi_k + \xi F_k |0\rangle$$
$$= \xi \langle 0| F_k |0\rangle$$

Unbroken SUSY implies

$$\boxed{\langle H \rangle = \langle F \rangle = 0}$$

 = 0

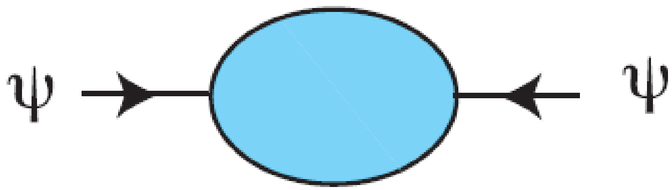
Vacuum energy vanishes!

An example

$$W = \frac{m}{2}\phi^2 + \frac{\lambda}{3}\phi^3$$

$$\begin{aligned}\mathcal{L} = \partial^m \phi^* \partial_m \phi + \psi^\dagger i\overline{\sigma}^m \partial_m \psi - |m\phi + \lambda\phi^2|^2 \\ -\frac{1}{2}(m + 2\lambda\phi)\psi^T c\psi + \frac{1}{2}(m + 2\lambda\phi)^* \psi^\dagger c\psi^*\end{aligned}$$

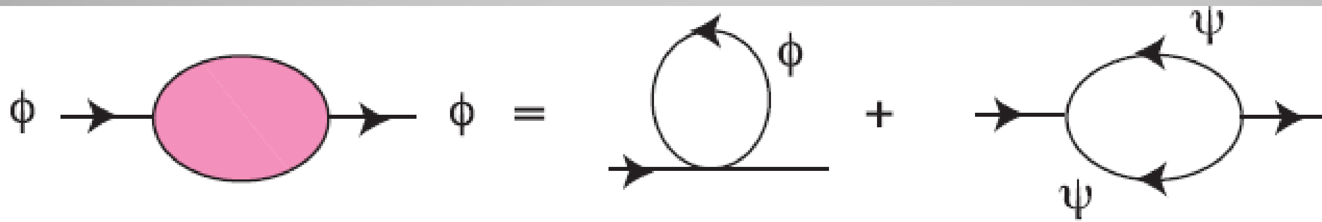
Radiative correction to fermion mass



= 0 (no diagrams)

$$\boxed{-\frac{m}{2}\psi^T c\psi}$$

Radiative correction to scalar mass

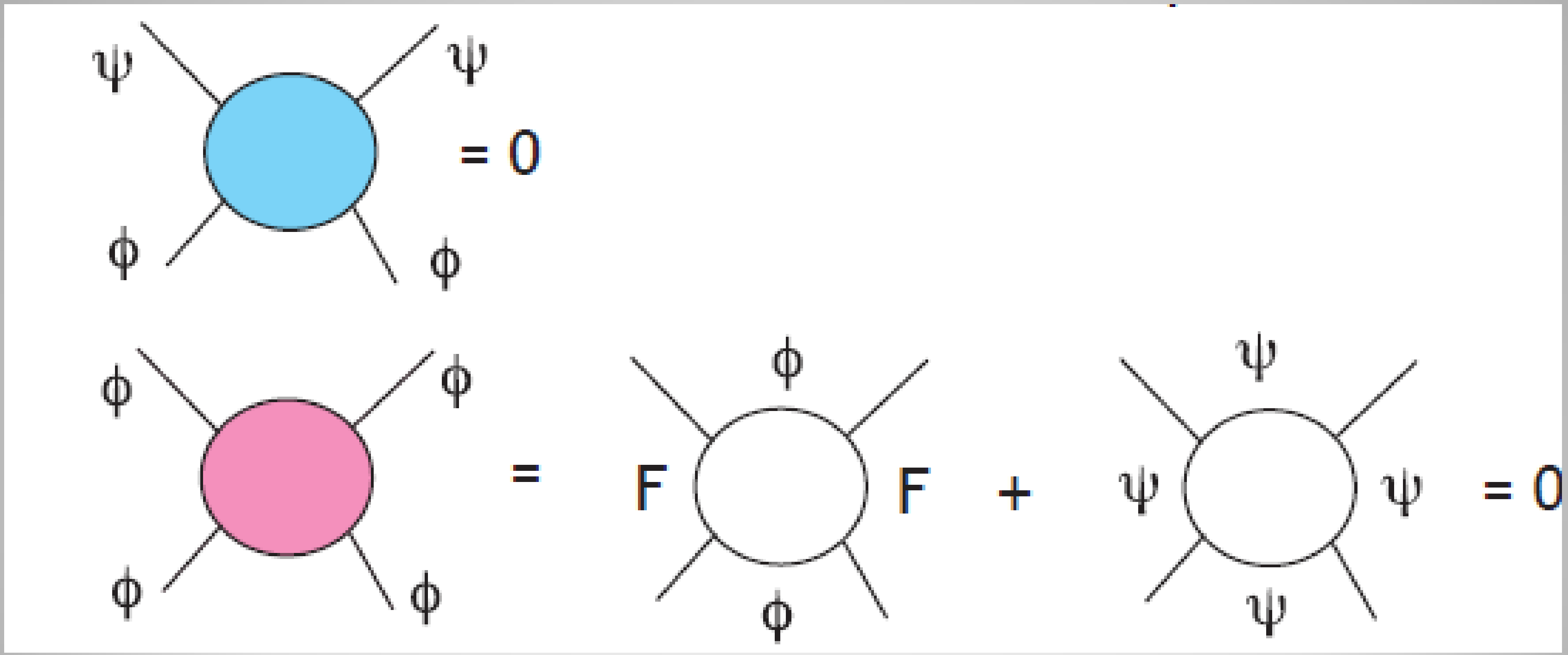


$$= -4i\lambda^2 \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2}$$

$$+\frac{1}{2}(-2i\lambda)(+2i\lambda)\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}[\frac{i\sigma\cdot p}{p^2}c\frac{i\sigma^T\cdot(-p)^T}{p^2}c]$$

$$= 0$$

Renormalization of other terms from super-potential

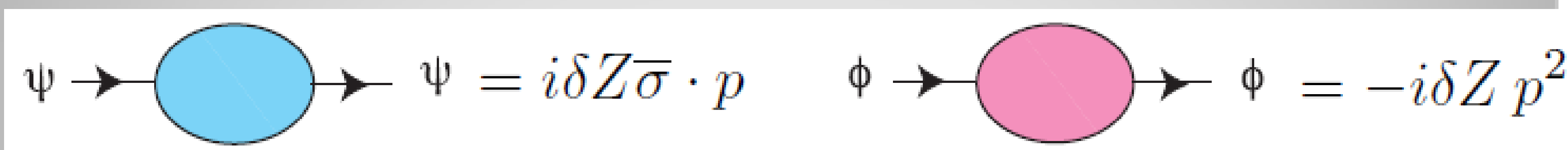


Non-renormalization theorem

- Super-potential does not get renormalized!

(As far as SUSY is exact!)

- However, field strength does



- We should then renormalize the fields.

Superspace

$$(x^\mu, \theta_\alpha, \bar{\theta}_\alpha)$$

Super-field $\Phi(x, \theta, \bar{\theta})$

How can we define susy transformation?

Simplest guess: $\theta \longrightarrow \theta + \xi$

But does not work $\{\delta_\xi, \delta_\eta\} = 0$

Supersymmetry transformation

$$\delta_\xi \Phi = \mathcal{Q}_\xi \Phi$$

$$\mathcal{Q}_\xi = \left(-\frac{\partial}{\partial\theta} - i\bar{\theta}\bar{\sigma}^m\partial_m\right)\xi + \xi^\dagger\left(\frac{\partial}{\partial\bar{\theta}} + i\bar{\sigma}^m\theta\partial_m\right)$$

$$[\mathcal{Q}_\xi, \mathcal{Q}_\eta] = -2i(\xi^\dagger\bar{\sigma}^m\eta - \eta^\dagger\bar{\sigma}^m\xi)\ \partial_m$$

- Let us define

$$D_\alpha = \frac{\partial}{\partial\theta_\alpha} - i(\overline{\theta}\sigma^m)_\alpha\partial_m$$

and

$$\overline{D}_\alpha = -\frac{\partial}{\partial\overline{\theta}_\alpha} + i(\sigma^m\theta)_\alpha\partial_m$$

$$[D_\alpha, \mathcal{Q}_\xi] = [\overline{D}_\alpha, \mathcal{Q}_\xi] = 0$$

Definition of chiral superfield $\overline{D}_\alpha\Phi = 0$

$$\bar{D}_\alpha(\Phi + \delta_\xi\Phi) = 0$$

Chiral superfield

$$\Phi(x,\theta,\overline{\theta}) = \Phi(x + i\overline{\theta}\overline{\sigma}^m\theta, \theta)$$

- Taylor expansion:

$$\Phi(x,\theta) = \phi(x) + \sqrt{2}\theta^T c\psi(x) + \theta^T c\theta\, F(x)$$

$$\int d^2\theta \; 1 = \int d^2\theta \; \theta_\alpha = 0$$

$$\int d^2\theta\theta^T c\theta$$

$$\int d^2\theta \; \Phi(x,\theta) = F(x)$$

$$\int d^2\theta \; W(\Phi) = F\frac{\partial W}{\partial \phi} - \frac{1}{2}\psi^T c\psi\frac{\partial^2 W}{\partial \phi^2}$$

$$\int d^2\theta d^2\overline{\theta} \; \Phi^\dagger\Phi = \partial^m\phi^*\partial_m\phi + \psi^\dagger i\overline{\sigma}\cdot\partial\psi + F^*F$$

Lagrangian

$$\int d^4\theta\ K(\Phi,\overline{\Phi})\ +\ \int d^2\theta\ W(\Phi)\ +\ \int d^2\overline{\theta}\ W(\overline{\Phi})$$

Kahler potetial

Supergraphs

- Feynman rules in superspace

Radiative correction

$$\int d^4\theta \; X(\Phi, \overline{\Phi})$$

Non-renormalizability of the superpotential