The Polarized Structure Function $g_2(x, Q^2)$

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Strong Interaction and QCD

- Strong interaction, running coupling ~1
 - -- QCD: accepted theory for strong interaction
 - -- asymptotic freedom (2004 Nobel) perturbation calculation works at high energy
- -- interaction significant at intermediate energy
 - quark-gluon correlations
- -- interaction strong at low energy (nucleon size)

confinement

- theoretical tools:
 pQCD, OPE, Lattice QCD, ...
- A major challenge in fundamental physics: Understand QCD in strong interaction region

→ Study and understand nucleon structure



What we "see" changes with spatial resolution



Nucleon Structure and Sum Rules

- 3 valence quarks carry ~50% of the nucleon momentum
- Spin=1/2, quarks contribute ~30%

Spin Sum Rule

Axial charge

Bjorken Sum Rule

Polarizabilities (Spin, Color)



Three Decades of Spin Structure

- 1980s: EMC (CERN) + early SLAC
- Quark contribution to proton is very small

 $\Delta \Sigma = \Delta \mathbf{u} + \Delta \mathbf{d} + \Delta \mathbf{s} \approx \mathbf{0.3}$ "spin crisis"

Violation of Ellis- Jaff Sum Rule



Three Decades of Spin Structure

1990s: SLAC, SMC(CERN), HERMES(DESY)

 $\Delta\Sigma = 20 - 30\%$

• The rest = gluon and quark orbital angular momentum

$$1/2 = (1/2)\Delta\Sigma + L_q + \Delta G + L_g$$

Three Decades of Spin Structure

 2000s: COMPASS(CERN), HERMESS, RHIC-Spin, JLab,...

 $\Delta\Sigma \approx 30\%$

 ΔG probably small

Orbital angular momentum probably significiant Transversity

Transverse-momentum dependent distribution Generalized Parton Distributions Much more work to do at next decade

Longitudinal Spin

Spin in Valence (high-x) Region Moments of Spin Structure Functions Spin Sum Rules

Some fundamental relations

The spin structure of the nucleon in Inclusive <u>DIS</u> scattering

Longitudinally electron in lab frame



definition of scattering & polarization angles The cross section difference between target polarized along $\alpha, \alpha + \pi$

$$\frac{d^3(\sigma(\alpha) - \sigma(\alpha + \pi))}{dx \, dy \, d\phi} = \frac{e^4}{4\pi^2 Q^2} \left\{ \cos(\alpha) \left\{ \left(1 - \frac{y}{2} - \frac{y^2}{4}\gamma^2\right) \, g_1(x, Q^2) - \frac{y}{2}\gamma^2 g_2(x, Q^2) \right\} - \sin(\alpha)\cos(\phi)\sqrt{\gamma^2(1 - y - \frac{y^2}{4}\gamma^2)} \left\{ \frac{y}{2} \, g_1(x, Q^2) + g_2(x, Q^2) \right\} \right\} .$$

Transverse Spin

g₂ Structure Function and Moments Burkhardt - Cottingham Sum Rule Color Polarizability d₂



Transversity

Three twist-2 quark distributions:

Momentum distributions: $q(x,Q^2) = q^{\uparrow}(x) + q^{\downarrow}(x)$ Longitudinal spin distributions: $\Delta q(x,Q^2) = q^{\uparrow}(x) - q^{\downarrow}(x)$ Transversity distributions: $\delta q(x,Q^2) = q^{\perp}(x) - q_{\perp}(x)$

$$\begin{aligned} f(x) &= \int \frac{\mathrm{d}\xi^{-}}{4\pi} \,\mathrm{e}^{\mathrm{i}xP^{+}\xi^{-}} \langle PS | \overline{\psi}(0)\gamma^{+}\psi(0,\xi^{-},0_{\perp}) | PS \rangle \\ \Delta f(x) &= \int \frac{\mathrm{d}\xi^{-}}{4\pi} \,\mathrm{e}^{\mathrm{i}xP^{+}\xi^{-}} \langle PS | \overline{\psi}(0)\gamma^{+}\gamma_{5}\psi(0,\xi^{-},0_{\perp}) | PS \rangle \\ \Delta_{T}f(x) &= \int \frac{\mathrm{d}\xi^{-}}{4\pi} \,\mathrm{e}^{\mathrm{i}xP^{+}\xi^{-}} \langle PS | \overline{\psi}(0)\gamma^{+}\gamma^{1}\gamma_{5}\psi(0,\xi^{-},0_{\perp}) | PS \rangle \end{aligned}$$

Spin structure functions

- Quark component of the nucleon helicity is much smaller than the naive QPM predictions
- $g_1(x,Q^2), g_2(x,Q^2)$ are important tools for testing QCD, models of nucleon structure, and sum rules
- *g*₁ is dominant when the target is polarized along the beam direction
- g₂ is dominant when longitudinally polarized leptons scatter from transversely polarized nucleons
- g₂: central to knowledge of nucleon structure but remained unmeasured at low Q² and still have limited statistical precision
- Properties of g₂ have been established using OPE within QCD

g₂ and Quark-Gluon Correlations

 $g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \bar{g}_2(x,Q^2)$

a twist-2 term (Wandzura & Wilczek, 1977):

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 g_1(x,Q^2) \frac{dy}{y}$$

a twist-3 term with a suppressed twist-2 piece (Cortes, Pire & Ralston, 1992):

$$\bar{g}_{2}(x,Q^{2}) = -\int_{x}^{1} \frac{\partial}{\partial y} \left[\frac{m_{q}}{M} \frac{h_{T}(y,Q^{2})}{\sqrt{1 + \xi(y,Q^{2})}} + \frac{\xi(y,Q^{2})}{\sqrt{1 + \xi(y,Q^{2})}} \right] \frac{dy}{y}$$
Transversity

 g₂ - g₂^{WW}: a clean way to access twist-3 contribution quantify q-g correlations expression of Wandzura-Wilczek(g₂^{tw.2})

can be derived from the OPE Sum rules for $g_1 \& g_2$

at fixed Q²

$$\int_0^1 x^n g_1(x, Q^2) dx = \frac{a_n}{2}, \quad n = 0, 2, 4, \dots,$$

$$\int_0^1 x^n g_2(x, Q^2) dx = \frac{1}{2} \frac{n}{n+1} (d_n - a_n), \quad n = 2, 4, \dots$$

by keeping a_n(twist-2) and neglecting the d_n(twist-3) matrix elements



Figure 2: Q^2 averaged (0.8–8.2 GeV²) xg_2 from E155x (solid circle), E143 (open diamond) and E155 (open square). Error bars are statistical. Also shown is g_2^{WW} (solid line) at the average Q^2 of E155x. Curves are the bag model calculations of Stratmann[30] (dash-dot) and Song[31] (dot) and the chiral soliton models of Weigel and Gamberg[32] (short dash) and Wakamatsu[33] (long dash). *Reproduced from [26].*











Q^2 dependence below 1 GeV²

JLab E97-103 (3He) DIS, Q2 dependence mainly below 1.4 GeV2

Spokespersons: T. Averett and W. Korsch

Student: K. Kramer

Phys. Rev. Lett. 95:142002, 2005.



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Moments of Structure Functions (continued)

$$\implies a_2(Q^2) \equiv 2 \int_0^1 dx \, x^2 \, g_1^{\text{twist}-2}(x,Q^2) \rightarrow \text{target mass correction term}$$

 \implies $d_2(Q^2) \rightarrow$ dynamical twist-3 matrix element

$$d_2(Q^2) = \int_0^1 dx \ x^2 \left[2\mathbf{g_1}(x, Q^2) + 3\mathbf{g_2}(x, Q^2) \right]$$

 \implies $f_2(Q^2) \implies$ dynamical twist-4 matrix element

$$f_2 M^2 S^{\mu} = \frac{1}{2} \sum_q e_q^2 < N |\bar{\psi}_q g \widetilde{F}^{\mu\nu} \gamma_{\nu} \psi_q | N >$$

• 2^{nd} moment of $g_2 - g_2^{WW}$

 $d_2(Q^2) \rightarrow$ dynamical twist-3 matrix element

$$d_{2}(Q^{2}) = 3\int_{0}^{1} x^{2} [g_{2}(x,Q^{2}) - g_{2}^{WW}(x,Q^{2})] dx$$
$$= \int_{0}^{1} x^{2} [2g_{1}(x,Q^{2}) + 3g_{2}(x,Q^{2})] dx$$

d₂ and g₂-g₂^{WW}: clean access of higher twist (twist-3) effect: q-g correlations

Color Polarizabilities χ_E , χ_B are linear of d_2 , f_2

$$d_{2} = \frac{1}{8} (\chi_{E} + 2 \chi_{B})$$
$$f_{2} = (\chi_{E} - \chi_{B})/2$$

Provide a benchmark test of Lattice QCD at high Q² at high Q², d₂ measure include color field by target spin





Induced Color Magnetic and Electric Fields

$$\mathbf{B}_C \sim \chi_B \mathbf{S}$$

 $\mathbf{E}_C \sim \chi_E \mathbf{S}$



- Spin structure study full of surprises and puzzles
- Past experiments on the neutron and proton suggest that the twist-3 and twist-4 are small but finite.
- Precision measurements of g1 and g2 in the range 1 < Q² < 4 GeV² are crucial for an improved extraction of the
 - Average color Lorentz force
 - "Color polarizabilities"
- We need greater Q^2 and x coverage, to have a better interpretation of g_2 s behavior and interesting physics.



- The behaviour of g₂(x,Q²) is more complicated in comparison with the other structure functions.
- > Within experimental precision, specially in DIS region, the g_2 data are well described by the twist-2 contribution, g_2^{WW} .
- > Twist-3 matrix elements are extracted and compared to theoretical predictions.
- More precise data on g₂ are needed to make any conclusions regarding possible twist-3.
- > *PSFs* at low Q² have physical interpretation. $g_2(x)$ provides access to q-g correlations in the nucleon. The x² moment of $\overline{g}_2(x)$ can be considered as an average transverse force on quarks in DIS scattering.



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