Gauge-flation

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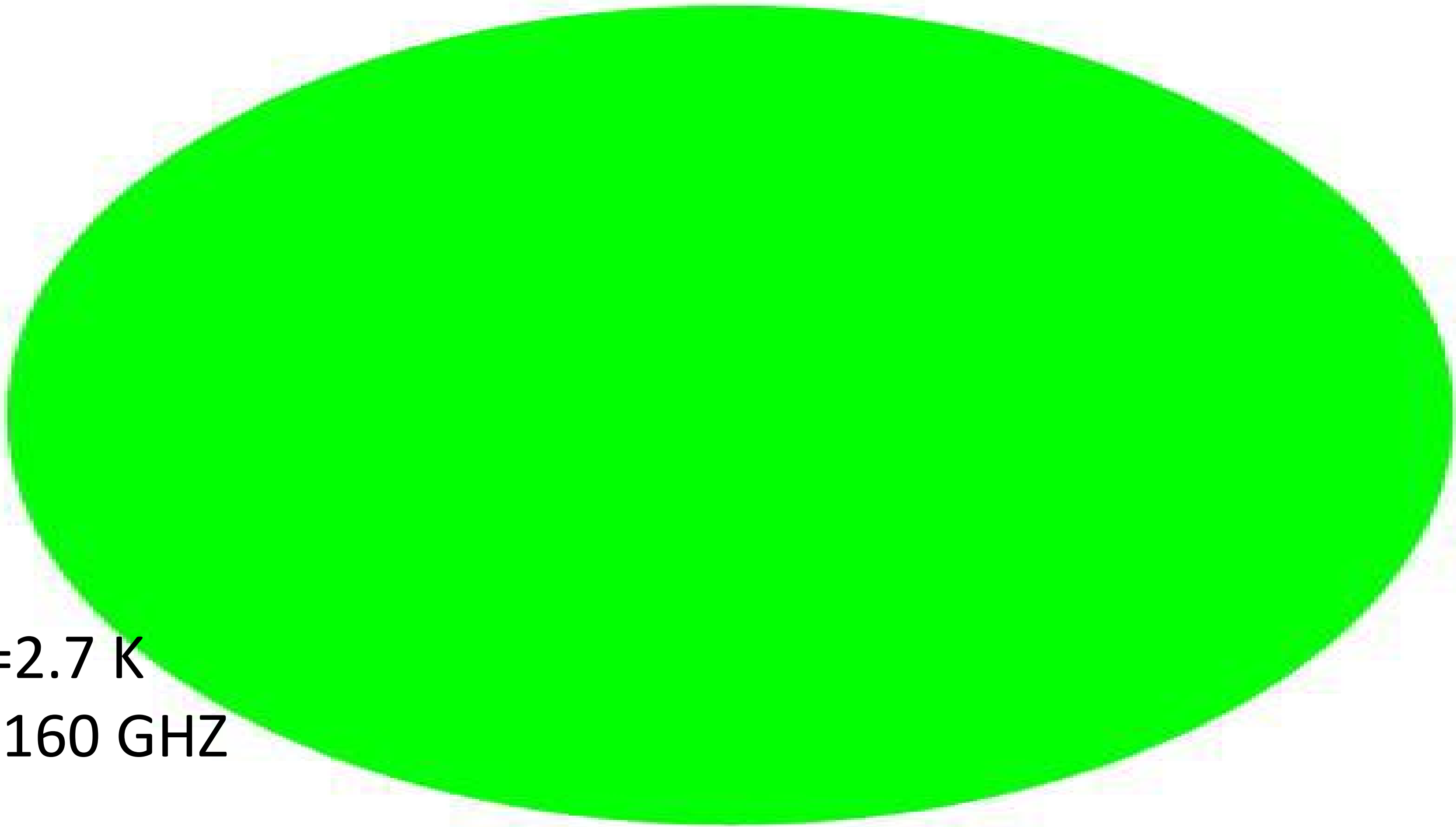
arXiv:1102.1513
&
arXiv:1102.1932

Gauge-flation

- The Story of Inflation
- Introducing Gauge-flation Framework
- Background Dynamics *(analytic & numerical)*
- Gauge-flation Cosmic Perturbation Analysis
- Contact with Observations
- Summary and Outlook

Cosmic Microwave Background

Wilson & Penzias   1964

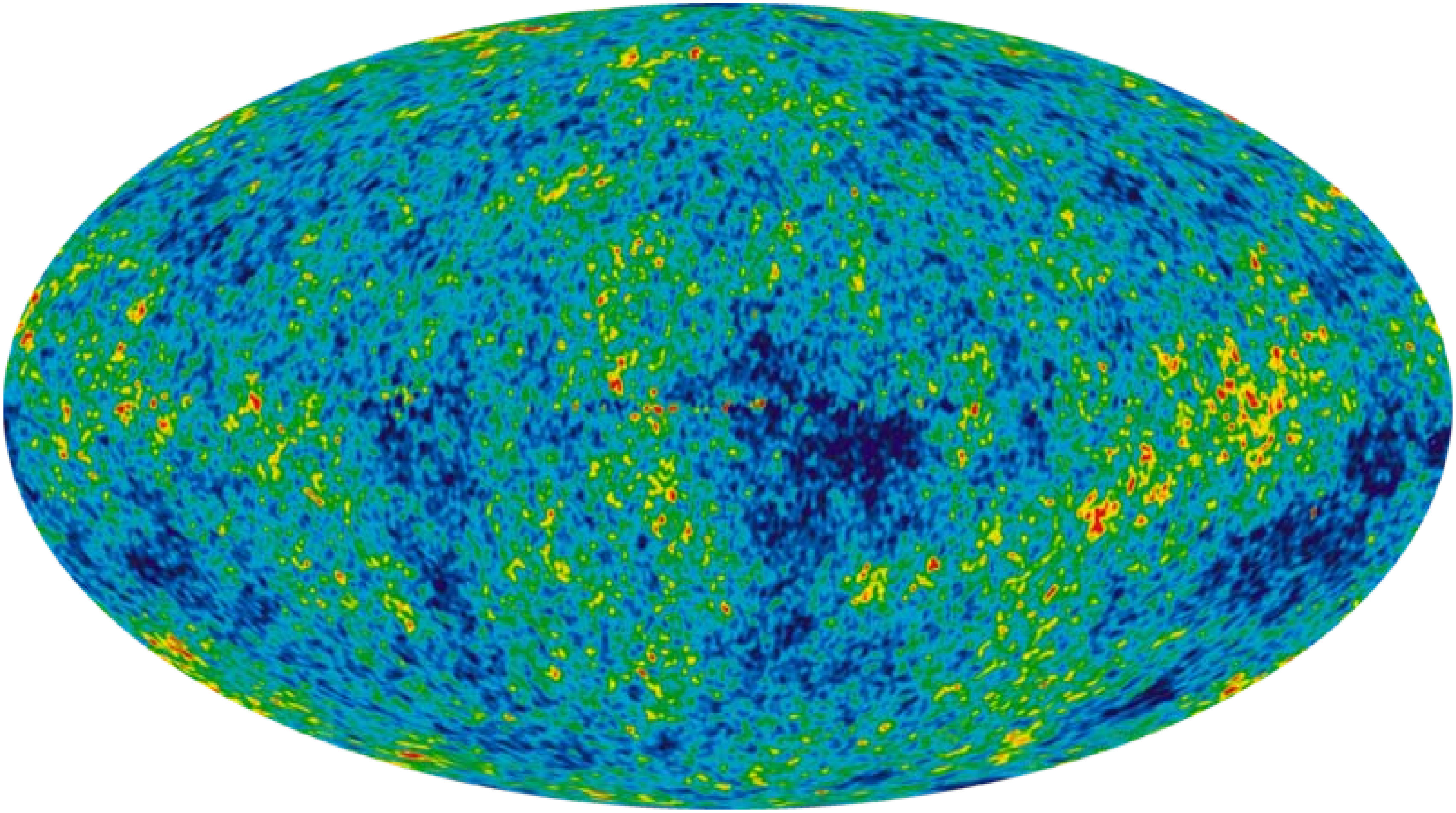


T=2.7 K

f=160 GHZ

Cosmic Microwave Background

CMB as we know today



CMB data & the inverse problem

$$\mathcal{P}_{\mathcal{R}} \simeq 2.5 \times 10^{-9} ,$$

$$n_{\mathcal{R}} = 0.968 \pm 0.012 ,$$

$$r < 0.24 .$$

$$\Delta^2_{\mathcal{R}}(k) \equiv \frac{k^3 P_{\mathcal{R}}(k)}{2\pi^2} = \Delta^2_{\mathcal{R}}(k_0) \left(\frac{k}{k_0}\right)^{n_s(k_0)-1}$$

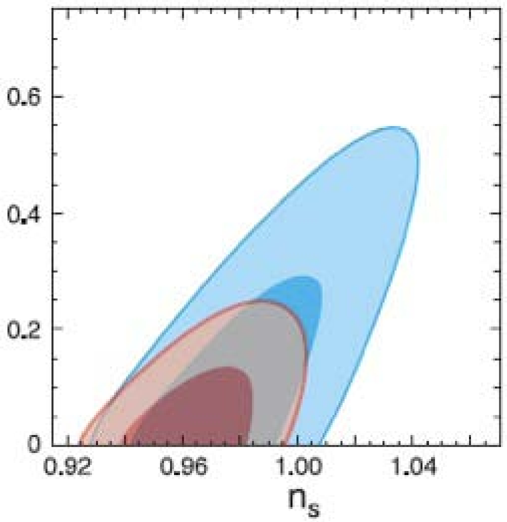
$$\Delta^2_h(k) \equiv \frac{k^3 P_h(k)}{2\pi^2} = \Delta^2_h(k_0) \left(\frac{k}{k_0}\right)^{n_t}$$

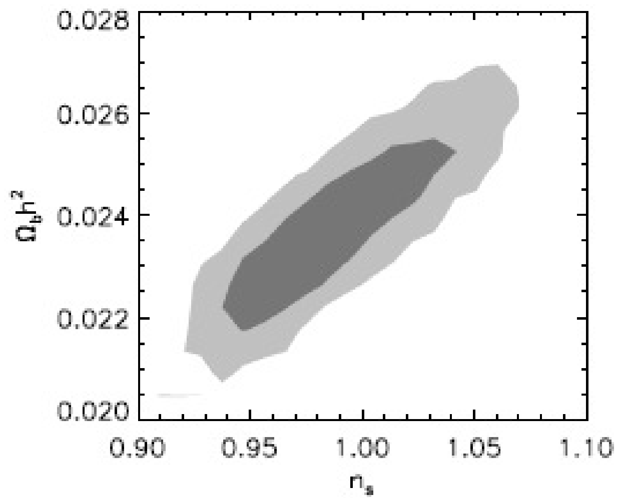
$$r \equiv \frac{\Delta^2_h(k_0)}{\Delta^2_{\mathcal{R}}(k_0)},$$

Komatsu, E. et al. 2010, Astrophys. J. Suppl., submitted, arXiv:1001.4538

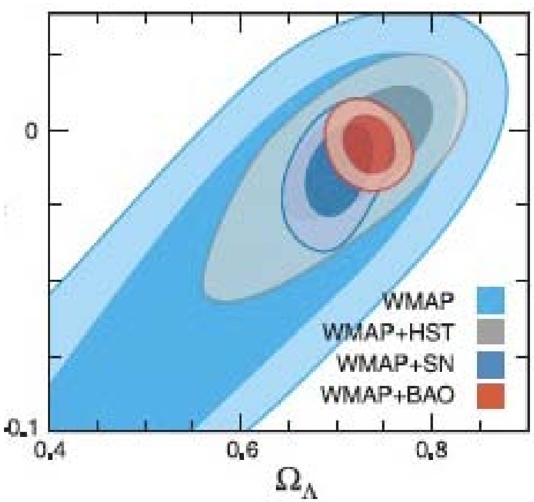
WMAP results

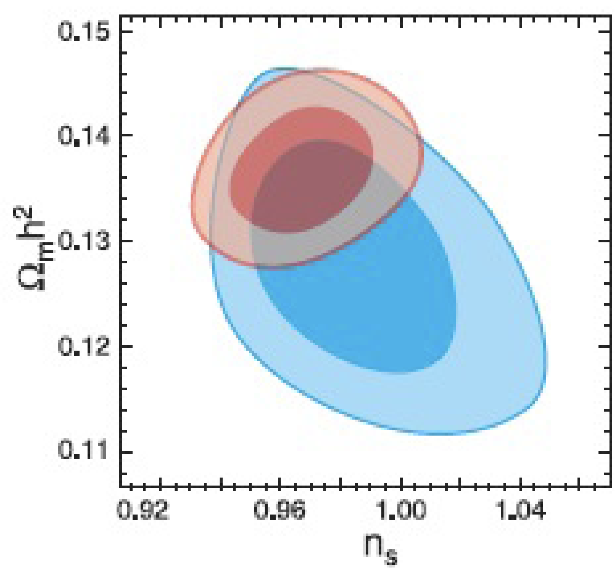
r





Ω_k





Initial condition puzzles in big bang cosmology

- **The Horizon Problem**: Why is the Universe at cosmic scales so homogeneous and isotropic? During its evolution, the Universe did not have enough time to become so isotropic and homogeneous.

- **The Flatness Problem:** Why is the Universe so flat? If today $\Omega \sim 1$, then extrapolating back to very early Universe at Planck time we find $|\Omega - 1| \sim 10^{-60}$

- There are tiny *fluctuations* at the level of 10^{-5} on the smooth CMB background, which are almost **scale invariant**, adiabatic and Gaussian. What mechanism can create these perturbations ?

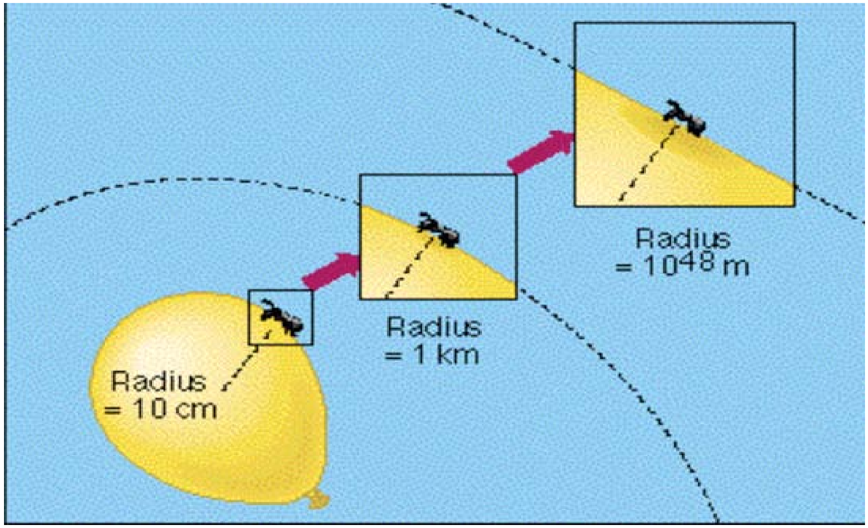
Inflation

- A period of acceleration in very early Universe will provide initial conditions and flattens the Universe.

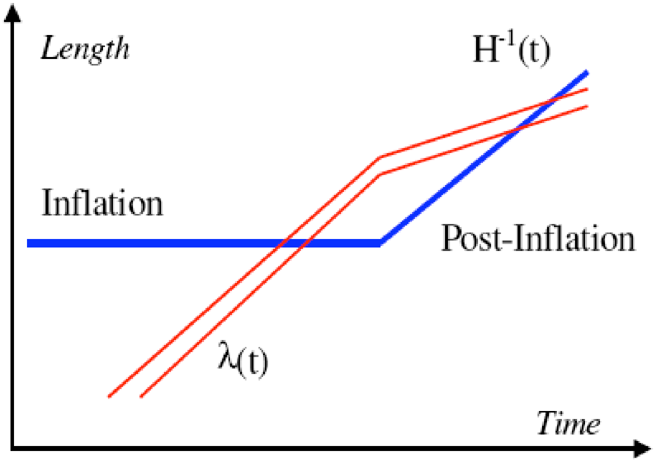
- Primordial quantum fluctuations during inflation seeds the observed almost scale invariant Gaussian perturbations in CMB.

- Quantum fluctuations initially inside the horizon are stretched by Inflation to super-horizon modes and *freeze out*.

- After inflation ends they re-enter the horizon as classical modes to seed the observed structures and induce the CMB fluctuations.



www.astro.princeton.edu/~tremaine/ast541/das.ppt

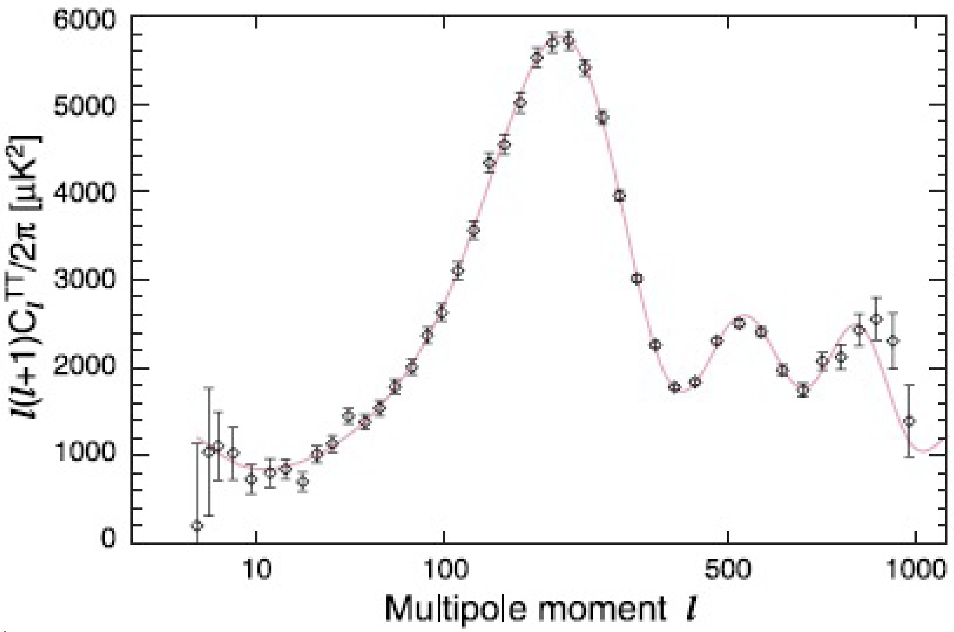


CMB observations and Inflation

- All observations WMAP, fit well within LCDM model and support inflation.

- Different inflationary models predict different values for cosmological parameters like scalar spectral index n_s and tensor-to-scalar ratio r which can be measured in CMB.

- Despite of many attempts to embed inflation within the context of string theory or particle physics, there is no compelling and theoretically well-motivated model of inflation.



WMAP results

Basics of Inflation, *Slow-roll inflation*

Given the FRW metric

$$ds^2 = -dt^2 + a(t)^2 dx_i dx_i$$

For a minimally coupled to a scalar field ϕ with

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi),$$

In the slow-roll regime where $\dot{\phi}^2 << V(\phi)$

Leads to $a(t) \sim e^{Ht},$ $\qquad H^2 = \frac{1}{3M_{pl}^2} V(\phi)$

Slow-roll Single-Field Inflation

- Slow-roll parameters

$$\varepsilon = \frac{T_{kin}}{V(\phi)} = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = M_{pl}^2 \frac{V''}{V}.$$

- In models of chaotic inflation, to have successful inflation we should have *super-Planckian fields and a hierarchy*:

$$\varepsilon, \eta \approx 0.01, \qquad H \approx 10^{-4} - 10^{-5} M_{pl}$$

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \longrightarrow m \approx 10^{-6} - 10^{-7} M_{pl}, \; \phi_i \sim 15 M_{pl}$$

$$V(\phi) = \frac{1}{4} \lambda \phi^4 \longrightarrow \lambda \approx 10^{-13} - 10^{-14}, \; \phi_i \sim 22 M_{pl}$$

Simple Inflationary models against the data

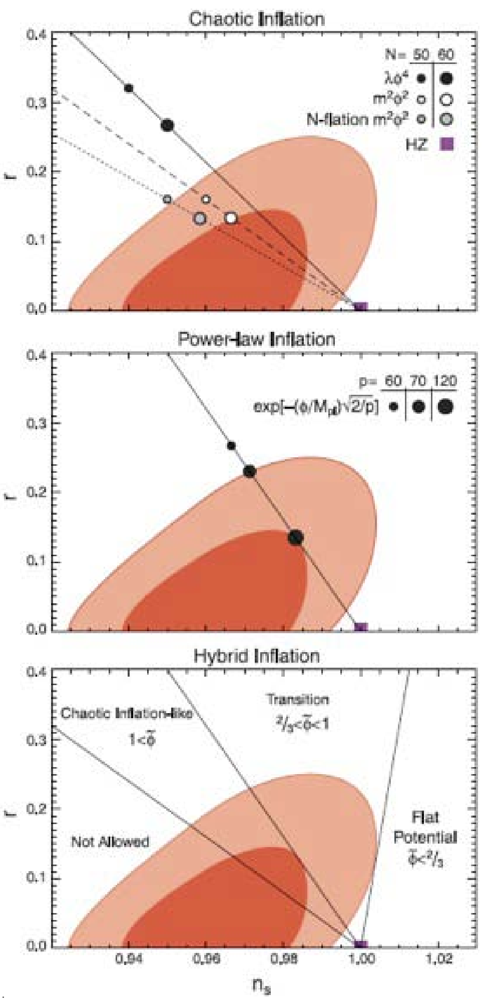
$$n_s - 1 = 2\eta - 6\varepsilon,$$

$$r = 16\varepsilon$$

$$1 - n_s = 3M_{pl}^2\left(\frac{V'}{V}\right)^2 - 2M_{pl}^2\frac{V''}{V},$$

$$r = 8M_{pl}^2\left(\frac{V'}{V}\right)^2,$$

$$r = \frac{8}{3}(1 - n_s) + \frac{16M_{pl}^2}{3}\frac{V''}{V}.$$



$$V(\phi) \propto \phi^n$$

$$V(\phi) = V_0 \exp\left(\frac{-\phi}{M_{pl}}\sqrt{2/p}\right)$$

Hybrid Inflation

Zoo of Inflationary Models

1980

- Old Inflation
- New Inflation
- Chaotic inflation
- Double Inflation
- Extended Inflation
- Power-law Inflation

1990

- Hybrid Inflation
- Natural- Inflation
- SUSY D-term Inf.
- SUSY F-term Inf.
- Brane-Inflation
- Assisted Inflation

2000

- K-Inflation
- DBI Inflation
- Tachyon Inflation
- Inflection point Inf.
- N-flation
- Racetrack Inflation
- Warped Brane-Inf.
- D3-D7-Inflation
- M-flation
- Galileon (Inflation)

2010

- **Gauge-flation**

Gauge-flation: General Framework

- Slow-roll inflation driven by non-Abelian gauge field

  A^a_μ e.g. in the SU(2) algebra a=1,2,3, with the Field Strength

$$F^a_{\ \mu\nu} = \partial_\mu A^a_{\ \nu} - \partial_\nu A^a_{\ \mu} - g\epsilon^a_{\ bc} A^b_{\ \mu} A^c_{\ \nu} \ .$$

- Minimally coupled to Einstein gravity

- Gauge and Lorentz invariant Lagrangians $\mathcal{L} = \mathcal{L}(F^a_{\mu\nu}; g_{\mu\nu})$,

Gauge-flation: General Framework

- FRW metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \,,$$

- Ansatz in the temporal gauge

$$A^a{}_\mu = \begin{cases} \phi(t)\delta^a_i \,, & \mu = i \\ 0 \,, & \mu = 0 \,. \end{cases}$$

- $\psi(t) = \dfrac{\phi(t)}{a(t)}$ is a scalar under diffeomorphism

$$F^a{}_{0i} = \dot{\phi}\delta^a_i \,, \qquad F^a{}_{ij} = -g\phi^2\epsilon^a{}_{ij} \,.$$

Restoring Rotational Symmetry

❖ Turning on gauge (vector) fields in the background breaks rotational symmetry:

$$A_i^a \to R_i^j A_j^a \qquad \text{(A)}$$

❖ Gauge fields are defined up to gauge transformations

$$A_\mu^a \to \partial_\mu \lambda^a - \varepsilon_{bc}^a \lambda^b A_\mu^c$$

➢ In the temporal gauge , we still can make ***time independent*** gauge transformations :

$$A_i^a \to \Lambda_b^a A_i^b \qquad \text{(B)}$$

Rotational non-invariance is compensated by global time independent gauge transformation

Consistency of Reduction to Rotation Invariant Sector

Gauge field Eq. $D_\mu \frac{\partial \mathfrak{L}}{\partial F_{\mu\nu}} = 0\,,$

i) Has such solution

ii) Evaluated on the anstaz, it's E.o.M is equivalent to the one obtained from $\mathfrak{L}_{\rm red}(\dot{\phi}, \phi; a(t))$,

$$\frac{d}{a^3 dt}(a^3 \frac{\partial \mathfrak{L}_{red.}}{\partial \dot{\phi}}) - \frac{\partial \mathfrak{L}_{red.}}{\partial \phi} = 0\,,$$

Consistency of Reduction to Rotation Invariant Sector

The energy-momentum tensor in the ansatz is

$$T^{\mu}_{\nu} = diag(-\rho, P, P, P) \,.$$

And equal to the T^{μ}_{ν} obtained from the reduced action:

$$\rho = \frac{\partial \mathfrak{L}_{red.}}{\partial \dot{\phi}} \dot{\phi} - \mathfrak{L}_{red.}$$

$$P = \frac{\partial (a^3 \mathfrak{L}_{red.})}{\partial a^3} \,.$$

A specific model of Gauge-flation

- An appropriate choice involving F^4

$$S = \int d^4x\sqrt{-g}\left[-\frac{R}{2} - \frac{1}{4}F^a{}_{\mu\nu}F_a{}^{\mu\nu} + \frac{\kappa}{384}(\epsilon^{\mu\nu\lambda\sigma}F^a{}_{\mu\nu}F^a{}_{\lambda\sigma})^2\right]$$

Reduced Lagrangian

$$\mathcal{L}_{red} = \frac{3}{2}\left(\frac{\dot{\phi}^2}{a^2} - \frac{g^2\phi^4}{a^4} + \kappa\frac{g^2\phi^4\dot{\phi}^2}{a^6}\right).$$

- Energy
- pressure

$$\rho = \frac{3}{2}\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4} + \kappa\frac{g^2\phi^4\dot{\phi}^2}{a^6}\right),$$

$$P = \frac{1}{2}\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4} - 3\kappa\frac{g^2\phi^4\dot{\phi}^2}{a^6}\right).$$

Background Dynamics (Analytic)

Friedmann Eq.s

$$H^2 = \frac{1}{2} \left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4} + \kappa \frac{g^2\phi^4\dot{\phi}^2}{a^6} \right),$$

$$\dot{H} = -\left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2\phi^4}{a^4}\right),$$

Field Eq.

$$\left(1 + \kappa\frac{g^2\phi^4}{a^4}\right)\frac{\ddot{\phi}}{a} + \left(1 + \kappa\frac{\dot{\phi}^2}{a^2}\right)\frac{2g^2\phi^3}{a^3} + \left(1 - 3\kappa\frac{g^2\phi^4}{a^4}\right)\frac{H\dot{\phi}}{a} = 0\,.$$

- Slow-roll parameters: $\epsilon = -\frac{\dot{H}}{H^2}$, & $\eta = -\frac{\ddot{H}}{2\dot{H}H}$,
- *NOTE: In our units* $M_{pl} = 1$

Background Dynamics (Analytic)

- Asking for small $\epsilon = \frac{2\rho_{YM}}{\rho_{YM} + \rho_\kappa}$. : $\rho_\kappa \gg \rho_{YM}$.
- To measure the slow-roll $\delta \equiv -\frac{\dot\psi}{H\psi}$,

$$\gamma = \frac{g^2\psi^2}{H^2} ,$$

$$\epsilon \simeq \psi^2(\gamma+1),$$

$$\eta \simeq \psi^2 \quad \Rightarrow \quad (3 + \frac{\dot\delta}{H\delta})\delta \simeq \frac{\gamma}{2(\gamma+1)}\epsilon^2 ,$$

$$\kappa \simeq \frac{(2-\epsilon)(\gamma+1)^3}{g^2\epsilon^3},$$

Background Dynamics (Analytic)

- The Hubble

$$H^2 \simeq \frac{g^2\psi^4}{\epsilon - \psi^2} = \frac{g^2\epsilon}{\gamma(\gamma+1)}\,.$$

- Number of e-folding

$$N_e = \int_{t_i}^{t_f} H dt \simeq \frac{\gamma_i + 1}{2\epsilon_i} \ln \frac{\gamma_i + 1}{\gamma_i}\,.$$

- #e-fold for a single scalar model

$$N_{ss} \simeq \int_{\phi_f}^{\phi} \frac{V}{V_\phi} d\phi$$

Background Dynamics (Numerical)

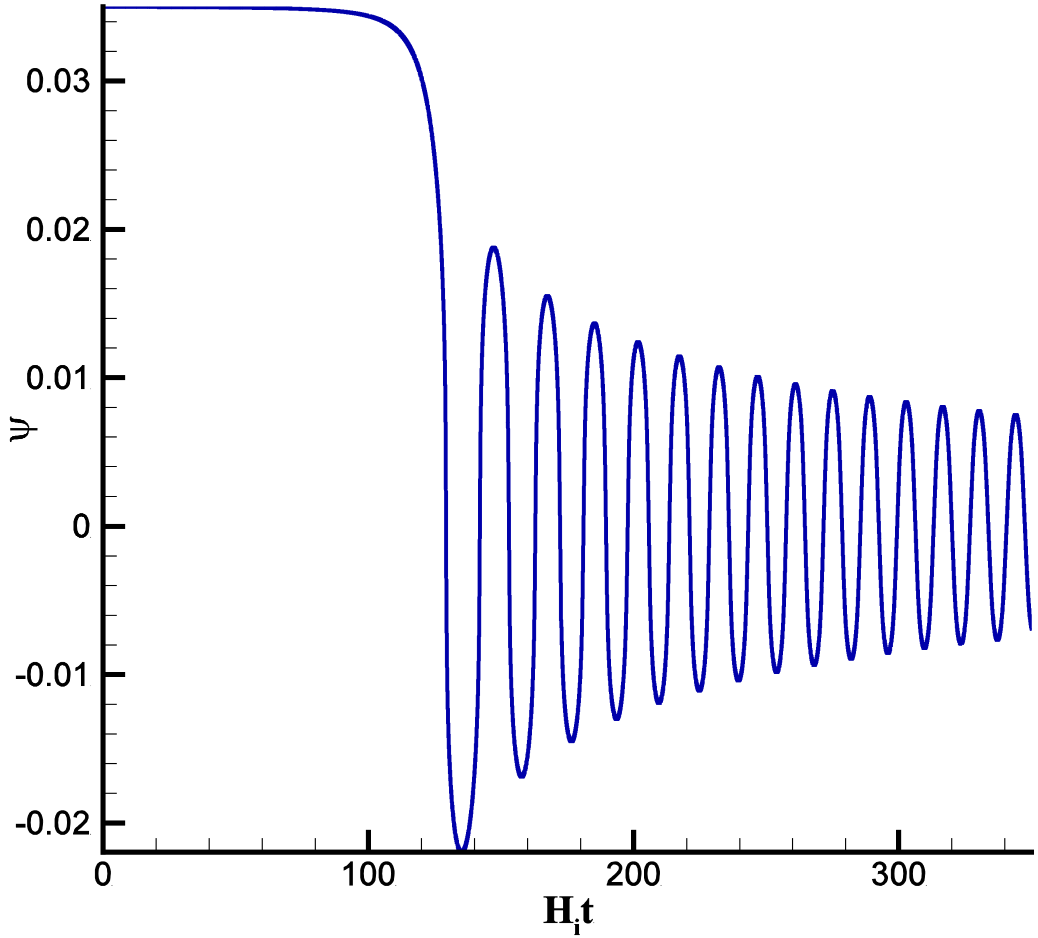
$$\psi_i = 0.035, \dot{\psi}_i = -10^{-10}; \; g = 2.5 \times 10^{-3}, \kappa = 1.733 \times 10^{14}.$$

$$H_i = 3.4 \times 10^{-5},$$

$$\gamma_i = 6.62,$$

$$\epsilon_i = 9.3 \times 10^{-3}$$

$$\delta_i = 8.4 \times 10^{-5}.$$



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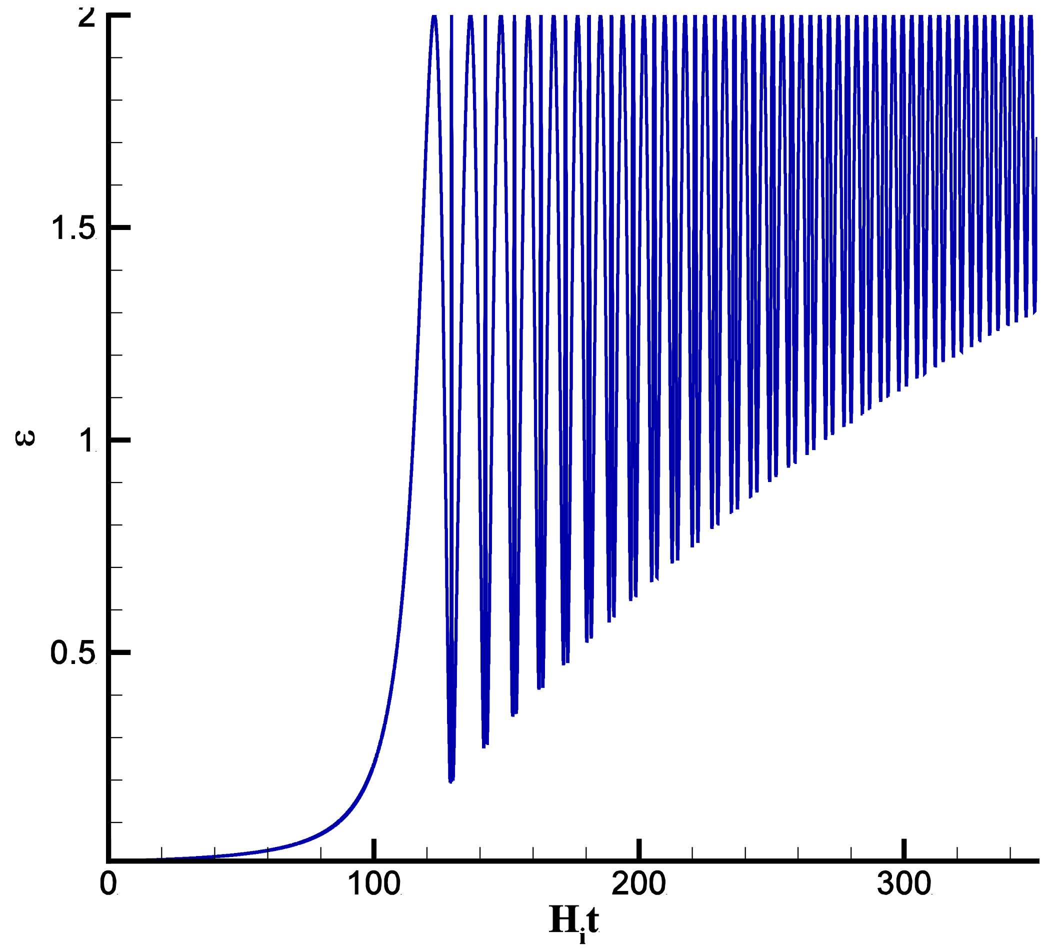

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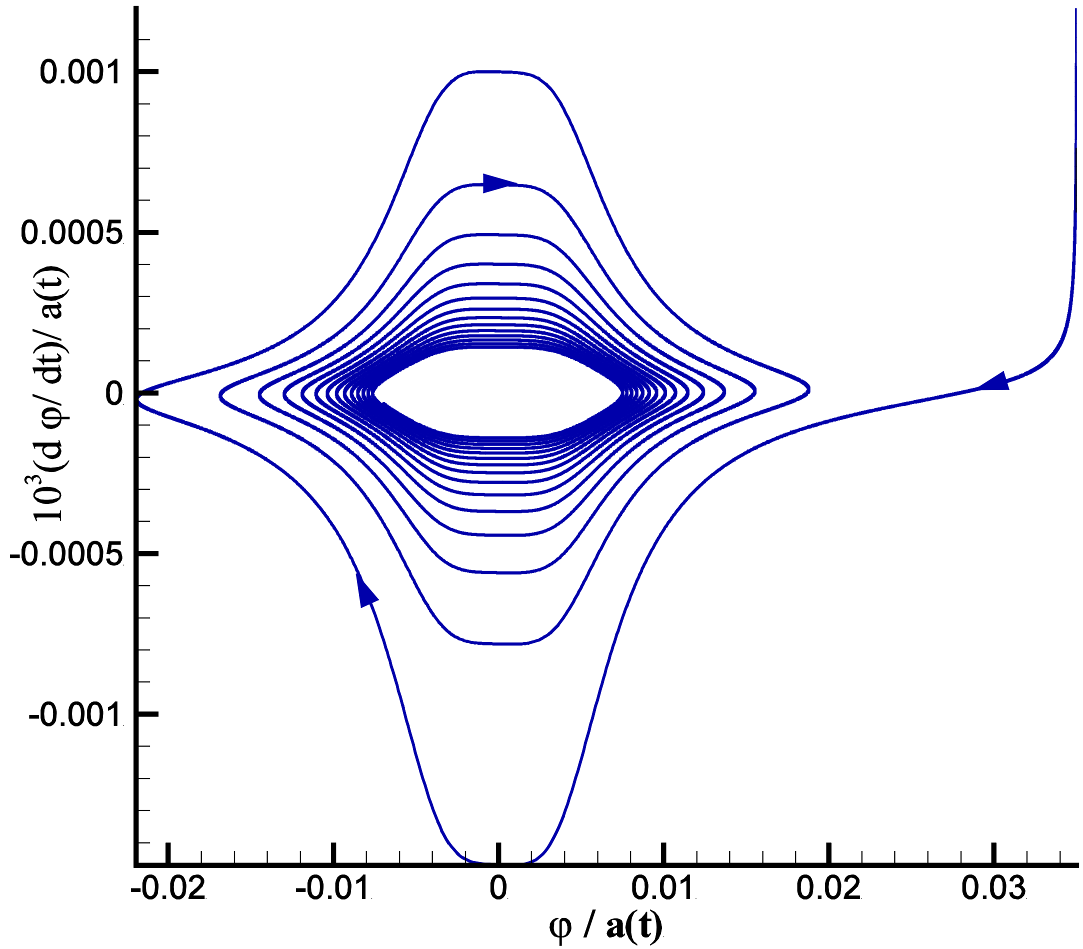

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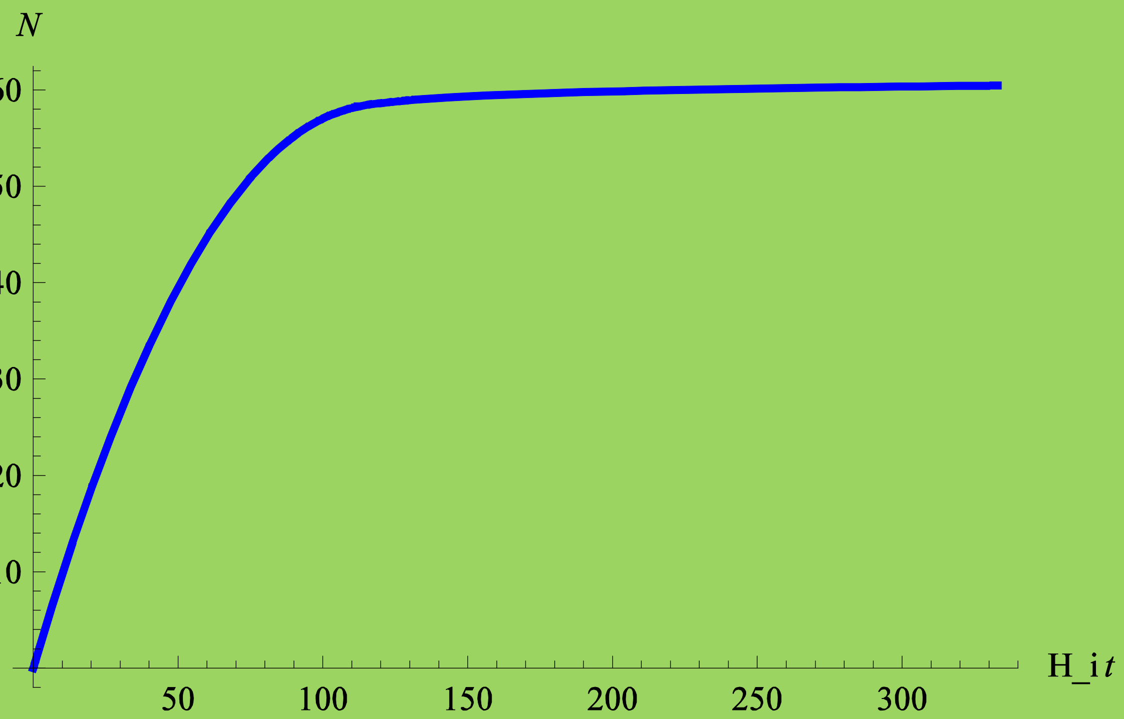

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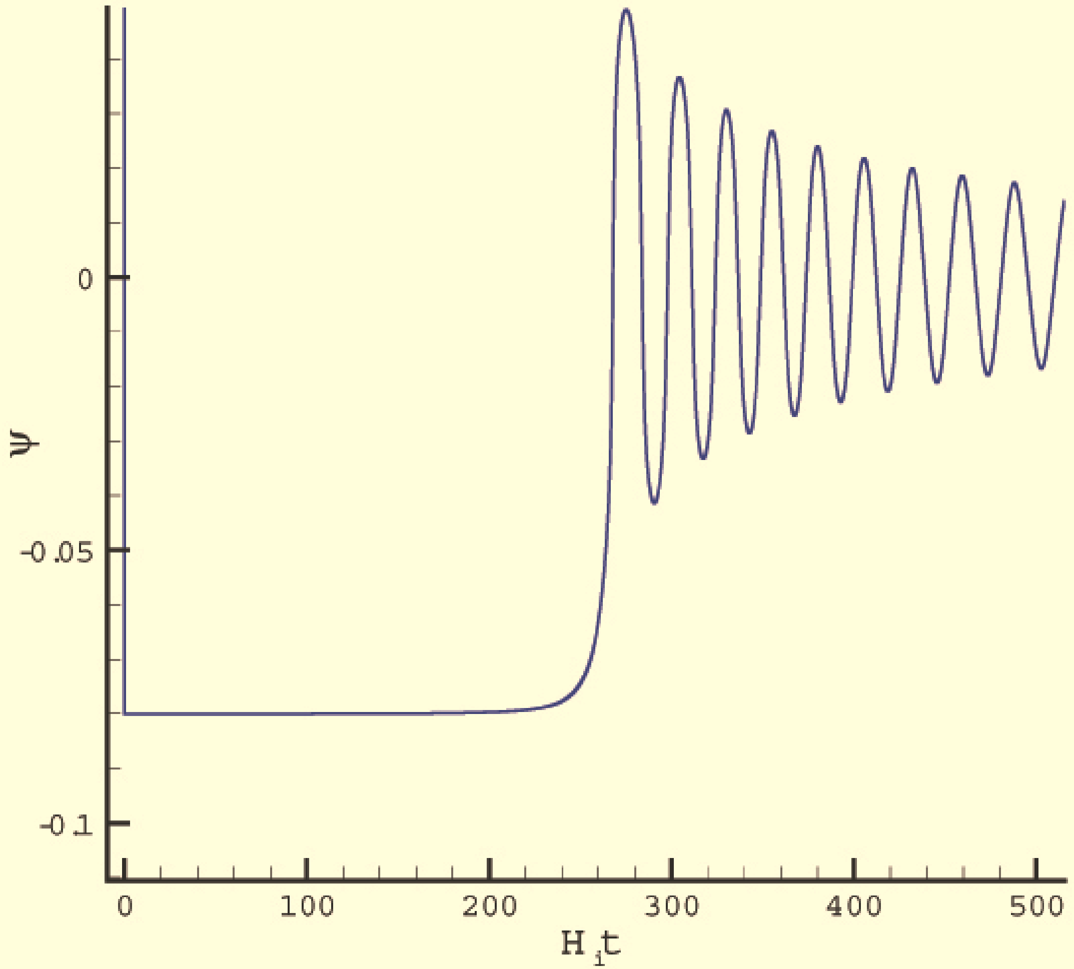
$$\psi_i = 8.0 \times 10^{-2}, \dot{\psi}_i = -10^{-4}; \, g = 4.004 \times 10^{-4} \;, \kappa = 4.73 \times 10^{13}.$$

$$H_i = 6.25 \times 10^{-4},$$

$$\epsilon_i = 6.4 \times 10^{-3}.$$

$$\delta \sim 2,$$



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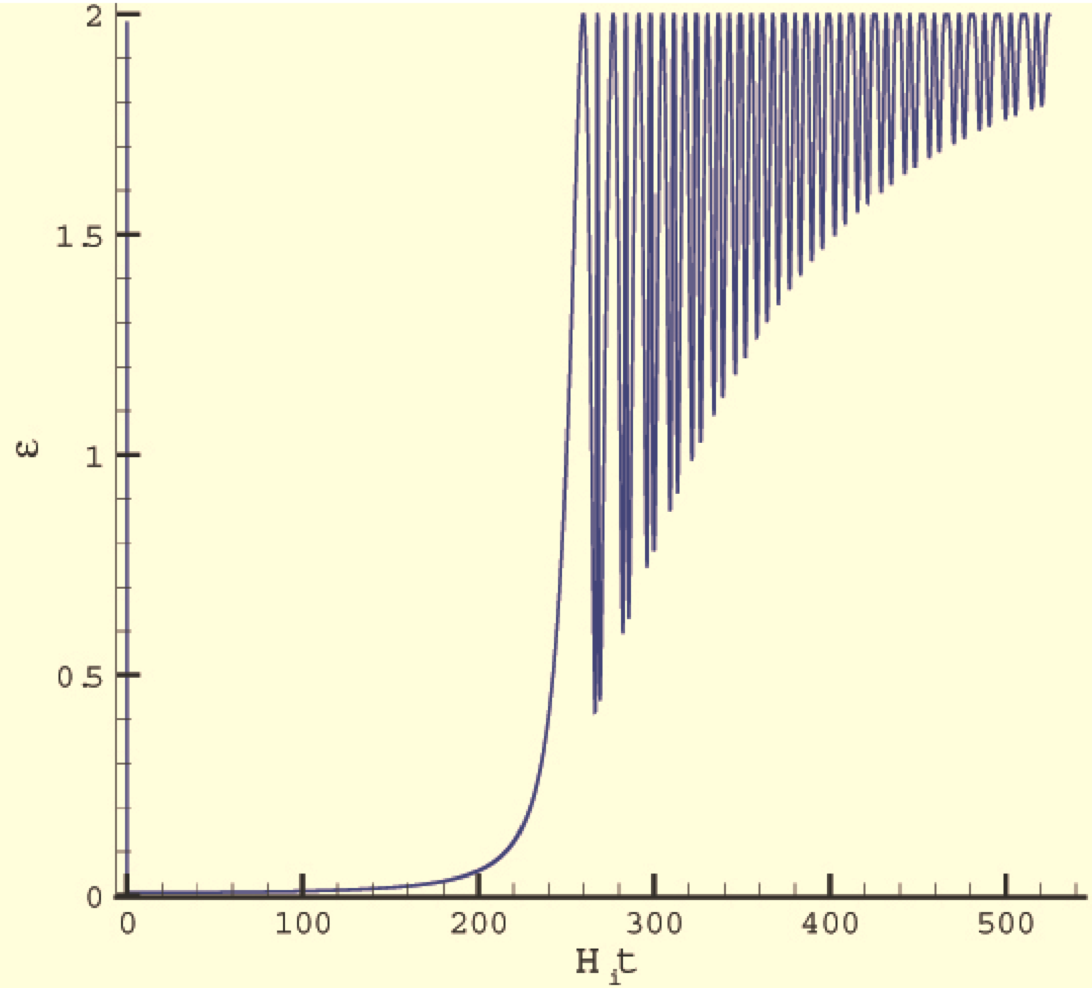

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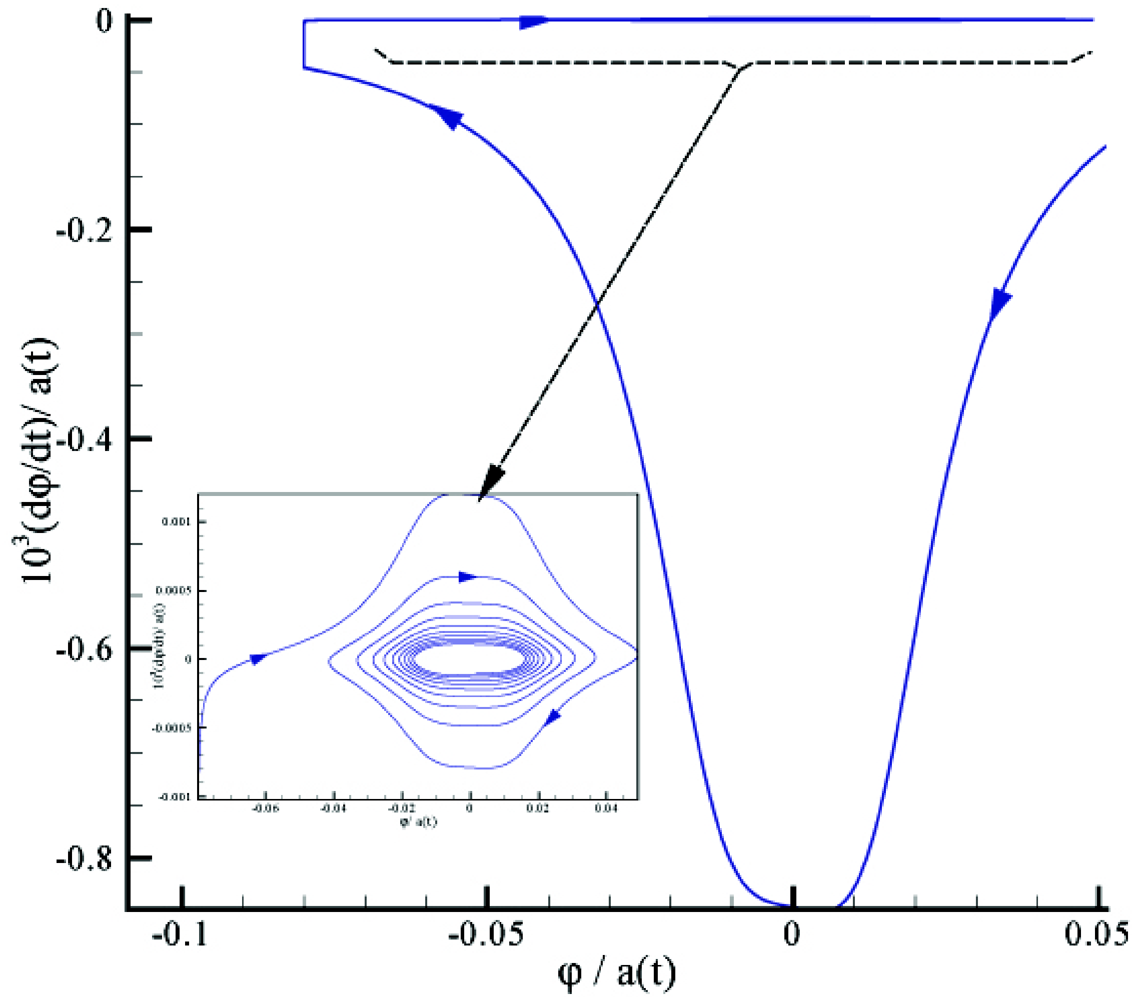

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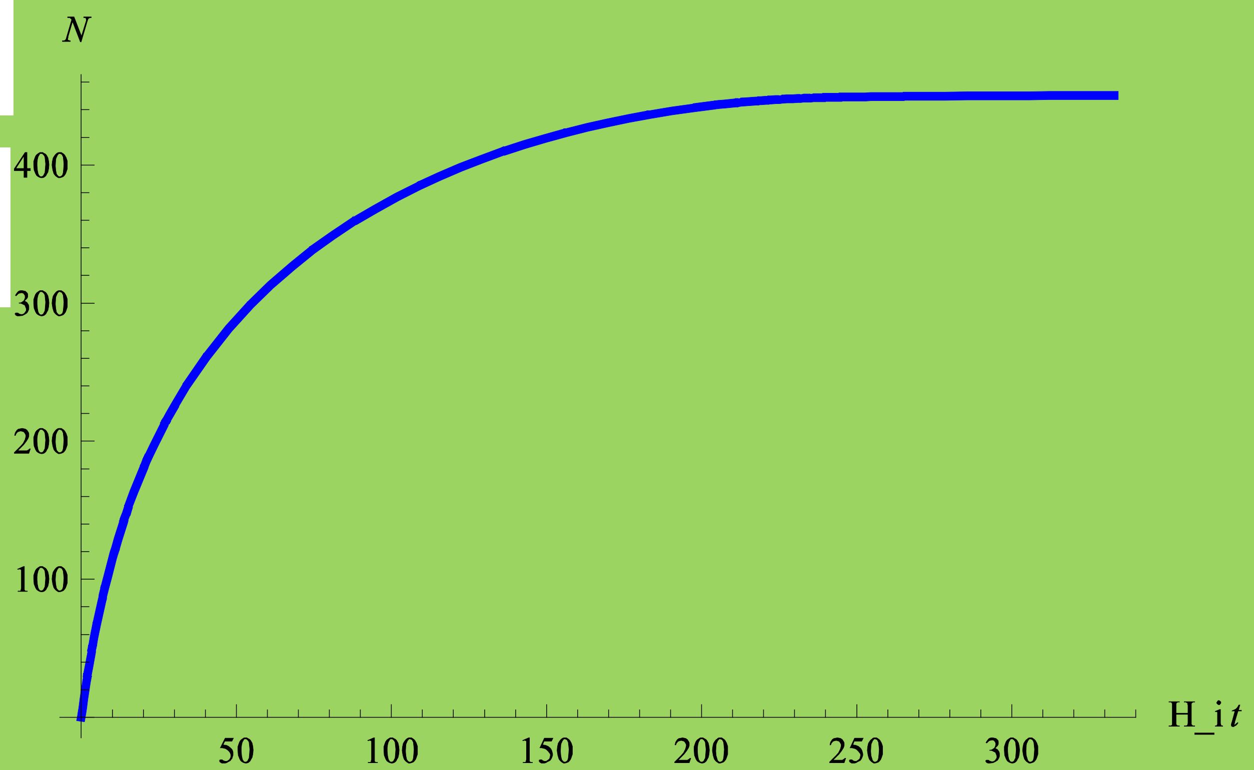


Gauge-flation Cosmic Perturbation Theory

Perturbed Metric

$$ds^2 = -(1+2A)dt^2 + 2a(\partial_i B - S_i)dx^i dt + a^2\left((1-2C)\delta_{ij} + 2\partial_{ij}E + 2\partial_{(i}W_{j)} + h_{ij}\right)dx^i dx^j ,$$

Gauge Field Perturbations

$$\delta A^a{}_0 = \delta^k_a \partial_k \dot{Y} + \delta^j_a u_j ,$$

$$\delta A^a{}_i = \delta^a_i Q + \delta^{ak}\partial_{ik}M + g\phi\epsilon^a{}_i{}^k\partial_k P + \delta^j_a\partial_i v_j + \epsilon^a{}_i{}^j w_j + \delta^{aj}t_{ij} ,$$

Coordinate transformations

$$t \to \tilde{t} = t + \delta t ,$$

$$x^i \to \tilde{x}^i = x^i + \delta^{ij}\partial_j x + \delta x^i_V ,$$

$$\lambda^a = \delta^{ai}\partial_i\lambda + \delta^a_i\lambda^i_V .$$

Gauge transformations

Gauge Invariant Perturbations

FIVE physical (gauge and diff invariant) scalar variables:

$$\Psi = C + a^2 H(\dot{E} - \frac{B}{a}),$$

$$\Phi = A - \frac{d}{dt}\left(a^2(\dot{E} - \frac{B}{a})\right)$$

$$\mathcal{Q} = Q - a^2\dot{\phi}(\dot{E} - \frac{B}{a}),$$

$$\mathcal{M} = M + P - \phi E,$$

$$\dot{\mathcal{Y}} = \dot{Y} + \dot{P} - \phi\dot{E}.$$

Three physical vectors,

TWO physical tensor variables: h_{ij} and t_{ij},

Dynamics of Perturbations, Scalars

❖ There are *four constraints* and *one dynamical* equation for the five physical scalar perturbations:

➢ *Three constraints and the dynamical* equation are coming from perturbed Einstein equations: $\delta G_{\mu\nu} = \delta T_{\mu\nu}$

➢ Constraint $\pi_s = 0$ comes from the gauge field equations of motion:

$$a^2\pi^s = 2\frac{\dot{\phi}}{a}(a\dot{\mathcal{Y}} - a\dot{\mathcal{M}}) + 2\frac{g^2\phi^3}{a^3}a\mathcal{M}\,.$$

Dynamics of Perturbations: Tensor and Vector Modes

❖ *Three vector modes* are exponentially damped, as in the usual inflationary models.

❖ The equations for *two tensor modes* can be decoupled,

➢ one of them is the same as usual tensor perturbations , becoming constant while

➢ the other shows exponential damping at super-horizon scales.

❖ One may then quantize these modes using *Bunch-Davis vacuum* state for infinite past.

❖ *Super-horizon modes* are those becoming classical and affecting the CMB.

Summary of Gauge-flatoin Cosmic Perturbation Theory Results

- *Scalar Power Spectrum*

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{(2\gamma+1)^2}{2\epsilon(\gamma+1)(\gamma+2)^2}(\frac{H}{\pi})^2|_{k=aH}$$

- *Scalar Spectral index*

$$n_{\mathcal{R}} - 1 \simeq -2\frac{3\gamma-1}{\gamma+1}\epsilon$$

- *Tensor Power Spectrum*

$$\mathcal{P}_T \simeq 2\left(\frac{H}{\pi}\right)^2 |_{k=aH} .$$

- *Tensor spectral index*

$$n_T \simeq -2\epsilon,$$

- *Tensor to Scalar ratio*

$$r \simeq \frac{4(\gamma+1)(\gamma+2)^2}{(2\gamma+1)^2}\epsilon.$$

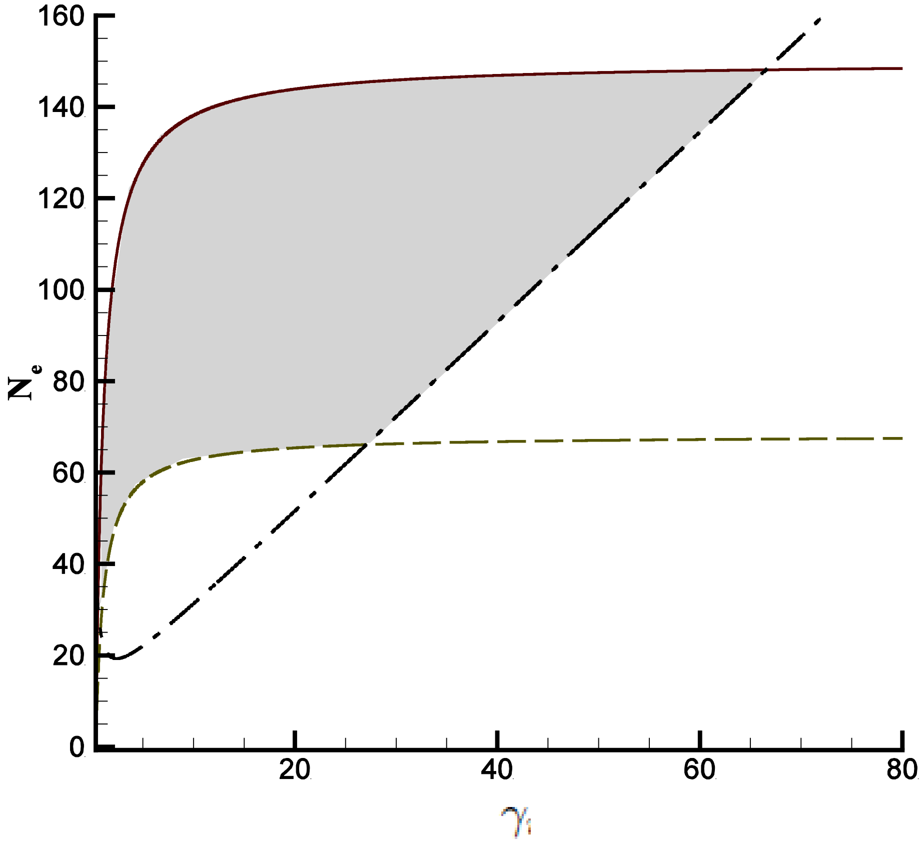
Contact with Observations

WMAP 7y data

$$\mathcal{P}_{\mathcal{R}} \simeq 2.5 \times 10^{-9}\,,$$
$$n_{\mathcal{R}} = 0.968 \pm 0.012\,,$$
$$r < 0.24\,.$$

directly, gives the value of the gauge coupling g

$$g^2 \simeq 5\pi^2 \times 10^{-9} \frac{\gamma(\gamma+1)^2(\gamma+2)^2}{(2\gamma+1)^2}$$



Contact with Observations

Demanding $N_e = 60$, the range of parameters are restricted as below

$$0.85 < \gamma < 6.35\,, \qquad \epsilon = (0.9 - 1.2) \times 10^{-2}\,.$$

$$\psi = (3.5 - 8.0) \times 10^{-2} M_{\rm pl}\,, \qquad H \simeq 3.5 \times 10^{-5} M_{\rm pl}\,.$$

$$\frac{g^2}{4\pi} = (0.13 - 5.0) \times 10^{-7}\,, \qquad \kappa = (4.6 - 17) \times 10^{13} M_{\rm pl}^{-4}\,.$$

with $r \simeq 0.1$, within the range to be probed by Planck.

Summary and Outlook

- Successful slow-roll inflation can be driven by non-Abelian gauge fields with gauge invariant actions minimally coupled to Einstein gravity.

- Our specific gauge-flation model has two parameters Yang-Mills coupling g and the dimensionful coefficient of $(F \wedge F)^2$ term κ .

- Cosmic data implies

$$g \cong 0.7 \times 10^{-3}$$

$$\kappa \cong (6 \times 10^{14} GeV)^{-4}$$

Summary and Outlook

- κ -term may be obtained by integrating out *massive axions* of the non-Abelian gauge theory, *if the axion mass is above Hubble scale H.*

- In this model, the axion coupling scale Λ is

$$\Lambda \approx 10H \cong 2\pi \times 10^{14} GeV$$

a reasonable value from particle physics viewpoint.

- Although a small field model, $\psi \approx 0.05M_{pl}$, we have sizeable gravity-wave power spectrum

$$r \cong 0.1$$

detectable by Planck.


Thank you for your attention