A Realization for the Cohen-Glashow Very Special Relativity

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Cohen-Glashow VSR, theoretical motivations

- Special Relativity (SR): physical theories are invariant under the *Poincaré group*, Lorentz transformation plus space-time translations.

- Extensions of Poincaré algebra? Maximal extension conformal group $so(4,2)$, not a symmetry of the particle physics models due to the presence of massive particles.

- There is of course extension of Poincaré by spinor generators, the SUSY...

Cohen-Glashow VSR, theoretical motivations

- Other extension of Poincaré group (or algebra): addition of the discrete symmetries of space and time inversion P, T,

- in particle physics, also with charge conjugation C, leading to $SL(2,\mathbb{C})$, to also incorporate antiparticles.

- Physics models and their Hilbert spaces are hence also taken to be *CPT* invariant.

Cohen-Glashow VSR, observational motivations

- No decisive observational or experimental signal for Lorentz symmetry violation as yet.
- Many precision tests are underway, see the review V. A. Kostelecky and N. Russell, Rev. Mod. Phys. **83**, 11 (2011).
- Various particle physics and gravity wave observations have been used to constrain the Lorentz violation parameters of LV SME: deformation of SM by all possible Lorentz violating, but gauge invariant, operators.

Cohen-Glashow VSR, observational motivations

- LV can happen in matter, photon, neutrino and gravity sectors.
- LV operators may be arranged by their scaling dimension and behavior under discrete symmetries, C, P, T or any combinations of them.
- Several works, for example by Glashow et al and Kostelecky et al have been devoted to studying tests of LV SME.
- But all are phenomenological models and not based on a rigorous theory.

Cohen-Glashow VSR, Motivations

- How to put these phenomenological high energy surveys and analysis in a firmer theoretical framework?!
- Formulation of these ideas?!
- One possibility:

  the Cohen-Glashow Very Special Relativity

Cohen-Glashow VSR, Motivations

- At very low energy (QED+QCD regime) P and T are conserved, while at higher energy, e.g. SM scale or above, P or T are violated.

- In a high energy theory Poincaré symmetry may also be violated.

- The main idea behind the VSR:

  P or T and possible Poincaré violations are caused by the same source.

Cohen-Glashow VSR, Motivations

- Other relevant ideas:
- Poincaré group is the symmetry, the isometry, of the Minkowski space.
- Poincaré symmetry could be lost (at some higher energy scale), however, *a part of Lorentz group plus translations* could still remain an exact symmetry of the more fundamental theory.

Plan of the Talk

- A Review on the Cohen-Glashow Very Special Relativity (VSR).
- Realization of VSR's as symmetry groups of a "deformed" (noncommutative) Minkowski space.
- Setting the stage for formulation of physical theories and models realizing VSR.
- Bounds on the scale of VSR deformation.

Cohen-Glashow VSR, A Brief Review

- VSR is a symmetry group involving spacetime translations + a proper subgroup of Lorentz group such that

upon addition of the space and time inversion P, T this subgroup is enhanced to the full Lorentz group $so(3,1)$.

Cohen-Glashow VSR, A Brief Review

- Smallest VSR is the two parameter Abelian subgroup of Lorentz $T(2)$, generated by

  $$T_1 = K_x + J_y \ , \qquad T_2 = K_y - J_x \ .$$

  Evidently, $[T_1, T_2] = 0$.

- T_1, T_2 together with four momenta P_μ form the smallest VSR.

- $T(2)$ VSR, has six generators (out of ten of the Poincaré).

$T(2)$ Cohen-Glashow VSR

- $T(2)$ is the translation group on a hypothetical two dimensional plane.
- Upon action of parity P,

$$T_1 \longrightarrow T_1^P = -K_x + J_y \,, \quad T_2 \longrightarrow T_2^P = -K_y - J_x \,.$$

- Algebra obtained from $T_1, \; T_2, T_1^P$ and T_2^P closes on the whole Lorentz group.
- Other VSR's obtained by adding one or more of the Lorentz generators to $T(2)$.

Other Cohen-Glashow VSR's

- There are only three other VSR's:
  - E(2) VSR: J_z added to T_1 and T_2.
  - HOM(2) VSR: K_z added to $T(2)$.
  - SIM(2) VSR: generated by $T_1,\ T_2,\ K_z,\ J_z$.
- Evidently by the action of parity and/or time reversal all of the above four VSR's enlarge to the whole Poincaré group.

Comments on Cohen-Glashow VSR's

- E(2) is the isometry group of two dim. Euclidian plane:

$$[T_1, T_2] = 0, \quad [J_z, T_1] = -iT_2, \quad [J_z, T_2] = +iT_1 \ .$$

- HOM(2) is the group of homotheties of two dimensional plane:

$$[T_1, T_2] = 0, \quad [K_z, T_1] = -iT_1, \quad [K_z, T_2] = -iT_2 \ .$$

- SIM(2) is the similitude group of 2d plane:

Comments on Cohen-Glashow VSR's

- As subgroups of Poincaré, VSR's keep the Minkowski metric $\eta_{\mu\nu}$ invariant.
- $T(2)$ VSR has an invariant vector $n_\mu = (1,0,0,1)$ & an invariant two form.
- $n_\mu = (1,0,0,1)$ is also invariant vector of the $E(2)$ VSR.
- E(2) does not have any invariant two form.
- $HOM(2)$ and $SIM(2)$ VSR's do not admit any invariant vector or tensors other than those of Lorentz algebra.

Difficulties with Formulating Cohen-Glashow VSR's

- All of irreps of the VSR are also reps of the Lorentz group but the converse is not true.
- The reps of VSR's are one dimensional.
- How to label the states in VSR QFTs?
- Spin statistics? notion of fermions and bosons? and CPT theorem?
- Does VSR symmetry remain anomaly free VSR QFT's?

Formulating Cohen-Glashow VSR

- To realize $T(2)$ or $E(2)$ VSR's: spontaneous Lorentz symmetry breaking and give VEV's to a vector or a tensor Cohen and Glashow [PRL**97**:021601].
- Formulation of $HOM(2)$ and $SIM(2)$ invariant theories should be done in some other ways.
- The $SIM(2)$ case as the largest VSR has been studies more
- General Ver Special relativity based on SIM(2) has also been discussed (with Finsler geometry approach).

VSR particle physics models

- Relaxing Lorentz and demanding only VSR invariance, we can have Lepton number conserving neutrino mass terms without the need for sterile (right-handed) states. Cohen & Glashow arXiv:0605036.
- In this model, Neutrinoless double beta decay is forbidden, and
- VSR effects can be significant near the beta decay endpoint where neutrinos are not ultra-relativistic.

VSR particle physics models

- VSR allows for particular electric dipole moment for charged leptons.
- SU(2) invariance may then relate such dipole moments to neutrino masses

$$d_{lepton} \sim (m_\nu/m_l)^2(e/m_l).$$

- With $m_\nu \sim 10^{-4}$eV we are close to current bounds....

Some reference on Cohen-Glashow VSR

J. Fan, W. Skiba, W. Goldberger, PLB**649** (2007) 186;

A. Cohen, D. Freedman, SUSY VSR, JHEP **0707**, 039.

A. Bernardini, VSR neutrino mass and modification in β-decay end points, Phys. Rev. D **75**, 097901 (2007).; O. Bertolami and A. Bernardini, PRD**77**:085032 (2008).

G. W. Gibbons, J. Gomis and C. N. Pope, General Very Special Relativity, Phys. Rev. D **76**, 081701 (2007).

Many papers in the midst of OPERA ....

The (exotic?!) ELKO dark matter, D. Ahluwalia, S. Horvath, JHEP **1011**, 078.

Problems with Cohen-Glashow proposal

- This does not answer questions we posed about reps and CPT.
- It is very phenomenological and introduces many parameters in the theory.
- More fundamental setup/theory?!

Realization of Cohen-Glashow VSR's, the notion of twist

- Given a Lie group, like Poincaré , one can construct irreps.
- In QFT actions we use not only these irreps, but also their products.
- One can *twist* product of two irreps by an element made out of the algebra itself, and construct a twisted co-product [V. Drinfel'd, 1983].

Realization of Cohen-Glashow VSR's, the notion of twist

- Given irreps $\mathcal{R}_1, \ \mathcal{R}_2$, i.e.

$$\mathcal{R}_1 \underset{twist}{\bigotimes} \mathcal{R}_2 = \mathcal{R}_1 e^{i\lambda_{ab} \overleftarrow{T}^a \otimes \overrightarrow{T}^b} \mathcal{R}_2,$$

  T^a are generators of the Lie algebra and λ_{ab} are twist parameters.

- Note that we have not deformed the algebra. We can still use the same basic irreps.

- Twisted co-product is specified by λ_{ab} which is a tensor in the original algebra.

The notion of twist, Cont'd

- The twist deformation reduces the symmetry to a subgroup which keeps λ_{ab} invariant, i.e. the stability group of twisted Poincaré algebra.

M. Chaichian, P. Kulish, K. Nishijima and A. Tureanu, PLB**604** (2004) 98;

See also M. Chaichian, P. Prešnajder and A. Tureanu, PRL**94** (2005) 151602 for physical implications of the twisted Poincaré .

The notion of twist, Cont'd

- Further discussions on the notion of twist and deformation may be found in
  - V. Chari and A. Pressley, Camb. Uni. Press, 1994;
  - S. Majid, Cambridge Uni. Press, 1995;
  - M. Chaichian and A. Demichev, World Scientific Singapore, 1996.
  - For twisted Poincaré algebras and their classification e.g. see A. Tureanu, arXiv:0706.0334 [hep-th] and references therein.

Realization of Cohen-Glashow VSR's on NC spaces

- Our idea:

  To realize VSRs through twist deformations of Poincaré algebra.

- If realized, it provides a very natural and consistent setting for formulation VSR's.
- Here we show that
  - VSR subgroups of Lorentz fit within the classification of the *twisted Poincaré* &
  - The VSR's can be realized as symmetries ("isometries") of Noncommutative Spaces.

Realization of Cohen-Glashow VSR's on NC spaces

- A noncommutative (NC) space can be defined by

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}(x)$$

  where $\theta^{\mu\nu}(x)$ is a two form.

- For a given $\theta^{\mu\nu}$ the Lorentz (and Poincaré) symmetry is broken to a subgroup which keeps $\theta^{\mu\nu}$ invariant (covariant, for the more general x-dependent case).

- For the special case of constant $\theta^{\mu\nu}$ we are dealing with the Moyal NC space.

De tour: Classification of NC spaces

- Depending on the values of the two Lorentz invariants made out of $\theta^{\mu\nu}$

$$\Lambda^4 \equiv \theta^{\mu\nu}\theta_{\mu\nu}, \qquad L^4 \equiv \epsilon_{\mu\nu\alpha\beta}\theta^{\mu\nu}\theta^{\alpha\beta}$$

  there are nine possibilities: either of Λ^4 and L^4 can be positive, zero or negative.

- It has been argued that (at least for constant $\theta^{\mu\nu}$ case) the $L^4 \neq 0$ does not lead to a unitary field theory [O. Aharony, J. Gomis and T. Mehen, JHEP**0009**:023 (2000)].

De tour: Classification of NC spaces, Cont'd

- However, it has been argued that the Doplicher-Fredenhagen-Roberts [hep-th/0303037] "quantum space-time" ($\Lambda = 0,\ L \neq 0$), leads to unitary QFT's [D. Bahns, S. Doplicher, K. Fredenhagen, G. Piacitelli, PLB**533** (2002) 178, hep-th/0201222; CMP**237** (2003) 221, hep-th/0301100].
- Focusing on the $L = 0$ case, we remain with three possibilities
  - $\Lambda^4 > 0$, the space-like noncommutativity,
  - $\Lambda^4 = 0$, the light-like noncommutativity,
  - $\Lambda^4 < 0$, the time-like noncommutativity.

De tour: Classification of NC spaces, Cont'd

- For constant $\theta^{\mu\nu}$ quantum field theories on spaces with time-like noncommutativity suffer from non-unitarity [M. Chaichian, A. Demichev, P. Presnajder and A. Tureanu, EPJC**20** (2001) 767; J. Gomis and T. Mehen, NPB **591** (2000) 265].
- However, the light-like noncommutative case is unitary [O. Aharony, J. Gomis and T. Mehen, JHEP**0009**:023 (2000); L. Alvarez-Gaumè, J. Barbòn and R. Zwicky, JHEP**0105**:057 (2001), ; MMSh-J. and A. Tureanu, Phys. Lett. B **697**, 63 (2011)] .

De tour: Classification of Twisted Poincaré algebras

There are only three choices for $\theta^{\mu\nu}$ which the deformation can also appear as a twist in the Poincaré co-algebra:

- *Constant* $\theta^{\mu\nu}$; the Moyal space:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

  where $\theta^{\mu\nu}$ is a constant tensor.

- *Linear* $\theta^{\mu\nu}$; the Lie-algebra type:

$$[x^\mu, x^\nu] = iC^{\mu\nu}_{\rho}\ x^\rho.$$

- *Quadratic* $\theta^{\mu\nu}$; the quantum group type:

$$[x^\mu, x^\nu] = \frac{1}{q} R^{\mu\nu}_{\rho\lambda}\ x^\rho x^\lambda.$$

Realization of Cohen-Glashow VSR's on NC spaces, Cont'd

- Formulation of QFT's on all the above NC space-times have originally been studied in [M. Chaichian, A. Demichev and P. Prešnajder , NPB **567** (2000) 360; J. Lukierski and M. Woronowicz, PLB **633** (2006) 116].
- The linear and quadratic $\theta^{\mu\nu}$ cases the translational invariance is lost and hence they do not lead to a VSR model. (Recall that by definition the VSR theory has a subgroup of Lorentz plus four translations.)

Realization of Cohen-Glashow VSR's on NC spaces, Cont'd

- Therefore, only the constant $\theta^{\mu\nu}$ Moyal space case is suitable for realization of VSR's.
- As argued
  - in the Moyal case the part of the Lorentz group which keeps the noncommutativity tensor $\theta^{\mu\nu}$ invariant is left over.
  - Moreover, only the $T(2)$ VSR admits invariant two tensor. Hence we have a natural setting to formulate $T(2)$ VSR invariant theories.

T(2) VSR on Light-like Moyal Space

- It is straightforward to show that the anti-symmetric two tensor which remains invariant under $T_1,\ T_2$ can only have the following non-zero components

$$\theta^{0i} = -\theta^{3i}, \qquad i = 1, 2.$$

- Note that invariance under $T_1,\ T_2$ does not restrict the x-dependence of $\theta^{\mu\nu}$.
- With the above we see that

$$\Lambda^4 = 0, \quad L^4 = 0$$

that is, *Invariance under T_1, T_2 demands a light-like noncommutativity.*

T(2) VSR on Light-like Moyal Space, Cont'd

- In the light-cone frame

$$x^{\pm} = (t \pm x^3)/2, \quad x^i, \quad i = 1, 2$$

θ^{-i} is the only non-zero component and

$$\theta^{-i} = \theta^{0i} = -\theta^{3i},$$

$(\theta^{+i} = \theta^{+-} = \theta^{ij} = 0)$.

- In the light-cone frame one may think of x^+ as light-cone time and x^- as light-cone space direction.
- Without any loss of generality one can still rotate the frame in the (x^1, x^2) plane to only keep the $\theta^{-1} \equiv \theta$ non-zero.

E(2), HOM(2) and SIM(2) Invariant NC spaces

- As discussed invariance under T_1, T_2 implies the light-like noncommutativity with $\theta^{-i} \neq 0$.
- Although one loses translational invariance when $\theta^{\mu\nu}$ has x-dependence, for specific x-dependent $\theta^{\mu\nu}$ we can have invariance under larger VSR Lorentz subgroups.
- The $E(2)$ and $SIM(2)$ Lorentz subgroups have such a realization on a NC space.

De tour to these cases......

E(2) Invariant NC space

- $E(2)$ is made up of T_1, T_2, J_z.
- $x^{\pm}$ are invariant under J_z.
- δ_{ij} and ϵ_{ij} are the (only) two invariant tensors under J_z.
- Therefore, $\theta^{-i} = \ell\epsilon_{ij}x^j$ and $\theta^{-i} = \ell x^i$ lead to $E(2)$ invariant space.
- That is a space with

$$[x^-, x^i] = i\ell\epsilon_{ij}x^j, \quad \text{OR} \quad [x^-, x^i] = i\ell x^i.$$

- With the above it is evident that translation along x^i plane is lost.

E(2) Invariant NC spaces, Cont'd

- The $[x^-, x^i] = i\ell\epsilon_{ij}x^j$ case is a noncommutative cylinder. To see this let us adapt the coordinates
$$\rho e^{\pm i\phi} \equiv x^1 \pm ix^2 \ .$$
- In this coordinate system we have
$$[x^-, \rho] = 0, \quad [x^-, e^{\pm i\phi}] = \pm\lambda e^{\pm i\phi}, \quad [\rho, e^{\pm i\phi}] = 0.$$
- At any given fixed ρ the above describes a NC cylinder of radius ρ with the axis along the x^- direction.
- λ is the deformation parameter and is the shortest length we can measure along the x^- direction.
- $[x^-, x^i] = i\ell x^i$ corresponds to a less familiar case and in the above cylindrical coordinates takes the form
$$[x^-, e^{\pm i\phi}] = 0, \quad [x^-, \rho] = i\ell\rho, \quad [\rho, e^{\pm i\phi}] = 0.$$

SIM(2) Invariant NC spaces

- Recalling that K_z is acting on x^- as scaling while keeping x^i invariant, and that $HOM(2)$ does not have an invariant vector, it is impossible to realize $HOM(2)$ in the NC setting as we did for $E(2)$ case.
- $SIM(2)$ is, however, possible with the following commutation relations

  $[x^-, x^i] = i \sin\xi \epsilon^i{}_j \{x^-, x^j\}$, or $[x^-, x^i] = i \sin\xi \, \{x^-, x^i\}$
- In the cylindrical coordinate system the above can be recast in the form of the Manin plane.
- Translational invariance in x^-, x^i directions is lost.

On T(2) VSR Invariant QFT's

- The only VSR which can be realized in the NC setting is the $T(2)$, associated with the light-like Moyal plane.
- There is a very well-established recipe for writing down general QFT's on the Moyal spaces:

take the commutative QFT action and replace the product between fields with the Moyal star-product:

$$(\phi * \psi)(x) = \phi(x)\, e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial_\mu}\overrightarrow{\partial_\nu}}\, \psi(x)\ ,$$

where $\theta^{\mu\nu}$ is the light-like noncommutativity parameter and we take its only non-vanishing component to be

$$\theta^{-1} = \theta.$$

On T(2) VSR Invariant QFT's, Cont'd

- Due to twisted Poincaré symmetry, fields carry representations of the full Lorentz group [M. Chaichian, P. Kulish, A. Tureanu, R. B. Zhang and Xiao Zhang, JMP**49**:042302 (2008); M. Chaichian, K. Nishijim, A. Tureanu, JHEP 0806 (2008) 078],
- but the theory is invariant only under transformations in the stability group of the above $\theta^{\mu\nu}$, $T(2)$.
- NCQFT's are CPT invariant while break C, P and T [M.M.Sh-J, PRL.**84**, 5265 (2000); M. Chaichian, K. Nishijima and A. Tureanu, PLB**568** (2003) 146].

On T(2) VSR scale

- The Lorentz violation scale is set by θ.
- One should make VSR model building and compare the results with the data.
- A bound comes from H-atom spectroscopy and the Lamb shift:

$$\theta \sim \Lambda_{NC}^{-2} \sim (1 - 10 \text{ GeV})^{-2}$$

M. Chaichian, M. M. Sheikh-Jabbari and A. Tureanu, Phys. Rev. Lett. **86** (2001) 2716.

- Stronger, less robust bounds suggest $\Lambda_{NC} \sim$ TeV.

On T(2) VSR Invariant QFT's, Cont'd

- A feature of any quantum field theory on NC space is the IR/UV mixing S. Minwalla, M. Van Raamsdonk and N. Seiberg, JHEP **0002**, 020 (2000).
- The "effective cut-off" of the theory is

$$\Lambda_{eff}^{-2} = \Lambda^{-2} + p \circ p$$

Λ: the usual UV cut-off, p: the external momentum and

$$p \circ p = p_\mu \theta^{\mu\nu} \theta_{\nu\rho} p^\rho .$$

- So, $\Lambda_{eff} \to \infty$ can take place when

$\Lambda \to \infty$ (*UV limit*) *or* $p \to 0$ (*the IR limit*).

On T(2) VSR Invariant QFT's, Cont'd

We showed in
M.M.Sh-J, A. Tureanu, Phys.Lett.B **697** 63 (2011),
that in the light-like noncommutativity and in $T(2)$ VSR QFT's

- IR/UV connection is not of the "standard" $p \circ p$ form and despite the fact that DLCQ introduces an IR cutoff and ameliorates IR/UV mixing, it does not remove the IR/UV mixing.
- Cutting rules are satisfied and perturbative unitarity is there.

Summary and Outlook

- The light-like Moyal NC space provides a consistent framework for $T(2)$ Cohen-Glashow VSR.
- Realization of VSR as noncommutative theories has several advantages:
  - In spite of the lack of full Lorentz symmetry, fields can still be labeled by the Lorentz representations.
  - For the NC QFTs we can rely on the basic notions of fermions and bosons, spin-statistics relation and CPT theorem. (For a more complete discussion on the spin-statistics theorem in NCQFT [A. Tureanu, PLB**638** (2006) 296].)

Summary and Outlook

- There is a simple recipe for constructing the NC version of any given QFT.
- Noncommutativity introduces a structure, fixing the form of the VSR-invariant action.
- This VSR symmetry is not anomalous.
- In the noncommutative setting we only deal with a single possible deformation parameter. Note that we can always choose the coordinates on (x^1, x^2)-plane such that θ^{-2} is zero.

Summary and Outlook

- In our realization of $T(2)$ invariant theories there is the scale, the noncommutativity scale

$$\Lambda_{NC}^2 = 1/\theta$$

  above which the absence of full Lorentz symmetry becomes manifest.

- Studying the spectrum of Hydrogen atom in a $T(2)$ invariant QED one can impose bounds on Λ_{NC}:

$$\Lambda \gtrsim 3\text{GeV}.$$

- Model building within our setup, still open......