A Realization for the Cohen-Glashow Very Special Relativity

M.M. Sheikh-Jabbari

IPM, Tehran-Iran


April 2013, IPM, Tehran
Special Relativity (SR): physical theories are invariant under the \textit{Poincaré group}, Lorentz transformation plus space-time translations.

Extensions of Poincaré algebra? Maximal extension conformal group $so(4, 2)$, not a symmetry of the particle physics models due to the presence of massive particles.

There is of course extension of Poincaré by spinor generators, the SUSY...
Other extension of Poincaré group (or algebra): addition of the discrete symmetries of space and time inversion $P, T$,

in particle physics, also with charge conjugation $C$, leading to $SL(2, \mathbb{C})$, to also incorporate antiparticles.

Physics models and their Hilbert spaces are hence also taken to be $CPT$ invariant.
No decisive observational or experimental signal for Lorentz symmetry violation as yet.

Many precision tests are underway, see the review V. A. Kostelecky and N. Russell, Rev. Mod. Phys. 83, 11 (2011).

Various particle physics and gravity wave observations have been used to constrain the Lorentz violation parameters of LV SME: deformation of SM by all possible Lorentz violating, but gauge invariant, operators.
LV can happen in matter, photon, neutrino and gravity sectors.

LV operators may be arranged by their scaling dimension and behavior under discrete symmetries, $C, P, T$ or any combinations of them.

Several works, for example by Glashow et al and Kostelecky et al have been devoted to studying tests of LV SME.

But all are phenomenological models and not based on a rigorous theory.
How to put these phenomenological high energy surveys and analysis in a firmer theoretical framework?!

Formulation of these ideas?!

One possibility:

*the Cohen-Glashow Very Special Relativity*
At very low energy (QED+QCD regime) \( P \) and \( T \) are conserved, while at higher energy, e.g. SM scale or above, \( P \) or \( T \) are violated.

In a high energy theory Poincaré symmetry may also be violated.

The main idea behind the VSR:

\( P \) or \( T \) and possible Poincaré violations are caused by the same source.
Other relevant ideas:

- Poincaré group is the symmetry, the isometry, of the Minkowski space.

- Poincaré symmetry could be lost (at some higher energy scale), however, a part of Lorentz group plus translations could still remain an exact symmetry of the more fundamental theory.
Plan of the Talk

• A Review on the Cohen-Glashow Very Special Relativity (VSR).

• Realization of VSR’s as symmetry groups of a “deformed” (noncommutative) Minkowski space.

• Setting the stage for formulation of physical theories and models realizing VSR.

• Bounds on the scale of VSR deformation.
VSR is a symmetry group involving spacetime translations + a proper subgroup of Lorentz group such that

upon addition of the space and time inversion $P, T$ this subgroup is enhanced to the full Lorentz group $so(3, 1)$. 
Smallest VSR is the two parameter Abelian subgroup of Lorentz $T(2)$, generated by

$$T_1 = K_x + J_y \ , \quad T_2 = K_y - J_x \ .$$

Evidently, $[T_1, T_2] = 0$.

$T_1, T_2$ together with four momenta $P_\mu$ form the smallest VSR.

$T(2)$ VSR, has six generators (out of ten of the Poincaré ).
\( T(2) \) Cohen-Glashow VSR

- \( T(2) \) is the translation group on a hypothetical two dimensional plane.

- Upon action of parity \( P \),

  \[
  T_1 \longrightarrow T_1^P = -K_x + J_y, \quad T_2 \longrightarrow T_2^P = -K_y - J_x.
  \]

- Algebra obtained from \( T_1, T_2, T_1^P \) and \( T_2^P \) closes on the whole Lorentz group.

- Other VSR’s obtained by adding one or more of the Lorentz generators to \( T(2) \).
There are only three other VSR’s:

- **E(2) VSR**: $J_z$ added to $T_1$ and $T_2$.
- **HOM(2) VSR**: $K_z$ added to $T(2)$.
- **SIM(2) VSR**: generated by $T_1$, $T_2$, $K_z$, $J_z$.

Evidently by the action of parity and/or time reversal all of the above four VSR’s enlarge to the whole Poincaré group.
E(2) is the isometry group of two dim. Euclidian plane:

\[ [T_1, T_2] = 0, \quad [J_z, T_1] = -iT_2, \quad [J_z, T_2] = +iT_1. \]

HOM(2) is the group of homotheties of two dimensional plane:

\[ [T_1, T_2] = 0, \quad [K_z, T_1] = -iT_1, \quad [K_z, T_2] = -iT_2. \]

SIM(2) is the similitude group of 2d plane:
As subgroups of Poincaré, VSR’s keep the Minkowski metric $\eta_{\mu\nu}$ invariant.

$T(2)$ VSR has an invariant vector $n_\mu = (1, 0, 0, 1)$ & an invariant two form.

$n_\mu = (1, 0, 0, 1)$ is also invariant vector of the $E(2)$ VSR.

$E(2)$ does not have any invariant two form.

$HOM(2)$ and $SIM(2)$ VSR’s do not admit any invariant vector or tensors other than those of Lorentz algebra.
Difficulties with Formulating Cohen-Glashow VSR’s

- All of irreps of the VSR are also reps of the Lorentz group but the converse is not true.
- The reps of VSR’s are one dimensional.
- How to label the states in VSR QFTs?
- Spin statistics? notion of fermions and bosons? and CPT theorem?
- Does VSR symmetry remain anomaly free VSR QFT’s?
To realize $T(2)$ or $E(2)$ VSR’s: spontaneous Lorentz symmetry breaking and give VEV’s to a vector or a tensor Cohen and Glashow [PRL97:021601].

Formulation of $HOM(2)$ and $SIM(2)$ invariant theories should be done in some other ways.

The $SIM(2)$ case as the largest VSR has been studies more

General Ver Special relativity based on $SIM(2)$ has also been discussed (with Finsler geometry approach).
Relaxing Lorentz and demanding only VSR invariance, we can have Lepton number conserving neutrino mass terms without the need for sterile (right-handed) states. Cohen & Glashow arXiv:0605036.

In this model, Neutrinoless double beta decay is forbidden, and

VSR effects can be significant near the beta decay endpoint where neutrinos are not ultra-relativistic.
VSR allows for particular electric dipole moment for charged leptons.

SU(2) invariance may then relate such dipole moments to neutrino masses

\[ d_{\text{lepton}} \sim \left( \frac{m_\nu}{m_l} \right)^2 \left( \frac{e}{m_l} \right). \]

With \( m_\nu \sim 10^{-4} \text{eV} \) we are close to current bounds....
Some reference on Cohen-Glashow VSR


A. Cohen, D. Freedman, SUSY VSR, JHEP $^{0707}$, 039.

A. Bernardini, VSR neutrino mass and modification in $\beta$-decay end points, Phys. Rev. D $^{75}$, 097901 (2007).; O. Bertolami and A. Bernardini, PRD$^{77}$:085032 (2008).


Many papers in the midst of OPERA ....

The (exotic?!?) ELKO dark matter, D. Ahluwalia, S. Horvath, JHEP $^{1011}$, 078.
Problems with Cohen-Glashow proposal

- This does not answer questions we posed about reps and CPT.

- It is very phenomenological and introduces many parameters in the theory.

- More fundamental setup/theory?!
Realization of Cohen-Glashow VSR’s, the notion of twist

- Given a Lie group, like Poincaré, one can construct irreps.

- In QFT actions we use not only these irreps, but also their products.

- One can *twist* product of two irreps by an element made out of the algebra itself, and construct a twisted co-product [V. Drinfel’d, 1983].
Given irreps $\mathcal{R}_1$, $\mathcal{R}_2$, i.e.

$$\mathcal{R}_1 \boxtimes \mathcal{R}_2 = \mathcal{R}_1 e^{i\lambda_{ab} \overset{\leftarrow}{T}^a \otimes \overset{\rightarrow}{T}^b} \mathcal{R}_2,$$

$T^a$ are generators of the Lie algebra and $\lambda_{ab}$ are twist parameters.

Note that we have **not** deformed the algebra. We can still use the same basic irreps.

Twisted co-product is specified by $\lambda_{ab}$ which is a tensor in the original algebra.
The twist deformation reduces the symmetry to a subgroup which keeps $\lambda_{ab}$ invariant, i.e. the stability group of twisted Poincaré algebra.


See also M. Chaichian, P. Prešnajder and A. Tureanu, PRL94 (2005) 151602 for physical implications of the twisted Poincaré.
Further discussions on the notion of twist and deformation may be found in

- S. Majid, Cambridge Uni. Press, 1995;

For twisted Poincaré algebras and their classification e.g. see
Our idea:

To realize VSRs through twist deformations of Poincaré algebra.

If realized, it provides a very natural and consistent setting for formulation VSR’s.

Here we show that

- VSR subgroups of Lorentz fit within the classification of the twisted Poincaré &
- The VSR’s can be realized as symmetries (“isometries”) of Noncommutative Spaces.
A noncommutative (NC) space can be defined by

\[ [x^\mu, x^\nu] = i\theta^{\mu\nu}(x) \]

where \( \theta^{\mu\nu}(x) \) is a two form.

For a given \( \theta^{\mu\nu} \) the Lorentz (and Poincaré) symmetry is broken to a subgroup which keeps \( \theta^{\mu\nu} \) invariant (covariant, for the more general \( x \)-dependent case).

For the special case of constant \( \theta^{\mu\nu} \) we are dealing with the Moyal NC space.
Depending on the values of the two Lorentz invariants made out of $\theta^{\mu\nu}$

$$\Lambda^4 \equiv \theta^{\mu\nu} \theta_{\mu\nu}, \quad L^4 \equiv \epsilon_{\mu\nu\alpha\beta} \theta^{\mu\nu} \theta^{\alpha\beta}$$

there are nine possibilities: either of $\Lambda^4$ and $L^4$ can be positive, zero or negative.

It has been argued that (at least for constant $\theta^{\mu\nu}$ case) the $L^4 \neq 0$ does not lead to a unitary field theory [O. Aharony, J. Gomis and T. Mehen, JHEP0009:023 (2000)].
However, it has been argued that the Doplicher-Fredenhagen-Roberts [hep-th/0303037] “quantum space-time” \((\Lambda = 0, \ L \neq 0)\), leads to unitary QFT’s [D. Bahns, S. Doplicher, K. Fredenhagen, G. Piacitelli, PLB533 (2002) 178, hep-th/0201222; CMP237 (2003) 221, hep-th/0301100].

Focusing on the \(L = 0\) case, we remain with three possibilities

- \(\Lambda^4 > 0\), the space-like noncommutativity,
- \(\Lambda^4 = 0\), the light-like noncommutativity,
- \(\Lambda^4 < 0\), the time-like noncommutativity.

De tour: Classification of Twisted Poincaré algebras

There are only three choices for $\theta^{\mu\nu}$ which the deformation can also appear as a twist in the Poincaré co-algebra:

- **Constant $\theta^{\mu\nu}$;** the Moyal space:
  \[ [x^\mu, x^\nu] = i\theta^{\mu\nu} \]
  where $\theta^{\mu\nu}$ is a constant tensor.

- **Linear $\theta^{\mu\nu}$;** the Lie-algebra type:
  \[ [x^\mu, x^\nu] = iC^{\mu\nu}_\rho x^\rho. \]

- **Quadratic $\theta^{\mu\nu}$;** the quantum group type:
  \[ [x^\mu, x^\nu] = \frac{1}{q} R^{\mu\nu}_{\rho\lambda} x^\rho x^\lambda. \]
Formulation of QFT’s on all the above NC space-times have originally been studied in [M. Chaichian, A. Demichev and P. Prešnajder, NPB567 (2000) 360; J. Lukierski and M. Woronowicz, PLB 633 (2006) 116 ].

The linear and quadratic $\theta^{\mu\nu}$ cases the translational invariance is lost and hence they do not lead to a VSR model. (Recall that by definition the VSR theory has a subgroup of Lorentz plus four translations.)
Therefore, only the constant $\theta^{\mu\nu}$ Moyal space case is suitable for realization of VSR’s.

As argued

- in the Moyal case the part of the Lorentz group which keeps the noncommutativity tensor $\theta^{\mu\nu}$ invariant is left over.

- Moreover, only the $T(2)$ VSR admits invariant two tensor. Hence we have a natural setting to formulate $T(2)$ VSR invariant theories.
It is straightforward to show that the anti-symmetric two-tensor which remains invariant under $T_1$, $T_2$ can only have the following non-zero components

$$\theta^{0i} = -\theta^{3i}, \quad i = 1, 2.$$  

Note that invariance under $T_1$, $T_2$ does not restrict the $x$-dependence of $\theta^{\mu\nu}$.

With the above we see that

$$\Lambda^4 = 0, \quad L^4 = 0$$

that is, *Invariance under $T_1$, $T_2$ demands a light-like noncommutativity.*
In the light-cone frame

\( x^\pm = (t \pm x^3)/2, \quad x^i, \quad i = 1, 2 \)

\( \theta^{-i} \) is the only non-zero component and

\[ \theta^{-i} = \theta^{0i} = -\theta^{3i}, \]

\( (\theta^{+i} = \theta^{+-} = \theta^{ij} = 0). \)

In the light-cone frame one may think of \( x^+ \) as light-cone time and \( x^- \) as light-cone space direction.

Without any loss of generality one can still rotate the frame in the \( (x^1, x^2) \) plane to only keep the \( \theta^{-1} \equiv \theta \) non-zero.
As discussed invariance under $T_1, T_2$ implies the light-like noncommutativity with $\theta^{-i} \neq 0$.

Although one loses translational invariance when $\theta^{\mu\nu}$ has $x$-dependence, for specific $x$-dependent $\theta^{\mu\nu}$ we can have invariance under larger VSR Lorentz subgroups.

The $E(2)$ and SIM(2) Lorentz subgroups have such a realization on a NC space.

De tour to these cases......
\( E(2) \) \textit{Invariant NC space}

- \( E(2) \) is made up of \( T_1, T_2, J_z \).
- \( x^\pm \) are invariant under \( J_z \).
- \( \delta_{ij} \) and \( \epsilon_{ij} \) are the (only) two invariant tensors under \( J_z \).

Therefore, \( \theta^{-i} = \ell \epsilon_{ij} x^j \) and \( \theta^{-i} = \ell x^i \) lead to \( E(2) \) invariant space.

That is a space with

\[
[x^-, x^i] = i \ell \epsilon_{ij} x^j, \quad \text{OR} \quad [x^-, x^i] = i \ell x^i.
\]

With the above it is evident that translation along \( x^i \) plane is lost.
The $[x^-, x^i] = i\ell \epsilon_{ij} x^j$ case is a noncommutative cylinder. To see this let us adapt the coordinates $\rho e^{\pm i\phi} \equiv x^1 \pm ix^2$.

In this coordinate system we have

$[x^-, \rho] = 0, \quad [x^-, e^{\pm i\phi}] = \pm \lambda e^{\pm i\phi}, \quad [\rho, e^{\pm i\phi}] = 0$.

At any given fixed $\rho$ the above describes a NC cylinder of radius $\rho$ with the axis along the $x^-$ direction.

$\lambda$ is the deformation parameter and is the shortest length we can measure along the $x^-$ direction.

$[x^-, x^i] = i\ell x^i$ corresponds to a less familiar case and in the above cylindrical coordinates takes the form

$[x^-, e^{\pm i\phi}] = 0, \quad [x^-, \rho] = i\ell \rho, \quad [\rho, e^{\pm i\phi}] = 0$. 

SIM(2) Invariant NC spaces

Recalling that $K_z$ is acting on $x^-$ as scaling while keeping $x^i$ invariant, and that $HOM(2)$ does not have an invariant vector, it is impossible to realize $HOM(2)$ in the NC setting as we did for $E(2)$ case.

$SIM(2)$ is, however, possible with the following commutation relations

$$[x^-, x^i] = i \sin \xi \epsilon^i_j \{x^-, x^j\}, \quad \text{or} \quad [x^-, x^i] = i \sin \xi \{x^-, x^i\}$$

In the cylindrical coordinate system the above can be recast in the form of the Manin plane.

Translational invariance in $x^-$, $x^i$ directions is lost.
The only VSR which can be realized in the NC setting is the $T(2)$, associated with the light-like Moyal plane.

There is a very well-established recipe for writing down general QFT’s on the Moyal spaces:

*take the commutative QFT action and replace the product between fields with the Moyal star-product:*

$$(\phi \ast \psi)(x) = \phi(x) e^{\frac{i}{2} \theta^\mu_\nu \partial_\mu \partial_\nu} \psi(x),$$

where $\theta^\mu_\nu$ is the light-like noncommutativity parameter and we take its only non-vanishing component to be

$$\theta^{-1} = \theta.$$
Due to twisted Poincaré symmetry, fields carry representations of the full Lorentz group [M. Chaichian, P. Kulish, A. Tureanu, R. B. Zhang and Xiao Zhang, JMP 49:042302 (2008); M. Chaichian, K. Nishijima, A. Tureanu, JHEP 0806 (2008) 078], but the theory is invariant only under transformations in the stability group of the above $\theta^{\mu\nu}, T(2)$.

NCQFT’s are CPT invariant while break $C, P$ and $T$ [M.M.Sh-J, PRL 84, 5265 (2000); M. Chaichian, K. Nishijima and A. Tureanu, PLB 568 (2003) 146].
The Lorentz violation scale is set by $\theta$. One should make VSR model building and compare the results with the data. A bound comes from H-atom spectroscopy and the Lamb shift:

$$\theta \sim \Lambda_{NC}^{-2} \sim (1 - 10 \text{ GeV})^{-2}$$


Stronger, less robust bounds suggest $\Lambda_{NC} \sim \text{TeV}$. 
A feature of any quantum field theory on NC space is the IR/UV mixing S. Minwalla, M. Van Raamsdonk and N. Seiberg, JHEP 0002, 020 (2000).

The “effective cut-off” of the theory is

\[ \Lambda_{\text{eff}}^{-2} = \Lambda^{-2} + p \circ p \]

\( \Lambda \): the usual UV cut-off, \( p \): the external momentum and

\[ p \circ p = p_\mu \theta^{\mu\nu} \theta_{\nu\rho} p^\rho. \]

So, \( \Lambda_{\text{eff}} \to \infty \) can take place when

\[ \Lambda \to \infty \text{ (UV limit)} \text{ or } p \to 0 \text{ (the IR limit)}. \]
We showed in M.M.Sh-J, A. Tureanu, *Phys. Lett. B* 697 63 (2011), that in the light-like noncommutativity and in $T(2)$ VSR QFT’s

- IR/UV connection is not of the “standard” $p \circ p$ form and despite the fact that DLCQ introduces an IR cutoff and ameliorates IR/UV mixing, it does not remove the IR/UV mixing.

- Cutting rules are satisfied and perturbative unitarity is there.
Summary and Outlook

- The light-like Moyal NC space provides a consistent framework for $T(2)$ Cohen-Glashow VSR.

- Realization of VSR as noncommutative theories has several advantages:
  - In spite of the lack of full Lorentz symmetry, fields can still be labeled by the Lorentz representations.
  - For the NC QFTs we can rely on the basic notions of fermions and bosons, spin-statistics relation and CPT theorem. (For a more complete discussion on the spin-statistics theorem in NCQFT [A. Tureanu, PLB638 (2006) 296].)
There is a simple recipe for constructing the NC version of any given QFT.

Noncommutativity introduces a structure, fixing the form of the VSR-invariant action.

This VSR symmetry is not anomalous.

In the noncommutative setting we only deal with a single possible deformation parameter. Note that we can always choose the coordinates on \((x^1, x^2)\)-plane such that \(\theta^{-2}\) is zero.
In our realization of $T(2)$ invariant theories there is the scale, the noncommutativity scale

$$\Lambda_{NC}^2 = 1/\theta$$

above which the absence of full Lorentz symmetry becomes manifest.

Studying the spectrum of Hydrogen atom in a $T(2)$ invariant QED one can impose bounds on $\Lambda_{NC}$:

$$\Lambda \gtrsim 3\text{GeV}.$$  

Model building within our setup, still open......