Dark matter direct detection with anisotropic halo models

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Based on work done with R. Catena and T. Schwetz [1310.0468]





Dark matter halo



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- "Standard Halo Model": isothermal sphere with an isotropic Maxwell-Boltzmann velocity distribution.
 - local DM density: $ho_{\chi} \sim 0.3 \ {
 m GeV} \ {
 m cm}^{-3}$
 - typical DM velocity: $ar{
 u} \simeq$ 220 km/s

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 NOTE: These kinds of plots assume the Standard Halo Model and a specific DM-nucleus interaction.

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▶ Minimum WIMP speed required to produce a recoil energy *E_R*:

$$v_m = \sqrt{rac{m_A E_R}{2 \mu_{\chi A}^2}}$$

The differential event rate

The differential event rate (events/keV/kg/day):

$$R(E_R, t) = \frac{\rho_{\chi}}{m_{\chi}} \frac{1}{m_A} \int_{v > v_m} d^3 v \frac{d\sigma_A}{dE_R} v f_{det}(\mathbf{v}, t)$$

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$$f_{\text{det}}(\mathbf{v}, t) = f_{\text{sun}}(\mathbf{v} + \mathbf{v}_{e}(t)) = f_{\text{gal}}(\mathbf{v} + \mathbf{v}_{s} + \mathbf{v}_{e}(t))$$

Sun's velocity wrt the Galaxy: $v_s \approx (0, 220, 0) + (10, 13, 7)$ km/s Earth's velocity: $v_e \approx 30$ km/s

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- In the SHM, a truncated Maxwellian velocity distribution is assumed

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- DM distribution could be very different from Maxwellian.
- Numerical N-body simulations of galaxy formation predict anisotropic dark matter velocity distributions.

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- We propose a new benchmark distribution function (DF) which is anisotropic, and constrain its parameters using different observations of the Milky Way properties.
- We analyze the data from various direct detection experiments using this DF.

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Dark matter mass density profile

$$ho(\mathbf{x}) = \int d^3 v F(\mathbf{x},\mathbf{v})$$

Dark matter velocity distribution at any point x₀

$$f_{\mathbf{x}_0}(\mathbf{v}) = \frac{1}{\rho(\mathbf{x}_0)} F(\mathbf{x}_0, \mathbf{v})$$

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The distribution function at the Sun's position is relevant for direct dark matter experiments.

Anisotropy of a DF is quantified by

 $_{\varkappa}$ tangential velocity dispersion

r: galactocentric distance

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Our analysis

- 1. Relate the dark matter DF to a parametric model for the Galaxy, solving $\rho(\mathbf{x}) = \int d^3 \mathbf{v} F(\mathbf{x}, \mathbf{v})$ for $F(\mathbf{x}, \mathbf{v})$. We account for possible anisotropies in the DF.
- Use current astronomical data and statistical tools to constrain the parameters of the assumed galactic model. ⇒ Determine the DF favored by the data and the associated uncertainties.
- **3.** Use this DF to find the regions in the dark matter cross section and mass plane, favored by current direct detection experiments for spin-independent elastic scattering.

- Anisotropic DFs (assuming spherical symmetry) can be written as a function of
 - dark matter relative energy (per unit mass) $\mathcal{E} \equiv \Psi (1/2)v^2$, where Ψ is the relative gravitational potential related to the total density ρ_{tot} through Poisson's eqn.
 - the modulus of the dark matter angular momentum L.

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 - the modulus of the dark matter angular momentum L.
- Solve for F the integral equation

$$\rho(x) = \int d^3 v \; F(\mathcal{E},L)$$

which relates *F* to the relative potential Ψ and ρ .

DF with constant $\beta(r)$:

$$F_{\gamma}(\mathcal{E},L) = G(\mathcal{E})L^{2\gamma}$$

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$$F_{\gamma}(\mathcal{E},L) \propto L^{2\gamma} rac{d}{d\mathcal{E}} \int_{0}^{\mathcal{E}} rac{d
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Major limitation: a constant β(r) seems a too crude approximation. N-body simulations predict β(r) growing with r, at least up to a certain value of r.

Osipkov-Merritt DF:

• An anisotropic DF associated with a growing $\beta(r)$:

$$F(\mathcal{E},L) = F_{\text{OM}}(Q);$$
 $Q = \mathcal{E} - \frac{L^2}{2r_a^2}$

where r_a is a reference radius. At $r > r_a$, this DF exhibits some degree of radial anisotropy.

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Major limitation: OM DF leads to a β(r) growing with a rate much larger than observed in N-body simulations.

• We propose the following anisotropic distribution function:

$$F(\mathcal{E}, L) = \omega \underbrace{F_{OM}(\mathcal{E}, L)}_{\text{Osipkov-Merritt DF}} + (1 - \omega) \underbrace{F_{\gamma}(\mathcal{E}, L)}_{\text{DF with constant } \beta(r)}$$

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- Advantage: this type of DF can reproduce the behavior of β(r) observed in N-body simulations without requiring complicated inversion procedures to relate the DF to Ψ and ρ.
- Obtain different functions of $\beta(r)$ by properly choosing the three free parameters (ω, r_a, γ) .

• $\beta(r)$ as obtained from our benchmark DF:



Blue and red curves bracket the uncertainties in the predictions of the N-body simulations, while the black curve provides a good approximation to the best fit. ⇒ Good benchmark for dark matter searches!

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- We employ the galactic model studied in Catena & Ullio 2010 and 2012.
 - It has 8 parameters describing the dark matter halo, the stellar disk and the bulge region.
 - The parameters are subject to a variety of constraints derived from different observations of the Milky Way properties.
- We perform a Bayesian analysis of this model using current astronomical data constraining the Galactic gravitational potential.

- The free parameters describing the luminous components:
 - $ightarrow R_d$: length scale in the radial direction of the stellar disk.
 - R_0 : the local galactocentric distance.
 - fb: fraction of collapsed baryons
 - Γ: ratio between the bulge and disk masses.
 - β_{*}: anisotropy parameter of a population of halo stars used in the analysis.
- The free parameters describing the **dark matter halo**:
 - α : parameter in the Einasto profile: $f_E(x) = \exp\left[-\frac{2}{\alpha}(x^{\alpha}-1)\right]$
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- Besides the 3 parameters controlling the degree of anisotropy, our benchmark DF depends on 8 galactic model parameters.

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From the galactic model to the DF

The Bayesian posterior PDF for the galactic parameters (left) translates into a DF favored by current astronomical data (right):



The turquoise bands encode the astrophysical uncertainties obtained ± 2σ to the mean DF.

Alternative possibilites for DF

- Consider 4 cases for the DF as motivated by the results of N-body simulations, and calculate the corresponding β(r).
- Keep the model for the Milky Way fixed but allow for anisotropy.



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The local velocity distribution is affected by the degree of anisotropy at radii up to the virial radius.

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The impact of anisotropy

Adopt the best fit model for the galaxy from the analysis of kinematical data, and compare an anisotropic case (solid/color) with an isotropic case (dashed/gray).



Anisotropy effects mainly the low WIMP mass region where experiments probe the high velocity tail of the distribution.

Including astrophysical uncertainties

Compare two anisotropic models taking into account astrophysical uncertainties. solid/color: large anisotropy, dashed/gray: small anisotropy.



Shift of the regions for large WIMP masses comes from the effect of changing the parameters of the Milky Way model.

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- Baryonic components contribute significantly to the gravitational potential for r < 10 kpc.</p>
- The larger gravitational potential increases the number of dark matter particles with high velocities.

Effect of Baryons

 Including baryons (solid/color) compared to the same dark matter halo but without baryonic component (dashed/gray)



The low WIMP mass region is affected by a shift of ~ 2 GeV, where the main effect is the larger population of the high-velocity tail of the distribution due to baryons.

- We proposed an anisotropic DF which features a degree of anisotropy such as suggested by N-body simulations.
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 - In general exclusion limits and allowed regions shift in the same way, and the compatibility cannot be improved.
 - Baryonic component of the MW plays an important role to determine the local velocity distribution and cannot be neglected when building self-consistent models for the DM halo.

Additional slides

Three different classes of constraints:

- Direct measurement of kinematical properties of different tracers of Milky Way gravitational potential
 - Terminal velocities observed from the motion of H and CO gas clouds along different lines of sight used to constrain the MW rotation curve at R₀ < 8 kpc.</p>
 - Observed radial dispersion of a population of halo stars.

Dynamical constraints

- Observation of "integrated properties" of the Milky Way obtained integrating the mass profiles of different galactic components along the line of sight.
 - the total mass of the Milky Way within 50 150 kpc measured observing the motion of the Milky Way satellites or the radial velocity of distant halo stars

$$M(< 50 \text{ kpc}) = (5.4 \pm 0.25) \times 10^{11} M_{\odot}$$

$$M(< 150 \text{ kpc}) = (7.5 \pm 2.5) \times 10^{11} M_{\odot}$$

total mean surface density within 1.1 kpc (Kuijken & Gilmore 1991)

$$\Sigma_{|z|<1.1 \rm kpc} = (71 \pm 6) \, M_{\odot} \, {
m pc}^{-2}$$

Iocal surface density corresponding to the visible component

$$\Sigma_{\star} = (48 \pm 8) \, M_{\odot} \, \mathrm{pc}^{-2}$$

Dynamical constraints

- Measurement of local properties of the Milky Way rotation curve
 - sum of Oort's constants proportional to the local slope of the galactic rotation curve

$$A + B = -\left(\frac{\partial v_c}{\partial R}\right)_{R=R_0}$$

$$A + B = (0.18 \pm 0.47) \,\mathrm{km \, s^{-1} \, kpc^{-1}}$$

difference of the constants gives

$$A-B=\frac{v_c(R_0)}{R_0}$$

$$A - B = (27.2 \pm 0.9) \,\mathrm{km \, s^{-1} \, kpc^{-1}}$$

Iocal circular velocity of the Sun

$$v_c(R_0) = (218 \pm 6) \,\mathrm{km \, s^{-1}}$$

Dynamical constraints

- Combination of Oort's constants A B very important in the determination of local quantities relevant for direct dark matter detection.
- A B positively correlated with the local dark matter density ρ_{loc}

