Bimaximal neutrino mixing and GUT's

Davide Meloni

Dipartimento di Matematica e Fisica
and
INFN RomaTre

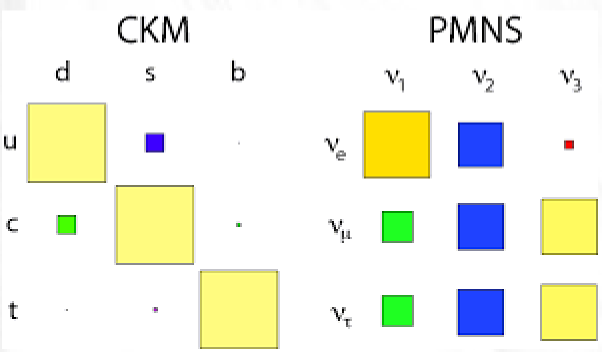
IPP2015

ROMA TRE
UNIVERSITÀ DEGLI STUDI

Standard oscillations

- Mixing matrix has the same structure in both contexts

$$U_{CKM\,,PMNS}=\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\times\begin{pmatrix}c_{13} & 0 & s_{13}e^{i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{-i\delta} & 0 & c_{13}\end{pmatrix}\times\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$



PMNS	CKM
all (but 1-3) matrix elements are of O(1)	matrix almost diagonal
one small and two large mixing angles	the three mixings are all small

in the Standard Model they do not talk to each other although the mechanism producing them is essentially the same

The need of New Physics

GOAL OF THIS TALK:

How to relate these two sectors ?

- Invoking GUT theories (different gauge groups):

  leptons and quarks sit in the same irreducible representations of the group

$$\Downarrow$$

Mass matrices are related

ex: SU(5)

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad m_d = m_e^T \qquad \Rightarrow$$

Not enough for producing the correct mixing

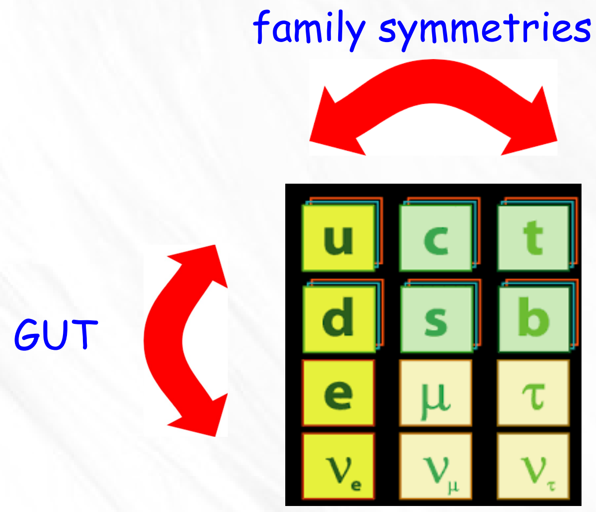
But see also Marzocca& Petcov& Romanino&Spinrath2011
Antusch&Maurer2011

The need of New Physics

- to improve predictability: Invoke family symmetries:

different families sit in the same irreducible representations of the group

Matrix elements of mass matrices are related



Being less ambitious...QLC

- Numerically, one sees that: $\theta_{12} + \theta_C \sim \pi/4$ → quark-lepton complementarity (QLC) Raidal2004, Minakata&Smirnov2004, Antusch&King&Mohapatra2005

  $\theta_{12} + O(\theta_C) \sim \pi/4$ is called
  weak complementarity

- Numerically, one also sees that: $\theta_{13} \sim \theta_C/\mathrm{sqrt}[2]$

this suggests that the Cabibbo is a key-role parameter

Where θ_C enters in the lepton sector?

Nature seems to help us !

- $m_\mu/m_\tau \sim \theta^2_C$
- $m_e/m_\mu \sim \theta_c^{3\text{-}4}$

we have to deal with **mass matrices !**

Introducing θ_c into the charged lepton masses

- for large fermion masses, we can use renormalizable operators (d=4):

$$\boxed{\overline{\psi_L} H \psi_R}$$

- to generate hierarchies, we can use non-renormalizable operators (d >= 5):

$$\boxed{\overline{\psi_L} H \psi_R \left(\frac{\varphi}{\Lambda}\right)^n}$$

new scalar fields, with vev = $\langle\phi\rangle$ transforming non-trivially under some flavor symmetry

cut-off of the theory

this number should be smaller than 1

breaking of the flavor symmetry

$$\frac{\langle\varphi\rangle}{\Lambda} \sim \theta_C$$

- $m_\mu/m_\tau \sim$ (d=6) / (d=4)

Natural assumption: the vevs of the new scalar fields are all of the same order of magnitude

Getting the QLC relation

- The strategy:

BM in the neutrino sector ← family symmetries

↓

Corrections from charged leptons:
QLC

↓

Connecting quarks and
leptons: obtaining Vus ~ λ_c

GUT

Getting the QLC

Start with a model whose LO prediction in the neutrino sector is $\theta_{12} = \pi/4$

An easy task with family symmetries

Plethora of models in the literature

Frampton, Petcov and Rodejohann, Nucl. Phys. B687 (2004) 31
T.Ohlsson, Phys.Lett.B622, 159 (2005)
Altarelli, Feruglio and Merlo, JHEP0905, 020 (2009)
D.Meloni, JHEP1110, 010 (2011)
Altarelli, Machado and Meloni, arXiv:1504.05514 [hep-ph]

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix} \xrightarrow{\text{diagonalization}} U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Maximal mixing

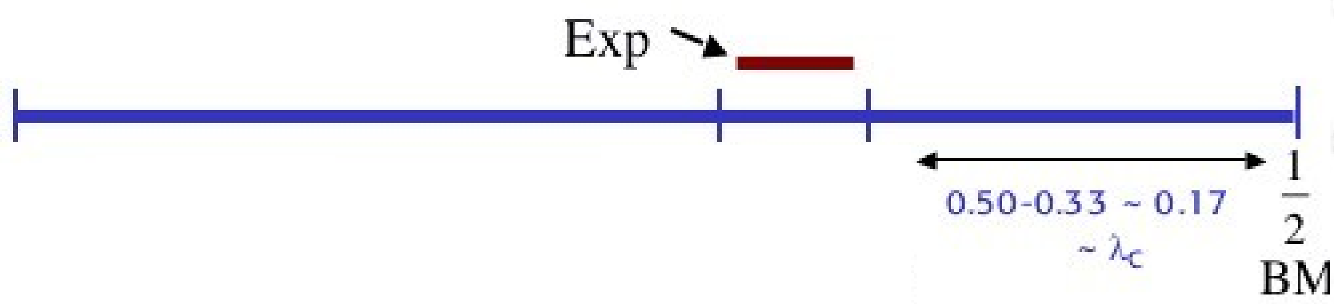
$$\sin^2\theta_{12} = \frac{1}{2} \quad \sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0$$

Corrections

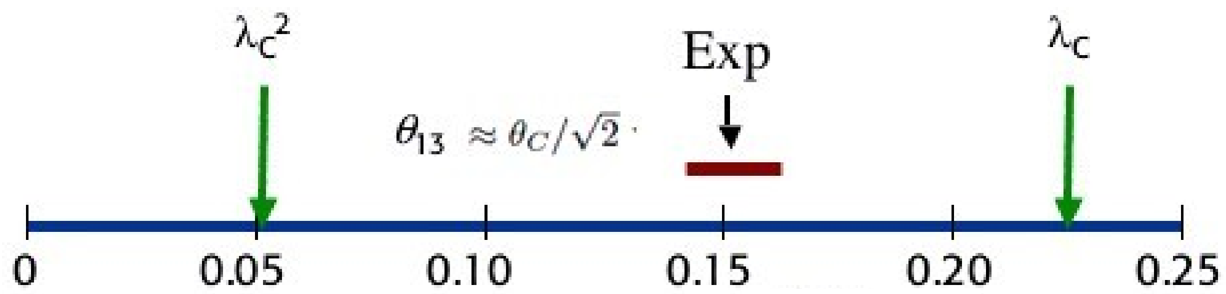
$$U_{PMNS} = U_l^+ U_{BM}$$

Now needs corrections to fall on the experimental value $\theta_{12} \sim 33^\circ$

$\sin^2\theta_{12}$



$\sin\theta_{13}$



Corrections provided by the diagonalization of the charged leptons

An explicit SU(5)xS4 example

- The discrete groups S4:

Meloni2011,
Altarelli, Machado, Meloni
arXiv:1504.05514

- S_4 is the group of permutations of 4 objects → 24 objects

- as usual, to generate all the group elements we need to identify " generators of the group" and their action on the elements of the group

these are called S and T

↓

One possible "representation": $S^2=T^3=1$ and $S\,T^2S=T$

- irreducible representations: two singlets 1_1 and 1_2 , one doublet 2 and two triplets 3_1 and 3_2

An explicit SU(5)xS4 example

Standard Model fields:

$$F = \begin{pmatrix} \bar{5}_e \\ \bar{5}_\mu \\ \bar{5}_\tau \end{pmatrix} \sim 3_1 \qquad 10_e \sim 1_1 \quad 10_\mu \sim 1_1 \quad 10_\tau \sim 1_1$$

Higgs fields: $H_5 \sim 1_1 \quad \bar{H}_5 \sim 1_1$

- Neutrinos of BM form from D=5 operators $F\,F\,H_5\,H_5$

An explicit SU(5)xS4 example

Additional ingredients for correct mass ratios :

- discrete Z_3 charges to suppress unwanted operators (for obtaining correct fermion mass ratios)
- formulation of the model in the extra dimension: 10_e and 10_μ propagate in the bulk, the other fermions are localized on the brane

the yukawa couplings of these two fields have additional suppression factors

Witten85,Kawamura2001,Faraggi2001,Hall&Nomura2001, Altarelli&Feruglio&Hagedorn2008

Additional ingredients for S_4 breaking

- set of additional scalars, called *flavons:* breaking in different subgroups in the neutrino and charged lepton sector

An explicit SU(5)xS4 example

- Charged lepton sector

$$m_e \sim \begin{bmatrix} a_{11}\lambda_C^5 & a_{12}\lambda_C^4 & a_{13}\lambda_C^4 \\ a_{21}\lambda_C^4 & -c\lambda_C^3 & 0 \\ a_{13}\lambda_C^4 & c\lambda_C^3 & a_{33}\lambda_C \end{bmatrix} \Rightarrow U_l \sim \begin{bmatrix} 1 & u_{12}\lambda_C & u_{13}\lambda_C \\ -u_{12}^*\lambda_C & 1 & 0 \\ -u_{13}^*\lambda_C & -u_{12}^* u_{13}^*\lambda_C^2 & 1 \end{bmatrix}$$

- a_{ij}, u_{ij} are O(1) coefficients
- u_{ij} is a linear combination of a_{ij}

$$U_{PMNS} = U_l^+ U_{BM}$$

this gives: $\sin^2\theta_{12} = \frac{1}{2} - u_{12}\lambda_C$ which is perfectly OK

this relation is of the *weak complementarity* form **IF** the model also generates Vus ~ $O(\lambda_c)$

⇒ link with GUT

The Vus matrix element

- the down sector

$$m_d = m_e^T \quad \Rightarrow \quad U_d \sim \begin{bmatrix} 1 & d_{12}\lambda_C & d_{13}\lambda_C^3 \\ -d_{12}^*\lambda_C & 1 & d_{23}^*\lambda_C^2 \\ (d_{12}^* d_{23}^* - d_{13}^*)\lambda_C^3 & -d_{23}^*\lambda_C^2 & 1 \end{bmatrix}$$

d_{ij} are a different combination of a_{ij}

so mixings are different but the off-diagonal (1-2) element is again of $O(\lambda_c)$

(obviously we have to be sure that the up-quark sector does not destroy the scheme)

weak complementarity is realized in the context GUT + family symmetry

What about BM and SO(10) ?

- no SO(10) singlets for right-handed neutrinos → more difficult explanation of the difference between charged fermions and neutrinos
- see-saw type II is an useful ansatz to separate the neutrino masses from the dominant contribution to the charged fermions (given by the Yukawa h)

$$M_\nu \sim f\langle 126\rangle_3 + \cancel{type-I}$$

f=Yukawa matrix of the fermion couplings to 126:
<u>can always be put on BM (or TBM) form</u>

$$\left.\begin{aligned} Y_u &\sim m_{top}(f+h) \\ Y_d &\sim m_b(f+h) \\ Y_e &\sim m_\tau(-3f+h) \end{aligned}\right\} \Rightarrow$$

Corrections from Ye are typically of the same order as the largest quark mixing angle, i.e. λc

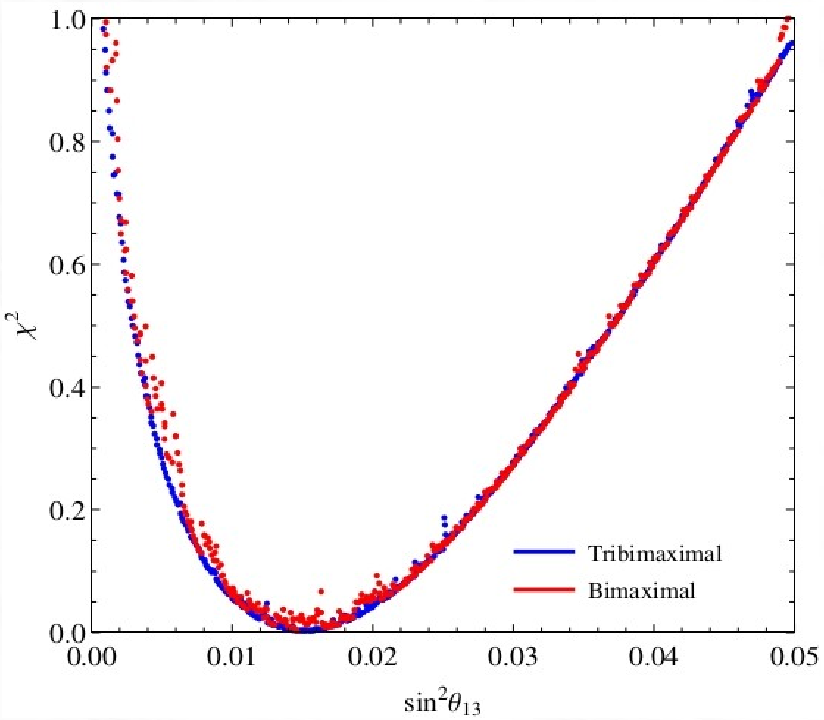
What about BM and SO(10) ?

The big question: which pattern (TBM or BM) is more favored by the data?

χ^2 analysis not very conclusive

in fact, we could have started from f of the TBM form and still obtain a good description of the data, i.e., of θ_{13}

the set of parameters used in one fit are functions of the parameters of the other fit, so the χ^2 in the two cases are simply related to each other

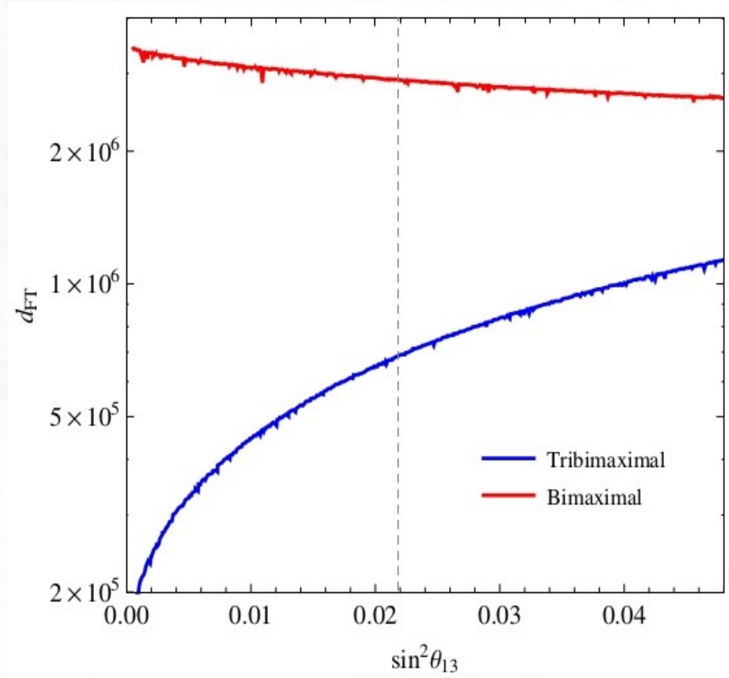


Better BM or TBM in SO(10) ?

- we have to use some estimator: the *fine-tuning parameter*

$$d_{FT} = \Sigma_i \left| \frac{par_i}{err_i} \right|$$

shift of the best-fit parameter
that changes the χ^2 by 1 unit



the small first family masses dominate the fine-tuning

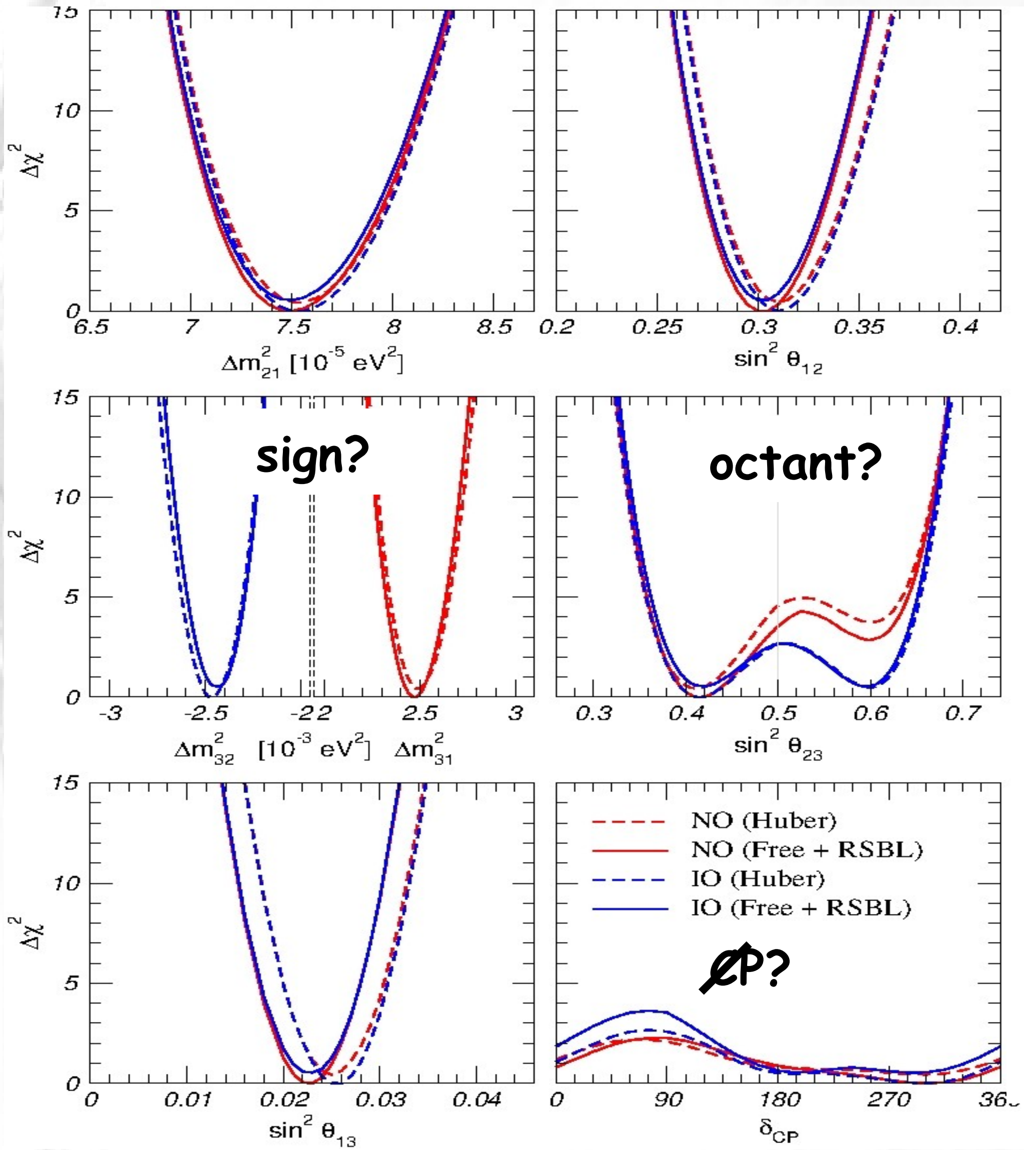
it turns out that the TBM fit to the data is slightly less fine-tuned than BM

Conclusions

- Weak form of complementarity can be easily implemented in GUT context
- BM is good starting point in SU(5) + family symmetry framework
- No clear preference in the description of the data emerged from SO(10)

Backup

Global fit on neutrino data



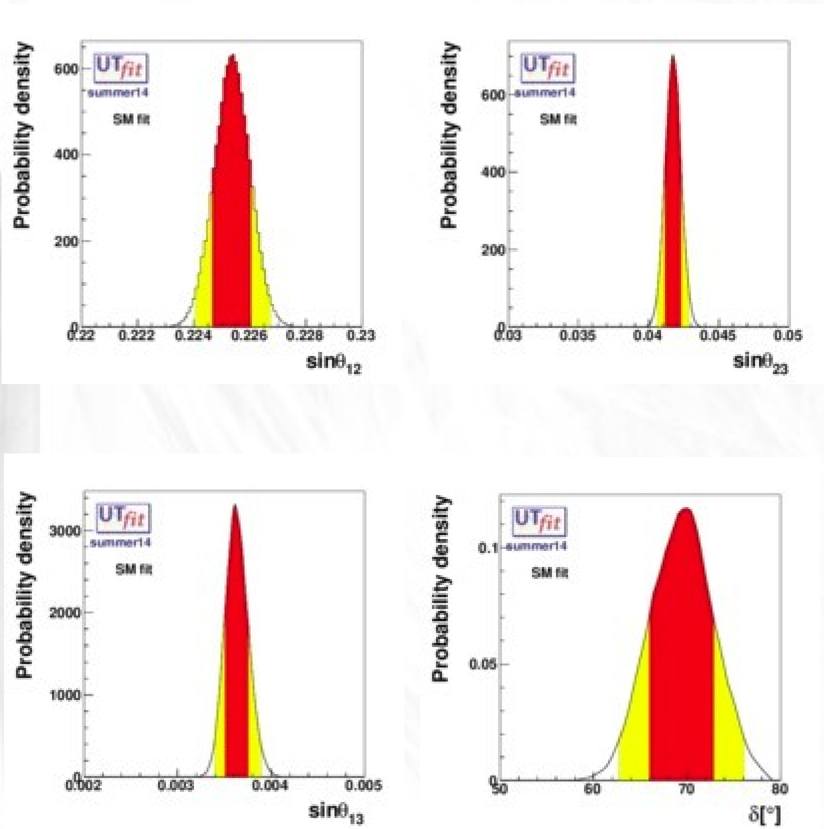
Gonzalez-Garcia et al. JHEP1212,(2012)123

Parameter	Result
θ_{12}	$33.36^{+0.81}_{-0.78}$
θ_{13}	$8.66^{+0.44}_{-0.46}$
θ_{23}	$40.0^{+2-1}_{-1.5}$
δ	300^{+66}_{-138}
Δm^2_{23} (10^{-3} eV^2)	$2.47^{+0.07}_{-0.07}$
Δm^2_{12} (10^{-5} eV^2)	$7.50^{+0.18}_{-0.19}$

$$|U| = \begin{pmatrix} 0.795 \to 0.846 & 0.513 \to 0.585 & 0.126 \to 0.178 \\ 0.205 \to 0.543 & 0.416 \to 0.730 & 0.579 \to 0.808 \\ 0.215 \to 0.548 & 0.409 \to 0.725 & 0.567 \to 0.800 \end{pmatrix}$$

PMNS mixing matrix

Global fit on quark data



http://www.utfit.org

Parameter	Result
$\sin\theta_{12}$	0.22523+-0.00065
$\sin\theta_{13}$	0.00363+-0.00012
$\sin\theta_{23}$	0.0417+-0.00057
$\delta^{(o)}$	69.4+-3.4

$$V_{CKM} = \begin{pmatrix} (0.97426 \pm 0.00015) & (0.22529 \pm 0.00061) & (0.00363 \pm 0.00012) e^{i(-69.3 \pm 3.3)^\circ} \\ (-0.22518 \pm 0.00066) e^{i(0.03509 \pm 0.00098)^\circ} & (0.97341 \pm 0.00015) e^{i(-0.00187 \pm 0.00005)^\circ} & (0.0417 \pm 0.00056) \\ (0.0088 \pm 0.00018) e^{i(-22.0 \pm 0.8)^\circ} & (-0.04092 \pm 0.00055) e^{i(1.069 \pm 0.042)^\circ} & (0.999119 \pm 0.000021) \end{pmatrix}$$

CKM mixing matrix

Mixing matrices

- U_{PMNS} and V_{CKM} have contributions from two different sectors

leptons

$$U_{PMNS} = U^{+l}_{j\alpha} U^{\nu}_{\alpha i}$$

from the diagonalisation of the charged lepton mass matrix

from the diagonalisation of the neutrino mass matrix

quarks

$$V_{CKM} = U^{+d}_{j\alpha} U^{u}_{\alpha i}$$

How to relate these two sectors ?

An explicit SU(5)xS4 example

Field	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$	φ_ν	ξ_ν	φ_ℓ	χ_ℓ	θ	θ'	φ_ν^0	ξ_ν^0	ψ_ℓ^0	χ_ℓ^0
SU(5)	$\bar{5}$	10	10	10	5	$\bar{5}$	1	1	1	1	1	1	1	1	1	1
S_4	3_1	1	1	1	1	1	3_1	1	3_1	3_2	1	1	3_1	1	2	3_2
Z_3	ω	ω	1	ω^2	ω^2	ω^2	1	1	ω	ω	1	ω	1	1	ω	ω

$$
\begin{aligned}
w_\ell \;=\;& FT_3H_{\bar{5}}\left[\frac{\alpha_b}{\Lambda^{3/2}}\varphi_\ell+\frac{\alpha_1}{\Lambda^{5/2}}(\varphi_\nu\varphi_\ell)_{3_1}+\frac{\alpha_2}{\Lambda^{5/2}}(\varphi_\nu\chi_\ell)_{3_1}+\frac{\alpha_3}{\Lambda^{5/2}}\varphi_\ell\xi_\nu+\right]+\\
& FT_2H_{\bar{5}}\theta\left[\frac{\beta_1}{\Lambda^3}\varphi_\nu+\frac{\beta_2}{\Lambda^4}(\varphi_\nu^2)_{3_1}+\frac{\beta_3}{\Lambda^4}\varphi_\nu\xi_\nu\right]+\\
& FT_2H_{\bar{5}}\theta'\left[\frac{\beta_4}{\Lambda^4}(\varphi_\ell^2)_{3_1}+\frac{\beta_5}{\Lambda^4}(\chi_\ell^2)_{3_1}+\frac{\beta_6}{\Lambda^4}(\chi_\ell\varphi_\ell)_{3_1}\right]+\\
& FT_1H_{\bar{5}}\theta^2\left[\frac{\gamma_1}{\Lambda^5}(\varphi_\ell^2)_{3_1}+\frac{\gamma_2}{\Lambda^5}(\chi_\ell^2)_{3_1}+\frac{\gamma_3}{\Lambda^5}(\varphi_\ell\chi_\ell)_{3_1}\right]+\\
& FT_1H_{\bar{5}}\theta\theta'\left[\frac{\gamma_4}{\Lambda^4}\varphi_\ell+\frac{\gamma_5}{\Lambda^5}(\varphi_\nu\varphi_\ell)_{3_1}+\frac{\gamma_6}{\Lambda^5}(\varphi_\nu\chi_\ell)_{3_1}+\frac{\gamma_7}{\Lambda^5}\xi_\nu\varphi_\ell\right]+\\
& FT_1H_{\bar{5}}\theta'^2\left[\frac{\gamma_8}{\Lambda^4}\varphi_\nu+\frac{\gamma_9}{\Lambda^5}(\varphi_\nu^2)_{3_1}+\frac{\gamma_{10}}{\Lambda^5}\xi_\nu\varphi_\nu\right],
\end{aligned}
$$

not so good

$$
\frac{m_u}{m_c}\sim\frac{m_d}{m_s}\sim\frac{m_s}{m_b}\sim\frac{m_e}{m_\mu}\sim\frac{m_\mu}{m_\tau}\sim\lambda^2
$$

$$
\frac{m_c}{m_t}\sim\lambda^4
$$

An explicit SU(5)xS4 example

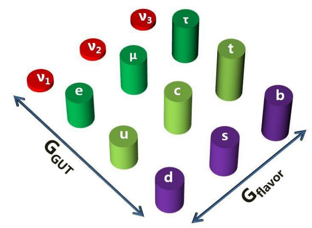
$$\begin{aligned} w_{up} &= \frac{\alpha_t}{\Lambda^{1/2}} T_3 T_3 H_5 + \frac{\delta}{\Lambda^4} T_2 T_3 H_5 \theta' (\varphi_\nu \varphi_\ell) + \frac{\sigma}{\Lambda^3} T_1 T_3 H_5 \theta \theta' + \\ & \frac{1}{\Lambda^{9/2}} T_1 T_2 H_5 \left(\tau_1 \theta^3 + \tau_2 \theta'^3 \right) + \\ & \frac{\rho}{\Lambda^{7/2}} T_2 T_2 H_5 \theta \theta' + \frac{\eta}{\Lambda^{11/2}} T_1 T_1 H_5 \theta^2 \theta'^2 \,. \end{aligned}$$

$$\begin{aligned} w_w &= \frac{y_w}{\Lambda^2} (FF)_1 H_5 H_5 + \frac{y_{w_1}}{\Lambda^3} (FF)_{3_1} H_5 H_5 \varphi_\nu + \frac{y_{w_2}}{\Lambda^3} (FF)_1 H_5 H_5 \xi_\nu + \frac{y_{w_3}}{\Lambda^4} (FF)_1 (\varphi_\nu \varphi_\nu)_1 H_5 H_5 \\ &+ \frac{y_{w_4}}{\Lambda^4} (FF)_2 (\varphi_\nu \varphi_\nu)_2 H_5 H_5 + \frac{y_{w_5}}{\Lambda^4} (FF)_{3_1} (\varphi_\nu \varphi_\nu)_{3_1} H_5 H_5 + \\ & \frac{y_{w_6}}{\Lambda^4} (FF)_1 \xi_\nu^2 H_5 H_5 + \frac{y_{w_7}}{\Lambda^4} (FF\varphi_\nu)_1 \xi_\nu H_5 H_5 \,. \end{aligned}$$

Reproducing fermion masses and mixing

Less free lagrangian parameters

Theory invariant under G_F



G_F mostly permutation groups

Residual symmetry in the neutrino sector G_ν: U_ν

Residual symmetry in the charged lepton sector G_l: U_l

$$U_{PMNS} = U_l^+ U_\nu$$

The role of family symmetries

- BM mixing corresponds to m in the basis where charged leptons are diagonal

$$m=\begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

x, y, z complex numbers

m is the most general matrix invariant under

$SmS = m$ and $A_{23}mA_{23} = m$

$$S=\begin{pmatrix} 0 & -\sqrt{1/2} & -\sqrt{1/2} \\ -\sqrt{1/2} & 1/2 & -1/2 \\ -\sqrt{1/2} & -1/2 & 1/2 \end{pmatrix}, \quad A_{23}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \Rightarrow$$

generators of the residual symmetry in the ν sector

The role of family symmetries

- superpotential

$$
\begin{aligned}
w_d = & \quad M_\varphi(\varphi_\nu^0\varphi_\nu) + g_1\left(\varphi_\nu^0(\varphi_\nu\varphi_\nu)_{3_1}\right) + g_2\left(\varphi_\nu^0\varphi_\nu\right)\xi_\nu + \\
& + \ \xi_\nu^0\left[M_\xi^2 + M_\xi'\xi_\nu + g_3(\varphi_\nu\varphi_\nu) + g_4\xi_\nu\xi_\nu\right] + \\
& + \ f_1\left(\psi_\ell^0(\varphi_\ell\varphi_\ell)_2\right) + f_2\left(\psi_\ell^0(\chi_\ell\chi_\ell)_2\right) + f_3\left(\psi_\ell^0(\varphi_\ell\chi_\ell)_2\right) + \\
& + \ f_4\left(\chi_\ell^0(\varphi_\ell\chi_\ell)_{3_2}\right) .
\end{aligned}
$$

$$
\langle\varphi_\ell\rangle = v_{\varphi_\ell}\begin{pmatrix}0\\1\\0\end{pmatrix} \qquad \langle\chi_\ell\rangle = v_\chi\begin{pmatrix}0\\0\\1\end{pmatrix}
$$

$$
\langle\varphi_\nu\rangle = v_{\varphi_\nu}\begin{pmatrix}0\\1\\-1\end{pmatrix} \qquad \langle\xi_\nu\rangle = v_\xi
$$