de Sitter Microstates from the $T\bar{T} + \Lambda_2$ deformation

arXiv:2110.14670 with Evan Coleman, Edward A. Mazenc, Vasudev Shyam, Ronak M Soni, Gonzalo Torroba, Sungyeon Yang

+ earlier works with Alishahiha et al, …,Dong, Gorbenko, Lewkowycz, Liu, Torroba…

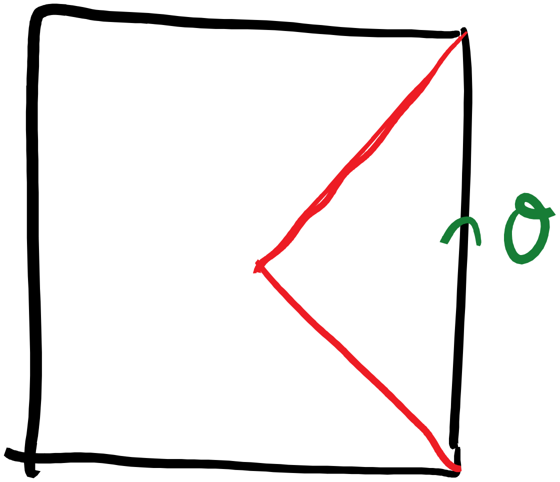
+ (time permitting):

dS_4 in M theory as uplift of AdS_4/CFT_3

arXiv 2104.13380 with B. De Luca and G. Torroba

Gravitational calculations suggest a thermodynamic interpretation of the de Sitter observer horizon, somewhat analogous to black hole thermodynamics

Gibbons-Hawking … Anninos et al (logarithmic corrections)



$$S = \underbrace{S_{GH}}_{\frac{A}{4G_N}} - \underbrace{3\log(S_{GH})}_{(A)\ dS_3\ \text{case}} + \ldots$$

$$S = S_{GH} - 3\log(S_{GH}) + \ldots$$

Suggests theory with a finite Hilbert space captures observer patch.

Today's talk: at the `pure gravity' level, we find that the *real dressed spectrum* of the universal and solvable

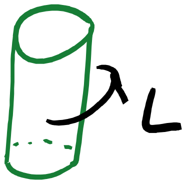
$T\bar{T} + \Lambda_2$ deformation

Zamalodchikov et al, Dubovsky et al, Cavaglia et al … Gorbenko ES Torroba '18

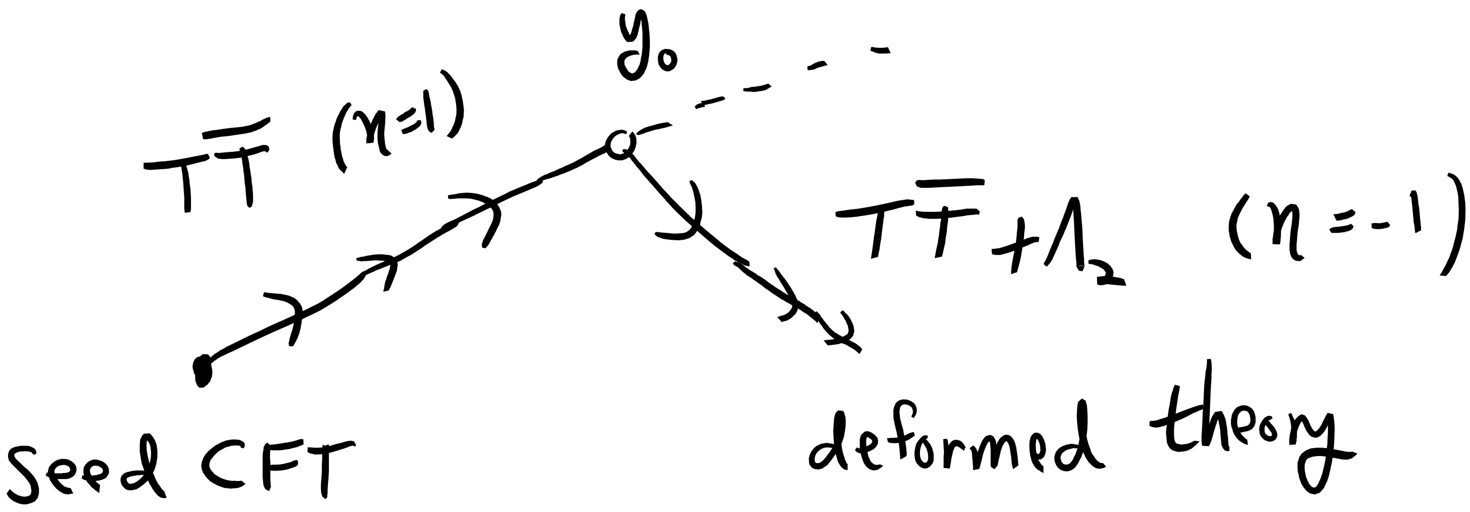
of a CFT on a cylinder captures the microstates and the geometry of the dS_3 observer patch

$$\frac{\partial}{\partial\lambda}\log Z = -2\pi\int d^2x\,\sqrt{g}\;T\bar{T} + \underbrace{\frac{1-\eta}{2\pi\lambda^2}}_{\Lambda_2}\int d^2x\,\sqrt{g}$$

e.g. cylinder



$$y \equiv \frac{\lambda}{L^2}$$



Deformed energy spectrum computed precisely:

Smirnov-Zamolodchikov, Cavaglia et al, Dubovsky et al…Gorbenko et al…

$$\frac{\partial}{\partial \lambda} \log Z = -2\pi \int d^2x \sqrt{g} \; T\bar{T}_{g} + \frac{1-\eta}{2\pi\lambda^2} \int d^2x \sqrt{g}$$

$\rightarrow \partial_\lambda \langle H \rangle \sim \langle T\bar{T} \rangle$, $\quad T^x_x \sim \frac{\partial E}{\partial L}$ (pressure), … $\rightarrow$

$$\mathcal{E} = EL$$

$$\pi y \mathcal{E}(y)\mathcal{E}'(y) - \mathcal{E}'(y) + \frac{\pi}{2}\mathcal{E}(y)^2 = \frac{1-\eta}{2\pi y^2} + 2\pi^3 J^2$$

$$\mathcal{E}\Big|_{y=0,\eta=1} = \mathcal{E}_{CFT} = 2\pi\left(\Delta - \frac{c}{12}\right)$$

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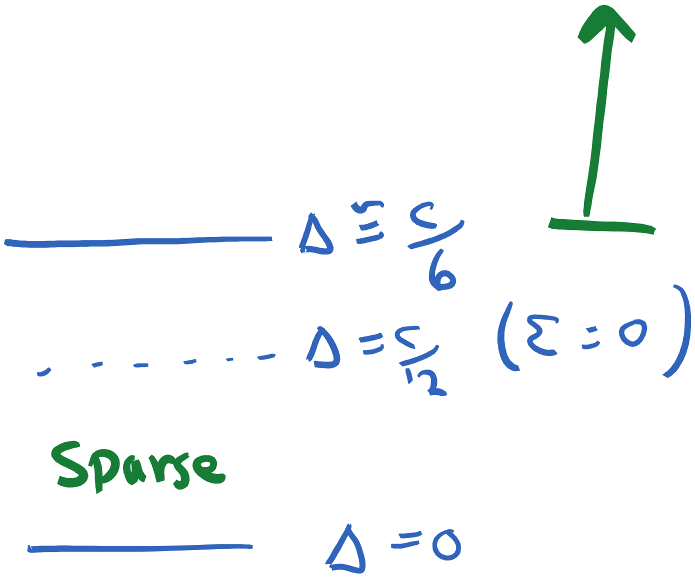
We will be interested in a seed CFT with a *sparse light spectrum* (in particular a holographic CFT)

Hartman Keller Stoica et al



$$S = S_{Cardy} = 2\pi\sqrt{\frac{c}{3}\left(\Delta - \frac{c}{12}\right)}$$

$\simeq \frac{A}{4G_N}$ for holographic CFTs, with

BTZ Black Holes for $\Delta > \frac{c}{12}$

General solution:

$$\mathcal{E}(y) = \frac{1}{\pi y}\left(1 \pm \sqrt{\eta - 4C_1 y + 4\pi^4 J^2 y^2}\right)$$

Fix integration constant and branch via appropriate boundary conditions for a given trajectory in theory space.

We will do two key examples, where the deformed energy matches the *Brown-York energy* for a given patch of dS, via a trajectory which is continuous for the corresponding band of energies.

Brown-York (quasilocal) stress-energy

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta I_{\text{on-shell}}}{\delta g^{\mu\nu}} = \frac{1}{8\pi G_N}\left(\underbrace{K_{\mu\nu} - g_{\mu\nu}K}_{\sqrt{\cdots}} + \frac{1}{\ell}g_{\mu\nu}\right)$$

Part of $\mathcal{E}$

← Dirichlet condition

$G_{ij}|_\partial = g_{ij}$ fixed

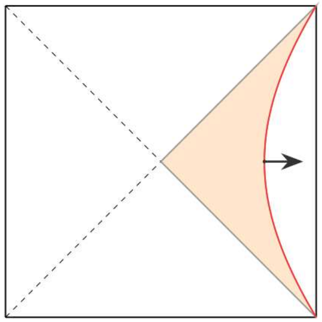
$\Pi^{\text{radial}}_{ij}|_\partial = K_{ij}$ free

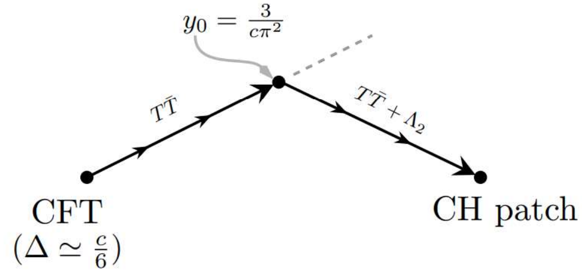
With cylinder slices, a subset of the Einstein equations imply the above differential equation for $\mathcal{E} = L\int_0^L dx\, T^t_t$

with dictionary $c = \dfrac{3\ell}{2G_N}, \quad \lambda = 8G_N\ell\,. \quad \Lambda_3 = -\dfrac{\eta}{\ell^2}$

Cosmic horizon patch

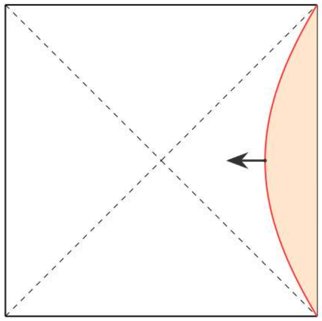
(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)

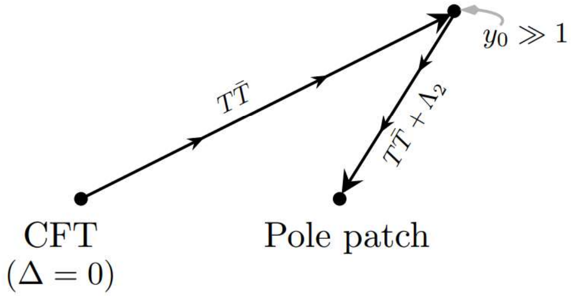




Pole patch

(Dressed $\Delta = 0$ vacuum)





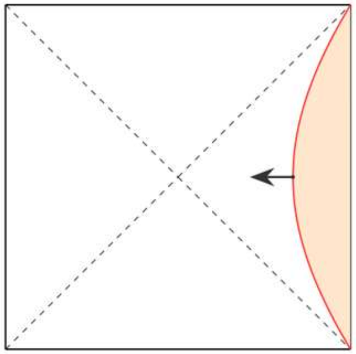
$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \dots}\right)$ $\longleftarrow$ related by $\pm\sqrt{}$ $\longrightarrow$ $\mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \dots}\right)$

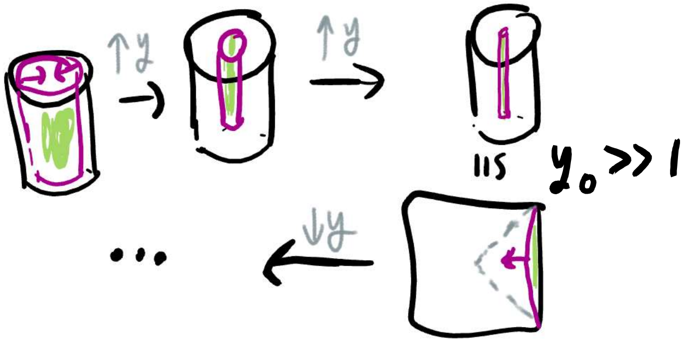
$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

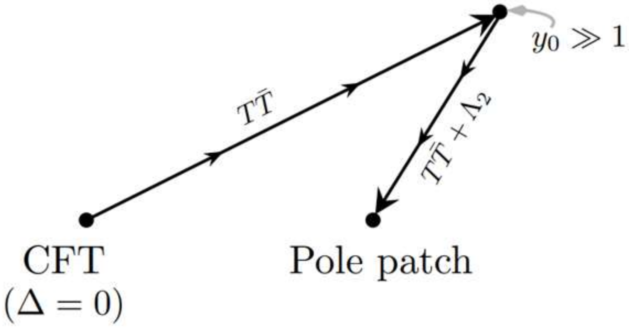
Pole patch

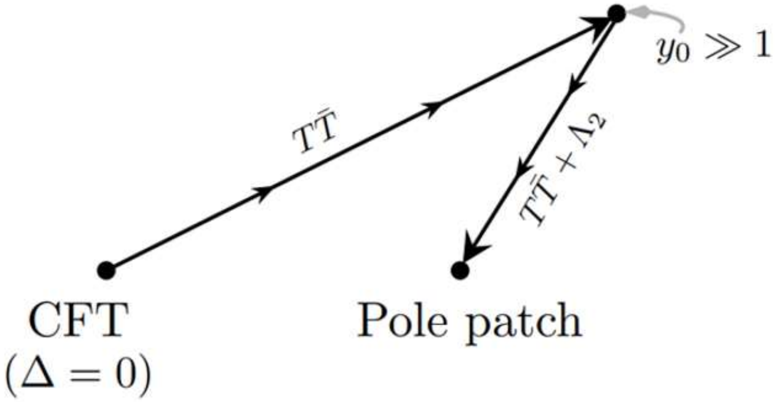
(Dressed $\Delta = 0$ vacuum)

Gorbenko ES Torroba ‘18



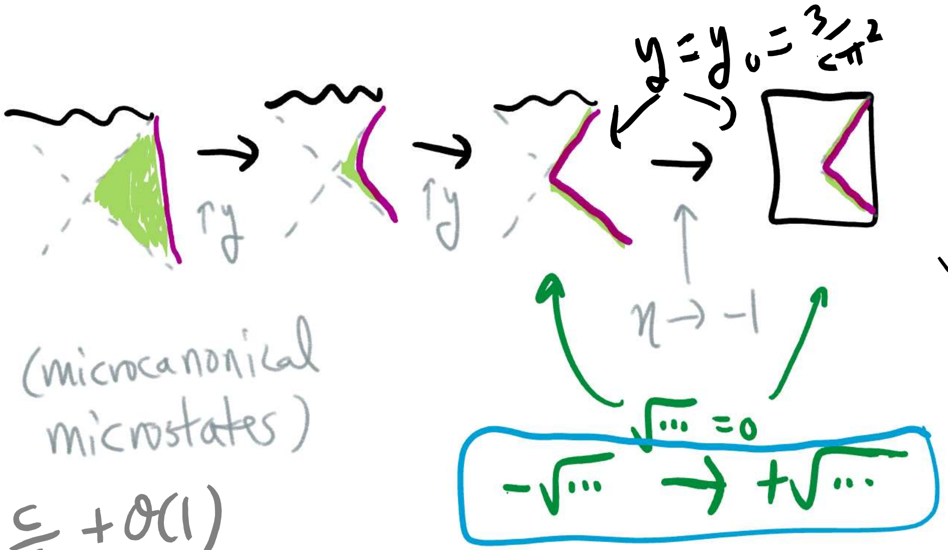






Cosmic horizon patch

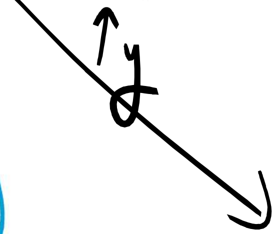
(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)

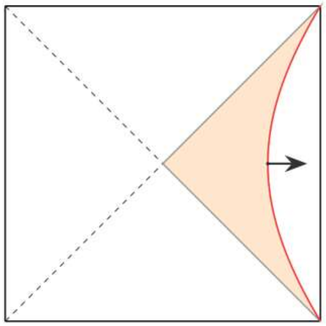


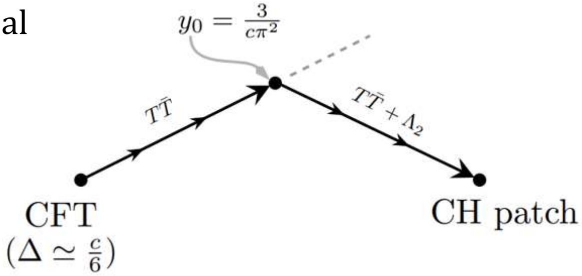
$$\Delta \leq \frac{c}{6} + \mathcal{O}(1)$$

$$ds_3^2 = -\frac{w^2}{\ell^2}d\tau^2 + dw^2 + (\ell^2 + \eta w^2)d\phi^2$$

At $y = y_0$, the near horizon patches are identical



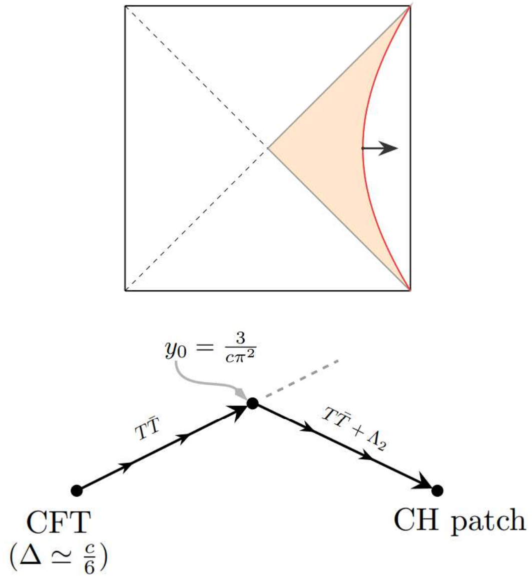




$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

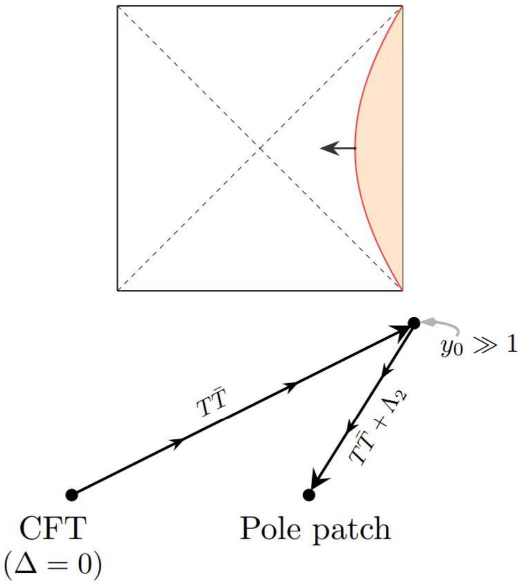
Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)



Pole patch

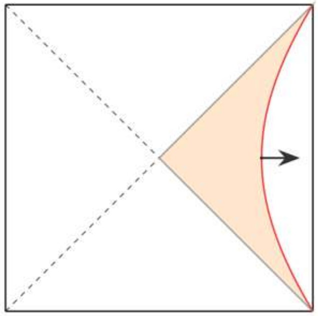
(Dressed $\Delta = 0$ vacuum)



$$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \ldots}\right) \quad \longleftarrow \text{ related by } \pm\sqrt{} \longrightarrow \quad \mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \ldots}\right)$$

As we vary y, we capture precisely the geometry of the dS patch

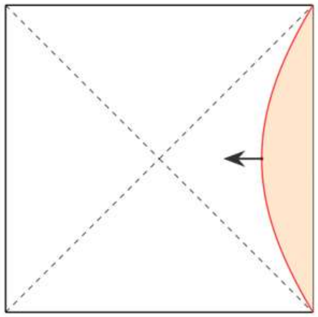
Propagation is causal in the bulk. Propagation between points on the boundary may be faster in the bulk or boundary:



Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)

Faster along the boundary
(subluminal in $T\bar{T}+\Lambda_2$ description)

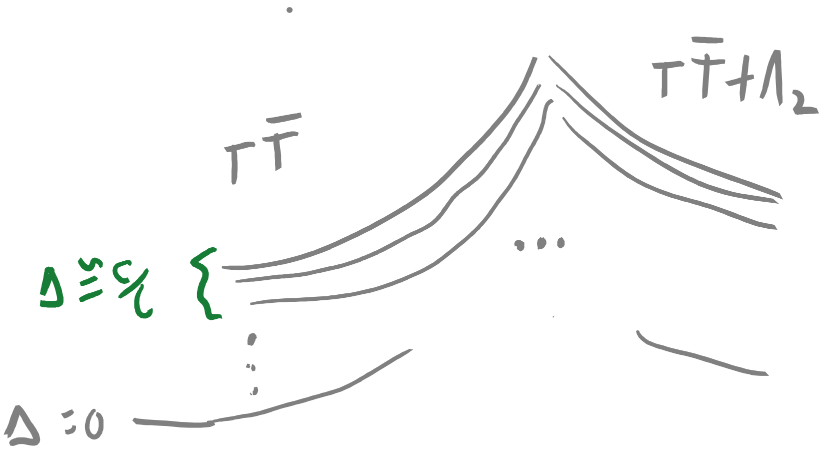


Pole patch

(Dressed $\Delta = 0$ vacuum)

Faster through bulk
(subluminal in $T\bar{T}+\Lambda_2$ description)

$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$



Count of states goes along for the ride
(`integrable deformation’)

$$\Delta < \frac{c}{12} : \text{ sparse spectrum} \quad \text{(particle states)}$$

$$\Delta \geq \frac{c}{6} : S \simeq S_{Cardy} = 2\pi\sqrt{\frac{c}{3}\left(\Delta - \frac{c}{12}\right)}$$

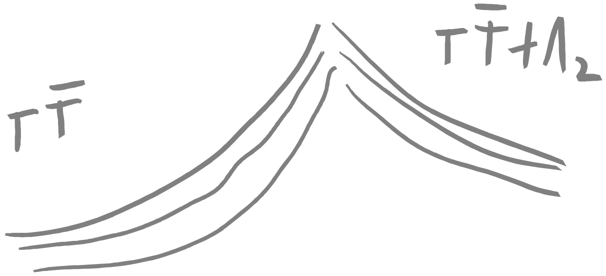
Other states:

$$\mathcal{E}=\frac{1}{\pi y}\left(1\mp\sqrt{\eta+\frac{y}{y_0}(1-\eta)-4\pi^2 y\left(\Delta-\frac{c}{12}\right)+4\pi^4 y^2 J^2}\right)$$

$\Delta > \frac{c}{6}$ states: dressed energies formally become complex, discarded in a unitary version of the theory => Finite dimensional Hilbert space with count of states agreeing with Gibbons-Hawking

$\Delta < \frac{c}{6}$ states: subdominant at large c, model-dependent (details require additions to the deformation)

=> Real spectrum of the $T\bar{T}+\Lambda_2$ deformation captures the finite dimensional Hilbert space (i) agreeing with Gibbons-Hawking and (ii) building up the geometry of the dS observer patch



Count of states goes along
for the ride (`integrable deformation'),
subleading check agrees:

$$S = A/4G_N - 3\log(A/4G_N)$$

Sen (AdS BTZ case) … Anninos Denef Law Sun (dS)

Further generalizations of $T\bar{T} + \Lambda_d$ valid at large c:

M. Taylor;   Hartman Kruthoff Shaghoulian Tajdini ’18

(1) Local bulk matter (model-dependent, subleading in entropy) requires similar term for each low-energy field: cf Guica et al

$\Pi_{\phi,\text{radial}} \sim \tilde{\mathcal{O}} \to$

$$T_i^i = -4\pi\lambda T\bar{T} - \frac{1}{\pi\lambda}\left(\frac{c\lambda}{L^2}\right)^{\Delta}\tilde{\mathcal{O}}^2 - \frac{cR^{(2)}}{24\pi} + \Lambda_2$$

(2) Higher dimensions & curvature:

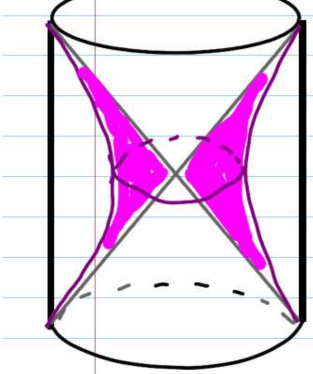
$$\tilde{T}^\mu_\mu = -4\pi G\ell\left(\tilde{T}_{\mu\nu}\tilde{T}^{\mu\nu} - \frac{1}{d-1}(\tilde{T}^\mu_\mu)^2\right) - \frac{\ell}{16\pi G}R^{(d)} - \frac{d(d-1)}{16\pi G\ell}(\eta - 1)$$

$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + a_d C_{\mu\nu}\,. \qquad C_{\mu\nu} = G_{\mu\nu} \text{ for } d \leq 4$$

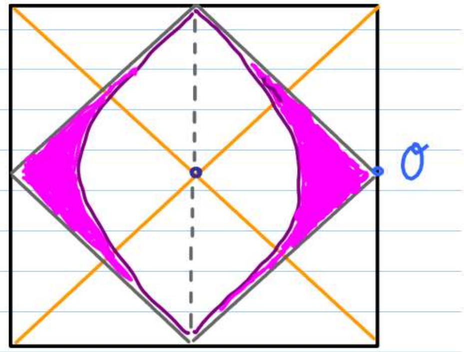
dS/dS case:

Alishahiha et al '04, …, Dong ES Torroba '18, …
Gorbenko ES Torroba '18, Shyam '21

$$
\begin{aligned}
ds^2_{(A)dS_{d+1}} &= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})ds^2_{dS_d} \\
&= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})\left[-d\tau^2 + \ell^2_{dS}\cosh^2\frac{\tau}{\ell_{dS}}d\Omega^2_{d-1}\right] .
\end{aligned}
$$



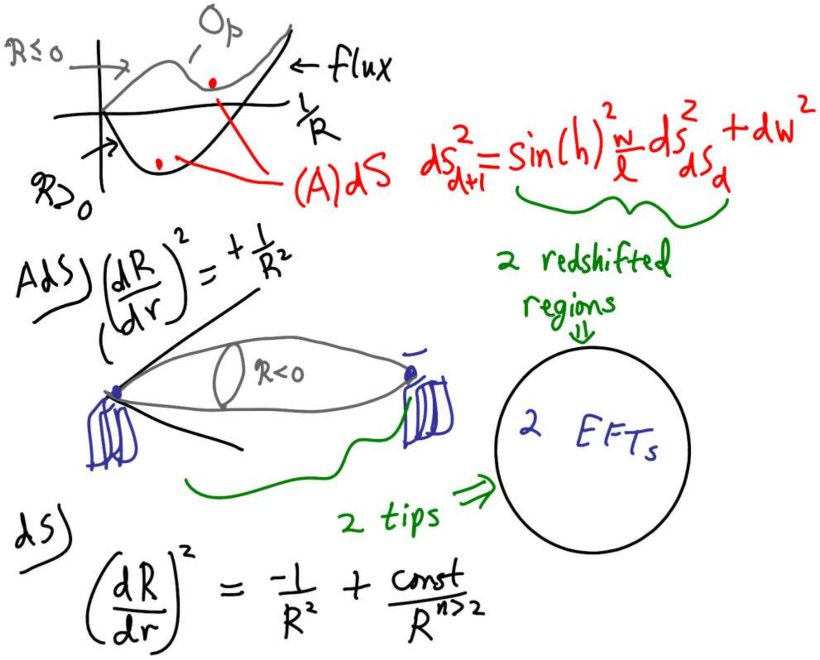
AdS/dS



dS/dS (each point is (d-1)-sphere)

Uplifting AdS/CFT => 2 sectors

Dong Horn ES Torroba '10



dS vs AdS brane construction:
independent derivation of the two sectors because of metastability.
Also true in dS/CFT

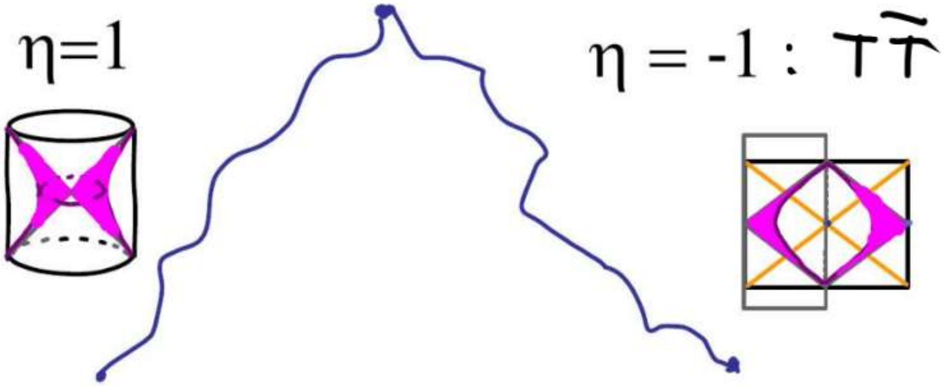
$$\begin{aligned} c &= \frac{3\ell}{2G} \\ \lambda &= 8G\ell \\ r &= \ell \sin\left(\frac{w_c}{\ell}\right) \\ L &= 2\pi\mu\ell \sin\left(\frac{w_c}{\ell}\right) \\ \Rightarrow \Lambda_2 = -\frac{1}{\pi\lambda} &= -\frac{c}{12\pi}\frac{1}{r^2} \quad (w_c = \ell\pi/2) \end{aligned}$$

$c\lambda/r^2 >> 1$

$T\bar{T}$

$\eta=1$

$\eta = -1$: $T\bar{T} + \Lambda_2$



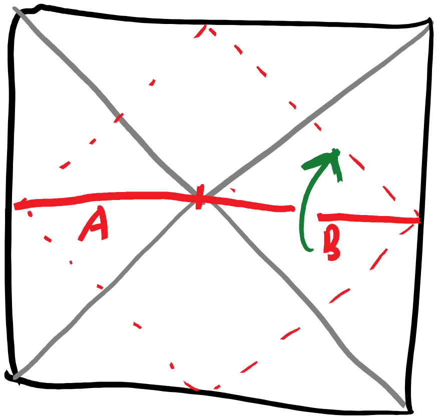
CFT

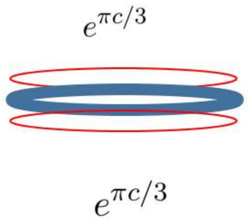
dS/dS warped throat

The static patch Hamiltonian is the Modular Hamiltonian K for dS/dS

$$K = -\log(\rho)$$

with ρ the reduced density matrix for 1 of the 2 dS/dS warped throats



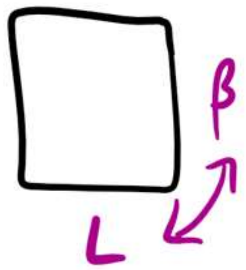


$$S_{GH} = -Tr\rho\log(\rho) = S_{EE} = \log(\dim H_{T\bar{T}+\Lambda_2})$$

Dong ES Torroba '18

Coleman Mazenc ES Shyam Soni Torroba Yang '21

In the canonical ensemble (fixed temperature ~ $1/\beta$ and L: Euclidean torus), our system exhibits an intriguing remnant of modular invariance



A 2d theory on a torus is invariant under $\beta \leftrightarrow L$ $\quad Z_\lambda(L,\beta) = Z_\lambda(\beta, L)$

But our theory, without the complex levels, is a 1d (quantum mechanics) theory, unitary but not fully local. Nonetheless, we find a remnant of modular invariance:

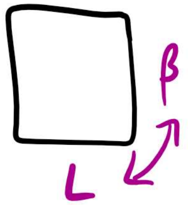
Seed CFT for c >> 1: $$\log Z \simeq \max\{\overbrace{-\beta E_{vac}(L)}^{\beta > L}\ ,\ \overbrace{-L E_{vac}(\beta)}^{\beta < L}\}$$ Hartman Keller Stoica et al

Deformation (β<L) : $$\log Z|_{\beta<L} \simeq -L E_{vac}(\beta) = S_{Cardy}(\Delta = c/6) - \beta E_{\Delta=c/6}(L)$$

The deformed $\Delta \simeq c/6$ levels propagate in the direct channel.

Shyam '21: this modular transformation starting from the pole patch spectrum yields S_{GH}

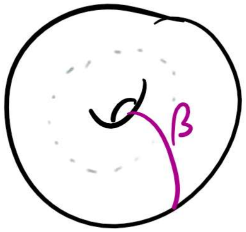
Relation/analogy to Hawking-Page:



AdS: $T < T_{HP}$

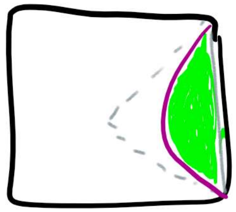
$\check{L} \to 0$
in interior

$T > T_{HP}$

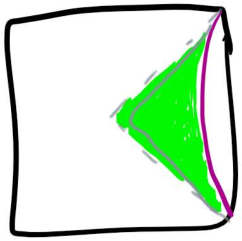


$\check{\beta} \to 0$ ($\check{T} \to \infty$)
in interior

dS:

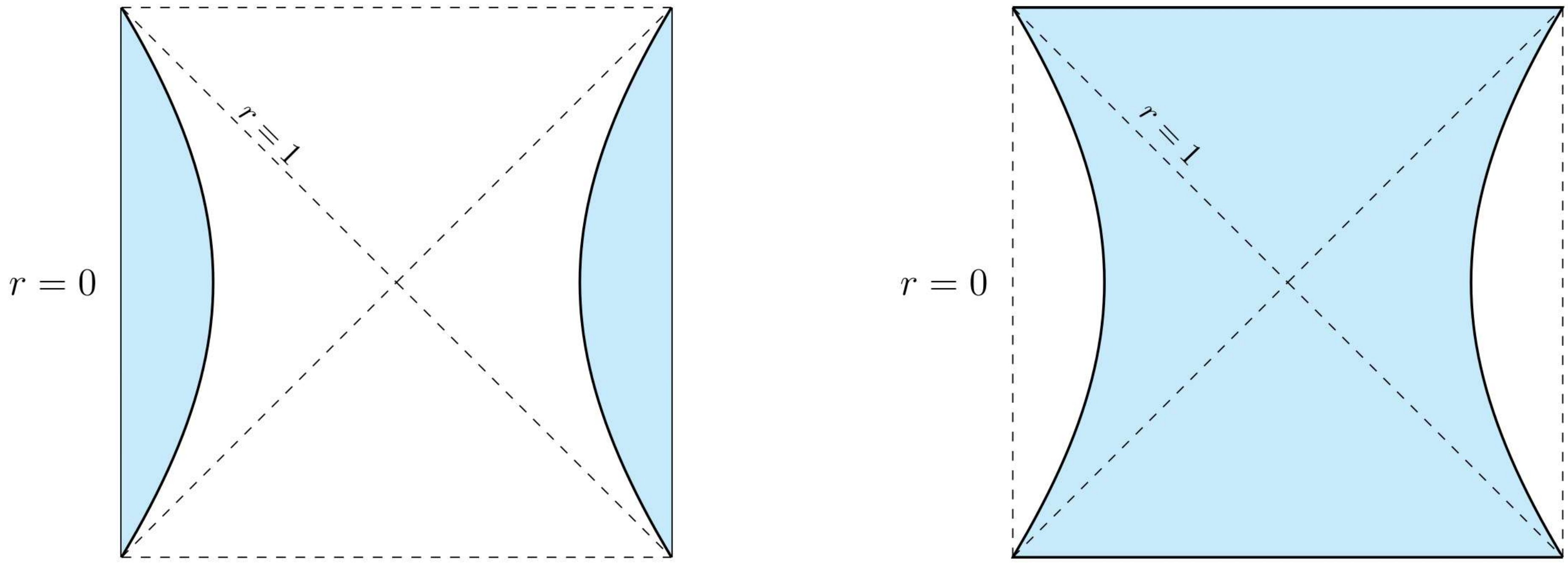


$\check{L} \to 0$
in interior



$\check{\beta} \to 0$
($\check{T} \to \infty$)
in interior

Can in principle double and glue the two patches together to get global dS:



$$|TFD\rangle = \sum_n e^{-\beta E_n/2} |E_{n,L}\rangle |E_{n,R}\rangle$$

Next steps/questions for this part:

- d+1=4 dimensional case and black hole states (at and below Hawking Page level)

- Chaos/complexity: square root with $\Lambda_2$cannot be expanded in fundamental (seed CFT) variables: not k-local cf Susskind '21

- Relation to string theoretic de Sitter(=dS quantum gravity)? Late time physics (metastable decay)?

dS examples:

Reviews of various aspects: Polchinski, Baumann/McAllister, Douglas/Kachru,
Denef, Frey, Hebecker; ES TASI '16, …

- Power-law stabilization

--(D-Dc), O-planes, flux, asymmetric orbifold (large-D expansion) '01-'02

(…other examples…)

--hyperbolic space, Casimir, flux '21

--including explicit uplifts of AdS/CFT

[D1-D5 theory -> dS3 '10,

M2 brane theory -> dS4 '21]

New example:

≥KK scale SUSY breaking

- Non-perturbative stabilization

--GKP '01/KKLT '03 and many followups, e.g.

--large volume scenario

Sub-KK scale SUSY breaking

(Weak-coupling EFT control. Ongoing studies of internal equations of motion in various cases & models, including ones with significant gradients e.g. Cordova et al, …)

M theory on hyperbolic space and dS Quantum Gravity

with G. Bruno De Luca and Gonzalo Torroba

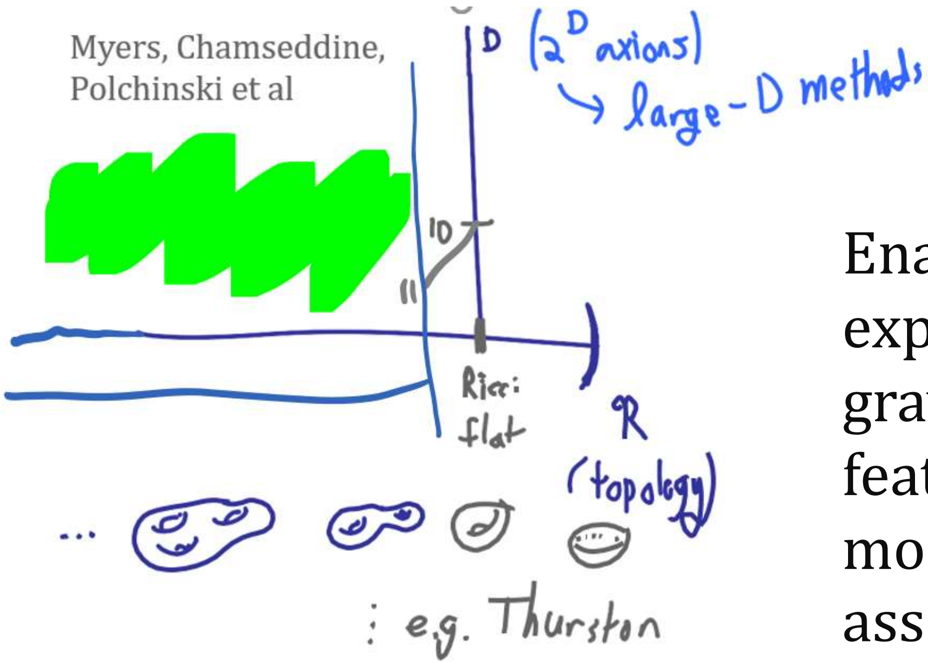
arXiv:2104.13380v1 [hep-th] 27 Apr 2021

(+ Scipost)

Simple motivation: Schematically $V_{4d} \sim \int \frac{(D-D_c)}{\alpha'} - R_{internal} + \cdots \Rightarrow D > D_c$ and $R < 0$ give leading positive potential energy & rigidity, and are generic in string theory (also dual to each other and connected to other limits)

Hellerman-Swanson, Green Lawrence McGreevy
Morrison Adams Saltman ES

Cordova De Luca Dodelson Dong Horn Maloney Saltman
ES Strominger Tomasiello Torroba ...



Myers, Chamseddine,
Polchinski et al

Enables simpler setups for more explicit analysis of dS quantum gravity (e.g. entropy counts, essential features of holographic dual) as well as more generic mathematical structure and associated phenomenology.

11d SUGRA (M-theory): No dilaton. Geometry and 6-form potential field C_6.
Compactify on hyperbolic space H_7/Γ with 7-form flux ⇒ Casimir energy *(inhomogeneous)*

Mostow rigidity for H_n/Γ *n>2, no flat directions unlike Riemann surface, Calabi-Yau which have moduli spaces.*

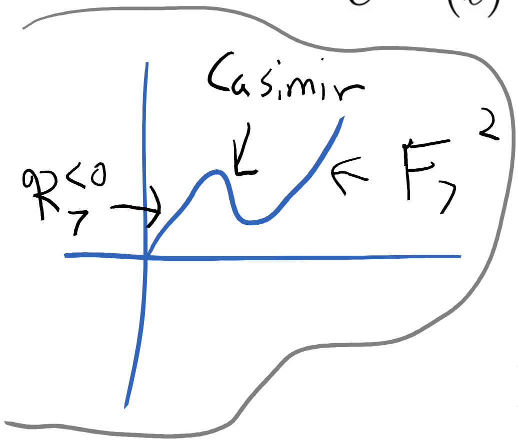
At the *homogeneous* level, 4d effective potential for the volume (equivalently curvature radius) from dimensionally reducing the 11d SUGRA Lagrangian:

$$U^{(0)}(\ell) \sim M_4^4 \frac{1}{v_7 \hat{\ell}^7}\left(\frac{1}{2}\frac{42}{\hat{\ell}^2} - \frac{\int_{H_7/\Gamma}\sqrt{-g}\,\rho_{Casimir}\,\ell_{11}^4}{v_7 \hat{\ell}^7} + \frac{N_7^2}{v_7^2 \hat{\ell}^{14}}\right) \quad + \text{warping}$$

$$= M_4^4 \frac{1}{v_7 \hat{\ell}^7}\left(\frac{21}{\hat{\ell}^2} - \frac{K}{\hat{\ell}^{11}} + \frac{N_7^2}{v_7^2 \hat{\ell}^{14}}\right), \qquad M_4^2 \sim v_7 \ell^7/\ell_{11}^9 = v_7 \frac{\hat{\ell}^7}{\ell_{11}^2}$$

$$\hat{\ell} \equiv \ell/\ell_{11}$$

+ warping



Einstein-Hilbert action: positive potential for $R^{(7)} < 0$

Casimir stress-energy:

Negative curvature (Kaloper et al, Saltman ES, Dong Horn ES Torroba, Cordova De Luca, Tomasiello, , …)
as well as Casimir energy Arkani-Hamed Dubovsky et al (Standard Model), Maldacena et al (SM wormholes)…
have been useful in stabilizing various solutions previously.

$$T^\mu_\nu = -\rho(R)\,\delta^\mu_\nu \quad , \quad T^y_y = -\rho(R) \quad , \quad T^a_b = -\left(\rho(R) + \frac{1}{n-1} R\rho'(R)\right)\delta^a_b$$

$$\rho(R) = -\frac{\rho_c}{R^{d+n}}$$

e.g. n small cycles of size R,
slowly varying

e.g. for bosonic field component

$$\begin{aligned}
\langle g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \rangle &= -d\rho(R) \\
\langle g^{yy}\partial_y\phi\partial_y\phi \rangle &= -\rho(R) \\
\langle g^{ab}\partial_a\phi\partial_b\phi \rangle &= (d+1)\rho(R)
\end{aligned}$$

Casimir term dominated by small circles in the geometry

e.g. filled hyperbolic cusps or other constructions with short systoles. I. Agol et al

$$ds^2_{local} = dy^2 + e^{-2y/\ell} ds^2_{T^6} \qquad y_0 < y < y_c$$

To end the cusp, incorporate Anderson's analogue of Dehn filling, one cycle smoothly contracts.

$$\frac{\int \sqrt{-g}\rho_{Casimir}\ell_{11}^4}{v_7\hat{\ell}^7} \sim \pm n_c \frac{v_T\hat{\ell}^6}{v_7\hat{\ell}^7}\int_{\hat{y}_0}^{\hat{y}_c} \frac{d\hat{y}}{\hat{\lambda}_c^{11}} e^{-6\hat{y}/\hat{\ell}} e^{11\hat{y}/\hat{\ell}} \simeq \pm \frac{v_T n_c}{5v_7\hat{\lambda}_c^{11}} e^{5y_c/\ell} \sim \pm \frac{1}{\hat{\ell}^{11}}\left(\frac{\hat{\ell}}{\hat{\ell}_c}\right)^5 \frac{Vol(T^6)}{\lambda_c^6}\frac{n_c}{v_7}$$

4d effective potential

Douglas ‘09

$$V_{eff}[g^{(7)}, C_6] = \frac{\ell_{11}^9}{2G_N^2} \frac{\int d^7y\,\sqrt{g^{(7)}}u^2|_c \left(\overbrace{[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2\Big|_c]}^{\text{net curvature term}} - \frac{1}{4}\ell_{11}^9 \overset{\ell_{11}^9\rho_c(R_c)\sim -\frac{\ell_{11}^9}{R_c^{11}}}{\overset{\uparrow}{T^{(Cas)\mu}_{\ \ \mu}}} + \frac{1}{2}|F_7|^2 \right)}{(\int d^7y\sqrt{g^{(7)}}u|_c)^2}$$

$$ds^2 = e^{2A(y)}ds^2_{dS_4} + e^{2B(y)}(g_{\mathbb{H}ij} + h_{ij})dy^i dy^j \qquad u(y) = e^{2A(y)}$$

u(y) satisfies GR constraint (its equation of motion):

$$\left(-\nabla^2 - \frac{1}{3}\left(-R^{(7)} - \frac{1}{4}\ell_{11}^9 T^{(\mathrm{Cas})\mu}_{\ \ \mu} + \frac{1}{2}|F_7|^2\right)\right)u = -\frac{C}{6}$$

Like a Schrodinger problem for

$$C\ell^2 \sim H^2\ell^2 \ll 1$$

$$\Rightarrow \quad V_{eff} = \frac{C}{4G_N} = \frac{R^{(4)}_{\mathrm{symm}}}{4G_N}\,.$$

Quantized flux solution:

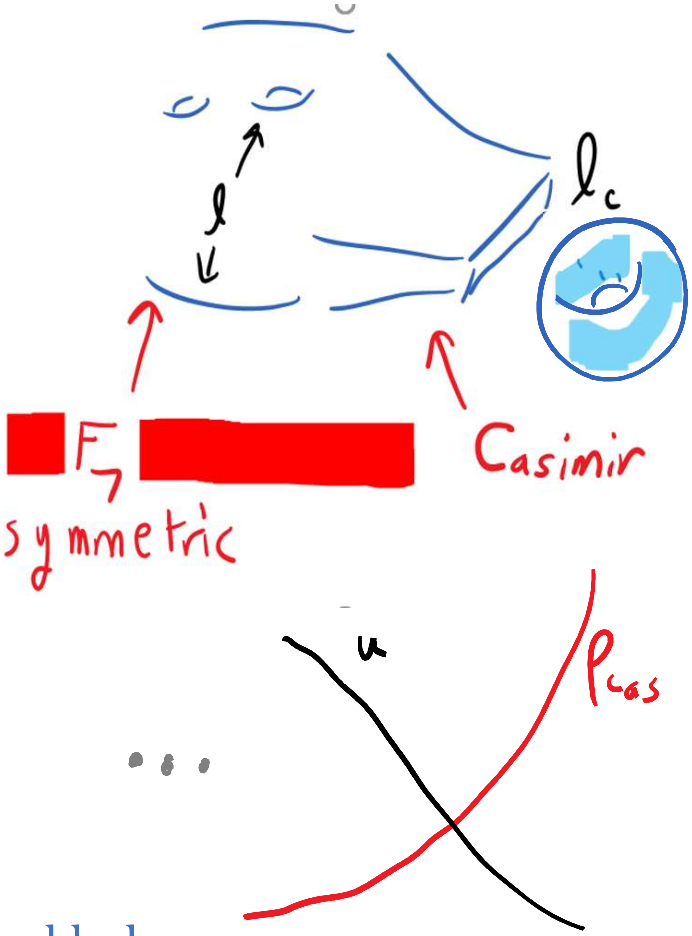
$$
\begin{aligned}
0 &= \partial_{j_1}\left(\sqrt{g^{(7)}}u^2 g^{(7)j_1 i_1}\dots g^{(7)j_7 i_7}F_{i_1,\dots,i_7}\right) \\
\bar{F}^{(7)}_{i_1,\dots,i_7} &= f_0\frac{\sqrt{g^{(7)}}}{u^2}\epsilon_{i_1,\dots,i_7}\,.
\end{aligned}
$$

$$
\frac{(2\pi)^{1/3}}{\ell_{11}^6}\int_{\Sigma_7}\bar{F}^{(7)} = 2\pi N_7 \Rightarrow f_0 \sim \ell_{11}^6\frac{N_7}{\int_{\Sigma_7}\frac{\sqrt{g^{(7)}}}{u^2}}
$$

i.e. $F_7^2 \sim \frac{N_7^2}{vol_7^2}$ + warping effects

Effective Schrödinger problem Douglas '09 :

$$-\frac{1}{6}C=\left(-\nabla^2+\frac{2}{3}U\right)u; \qquad U=\frac{1}{2}R-\frac{1}{4}\sum_p |F^{(p)}|^2+\frac{1}{4}T^{(d)}_{string}$$



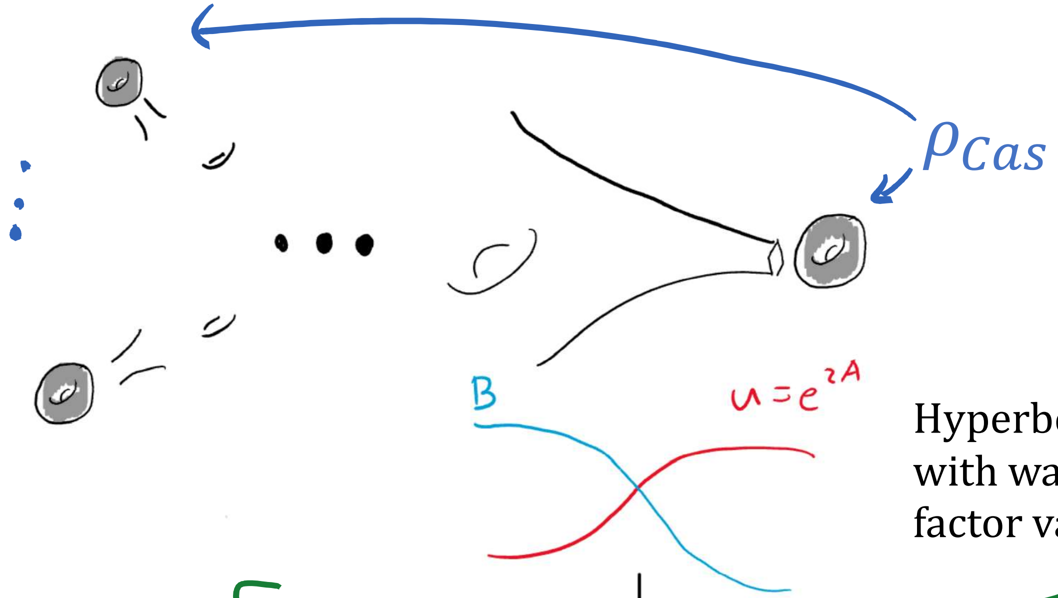
`Wavefunction' (warp factor) u is supported where the contributions to the naïve 4d potential -U are positive: u redshifts away runaway instabilities like the conformal factor.

$$g_{mn}\to e^{2B}g_{nm}$$

$$V_{\{R,naive\}}\propto -\int\sqrt{g}R\to -\int\sqrt{g}\,(e^{5B}R+30(\nabla B)^2)$$

Douglas '09 conjecture: the properly defined V_{eff} is bounded below.

cf Yamabe problem, see also Hertog Horowitz Maeda



Hyperbolic manifold dressed with warp and conformal factor variations

Tune small to compete with Casimir with $\ell_{11} \ll R_c \ll \ell$ →

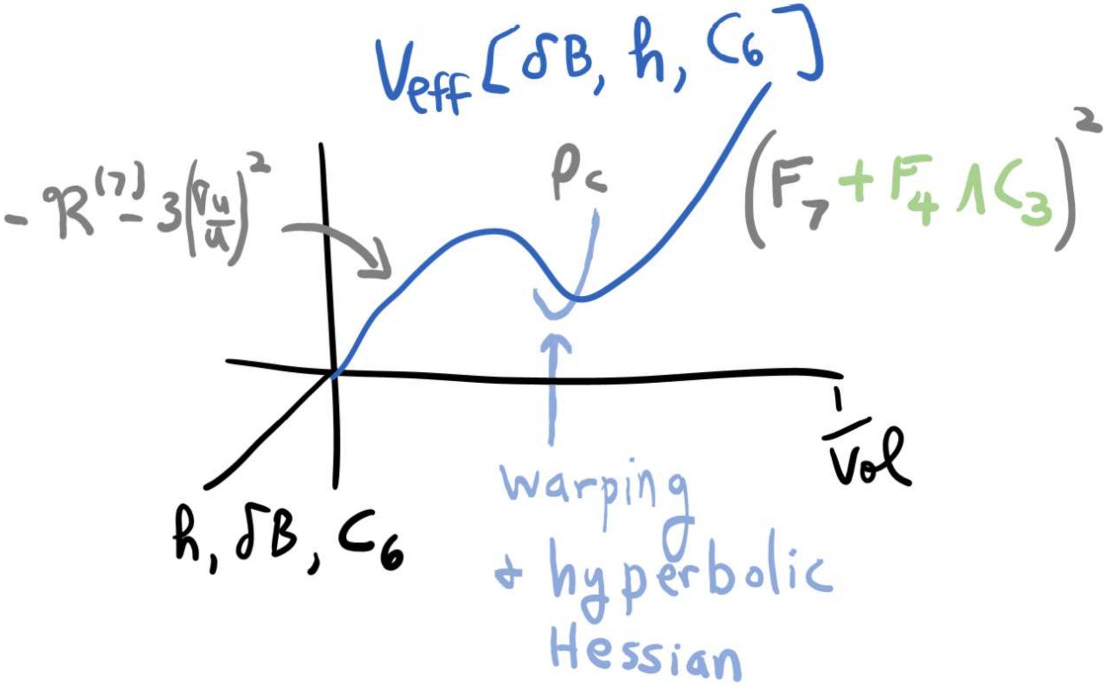
$$\left[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 < 0 \;\middle|\; -R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 > 0 \right]$$

warp & conformal factor eoms ⇒

$$-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 = 4\ell_{11}^9|\rho_C| - \cancel{\frac{C}{u}} - \frac{5}{2}F_7^2$$

Douglas
Kallosh '10

$$ds^2 = e^{2A(y)} ds^2_{dS_4} + e^{2B(y)}(g_{\mathbb{H}ij} + h_{ij})dy^i dy^j \qquad u(y) = e^{2A(y)}$$



$$a = \frac{\int \sqrt{g^{(7)}}u^2|_c[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2\Big|_c]}{\int \sqrt{g^{(7)}}u^2|_c\, 42/\ell^2} \quad << 1 \ :$$

$$-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 = 4\ell_{11}^9|\rho_C| - \frac{C}{u} - \frac{5}{2}F_7^2$$

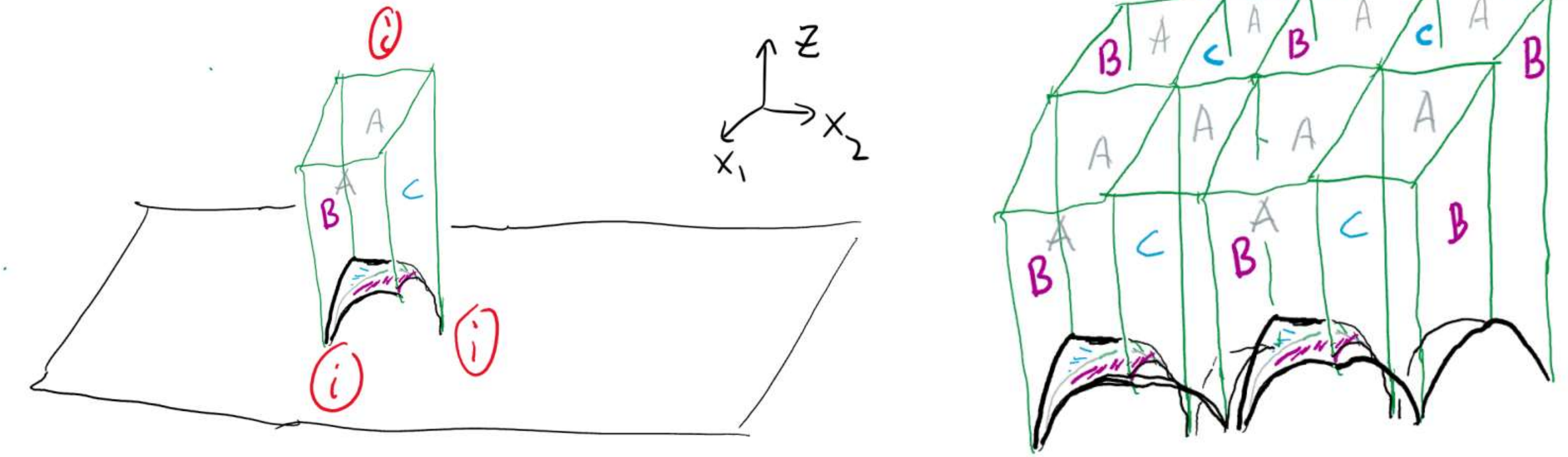
- If *a* is too large, increase volume of non-Casimir regions (e.g. via short filled cusps or covers k-fold -> (k+1)-fold)

- If *a* is too small, reduce flux quantum number

Work with concrete hyperbolic manifolds with comparable cusp and bulk volumes. Explicit radial solution below illustrates $a << 1$.

Finite volume hyperbolic space H_7/Γ constructed from gluing polytopes

Ratcliffe text; e.g. e.g. right angled polytopes Italiano et al arXiv:2010.10200[math.GT].



Gluing prescription ensures nonsingular finite volume manifold. Resulting cusp volume of order Vol(H_7/Γ), ensuring comparable contributions (positive and negative) to a.

Balance Terms in U =>

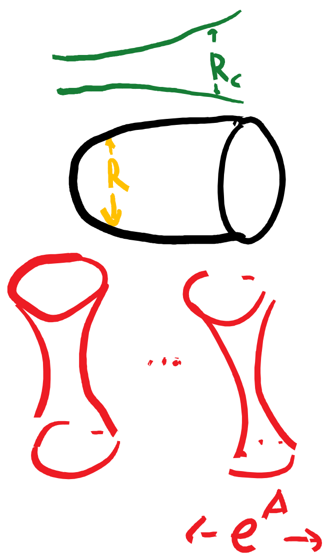
$$\hat{\ell}^4 \sim \frac{1}{\hat{\ell}_c^5} \frac{n_c}{v_7} \frac{Vol(T^6)}{\lambda_c^6} \cdot \frac{1}{a}$$

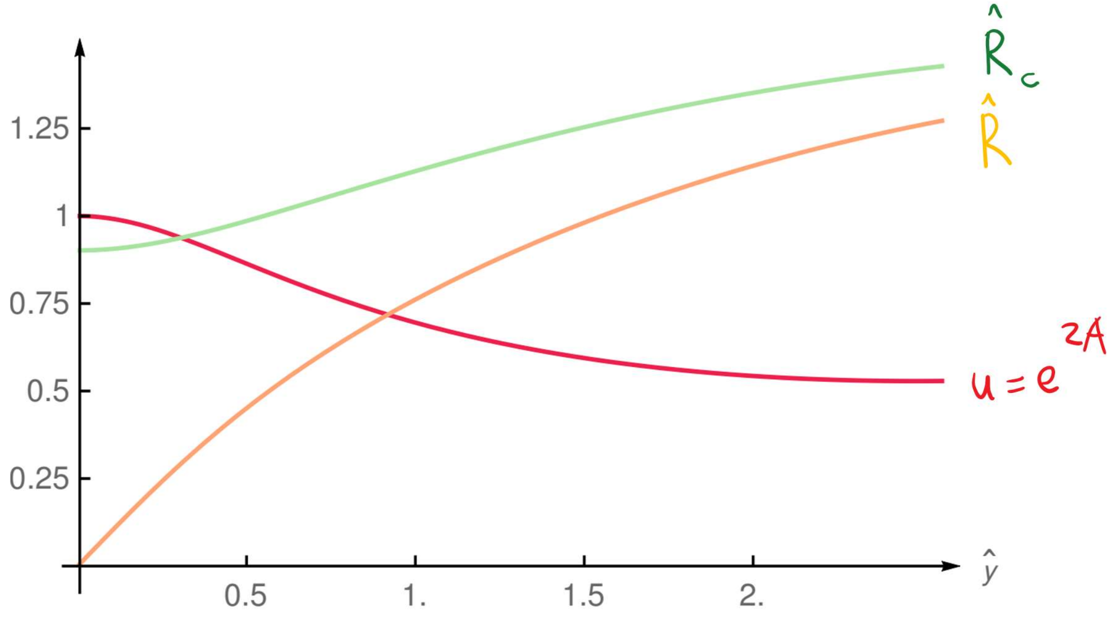
If *a* sufficiently small, then all length scales large:

$$\ell \gg \ell_c \gg \ell_{11}$$

a here is analogous to D>>Dcrit in supercritical compactification to dS '01, W_0 in KKLT '03, topological quantum numbers in large volume scenario and Riemann surface compactifications, etc.

Radial (ODE) solution illustrating background (rescaled in fig) and 0< a < 1

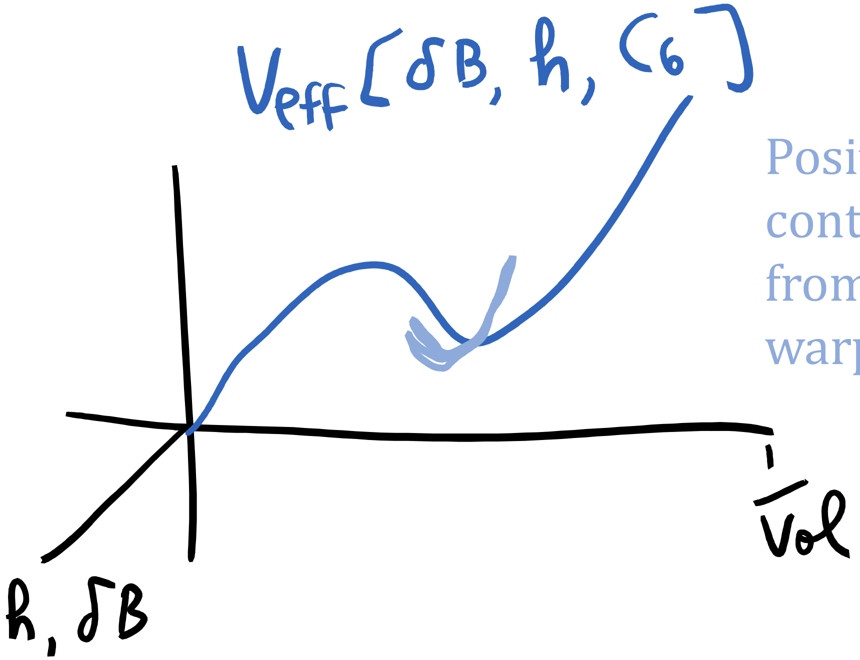




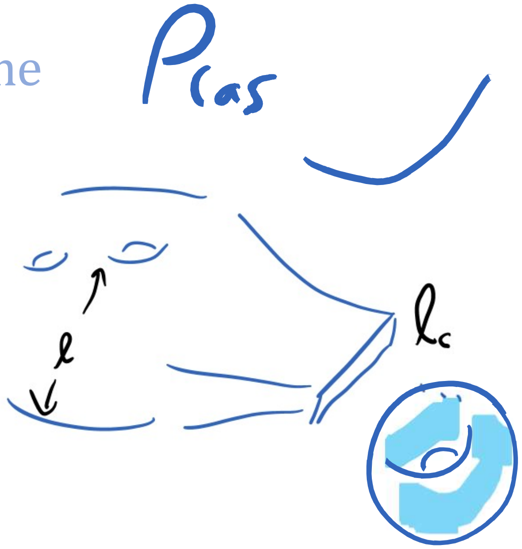
$$\frac{\int \sqrt{g^{(7)}}u^2\left[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 + \frac{1}{2}|F_7|^2\right]}{-\int \sqrt{g^{(7)}}u^2\,\ell_{11}^9\,R_c^{-11}} \sim -1.06$$

Positive and negative sources compete at large radius. Joining to full manifold requires PDEs: gluing of hyperbolic polytopes.

Directions transverse to volume: small tadpoles away from dressed background.



Positive Hessian contributions come from rigidity and warping effects



$$ds^2 = e^{2A(y)} ds^2_{dS_4} + e^{2B(y)} (g_{\mathbb{H}ij} + h_{ij}) dy^i dy^j$$

Summary:

- dS_3 observer patch geometry and microstates captured precisely by the solvable and universal $T\bar{T} + \Lambda_2$ deformation; other subleading effects go beyond pure gravity

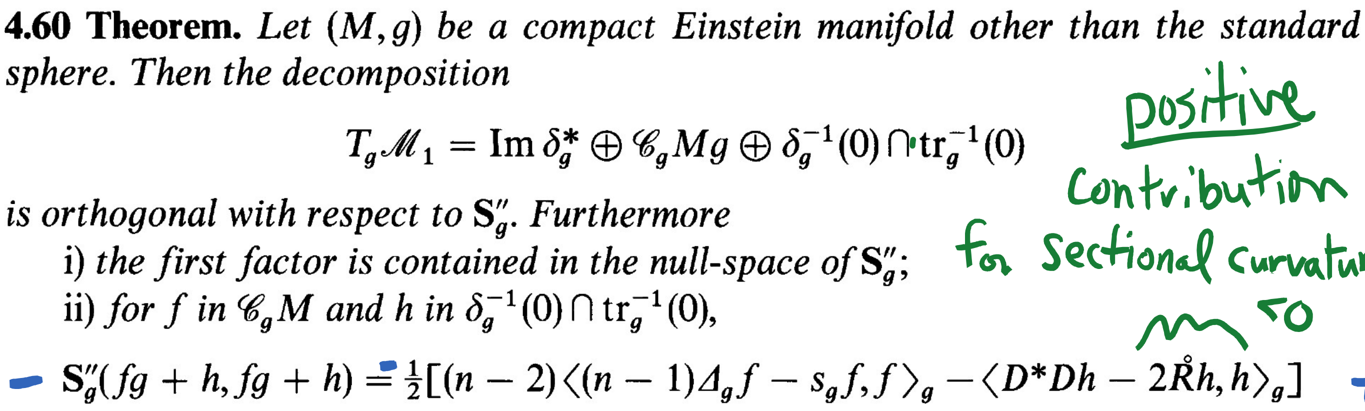
↕ relation? $\underline{\text{both}}$ AdS/CFT → dS

- dS_4 from uplift of M2-brane theory

Many near-future directions to pursue on both fronts…

Extra Slides

Hessian: two generally positive contributions. First, rigidity



4.60 Theorem. *Let (M, g) be a compact Einstein manifold other than the standard sphere. Then the decomposition*

$$T_g\mathscr{M}_1 = \mathrm{Im}\,\delta_g^* \oplus \mathscr{C}_g M g \oplus \delta_g^{-1}(0) \cap \mathrm{tr}_g^{-1}(0)$$

is orthogonal with respect to $\mathbf{S}''_g$. Furthermore

i) *the first factor is contained in the null-space of $\mathbf{S}''_g$;*

ii) *for f in $\mathscr{C}_g M$ and h in $\delta_g^{-1}(0) \cap \mathrm{tr}_g^{-1}(0)$,*

$$V''_{eff} \propto -\mathbf{S}''_g(fg+h, fg+h) = \tfrac{1}{2}[(n-2)\langle (n-1)\Delta_g f - s_g f, f\rangle_g - \langle D^*Dh - 2\mathring{R}h, h\rangle_g] + \text{Flux, Cas}$$

positive contribution for sectional curvatures > 0

Besse, *Einstein Manifolds*
+
Douglas '09:

Conformal mode would be negative but with warping effects, it turns positive:

$$V_{eff} = \int -\sqrt{g} e^{dA+(k-2)B}\left(R^{(k)} - 2(k-1)\nabla^2 B - (k-2)(k-1)(\nabla B)^2 + d(d-1)(\nabla A)^2\right)$$
$$= -\int \sqrt{g} e^{dA+(k-2)B}\left(R^{(k)} + 2d(k-1)\nabla A \nabla B + (k-2)(k-1)(\nabla B)^2 + d(d-1)(\nabla A)^2\right)$$

Warping contribution to Hessian positive for small c.c.:

Schrodinger problem:
u `wavefunction’

$$2\ell_{11}^9\, V_{eff} = -u^I \mathcal{H}_{IJ} u^J = -\langle u|\mathcal{H}|u\rangle$$

$$\mathcal{H} = \sqrt{g^{(7)}}\left(R^{(7)} - \ell_{11}^9 \rho_C - \frac{1}{2}F_7^2 - 3\nabla^2\right) = 3\sqrt{g^{(7)}}\,\hat{H}/\ell^2$$

In a solution:

$$2\ell_{11}^9\,\frac{\delta^2 V_{eff}}{\delta\gamma^2} = -\langle u|(\partial_\gamma^2\mathcal{H})|u\rangle - 2\langle\partial_\gamma u|\mathcal{H}|\partial_\gamma u\rangle - 4\langle\partial_\gamma u|(\partial_\gamma\mathcal{H})|u\rangle$$

Small c.c.: $\mathcal{H}|u_0\rangle \approx 0$, $\langle u|(\partial_\gamma\mathcal{H})|u\rangle = 0$ at fixed $u = u_0$ $\qquad \frac{\delta}{\delta\gamma}(\mathcal{H}|u\rangle) \simeq 0 \;\Rightarrow\; \mathcal{H}|\partial_\gamma u\rangle \simeq -(\partial_\gamma\mathcal{H})|u\rangle$

$$2\ell_{11}^9\,\frac{\delta^2 V_{eff}}{\delta\gamma^2} \approx \left\langle u_0\left|\left(-\partial_\gamma^2\mathcal{H} + 2\partial_\gamma\mathcal{H}\,\overbrace{\left(\sum_{i\neq 0}\frac{1}{\lambda_i}|u_i\rangle\langle u_i|\right)}^{\mathcal{H}^{-1}}\partial_\gamma\mathcal{H}\right)\right|u_0\right\rangle$$

$$= \langle u_0\,|(-\partial_\gamma^2\mathcal{H})|\,u_0\rangle + \boxed{\sum_{i\neq 0}\frac{1}{\lambda_i}|\langle u_0|\partial_\gamma\mathcal{H}|u_i\rangle|^2\,.}$$

automathically
positive

$$\langle u_j | \left\{ \Delta H \, |u^{(0)}\rangle + H^{(0)} \, |\Delta u\rangle \right\}$$

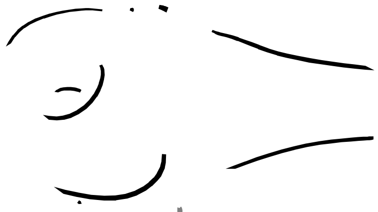
$$= \Delta H_{j0} + \langle u_j | H^{(0)} \left(\sum_{i \neq 0} \frac{\Delta H_{i0}}{-(\lambda_i - \lambda_0)} |u_i\rangle \right)$$

$$= \Delta H_{j0} + \langle u_j | \sum_{i \neq 0} \frac{\Delta H_{i0} \, \lambda_i}{-(\lambda_i - \underbrace{\lambda_0}_{\simeq 0})} |u_i\rangle$$

$$\simeq \Delta H_{j0} - \Delta H_{j0} \simeq 0 \quad \checkmark$$

Trial `wavefunction' analysis

$$\langle u_t|(-\hat{H})|u_t\rangle \leq \langle u_0|_c|(-\hat{H})|u_0|_c\rangle \simeq 2\ell_{11}^9\ell^2\, V_{eff}$$



$$A_{t0} = -2B$$

$\delta V_{eff} > 0$

$$A_t = A_{t*}e^{(y-y_*)/\ell} = -2B(y_*)e^{(y-y_*)/\ell}$$

$u \sim e^{2A}$ exp suppressed $y >> y_*$

Crude estimates suggest overall positive Hessian, as suggested by positive rigidity and warping contributions and some tests in radial solution.

(Not a complete calculation of the Hessian.)

General tadpole estimate: $\partial_I \partial_J V_{eff} \Delta\sigma_J = \partial_I V_{eff} \Rightarrow \Delta\sigma_J = \mathcal{H}^{-1} \partial V_{eff}$

$$V_{eff} = V_{eff,\mathbb{H}} + \frac{1}{\ell_{11}^9} \int d^7 y \sqrt{g_{\mathbb{H}}} \left\{ \frac{1}{2} \tilde{h}_n^m \Delta_{(Total)}{}_{mp}^{nq} \tilde{h}_q^p - \ell_{11}^9 \tilde{h}_m^n t_C{}_n^m \right\} + \mathcal{O}(\tilde{h}^3) \,.$$

$$t_C = \sum_I \tau_I \varphi_I, \qquad \Delta_{(Total)} \varphi_I = \lambda_I \varphi_I$$

$\tilde{h} \sim \frac{\ell_{11}^9 \tau_0}{\lambda_0} \varphi_0 \sim \frac{\ell_{11}^9 \ell^2}{R_c^{11}} \varphi_0$ $\ll$ 1 in our stabilization mechanism

Warp and conformal factor equations of motion

1u $$-\frac{\nabla_v^2 u}{u} + \frac{R_v^{(7)}}{3} - \frac{1}{6}\frac{\tilde{\alpha}_7}{u^4} + \frac{1}{3}\frac{|\rho_C(y)|}{v^{22/5}} = -\frac{C}{6u}$$ ← constraint equation

1v $$\frac{\delta V_{eff}}{\delta v} = u^2 v^{1+4/5}\left(-R_v^{(7)} + \frac{9}{5}(\frac{\nabla_v u}{u})^2 - \frac{7}{10}\frac{\tilde{\alpha}_7}{u^4} + \frac{24}{5}\frac{\nabla_v^2 u}{u} - \frac{7}{5}\frac{C}{u} + \frac{4}{5}\frac{|\rho_C(y)|}{v^{22/5}}\right) = 0$$

$$\tilde{\alpha}_7 \propto \frac{1}{\hat{\ell}^{14}} \times [\frac{N_7^2}{(\int_{\mathbb{H}_7/\Gamma}\frac{\sqrt{g_{\mathbb{H}_7}}e^{7B}}{u^2})^2}]$$

2 $$R_v^{(7)} - \frac{5}{2}\frac{\tilde{\alpha}_7}{u^4} + 3(\frac{\nabla_v u}{u})^2 + 4\frac{|\rho_C(y)|}{v^{22/5}} = \frac{C}{u} \qquad (1u)+(1v)$$

3a $$V_{eff} = \frac{1}{2}\int\sqrt{g_v}u^2\left[-R_v^{(7)} - \frac{|\rho_C(y)|}{v^{22/5}} + \frac{1}{2}\frac{\tilde{\alpha}_7}{u^4} - 3(\frac{\nabla_v u}{u})^2\right]$$

$$= \int\sqrt{g_v}u^2\left[-2\frac{\tilde{\alpha}_7}{u^4} - \frac{C}{u} + 3\frac{|\rho_C(y)|}{v^{22/5}}\right] \qquad (2)+(3a)$$

$$= \frac{C}{4}\int\sqrt{g_v}u[v,y] \qquad (1u)+V_{eff}+1/G_N$$

$$\begin{aligned} ds^2 &= e^{2A}ds^2_{dS_4} + e^{2B}ds^2_{\mathbb{H}_k/\Gamma}, \qquad ds_v^2 = e^{2B}ds^2_{\mathbb{H}_k/\Gamma} = v^{\frac{4}{k-2}}ds^2_{\mathbb{H}_k/\Gamma} \\ u &= e^{2A}, \qquad v = e^{\frac{k-2}{2}B} = e^{\frac{5}{2}B} \\ e^{2B} &= v^{\frac{4}{k-2}} = v^{\frac{4}{5}} \\ R_v^{(k)} &= v^{-\frac{4}{k-2}}(R_{\mathbb{H}}^{(k)} - \frac{4(k-1)}{k-2}\frac{\nabla^2_{\mathbb{H}}v}{v}) \Rightarrow R_v^{(7)} = v^{-4/5}(R_{\mathbb{H}}^{(k)} - \frac{24}{5}\frac{\nabla^2_{\mathbb{H}}v}{v}) \\ \nabla_v^2 f &= \frac{1}{\sqrt{g_v}}\partial_a(\sqrt{g_v}v^{-\frac{4}{k-2}}\partial^a f) = \frac{1}{\sqrt{g_v}}\partial_a(v^2\partial^a f) = v^{-\frac{4}{k-2}}(\nabla^2_{\mathbb{H}}f + 2\frac{\nabla_{\mathbb{H}}v}{v}\nabla_{\mathbb{H}}f) \end{aligned}$$

→ General **No Go** for **AdS** extremum ☺ in range:

Cf Gibbons, Maldacena-Nunez, Douglas…

For $\int 3u^2|\rho_c|v^{-22/5} > \int 2u^2 F_7^2$

$C < 0$ is incompatible with last 2 equations

With $F_4 \neq 0$: a new realization of axion monodromy inflation

$$c^I = \int_{\Sigma_I^{(3)}} \frac{C_3}{\ell_{11}^3}, \quad I = 1, \ldots, b_3 \qquad\qquad \tilde{F}_7 = F_7 + C_3 \wedge F_4$$

$$\tilde{V}_7 = \frac{\ell_{11}^9}{2G_N^2} \frac{\int d^7y \sqrt{g^{(7)}} u^2|_c \left(\frac{1}{2}|\tilde{F}_7|^2\right)}{\left(\int d^7y \sqrt{g^{(7)}} u|_c\right)^2} \sim M_P^4 \frac{(N_7 + cN_4)^2}{\hat{\ell}^{21}} + \cdots$$