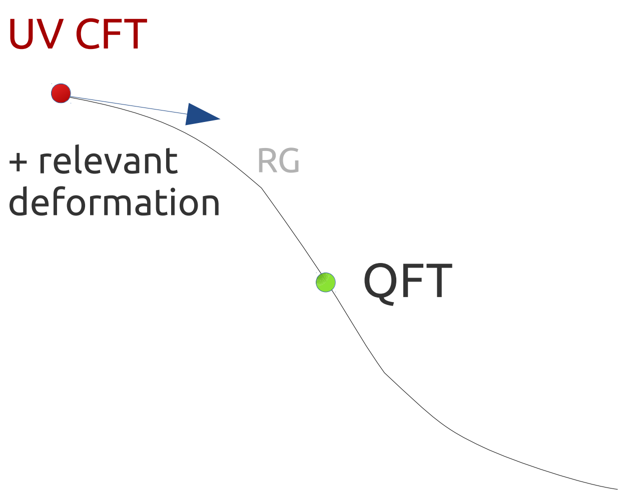
Irrelevant current-current deformations and holography

Monica Guica

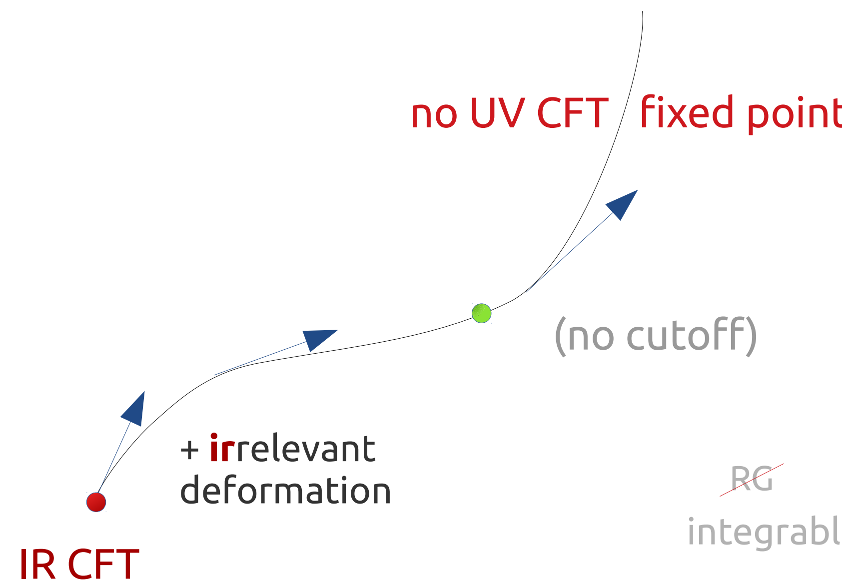
IphT, CEA Saclay

Motivation

Usual framework: local, UV complete QFTs

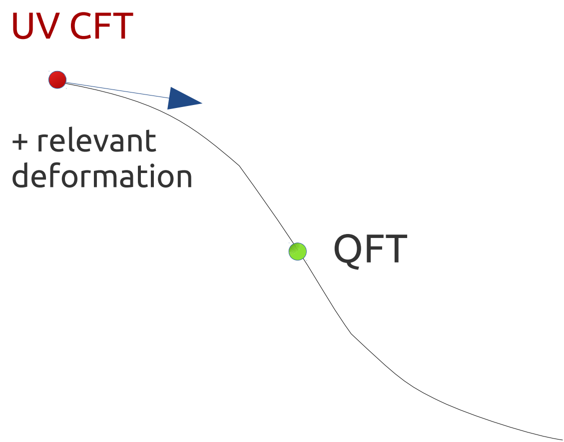


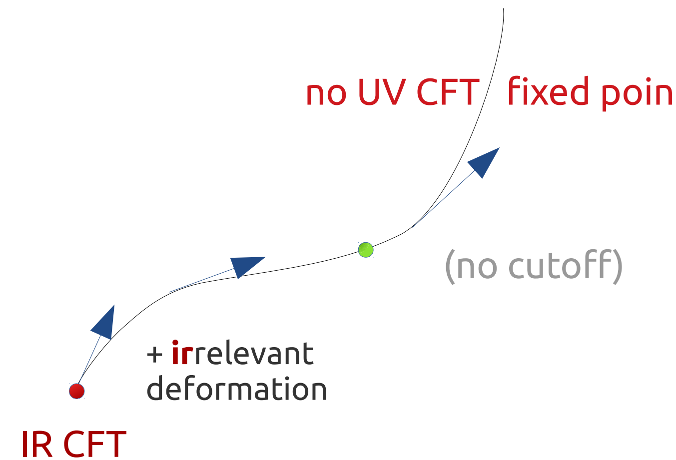
This talk: non-local, UV complete QFTs



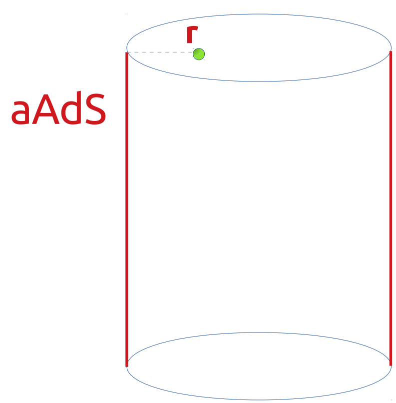
- novel UV behaviour in QFT relevant to e.g. QCD flux tubes Dubovsky et al.
- holography beyond AdS → near-horizon dynamics of generic black holes

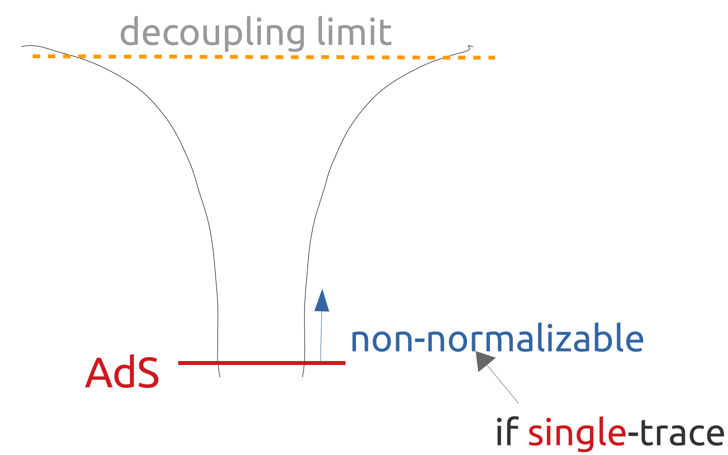
Motivation





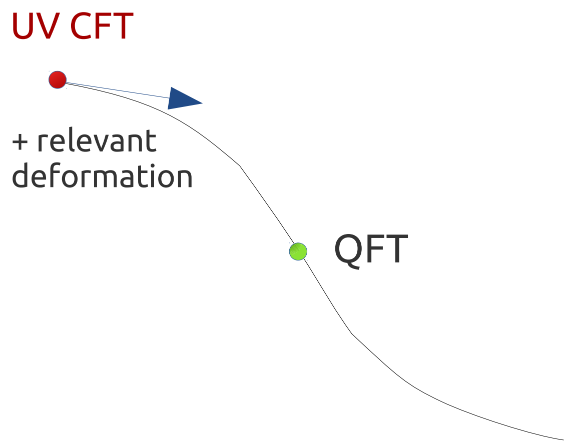
Holography

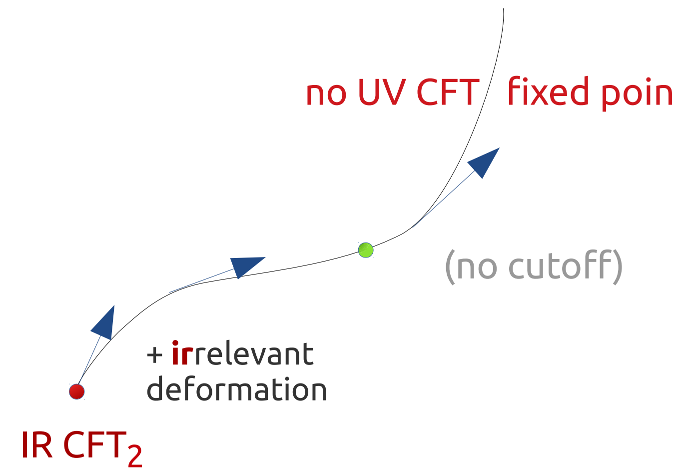




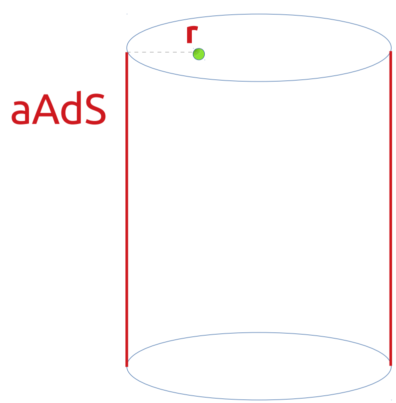
- e.g. dipole-deformed N=4 SYM

Motivation



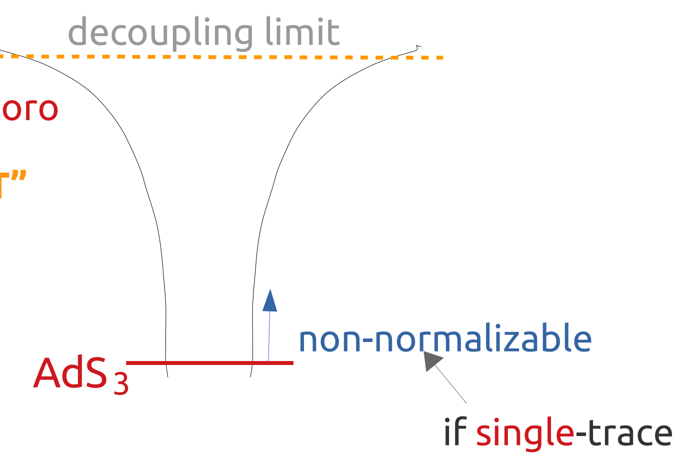


Holography



ASG : Virasoro x Virasoro

"non-local CFT"



Plan

- Smirnov-Zamolodchikov deformations ($T\bar{T}$ & $J\bar{T}$) and their properties

- holographic interpretation of $T\bar{T}$ & $J\bar{T}$ – deformed CFTs (double-trace)

  → mixed boundary conditions for dual AdS_3 metric

  → concrete field theory realisation of alternate boundary conditions in AdS_3 (test ASG)

- single-trace $T\bar{T}$ and $J\bar{T}$ – deformed CFTs and their holographic interpretation

  → non-AdS holography

- infinite symmetries of $T\bar{T}$ and $J\bar{T}$ – deformed CFTs

Smirnov-Zamolodchikov deformations

- irrelevant deformations of 2d QFTs → bilinears of two (higher spin) conserved currents $J^A,\ J^B$
- define $\mathcal{O}_{J^A J^B}$:

$$\boxed{\lim_{y\to x} \epsilon^{\alpha\beta} J^A_\alpha(x) J^B_\beta(y) = \mathcal{O}_{J^A J^B} +} \text{ derivative terms}$$

Zamolodchikov '04

SZ '16

nice factorization properties

- deformation :

$$\boxed{\frac{\partial S(\mu)}{\partial \mu} = \int d^2x\, \mathcal{O}_{J^A J^B}(\mu)}$$

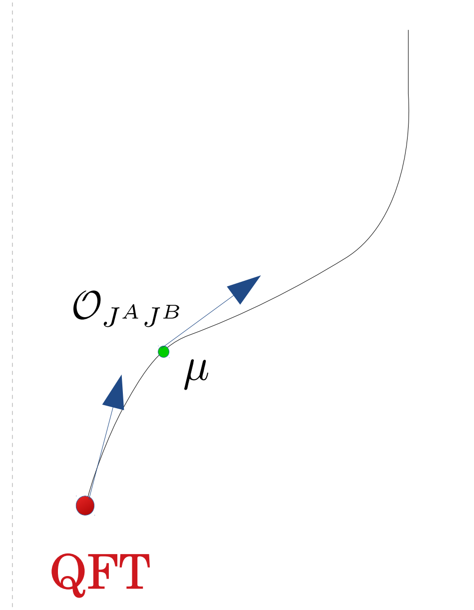
- examples:

universal

$$\begin{cases} T\bar{T}: \quad J^A_\alpha = T_\alpha{}^A \,, \quad J^B_\beta = T_\beta{}^B \quad (\times \epsilon_{AB}) \\ J\bar{T}: \quad J^A_\alpha = J_\alpha \,, \quad J^B_\beta = T_{\beta\bar{z}} \end{cases}$$

~~Lorentz~~

- highly tractable : exact finite -size spectrum, S-matrix, preserves integrability
- deformed theory non-local (scale $\mu^{\#}$) but argued UV complete

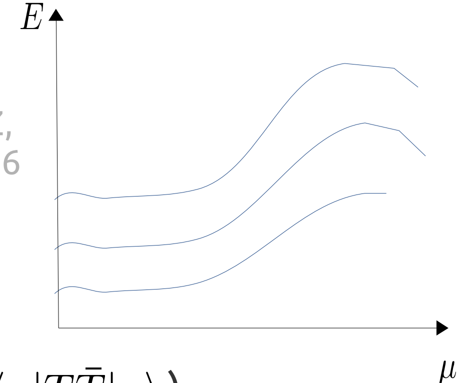


Sample results in $T\bar{T}$

- universal deformation of 2d QFTs $$\frac{\partial S}{\partial \mu} = \int d^2 z \, \underbrace{(T_{zz}T_{\bar{z}\bar{z}} - T_{z\bar{z}}^2)}_{\text{“}T\bar{T}\text{”}}{}_{\mu}$$

$$[\mu] = (length)^2$$

SZ, Cavaglia et al. '16



- in compact space (R) → energy levels continuously deformed

- deformed energies $E_\mu(R)$ determined solely by initial spectrum ($\partial_\mu E_n = \langle n|T\bar{T}|n\rangle$)

e.g. seed CFT

$$\boxed{E_\mu(R) = \frac{R}{2\mu}\left(\sqrt{1 + \frac{4\mu E_0}{R} + \frac{4\mu^2 P^2}{R^2}} - 1\right)}$$

$\mu > 0$: ground state energy $E_0 = -\dfrac{c}{12R}$ becomes complex for $R < R_{min} = \#\sqrt{\mu c}$

- Hagedorn behaviour $S \propto E$ at high energy

$$\boxed{S(E) = \sqrt{\frac{2\pi c E_0 R}{3}} = \sqrt{\frac{2\pi c(ER + \mu E^2)}{3}}}$$

$$T_H = R_{min}^{-1}$$

$\mu < 0$: all states with $E_0 > \dfrac{R}{4|\mu|}$ acquire imaginary energies → no sense in compact space (CTC)

Cooper, Dubovsky, Moshen

not a UV cutoff → mixed UV-IR effect

Sample results in $T\bar{T}$

- S-matrix: $\mathcal{S}_\mu = e^{i\mu \sum_{i,j} \epsilon_{\alpha\beta} p_i^\alpha p_j^\beta} \mathcal{S}_0$ related to deformed spectrum via TBA Dubovsky et al.

  Tateo et al., SZ

- $n\, T\bar{T}$ - deformed free bosons $=_{cls}$ Nambu-Goto action for string in $n+2$ - dim'l target space in static gauge

  $X^0 = t$

  $X^1 = \sigma$

  → $n=1$ $(\partial\phi)^2$ → $\frac{1}{\mu}(\sqrt{1+\mu(\partial\phi)^2} - 1)$

  → $T\bar{T}$ deformation = change of gauge in the NG action (conformal → static)

  → deformed and undeformed theories related by a field-dependent coordinate transformation

  Dubovsky et al.

- non-perturbative definition of the $T\bar{T}$ deformation in terms of coupling QFT to topological (JT) gravity

- does $T\bar{T}$ yield a theory of 2d quantum gravity ?

  off-shell observables

  - ✓ rel. to worldsheet of the bosonic string
  - ✓ $\mathcal{S} = e^{i\mu s}$ gravitational → minimum length

  Dubovsky et al.

  - ✗ flow eqn for correlation functions Cardy
  - ✗ Virasoro symmetry

  ≈ non-local QFT (CFT)

Sample results in JT̄ - deformed CFTs

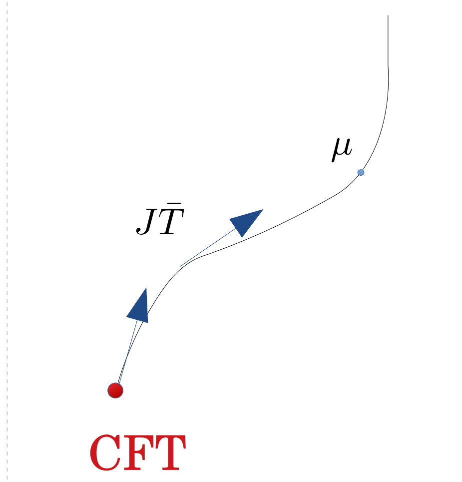
- universal deformation of 2d QFTs/CFTs with a $U(1)$ current MG '17

$$\frac{\partial S_{J\bar{T}}}{\partial \lambda} = \int d^2 z \, (J_z T_{\bar{z}\bar{z}} - J_{\bar{z}} T_{z\bar{z}})_{\lambda}$$

"$J\bar{T}$" $(1,2)$

$[\lambda] = length$

$\lambda^\mu \partial_\mu \propto \partial_{\bar{z}}$



- breaks Lorentz invariance $T_{z\bar{z}} \neq T_{\bar{z}z} (= 0)$

- preserves $\underbrace{SL(2,\mathbb{R})_L}_{\text{local \& conformal}} \times \underbrace{U(1)_R}_{\text{non-local!}}$ ← simpler than TT̄

$\boxed{CFT_1!}$

Im!

$$E_{L,R} = \frac{1}{2}(E \pm P)$$

- finite-size spectrum $E_R = \frac{4\pi}{\lambda^2 k}\left(R - \lambda Q_0 + \sqrt{(R-\lambda Q_0)^2 - \lambda^2 k R \, E_R^{(0)}}\right)$

$$Q(\lambda) = Q_0 + \frac{\lambda k}{2} E_R$$

Giveon et al. '18

- off-shell observables: correlation functions of operators in mixed basis $\mathcal{O}(z,\bar{p})$ CFT$_1$ correlators with

non-local QFT

$$h(\lambda) = h + \lambda q \bar{p} + \frac{k}{4}\lambda^2 \bar{p}^2 \qquad q(\lambda) = q + \frac{k}{2}\lambda \bar{p}$$

MG'19

→ spectral flow with momentum-dependent parameter

Holographic interpretation

Double-trace deformations in AdS/CFT

- in AdS/CFT parlance, the Smirnov-Zamolodchikov deformations are double-trace

- mixed boundary conditions for dual bulk fields

- e.g. scalar $$\Phi = \phi_{(0)}\, z^{d-\Delta} + \ldots + \phi_{(\Delta)}\, z^{\Delta} + \ldots$$

- undeformed CFT : source $\mathcal{J}$ (fixed) vev $\langle \mathcal{O} \rangle$ (fluctuates)

- $I_\mu = I_{CFT} + \mu \int \mathcal{O}^2$

only uses large N field theory

1. variational principle (equivalent to Hubbard-Stratonovich at large N)

$$\delta S_\mu = \delta S_{CFT} - \delta\,(\mu \int \mathcal{O}^2) = \int \mathcal{O}\delta\mathcal{J} - \mu \int \delta\mathcal{O}^2 = \int \mathcal{O}\delta\underbrace{(\mathcal{J} - 2\mu\mathcal{O})}_{\text{new source } \tilde{\mathcal{J}}}$$

new vev $\tilde{\mathcal{O}}$

2. interpret result in terms of bulk field data

$\tilde{\mathcal{J}} = \phi_{(0)} - 2\mu\phi_{(\Delta)}$ = fixed (mixed b.c.) $\langle \tilde{\mathcal{O}} \rangle = \phi_{(\Delta)}$

Holographic dictionary for $T\bar{T}$ - deformed CFTs

- variational principle (incrementally in μ) → relation between new and old sources and vevs

$$\gamma_{\alpha\beta}(\mu) = \gamma_{\alpha\beta}(0) - \mu \hat{T}_{\alpha\beta}(0) + \mu^2 \hat{T}_\alpha{}^\gamma \hat{T}_{\gamma\beta}(0)$$

$$\underbrace{\hat{T}_{\alpha\beta}}_{T_{\alpha\beta} - \gamma_{\alpha\beta} T}(\mu) = \hat{T}_{\alpha\beta}(0) - \mu \hat{T}_\alpha{}^\gamma \hat{T}_{\gamma\beta}(0)$$

  - both signs of μ
  - other (matter) vevs can be on
  - **only uses large N field theory**

- holographic interpretation (large N, large gap) → Fefferman - Graham expansion for AdS3 metric

$$ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \Big(\underbrace{\frac{g^{(0)}_{\alpha\beta}}{\rho} + g^{(2)}_{\alpha\beta}}_{\text{universal}} + \underbrace{\ldots}_{\text{non-univ}} \Big) dx^\alpha dx^\beta$$

$$g^{(0)}_{\alpha\beta} \leftrightarrow \gamma_{\alpha\beta}(0)\,, \qquad g^{(2)}_{\alpha\beta} \leftrightarrow 8\pi G \ell\, \hat{T}_{\alpha\beta}(0)$$

- $\gamma_{\alpha\beta}(\mu)$ fixed ↔ mixed non-linear boundary conditions for the AdS3 metric — MG, Monten '19

$$\gamma_{\alpha\beta}(\mu) = g^{(0)}_{\alpha\beta} - \frac{\mu}{4\pi G\ell} g^{(2)}_{\alpha\beta} + \frac{\mu^2}{(8\pi G\ell)^2} g^{(2)}_{\alpha\gamma} g^{(0)\gamma\delta} g^{(2)}_{\delta\beta}$$

  - only depend on asymptotics ✔

- $\langle T_{\alpha\beta}(\mu)\rangle$ depends non-linearly on $g^{(0)}$, $g^{(2)}$ → compute deformed energy spectrum

  → perfect match to field-theory formula (both signs of μ , matter field vevs on → universal!)

Comments

- precision holography, despite the deformation being irrelevant

- mixed metric boundary conditions keep full dynamics of matter fields → unchanged b.c.

- change bnd. conditons on AdS3 metric → radical modification of the dual theory: local → non-local

- asymptotic symmetries

  → expect : $T\bar{T}$ deformation breaks CFT conformal symmetries to $U(1)_L \times U(1)_R$

  → find: $\boxed{Virasoro(u) \times Virasoro(v)}$ with same **c** as in the undeformed CFT

  u, v → field-dependent coordinates $U = u - \mu \int T_{vv} dv$

  → suggest $T\bar{T}$ – deformed CFTs possess Virasoro symmetry, despite being non-local NEW!

Pure gravity

- the FG expansion terminates $ds^2 = \frac{\ell^2 d\rho^2}{4\rho^2} + \frac{g^{(0)}_{\alpha\beta} + \rho\, g^{(2)}_{\alpha\beta} + \rho^2/4\, g^{(2)\gamma}_{\alpha} g^{(2)}_{\gamma\beta}}{\rho} dx^\alpha dx^\beta$
- $\gamma_{\alpha\beta}(\mu)$ coincides with the induced metric at $\boxed{\rho_c = -\frac{\mu}{4\pi G\ell}}$ $\mu < 0$ $\approx$ Dirichlet at ρ_c
- $\langle T_{\alpha\beta}(\mu)\rangle$ coincides with the Brown-York stress tensor at $\rho_c:\ -\frac{1}{8\pi G}(K_{\alpha\beta} - K g_{\alpha\beta} - \ell^{-1} g_{\alpha\beta})$
- agrees with observation that $T\bar{T}$ – deformed energies coincide with energy of ``black hole in a box''

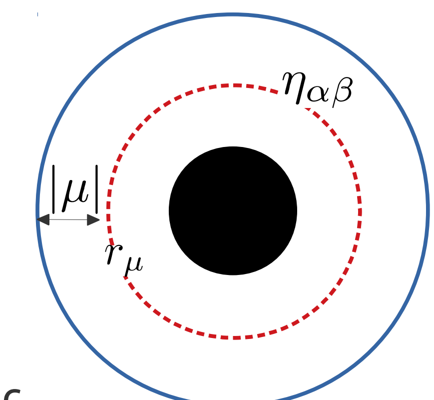
McGough, Mezei, Verlinde '16

$$E(\mu) = \frac{R}{2|\mu|}\left(1 - \sqrt{1 - \frac{4|\mu| M}{R} + \frac{4\mu^2 J^2}{R^2}}\right)$$

  - at $M_{max} = \frac{R}{4|\mu|}$ horizon reaches the r_μ surface

    → restrict to $M < M_{max}$ by imposing a hard cutoff $r < r_\mu$

  - coincidences only hold in pure gravity
  - M_{max} is not a UV cutoff & generally $\nexists\, r_\mu$ associated to the mixed b.c.



The $J\bar{T}$ holographic dictionary

MG, Bzowski '18

- introduce sources: $J^\alpha \leftrightarrow \mathrm{a}_\alpha \qquad T^a{}_\alpha \leftrightarrow e^a{}_\alpha$

- variational principle:

$$\delta S_\mu = \delta S_{CFT} - \delta S_{J\bar{T}} = \int d^2x \, [\overbrace{e T^a{}_\alpha \delta e^\alpha_a + e J^\alpha \delta \mathrm{a}_\alpha}^{\text{CFT}} - \delta(\overbrace{\mu_a T^a{}_\alpha J^\alpha e}^{\text{deformation}})] = \int d^2x \, \tilde{e} (\overbrace{\tilde{T}^a{}_\alpha \delta \tilde{e}^\alpha_a + \tilde{J}^\alpha \delta \mathrm{a}_\alpha}^{\text{new sources \& vevs}})$$

- new sources $\quad \tilde{e}^\alpha_a = e^\alpha_a - \mu_a \langle J^\alpha \rangle \,, \qquad \tilde{\mathrm{a}}_\alpha = \mathrm{a}_\alpha - \mu_a \langle T^a_\alpha \rangle$

- new vevs $\quad \tilde{T}^a{}_\alpha = T^a{}_\alpha + (e^a_\alpha + \mu_\alpha J^a)\, \mu_b T^b{}_\beta J^\beta \,, \qquad \tilde{J}^\alpha = J^\alpha$

large N field theory

Holography:

$(T^a{}_\alpha, e^a{}_\alpha)$ modelled by 3d Einstein gravity

$(J^\alpha, \mathrm{a}_\alpha)$ $U(1)$ Chern-Simons gauge field

non-dynamical

- AdS_3 gravity with mixed boundary conditions (CSS-like, but allowing full dynamics)

- perfect match between energies of black holes and the defomed CFT spectrum ✓

- asymptotic symmetry group:

$$\underbrace{SL(2,\mathbb{R})_L \times U(1)_L}_{\text{Virasoro - Kac-Moody}} \times U(1)_R$$

J

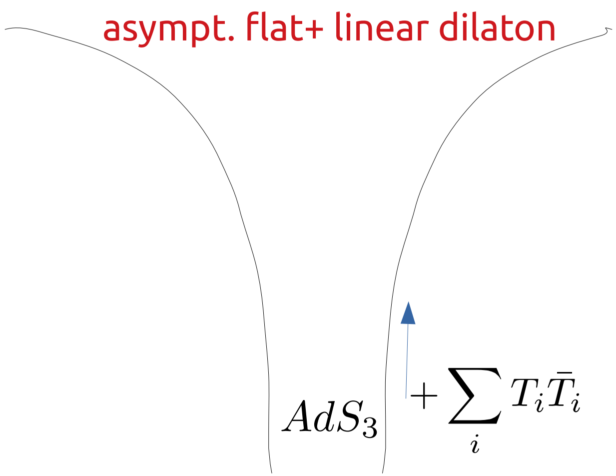
non-local

Virasoro - Kac-Moody $\times$ Virasoro$_R$ $\leftarrow f(x^- - \lambda \int J)$

non-local CFT!

Single-trace analogues of $T\bar{T}$ and $J\bar{T}$

The NS5 – F1 system



N_5 NS5 and N_1 F1 strings in the NS5 decoupling limit

$$g_s \to 0\,, \quad \alpha' \qquad \text{fixed}$$

UV: Little String Theory

non-gravitational, non-local theory with Hagedorn growth

IR: AdS_3 dual to $(\mathcal{M}_{6N_5})^{N_1}/S_{N_1}$ symmetric orbifold CFT

- can be obtained via TsT of near norizon AdS

- worldsheet σ - model : exactly marginal deformation of the $SL(2,\mathbb{R}) \times SU(2) \times U(1)^4$ WZW model that describes the near-horizon AdS_3 by $J^-\bar{J}^-$

- expand infinitesimally around IR AdS_3 → source for (2,2) single-trace operator $\sum_i T_i\bar{T}_i$

Proposed holographic duality

$$Z_{string}[\text{NS5- F1}] = Z\left[(T\bar{T} - \text{def. CFT}_{6N_5})^{N_1}/S_{N_1}\right]$$

Giveon, Itzhaki, Kutasov '17

+

- RHS is well-defined at finite deformation
- spectrum of string excitations exactly matches $T\bar{T}$ spectrum
- black hole entropy (Hagedorn)
- correlation functions $\langle O(p)O(-p)\rangle$ using worldsheet

−

- uses free product structure in an essential way
- not clear how to deform away from this (singular) point in moduli space
- naively different behaviour from $T\bar{T}$ correlator

more checks?

- similar story holds for J$\bar{\text{T}}$: pure NS-NS string background obtained from $AdS_3 \times S^3 \times T^4$ + TsT on one AdS and one angular direction → warped AdS_3 Apolo, Song '18, Chakraborty et al. '18

field-dependent?

- universal near-horizon geometry of extremal black holes, with Virasoro x Virasoro ASG

$T\bar{T}$ and $J\bar{T}$ - deformed CFTs as non-local CFTs

Abstract proof of Virasoro symmetries

MG'21

- ASG suggest $\exists$ Virasoro x Virasoro symmetry in $T\bar{T}/J\bar{T}$ → build it directly in field theory?

- both deformations smoothly deform energy eigenstates on a cylinder w/o changing Hilbert space

$$\partial_\lambda |n\rangle_\lambda = \mathcal{O}|n\rangle_\lambda$$

- define L_m^λ via $\partial_\lambda L_m^\lambda = [\mathcal{O}, L_m^\lambda]$, and which coincide with the CFT generators at $\lambda = 0$

  → satisfy Virasoro algebra by construction, same **c** as undeformed CFT

  → $L_0^\lambda |n\rangle_\lambda = h|n\rangle_\lambda$, where h are the undeformed conformal dimensions

- Virasoro algebra $\not\Rightarrow$ Virasoro symmetry → need conservation!

- universal $T\bar{T}$ – deformed spectrum → $L_0^\lambda = f(H,P)$

$$\underbrace{E_\lambda}_{H} = \frac{1}{2\lambda}\left(\sqrt{1+4\lambda(\underbrace{h+\bar{h}}_{L_0^\lambda})+4\lambda^2(h-\bar{h})^2}-1\right)$$

LeFloch, Mezei '19

- can show that $[L_m^\lambda, H] = \alpha_m(H,P) L_m^\lambda$ $\Rightarrow$ $L_{m,S}^\lambda(t) \equiv e^{i\alpha_m(H,P)t} L_{m,S}(0)$

$$\Rightarrow \left(\frac{\partial L_{m,S}^\lambda(t)}{\partial t}\right)_H + \frac{i}{\hbar}[H, L_{m,H}^\lambda] = 0$$

-» conserved vev -» **Virasoro symmetries**

Field-dependent symmetries

MG, Monten '20

- consider a 2d classical field theory with null coordinates $U, V = \sigma \pm t$

- consider the coordinate shifts: $U \to U + \epsilon\, f(u)$ $V \to V - \epsilon\, \bar{f}(v)$ where $u(U,V),\, v(U,V)$ are some possibly field-dependent coordinates

- the variation of the action is $\delta_f S = -\int dU dV \epsilon\, f'(u)\, (T_{UU}\partial_V u + T_{VU}\partial_U u)$

- in a 2d CFT $T_{VU} = 0$ off-shell → for $u = U$, $\delta_f S = 0\ \forall f(u)$ → infinite conformal symmetries

- in $T\bar{T}$, $J\bar{T}$ – deformed CFTs , still only two independent components of the stress tensor off-shell

  → choose $u(U,V)$ $\ni$ $T_{UU}\partial_V u + T_{VU}\partial_U u = 0$ $\Rightarrow$ infinite field-dependent symmetries

- special structure of $T\bar{T}$, $J\bar{T}$ → universal form for $u(U,V),\, v(U,V)$
  = coordinates in terms of which the deformed dynamics trivializes to that of the original CFT
  → field-dependent symmetries = original CFT symmetries (∞) seen through the prism of these coord.

Example: classical $J\bar{T}$ - deformed CFTs

field-dependent coordinate

- $\boxed{u = U}$ (due to $SL(2,\mathbb{R})$), $\boxed{v = V - \lambda\phi}$ $J\,:\;U(1)$ shift current for ϕ

- conserved charges $\bar{Q}_{\bar{f}} = -\int d\sigma \bar{f}(v) T_{tV} = \int d\sigma \bar{f}(v)\mathcal{H}_R$ + affine U(1) $\bar{P}_{\bar{\eta}}$ $\left\{ \begin{array}{l} \mathcal{H}_{L,R} = \dfrac{\mathcal{H} \pm \mathcal{P}}{2} \\ \mathcal{J}_\pm = \dfrac{\pi \pm \phi'}{2} \end{array} \right.$

- charge algebra $\{\bar{Q}_{\bar{f}}, \bar{Q}_{\bar{g}}\} = \bar{Q}_{\bar{f}'\bar{g} - \bar{f}\bar{g}'} \quad \ldots$ MG, Monten '20

  $\rightarrow$ functional ('= ∂_σ) Witt – Kac-Moody algebra charges seen by ASG

- compact space $\rightarrow$ these charges are inconsistent with $U(1)$ charge quantization $\leftarrow$ z.m. of ϕ

- fix: $v \rightarrow v_{imp}$ $\leftarrow$ v with ϕ zero mode removed MG'20

- new (consistent) cons. charges $\bar{\mathcal{Q}}_{\bar{f}} = \int d\sigma \bar{f}(v_{imp})\mathcal{H}_R$ & $\bar{\mathcal{P}}_{\bar{\eta}}$ $\rightarrow$ nonlinear modification of Witt-KM

- however, the non-linear combinations $R_v \bar{\mathcal{Q}}_{\bar{f}} - \lambda E_R \bar{\mathcal{P}}_{\bar{f}} + \dfrac{\lambda^2 E_R^2}{4}\delta_{\bar{f}=I}$ & $\bar{\mathcal{P}}_{\bar{f}} - \dfrac{\lambda E_R}{2}\delta_{\bar{f}=I}$

  do satisfy Witt - Kac-Moody $\Rightarrow$ $\bar{L}_0^\lambda$

Conclusions

- $T\bar{T}$, $J\bar{T}$ are a set of well-defined and highly tractable irrelevant deformations of 2d QFTs

  → deformed spectrum, S-matrix, ~ correlators → UV complete non-local QFTs

  → precision holography : AdS_3 with mixed bnd. conditions → neat testing ground for ASG analyses

- there exist closely related single-trace analogues of $T\bar{T}$, $J\bar{T}$

  → relevant for non-AdS holography (near-horizon dynamics of general back holes)

  → suggest larger set of theories similar to $T\bar{T}$, $J\bar{T}$ - UV completeness, symmetries?

  but more general ? - ~~universal~~ spectrum

- $T\bar{T}$, $J\bar{T}$ - deformed CFTs correspond to non-local CFT s ← field-dependent Virasoro symmetries

  → other observables, e.g. correlation functions?

  → what are the most general non-local CFTs (at large N)?

Thank you !

Resolution

1. Solution for v determined up to a constant → fix such that charge quantization is respected

$$v_{new} = \sigma - \lambda\phi + \frac{\lambda R_v}{R - \lambda Q_K}\widetilde{\phi}_0 \qquad \widetilde{\phi}_0 = \phi_0 - \frac{\lambda}{R_v}\int d\sigma\hat{\phi}(\mathcal{J}_- + \frac{\lambda}{2}\mathcal{H}_R)$$

generator of spectral flow in $J\bar{T}$

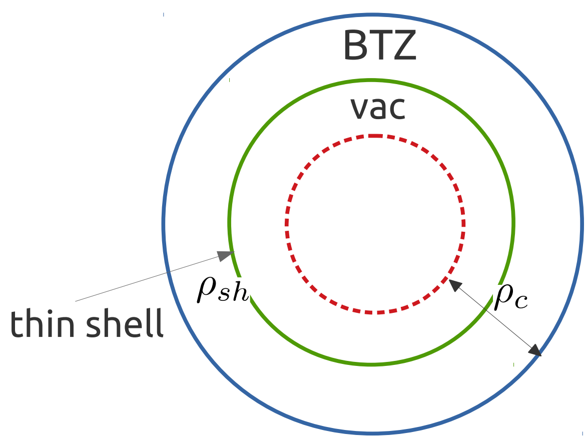
- modified charges $\bar{\mathcal{Q}}_n = \int d\sigma e^{-inv_{new}/R_v}\mathcal{H}_R$ are conserved and have Poisson brackets that are consistent with semiclassical quantization
- new charge algebra has quadratic terms on the RHS
- the combinations
$$\begin{cases} \tilde{Q}_n = R\,Q_n - \lambda E_R K_n + \dfrac{\lambda^2 E_R^2}{4}\delta_{n,0}\,, \quad K_n - \dfrac{\lambda E_R}{2}\delta_{n,0} \\ \tilde{\bar{\mathcal{Q}}}_n = R_v\bar{\mathcal{Q}}_n - \lambda E_R\bar{\mathcal{K}}_n + \dfrac{\lambda^2 E_R^2}{4}\delta_{n,0}\,, \quad \bar{\mathcal{K}}_n - \dfrac{\lambda E_R}{2}\delta_{n,0} \end{cases}$$
do satisfy Witt-Kac-Moody[2]

2. $\tilde{Q}_0, \tilde{\bar{\mathcal{Q}}}_0$ coincide with the undeformed CFT energies $E_{L,R}^{(0)}$ ← integer-spaced spectrum

→ not the left/right energies in the JT – deformed CFT!

Adding matter

- difference between mixed at infinity and Dirichlet at finite radial distance for $\mu < 0$



- shell outside ρ_c $\rightarrow$ only mixed bnd. cond. give correct energy

  → configurations outside this surface ✓

  → 2d $T\overline{T}$ describes entire spacetime : UV completeness

  integrability

- imaginary energies → breakdown of coordinate transformation
- used to make $\gamma_{\alpha\beta}(\mu) = \eta_{\alpha\beta}$, which only depends on the asymptotic value of the metric

Take-home: universal formula for energy ↔ universal asymptotic behaviour

- McGough et al picture still holds in typical high energy states