Non-Markovian open quantum system approach to the early universe

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Based on:
1) M. Zarei, N. Bartolo, D. Bertacca, A. Ricciardone and S. Matarrese, ``Non-Markovian open quantum system approach to the early Universe: Damping of gravitational waves by matter,'' Phys. Rev. D **104,** no.8, 083508 (2021)
2) H. Manshouri, A. Hoseinpour and M. Zarei, ``Quantum Boltzmann equation for fermions: An attempt to calculate the NMR relaxation and decoherence times using quantum field theory techniques,'' Phys. Rev. D **103**, no.9, 096020 (2021)
3) A. Hoseinpour, M. Zarei, G. Orlando, N. Bartolo and S. Matarrese, ``CMB V modes from photon-photon forward scattering revisited,'' Phys. Rev. D **102**, no.6, 063501 (2020)
4) M. Zarei, M. Abdi, M. Sharifian, N. Bartolo, S. Matarrese, and M. Peloso, `` Decoherence induced by GW: a non-Markovian open quantum system approach,'' work in progress.

IRCHEP 1400 8-10 November, 2021 (17-19 Aban,1400)

Outline:

1- Open quantum systems
2- Review of the Quantum Boltzmann Equation
3- Applications of QBE
4- Applications of non-Markovian QBE:
Damping of gravitational waves by matter
5- Applications of non-Markovian QBE:
Decoherence induced by GW noise

1-1) Open Quantum System (OQS)

The theory of OQS describes the interaction of a quantum system with its environment

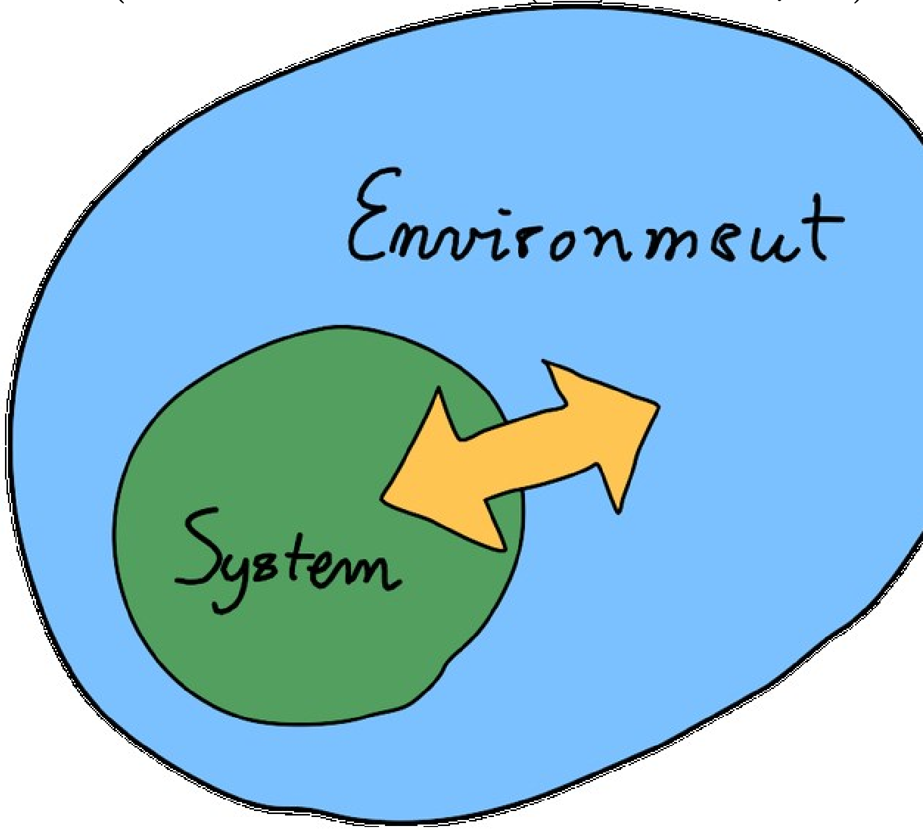
$$\frac{d}{dt}\rho(t) = -i[\hat{H}(t), \rho(t)]$$

Liouville-von Neumann equation

The reduced density matrix

$$\rho_S = Tr_E(\rho)$$

$(\text{Environment } (E), H_E, \rho_E)$



$(\text{System } (S), H_S, \rho_S)$

The full Hamiltonian for the system and environment:

$$\hat{H}(t) = H_S + H_E + H_{int}$$

interaction picture:

$$\rho_I(t) = e^{i(H_S+H_E)t}\rho(t)e^{-i(H_S+H_E)t}$$

$$H_{int}(t) = e^{i(H_S+H_E)t}H_{int}e^{-i(H_S+H_E)t}$$

$$\downarrow$$

$$\frac{d}{dt}\rho_I(t) = -i[H_{int}(t), \rho_I(t)]$$

The first step in solving the Liouville- von Neumann :

$$\rho_I(t) = \rho_I(0) - i \int_0^t dt' [H_{int}(t), \rho_I(t)]$$

substituting the integrated equation back into the Liouville- von Neumann equation:

$$\frac{d}{dt}\rho_I(t) = -i[H_{int}(t), \rho_I(0)] - \int_0^t dt' [H_{int}(t), [H_{int}(t'), \rho_I(t')]]$$

Approximations:

1- no entanglement between the system and the environment are present at t=0:

$$\rho_I(0) = \rho(0) = \rho_S(0) \otimes \rho_E(0)$$

$$\downarrow$$

$$\frac{d}{dt}\rho_I(t) = -\int_0^t dt' Tr_E[H_{int}(t), [H_{int}(t'), \rho_I(t')]]$$

2- Born approximation:

$$\rho_I(t) \approx \rho_S(t) \otimes \rho_E(0)$$

Employing the Born approximation we may rewrite the equation as:

↓

$$\frac{d}{dt}\rho_S(t) = -\int_0^t dt' Tr_E[H_{int}(t), [H_{int}(t'), \rho_S(t') \otimes \rho_E(0)]]$$

3- Markov approximation: No memory

$$\frac{d}{dt}\rho_S(t) = -\int_0^t dt' Tr_E[H_{int}(t), [H_{int}(t'), \rho_S(t) \otimes \rho_E(0)]]$$

This simply corresponds to replacing: $\rho_S(t') \to \rho_S(t)$

$$\frac{d}{dt}\rho_S(t) = -\int_0^t dt' Tr_E[H_{int}(t), [H_{int}(t'), \rho_S(t) \otimes \rho_E(0)]]$$

Markov master equation

In general, the system environment interaction can be presented as:

$$H_{int}(t) = \sum_i S_i(t) \otimes E_i(t)$$

S_i : operators acting only in the system's Hilbert space, E_i: operators that only act in the Hilbert space of the environment

$$\frac{d}{dt}\rho_S(t) \quad = \quad -\sum_{i,j}\int_0^t dt' \Big[(S_i(t)S_j(t')\rho_S(t) - S_j(t')\rho_S(t)S_i(t))]Tr_E[E_i(t)E_j(t')\rho_E(0)]$$
$$-(S_i(t)\rho_S(t)S_j(t') - \rho_S(t)S_j(t')S_i(t))]Tr_E[E_j(t')E_i(t)\rho_E(0)]\Big]$$

Markov approximation holds when the correlation function for the bath is

$$\langle E_i(t)E_j(t')\rangle = Tr_E[E_i(t)E_j(t')\rho_E(0)] \approx 0 \qquad \text{when} \qquad t - t' > \tau$$

correlation time tau defines the memory retention time of the environment. Therefore, as long as changes in the system occurs on a timescale much greater than tau the Markov approximation holds.

2- Review of the Quantum Boltzmann Equation

The density operator describing the system:

$$\hat{\rho}_S = \int \frac{d^3\mathbf{p}'}{(2\pi)^3} \rho_{ij}^S(\mathbf{p}') a_i^\dagger(\mathbf{p}') a_j(\mathbf{p}')$$

density matrix.

Another particular function, the number operator: $\hat{N}_{ij}^S(k,t) = a_i^\dagger(k,t) a_j(k,t)$

The expectation value of N is proportional to the density matrix:

$$\left\langle \hat{N}_{ij}^S(k,t) \right\rangle = tr[\hat{\rho}^{(S)} \hat{N}_{ij}^S(k,t)] = (2\pi)^3 \delta^3(0) 2k^0 \rho_{ij}^S(k,t)$$

In the Born- Markov approximation, the time evolution of this system is given by the following master equation

$$\frac{d}{dt_{mes}}\hat{N}^S_{ij}(k,t_{mes}) = i\left[H^0_{int}(t_{mes}),\hat{N}^S_{ij}(k,t_{mes})\right] - \int_0^{t_{mes}} dt_{mic}\left[H^0_{int}(t_{mes}),\left[H^0_{int}(t_{mes}-t_{mic}),\hat{N}^S_{ij}(k,t_{mes})\right]\right]$$

the time integration can be extended to infinity due to the Born-Markov approximation:

$$(2\pi)^3\delta^3(0)2k^0\frac{d}{dt_{mes}}\rho^S_{ij}(k,x,t_{mes}) = i\left\langle\left[H^0_{int}(t_{mes}),\hat{N}^S_{ij}(k,t_{mes})\right]\right\rangle_c - \int_0^{\infty} dt_{mic}\left\langle\left[H^0_{int}(t_{mes}),\left[H^0_{int}(-t_{mic}),\hat{N}^S_{ij}(k,t_{mes})\right]\right]\right\rangle_c$$

connected part of the correlation functions.

It is also assumed that the process obeys the time-reversal symmetry $H^0_{int}(-t_{mic}) = H^0_{int}(t_{mic})$

$$(2\pi)^3\delta^3(0)2k^0\frac{d}{dt_{mes}}\rho^S_{ij}(k,x,t_{mes}) = i\left\langle\left[H^0_{int}(t_{mes}),\hat{N}^S_{ij}(k,t_{mes})\right]\right\rangle_c - \frac{1}{2}\int_{-\infty}^{\infty} dt_{mic}\left\langle\left[H^0_{int}(t_{mes}),\left[H^0_{int}(t_{mic}),\hat{N}^S_{ij}(k,t_{mes})\right]\right]\right\rangle_c$$

the equation that deals with the reversible scattering processes.

The first term on the right side: forward scattering term
The second term is: the usual collision term

Extension to Markovian irreversible processes

There are a variety of examples of irreversible processes in the early universe. The question of how macroscopic irreversibility emerges from microscopic processes has always been a fundamental question. The root of this problem is that we still do not know exactly how to reconcile the second law of thermodynamics with its intrinsic arrow of time, with the microscopic time-reversible dynamical equations. In order to use the QBE to explain this phenomenon, we must first generalize it to irreversible phenomena.

In our formalism the phenomenon emerging on the macroscopic scale is not invariant under time reversal, whereas we assume that the microscopic interactions are invariant under time reversal. The emergent irreversible process must occur on macroscopic scales.

For a microscopic irreversible process, like absorption or emission the effective interaction Hamiltonian doesn't satisfy the relation $H(-t_{mic}) = H(t_{mic})$. The effective interaction Hamiltonian for a specific irreversible process is written in terms of the
creation and the annihilation operators. For such a process, the action of the time-reversal transformation on H is equivalent to the action of Hermitian conjugation:

$$H^0_{\rm int}(-t_{\rm mic}) \to H^{0\dagger}_{\rm int}(t_{\rm mic})$$

By doing this, the collision term is modified as:

$$-\int_0^\infty dt_{\rm mic} \left\langle \left[H^0_{\rm int}(t_{\rm mes}), \left[H^{0\dagger}_{\rm int}(t_{\rm mic}), \hat{\mathcal{N}}^{\mathcal{S}}_{ij}(\mathbf{k}, t_{\rm mes})\right]\right]\right\rangle_{\rm c}$$

Extension to non-Markovian (ir)reversible processes

In the remainder of this talk, I discuss deviations from the Markovian approximation. As discussed above, in most situations the non-Markovianity appears to be relevant for time-scales smaller than, or of the order of, the environment correlation time τ_E .

If memory effects in the environment are substantial, then the evolution of the reduced density matrix will depend on the past history of the system and the environment. In this condition, information can also flow back from the environment to the system, resulting in a back-reaction effect of the environment. The microscopic description of non-Markovian dynamics is much more complicated than the Markovian one. The precise details of the non-Markovian dynamics has still not fully worked out, partly because of the complexity behind such phenomena. Here, we will generalize the QBE as a new tool to deal with the non-Markovian processes. The generalized QBE is derived as follows: First, it is important to note that we still consider the Born approximation.

The equation we will ultimately work with to describe an irriversal and non-Markovian process is:

$$(2\pi)^3\delta^3(0)2k^0\frac{d}{dt_{mes}}\rho^S_{ij}(k,x,t_{mes}) = D_{ij}[\rho^S(k,x,t_{mes})]$$

Dissipator:

$$D_{ij}[\rho^S(k,x,t_{mes})] = -\int_0^{t_{mes}} dt_{mic}\left\langle\left[H^0_{int}(t_{mes}),\left[H^{0\dagger}_{int}(t_{mic}),\hat{N}^S_{ij}(k,t_{mes}-t_{mic})\right]\right]\right\rangle_c$$

3- Applications of QBE: CMB

M. Zarei, N. Bartolo, D. Bertacca, A. Ricciardone and S. Matarrese, ``Non-Markovian open quantum system approach to the early Universe: Damping of gravitational waves by matter,'' Phys. Rev. D **104,** no.8, 083508 (2021)

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N. Bartolo, A. Hoseinpour, G. Orlando, S. Matarrese and M. Zarei, ``Photon-graviton scattering: A new way to detect anisotropic gravitational waves?,'' Phys. Rev. D **98**, no.2, 023518 (2018)

4- Applications of non-Markovian QBE: Damping of gravitational waves by matter

The seminal work of Hawking has revealed that GWs do not interact with a perfect fluid in the absence of dissipative processes. However, in a viscous medium, the energy of GWs is converted into heat, without provoking macroscopic motions of the medium . A medium with a dynamical viscosity coefficient η could absorb the GW at a rate of absorption

$$\Gamma_{GW} = 16\pi G\eta$$

A well-known effect given by decoupled relativistic neutrinos on the CMB angular power-spectrum is the damping due to their anisotropic stress of the amplitude of the GW spectrum by 35%>

S. W. Hawking, "Perturbations of an expanding universe," Astrophys. J. **145**, 544 (1966).

S. Weinberg, "Damping of tensor modes in cosmology," Phys. Rev. D **69**, 023503 (2004) doi:10.1103/PhysRevD.69.023503 [arXiv:astro-ph/0306304 [astro-ph]].

Open quantum system components

System

We consider a quantum system of soft graviton degrees of freedom that is affected by its coupling to the environment.

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

The dynamics of free gravitons in the transverse-traceless gauge is given by the following Lagrangian density:

$$L_g = \frac{1}{4}\left[\dot{h}_{\mu\nu}\dot{h}^{\mu\nu} + \partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu}\right]$$

$$h_{\mu\nu}(x) = h^{+}_{\mu\nu}(x) + h^{-}_{\mu\nu}(x)$$

Fourier transforms of the fields are expressed by the following conventions:

$$h^{+}_{\mu\nu}(x) = \int dp \sum_{s=+,\times} a_s(p,t)\, h^{s}_{\mu\nu}(p)\, e^{-i(p^0 t - p\cdot x)}$$

$$h^{-}_{\mu\nu}(x) = \int dp \sum_{s=+,\times} a^{\dagger}_{s'}(p,t)\, h^{s*}_{\mu\nu}(p)\, e^{i(p^0 t - p\cdot x)}$$

$$\left[a_s(p,t), a^{\dagger}_{s'}(p',t)\right] = (2\pi)^3 2p^0 \delta^3(p - p')\delta_{ss'}$$

The graviton density operator:

$$\hat{\rho}^{(g)}(x,t) = \int \frac{d^3p}{(2\pi)^3} \rho^{(g)}_{ij}(x,t) a^{\dagger}_i(p,t) a_j(p,t)$$

polarization matrices for a system of gravitons:

$$\rho^{(g)}(x,t) = \frac{1}{2}\begin{pmatrix} I^{(g)}(x,t) + Q^{(g)}(x,t) & U^{(g)}(x,t) - iV^{(g)}(x,t) \\ U^{(g)}(x,t) + iV^{(g)}(x,t) & I^{(g)}(x,t) - Q^{(g)}(x,t) \end{pmatrix}$$

The Stokes parameters for monochromatic plane GWs are defined by

$$I^{(g)} = \left(h^{+}\right)^2 + \left(h^{\times}\right)^2 ,$$

$$Q^{(g)} = \left(h^{+}\right)^2 - \left(h^{\times}\right)^2 ,$$

$$U^{(g)} = 2\cos\alpha\, h^{+} h^{\times} ,$$

$$V^{(g)} = 2\sin\alpha\, h^{+} h^{\times} ,$$

Environment

The environment contains decoupled ultra-relativistic fermionic degrees of freedom and behaves like a thermal fermion bath.

$$\bar{\psi}_f^-(x) = \int dq \sum_r b_r^\dagger(q,t)\bar{u}_r(q)e^{i(q^0t - q\cdot x)},$$

$$\psi_f^+(x) = \int dq \sum_r b_r(q,t)u_r(q)e^{-i(q^0t - q\cdot x)}$$

The creation and the annihilation operators of fermions obey the following equal-time canonical anti-commutation relation

$$\left\{b_r(q,t), b_{r'}^\dagger(q',t)\right\} = (2\pi)^3\delta^3(q - q')\delta_{rr'}$$

fermionic density operators:

$$\hat{\rho}^{(f)}(x,t) = \int \frac{d^3q}{(2\pi)^3}\rho_{ij}^{(f)}(x,q,t)b_i^\dagger(q,t)b_j(q,t) \qquad \rho^{(f)}(x,t) = \frac{1}{2}\begin{pmatrix} n^{(f)}(x,t) & 0 \\ 0 & n^{(f)}(x,t)\end{pmatrix}$$

Interaction Hamiltonian

$$H_{int}^{0}(t) = -\frac{i}{2}\kappa \int d^3x\, h_{\mu\lambda}^{+} \overline{\psi}^{-} \gamma^{\lambda} \partial^{\mu} \psi^{+}$$

Fourier transforms

$$H_{int}^{0}(t) = -\frac{i}{2}\kappa \int dp\,dq\,dq' \sum_{s,r,r'} h_{\mu\lambda}^{s}(p)\overline{u}_{r'}(q')(-iq^{\mu})\gamma^{\lambda} u_r(q)(2\pi)^3\delta^3(q'-q-p)$$
$$\times\, e^{i(-p^0+q'^0-q^0)t} a_s(p,t) b_{r'}^{\dagger}(q',t) b_r(q,t)\ .$$

covariant bilinear transforms under time reversal

$$\overline{\psi}_a \gamma^{\lambda} \partial^{\lambda} \psi_b \xrightarrow{T} \overline{\psi}_b \gamma^{\lambda} \partial^{\lambda} \psi_a$$

N. A. Voronov, "Gravitational Compton effect and photoproduction of gravitons by electrons," Sov. Phys. JETP 37, 953 (1973) [Zh. Eksp. Teor. Fiz. 64, 1889 (1973)].

$$H^{0}_{int}(-t) = -\frac{1}{2}\kappa \sum_{s,r,r'} \int d^3x\, dp\, dq\, dq'\, h^{*s}_{\mu\lambda}(p)\bar{u}_r(q) q'^{\mu}\gamma^{\lambda} u_{r'}(q') e^{-i(-p^0+q'^0-q^0)t}$$

$$\times\ e^{-i(p+q-q')\cdot x} a^{\dagger}_s(p,t) b^{\dagger}_r(q,t) b_{r'}(q',t)$$

$$= H^{0\dagger}_{int}(t)\ ,$$

At the end of this part, we briefly discuss the action of parity operator on interaction Hamiltonian. In general, the covariant bilinear is transformed under a parity transformation as

$$\bar{\psi}_a \gamma^{\lambda} \partial^{\lambda} \psi_b \xrightarrow{P} \bar{\psi}_a \gamma^{\lambda} \partial^{\lambda} \psi_b$$

$$h^{R}_{\mu\nu} \rightarrow h^{L}_{\mu\nu}$$

Therefore, the interaction Hamiltonian is not totally invariant under a parity transformation.

$$h^{(s)}_{\mu\nu} \bar{\psi}_a \gamma^{\lambda} \partial^{\lambda} \psi_b$$

Calculation of the collision term

$$D_{ij}[\rho^{(g)}(k,x,t_{mes})] = -\frac{\kappa^2}{4}\int_0^{t_{mes}} dt_{mic}\int d^3x d^3x' dp_1 dq_1 dq_1' dp_2 dq_2 dq_2' \, e^{i(p_2^0+q_2^0-q_2'^0)t_{mic}}$$
$$\times \sum_{s_1,r_1,r_1'}\sum_{s_2,r_2,r_2'} h^{s_1}_{\mu_1\lambda_1}(p_1)\bar{u}_{r_1'}(q_1')q_1^{\mu_1}\gamma^{\lambda_1}u_{r_1}(q_1)h^{*s_2}_{\mu_2\lambda_2}(p_2)\bar{u}_{r_2}(q_2)q_2'^{\mu_2}\gamma^{\lambda_2}u_{r_2'}$$
$$\times \; e^{-i(q_1'-q_1-p_1)\cdot x_1}e^{-i(p_2+q_2-q_2')\cdot x_2}\Big\langle\Big[a_{s_1}(p_1,t_{mes})b^\dagger_{r_1'}(q_1',t_{mes})b_{r_1}(q_1,t_{mes}$$
$$,\Big[a^\dagger_{s_2}(p_2,t_{mic}))b^\dagger_{r_2}(q_2,t_{mic}))b_{r_2'}(q_2',t_{mic}))$$
$$,a_i^\dagger(k,t_{mes}-t_{mic}))a_j(k,t_{mes}-t_{mic})\Big]\Big]\Big\rangle_c \; .$$

$$q = |q|(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

The relativistic fermions are described by the following right-handed and left-handed helicity eigenstates

$$u_+(\hat{q}) = \frac{1}{\sqrt{2}}\begin{pmatrix}\cos(\theta/2)\\ \sin(\theta/2)e^{i\varphi}\\ \cos(\theta/2)\\ \sin(\theta/2)e^{i\varphi}\end{pmatrix}, \qquad u_-(\hat{q}) = \frac{1}{\sqrt{2}}\begin{pmatrix}-\sin(\theta/2)e^{-i\varphi}\\ \cos(\theta/2)\\ \sin(\theta/2)e^{-i\varphi}\\ -\cos(\theta/2)\end{pmatrix}$$

$$D_{ij}[\rho^{(g)}(k,x,t_{mes})] = -\delta^3(0)\frac{\kappa^2}{4}\int_0^{t_{mes}} dt_{mic}\int d^3q \sin^2\theta\left(q\cdot e^{(j)}\right)^2$$
$$\times\ g_f\left[n^{(f)}(|q|,t_{mes},t_{mic}) - n^{(f)}(|q|+|k|,t_{mes},t_{mic})\right]$$
$$\times\left[\cos 2\varphi\rho_{1i}^{(g)}(k,x,t_{mes}-t_{mic}) + \sin 2\varphi\rho_{2i}^{(g)}(k,x,t_{mes}-t_{mic})\right]$$

$$\frac{d}{dt_{mes}}\rho_{ij}^{(g)}(k,x,t_{mes}) = \frac{\kappa^2 g_f}{8|k|}\int_0^{t_{mes}} dt_{mic}\int\frac{d^3q}{(2\pi)^3}\sin^2\theta\left(q\cdot e^{(j)}\right)^2|k|\frac{\partial}{\partial|q|}n^{(f)}(|q|,t_{mes},t_{mic}$$
$$\times\left[\cos 2\varphi\,\rho_{1i}^{(g)}(k,x,t_{mes}-t_{mic}) + \sin 2\varphi\,\rho_{2i}^{(g)}(k,x,t_{mes}-t_{mic})\right],$$

g_f: denotes the number of helicity states for fermions

$$\frac{d}{dt_{mes}}\rho_{ij}^{(g)}(k,x,t_{mes}) = \frac{\kappa^2 g_f}{8|k|}\int_0^{t_{mes}} dt_{mic}\int\frac{d^3q}{(2\pi)^3}\sin^2\theta\left(q\cdot e^{(j)}\right)^2|k|\frac{\partial}{\partial|q|}n^{(f)}(|q|,t_{mes},t_{mic}$$
$$\times\left[\cos 2\varphi\,\rho_{1i}^{(g)}(k,x,t_{mes}-t_{mic}) + \sin 2\varphi\,\rho_{2i}^{(g)}(k,x,t_{mes}-t_{mic})\right],$$

$$I^{(g)} = \rho_{11}^{(g)} + \rho_{22}^{(g)}$$

$$I^{(g)} = \rho_{11}^{(g)} + \rho_{22}^{(g)}$$

$$\mathbf{k} = (0,0,1) \,,$$
$$\mathbf{e}^{(1)} = (1,0,0) \,,$$
$$\mathbf{e}^{(2)} = (0,1,0) \,.$$

The relativistic fermions are also described by an unpolarized Fermi-Dirac distribution

$$n^{(f)}(|\mathbf{q}|, t_{mes}, t_{mic}) = U_f(t_{mes}, t_{mic}) n_f(|\mathbf{q}|, t_{mes}) \qquad n^{(f)}(|\mathbf{q}|, t_{mes}) = \left[e^{\frac{|\mathbf{q}|}{T_f}} + 1\right]^{-1}$$

$$\dot{I}^{(g)}(\mathbf{k}, \mathbf{x}, t_{mes}) = -\frac{\kappa^2 \bar{\rho}_f}{4} \int_0^{t_{mes}} dt_{mic} \int d(\cos\theta) \sin^4\theta \, U_f(t_{mes}, t_{mic}) I^{(g)}(\mathbf{k}, \mathbf{x}, t_{mes} - t_{mic})$$

$$U_f(t_{mes}, t_{mic}) = e^{-i \int_{t_{mic}}^{t_{mes}} dt' \frac{|\mathbf{k}|\mu}{a(t')}}$$

the total energy of fermions per proper volume,

$$\bar{\rho}_f = g_f \frac{4\pi}{(2\pi)^3} \int d|\mathbf{q}| |\mathbf{q}|^3 \left[e^{\frac{|\mathbf{q}|}{T_f}} + 1\right]^{-1} = \frac{7}{8} \frac{\pi^2}{30} g_f \, T_f^4$$

$$\dot{I}^{(g)}(k, x, t_{mes}) = -4\kappa^2 \overline{\rho}_f \int_0^{t_{mes}} dt_{mic} \frac{j_2(s)}{s^2} I^{(g)}(k, x, t_{mes} - t_{mic})$$

$$s = |k| \int_{t_{mic}}^{t_{mes}} \frac{dt'}{a(t')}$$

The full time derivative of I can be expanded into partial derivatives as

$$\frac{\partial}{\partial t_{mes}} I^{(g)}(k, x, t_{mes}) - H k \frac{\partial}{\partial k} I^{(g)}(k, x, t_{mes}) + \frac{1}{a} \hat{k} \cdot \nabla I^{(g)}(k, x, t_{mes}) = -4\kappa^2 \overline{\rho}_f (t_{mes}) \int_0^{t_{mes}} dt_{mic} \frac{j_2(s)}{s^2} \times I^{(g)}(k, x, t_{mes} - t_{mic}) \, .$$

To compare this result with Weinberg, it should be noted that for left-handed relativistic neutrinos, we have $g_f = 1$. Considering neutrino and antineutrino, the neutrino energy density is given by

$$\ddot{h}_{ij}(x, t_{mes}) + 3H\dot{h}_{ij}(x, t_{mes}) - \frac{\nabla^2}{a^2(t_{mes})} h_{ij}(x, t_{mes}) = \kappa^2 \pi_{ij}(x, t_{mes})$$

$$\pi_{ij}(x, t_{mes}) = -2\bar{\rho}_f(t_{mes}) \int_0^{t_{mes}} dt_{mic} \frac{j_2(s)}{s^2} \dot{h}_{ij}(x, t_{mes} - t_{mic})$$

Considering neutrino and antineutrino, the neutrino energy density is given by

$$\bar{\rho}_f = 6\bar{\rho}_\nu = 2 \times \frac{7}{8}\frac{\pi^2}{30} T_\nu^4$$

$$\pi_{ij}(x, t_{mes}) = -4\bar{\rho}_\nu(t_{mes}) \int_0^{t_{mes}} dt_{mic} \frac{j_2(s)}{s^2} \dot{h}_{ij}(x, t_{mes} - t_{mic})$$

Calculation of GW damping in the radiation-dominated era

$$a(t_{mes}) \simeq \Omega_R^{1/4}(2H_0 t_{mes})^{1/2} \qquad H(t_{mes}) = \frac{1}{2t_{mes}}$$

$$\kappa^2 \overline{\rho}_\nu(t_{mes}) \simeq 6 f_\nu H^2$$ for three neutrino species, $f_\nu = \Omega_\nu / \Omega_R \simeq 0.4$

$$I^{(g)}(k, x_0 = 0, t_{mes}) = \left(\sqrt{\frac{t_{end}}{t_{mes}}}\right)^{4(1+\delta)} \sum_{LM\ell} \int \frac{d^3K}{(2\pi)^3} I^{(g)}(k, K, t_{end}) c^{I}_{LM} Y^{M}_{L}(\hat{K}) i^{\ell}(2\ell + 1)$$

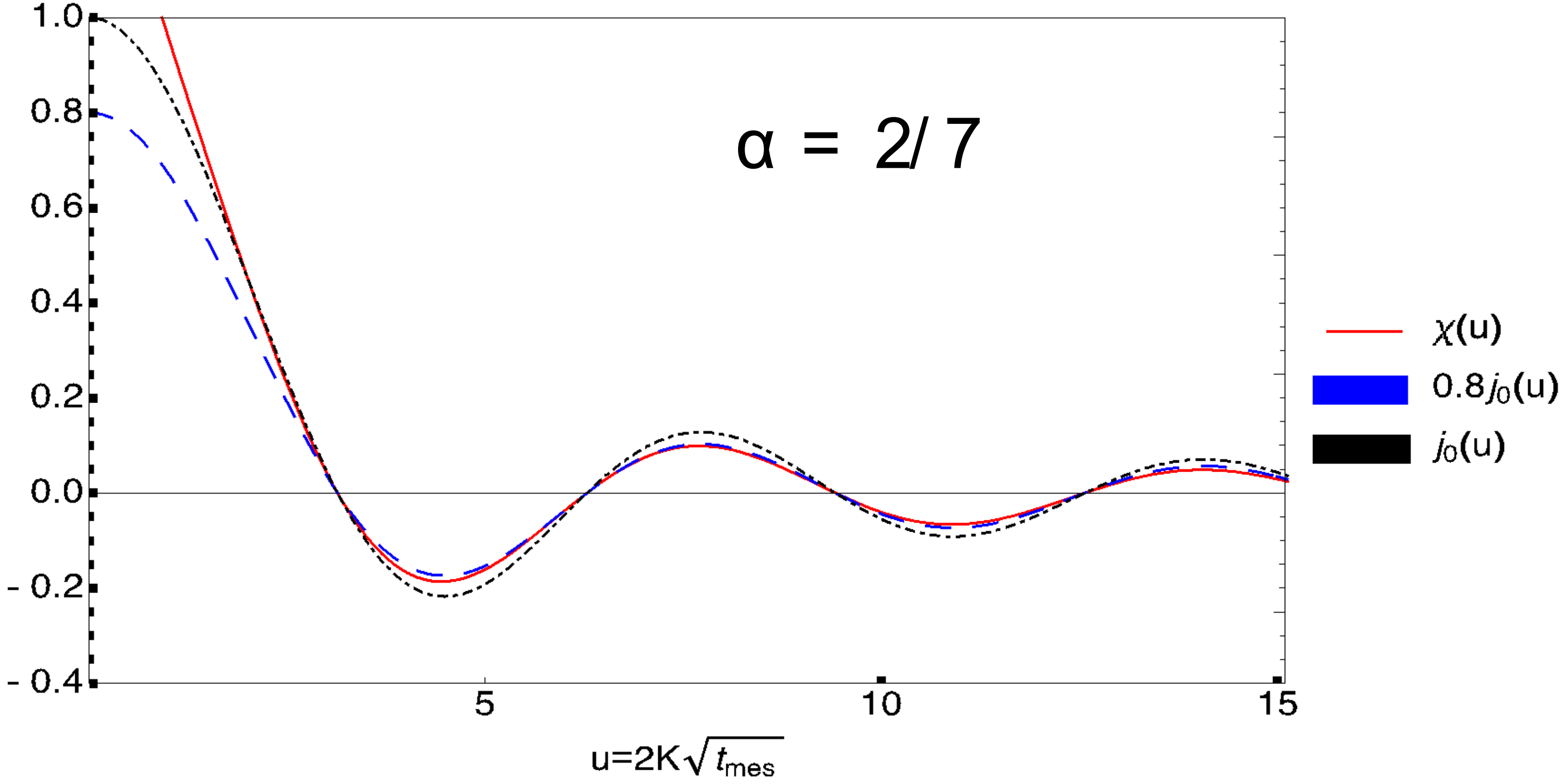
$$\times \; j_{\ell}\left(\tilde{K}\left(\sqrt{t_{mes}} - \sqrt{t_{end}}\right)\right) P_{\ell}(\mu') \; .$$

$$\delta = 8\alpha f_{\nu} / 5$$

$$\ddot{h}_{ij}(x, t_{mes}) + \left(\frac{3}{2} + \frac{8\alpha f_v}{5}\right)\frac{1}{t_{mes}}\dot{h}_{ij}(x, t_{mes}) - \frac{\nabla^2}{a^2(t_{mes})}h_{ij}(x, t_{mes}) = 0$$

$$h_{ij}(u) = h_{ij}(u_0)\chi(u) \qquad u = 2K\sqrt{t_{mes}}$$

$$\chi(u) = \chi(u_0)\frac{\Gamma(2\delta + 3/2)}{\pi^{1/2}}\left(\frac{2}{u}\right)^{2\delta+1}\sin(u)$$



Influence on the polarization of GW

$$P^{(g)}, = \left(I^{(g)}, Q^{(g)}, U^{(g)}, iV^{(g)}\right)$$

$$\frac{\partial}{\partial t_{mes}} P^{(g)}(k, K, t_{mes}) + 4H\, P^{(g)}(k, K, t_{mes}) - \frac{i}{a(t_{mes})} \mu' K\, P^{(g)}(k, K, t_{mes}) = -8\kappa^2 \overline{\rho}_v(t_{mes}$$

$$\times \int_0^{t_{mes}} dt_{mic} \frac{j_2(s)}{s^2} M\, P^{(g)}(k, K, t_{mes}, t_{mic}) \,,$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

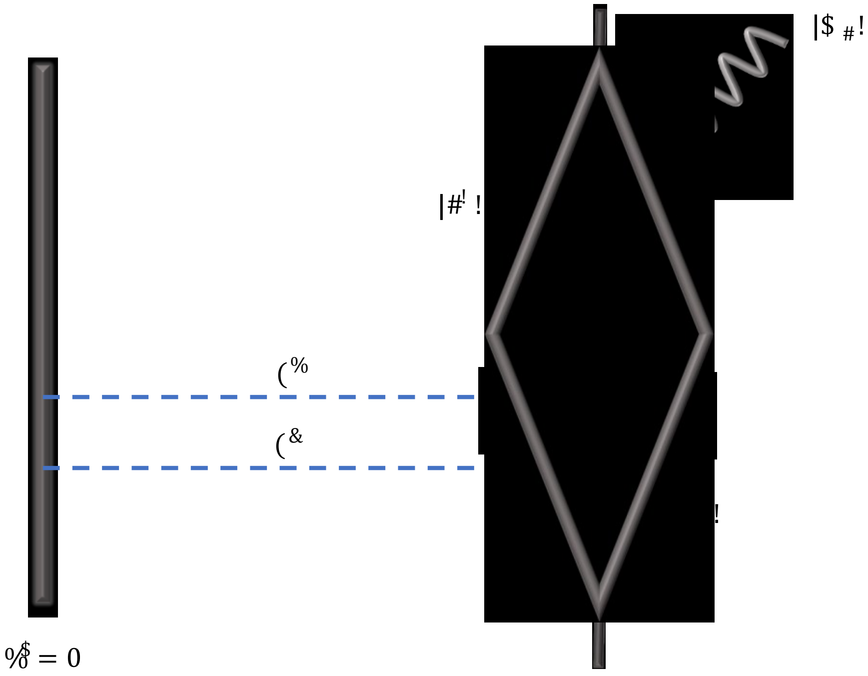
$$V^{(g)}(k, x_0 = 0, t_{mes}) = \left(\sqrt{\frac{t_{end}}{t_{mes}}}\right)^{4(1-\delta)} \sum_{LM\ell} \int \frac{d^3K}{(2\pi)^3} V^{(g)}(k, K, t_{end}) c^V_{LM} Y^M_L(\hat{K}) i^\ell (2\ell + 1$$

$$\times \; j_\ell\left(\widetilde{K}\left(\sqrt{t_{mes}} - \sqrt{t_{end}}\right)\right) P_\ell(\mu') \;,$$

$$V = V_R - V_L \xrightarrow{P} V_L - V_R = -V$$

Decoherence induced by graviton noise:

Indirect detection of gravitons, the quantum counterparts of the Stochastic Gravitational Wave Background has been recently proposed by making use of induced decoherence through the noise of gravitons



S. Kanno, J. Soda and J. Tokuda, "Noise and decoherence induced by gravitons," Phys. Rev. D **103**, no.4, 044017 (2021) doi:10.1103/PhysRevD.103.044017 [arXiv:2007.09838 [hep-th]].

M. Parikh, F. Wilczek and G. Zahariade, "The Noise of Gravitons," Int. J. Mod. Phys. D **29**, no.14, 2042001 (2020) doi:10.1142/S0218271820420018 [arXiv:2005.07211 [hep-th]].

Decoherence is a phenomenon of the quantum theory where the loss of coherence occurs and the interference effects are lost. By evaluating the reduced density operator with a mixed state, containing states of the system as well as environmental states one sees that the state of the system of interest is dephased and the off-diagonal terms decay.

$$(2\pi)^3\delta^3(0)2k^0\frac{d}{dt_{\mathrm{mes}}}\rho^{\mathcal{S}}_{ij}(\mathbf{k},\mathbf{x},t_{\mathrm{mes}}) = D_{ij}[\rho^S(\mathbf{k},\mathbf{x},t_{\mathrm{mes}})]$$

$$\hat{\rho}^S = \rho_{11}|\vec{\xi}^1\rangle\langle\vec{\xi}^1| + \rho_{12}|\vec{\xi}^1\rangle\langle\vec{\xi}^2| + \rho_{21}|\vec{\xi}^2\rangle\langle\vec{\xi}^1| + \rho_{22}|\vec{\xi}^2\rangle\langle\vec{\xi}^2|$$

$$\rho^S = \frac{1}{2}\begin{pmatrix} 1+Q^S & U^S - iV^S \\ U^S + iV^S & 1-Q^S \end{pmatrix}$$

Environment of gravitons

$$u_k^{\mathrm{sq}}(t) \equiv u_p^{\mathrm{M}}(t) \cosh r_p - e^{-i\varphi_k} u_p^{\mathrm{M}*}(t) \sinh r_p$$

$$h_{ij}(\mathbf{x}, t) = \sum_s \int \frac{d^3p}{(2\pi)^3 2p^0} \left[u_{\mathbf{p}}^s(t)\, e^{i\mathbf{p}\cdot\mathbf{x}} e_{ij}^{(s)}(\mathbf{p}) b^{(s)}(\mathbf{p}) + u_{\mathbf{k}}^{s*}(t)\, e^{-i\mathbf{p}\cdot\mathbf{x}} e_{ij}^{(s)}(\mathbf{p}) b^{\dagger(s)}(\mathbf{p}) \right]$$

Interaction

$$H_{\rm int}(t) = \frac{m_S}{4}\kappa\,\ddot{h}^{+}_{ij}(0,t)\,\hat{\xi}^{i-}\hat{\xi}^{j+}$$

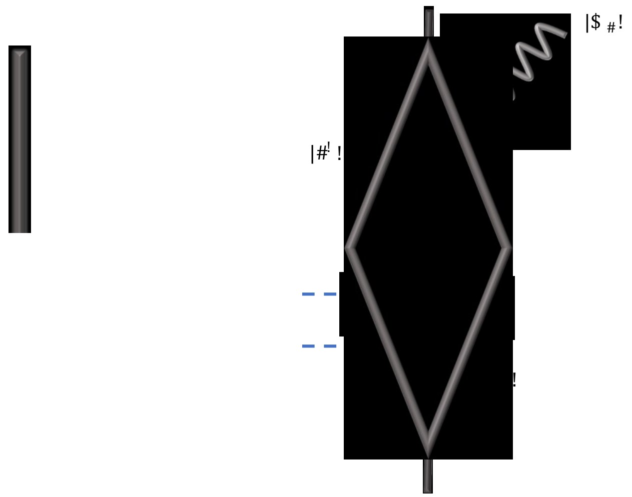
$$H_{\rm int}(t) = \frac{\kappa m_S}{4}\sum_{s,r,r'}\int \frac{d^3\mathrm{p}}{(2\pi)^3 2p^0}\,\ddot{u}^{s}_{\mathrm{p}}(t)\,e^{(s)}_{ij}(\mathrm{p})\,\xi^{i}_{r}(t)\,\xi^{j}_{r'}(t)\,b^{(s)}(\mathrm{p})\,a^{(r)\dagger}a^{(r')}$$

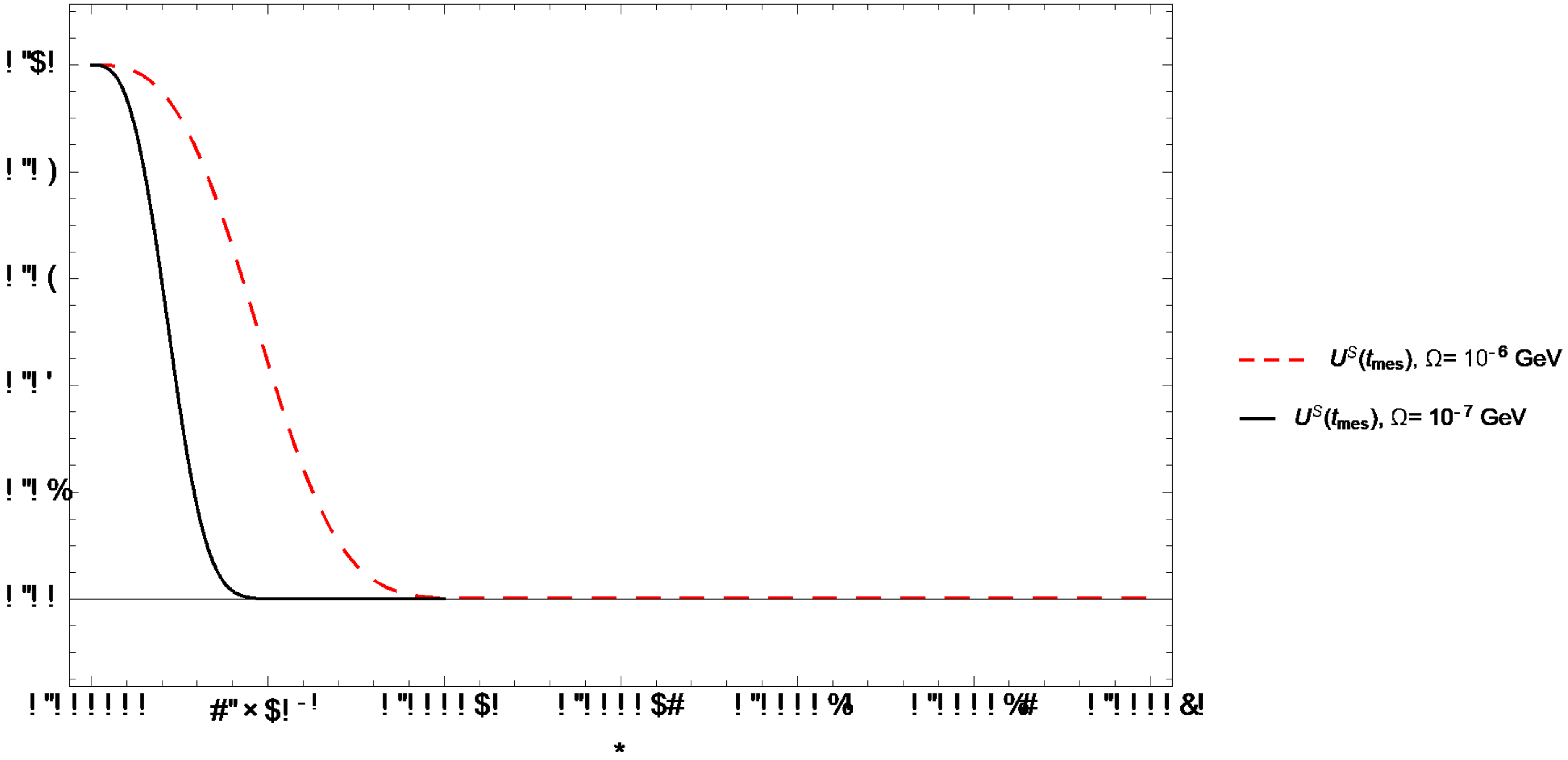
$$\frac{d}{dt_{\rm mes}} Q^S(t_{\rm mes}) \;=\; \Gamma_{QQ} Q^S(t_{\rm mes}) + \Gamma_{QU}\, U^S(t_{\rm mes})$$

$$\frac{d}{dt_{\rm mes}} U^S(t_{\rm mes}) \;=\; \Gamma^{\rm dec}_{UU} U^S(t_{\rm mes}) + \Gamma_{UQ}\, Q^S(t_{\rm mes})$$

$$\rho^S = \frac{1}{2} \begin{pmatrix} 1+Q^S & U^S - iV^S \\ U^S + iV^S & 1 - Q^S \end{pmatrix}$$

$$\begin{aligned}\Gamma^{\rm dec}_{UU} \;&=\; -\frac{\kappa^2 m_S^2}{32} \int \frac{d^3\mathbf{p}_2}{(2\pi)^3 p_2^0} \int_0^{t_{\rm mes}} dt_{\rm mic}\, \mathrm{Re}\left[\ddot{u}^{\rm sq}_{\mathbf{p}_2}(t_{\rm mic}) \ddot{u}^{{\rm sq}\,*}_{\mathbf{p}_2}(t_{\rm mes})\right] I^g(\mathbf{p}_2) \\ &\times \left[(\xi_1(t_{\rm mes}))^2 - (\xi_2(t_{\rm mes}))^2\right] \left[(\xi_1(t_{\rm mic}))^2 - (\xi_2(t_{\rm mic}))^2\right] \;.\end{aligned}$$





M. Zarei, M. Abdi, M. Sharifian, N. Bartolo, S. Matarrese, and M. Peloso, `` Decoherence induced by GW: a non-Markovian open quantum system approach,'' work in progress.

Conclusion:

1- QBE is a powerful to study reversible processes in the early universe.

2- We applied non-Markovian extension of QBE to explain the damping Of GWs due to interaction with a medium.

3- The non-Markovian QBE can be sued to study decoherence effects such Decoherence of a quantum system due to interaction with graviton noise.

Thank you for your attention!