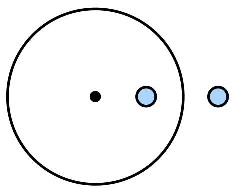
Resolution of the black hole information paradox: the fuzzball paradigm

Samir D. Mathur

The Ohio State University

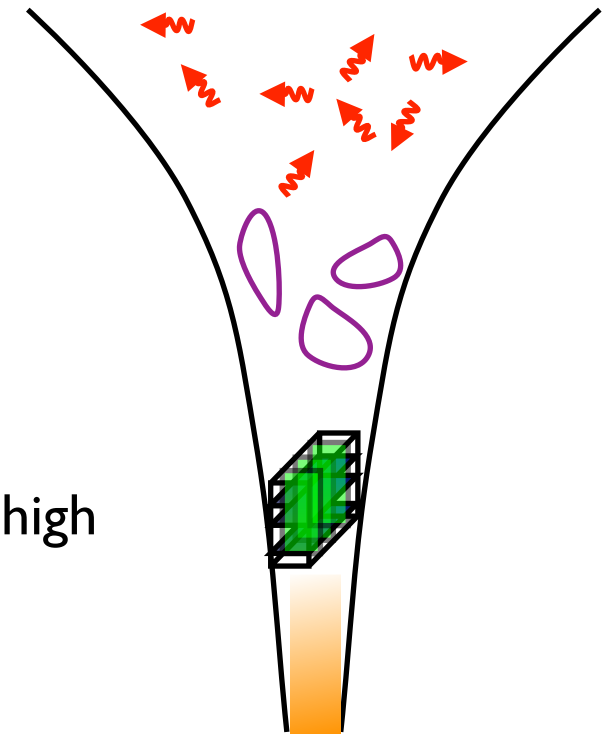
(arxiv: 1703.03042,1705.06407, 1805.09852, 1812.11641, 1905.12004, 2001.11057, 2009.09832, 2105.06963)

Black holes present us with a sharp question:



What is the resolution of the black hole information paradox ?

Cosmology presents us with many puzzles



(i) What is the cosmological constant so small?

(ii) What gives the energy needed to drive inflation?

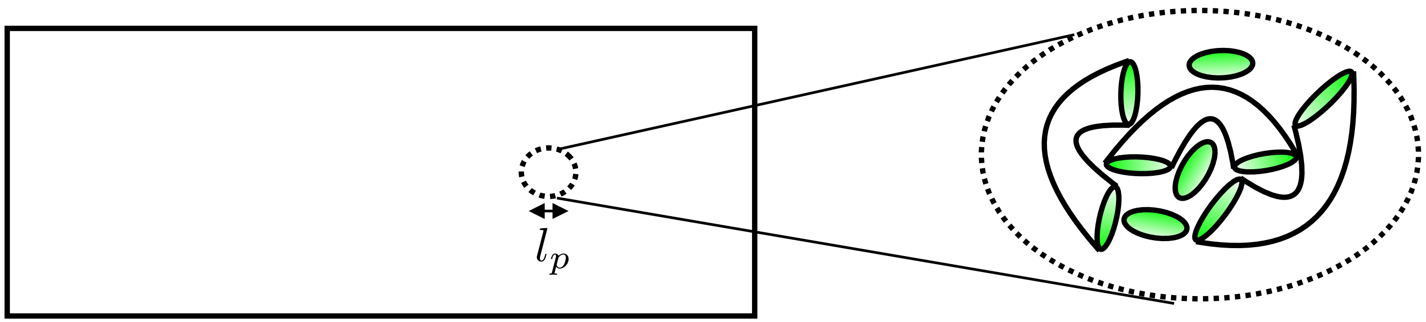
(iii) Why are the Hubble constant values between low and high redshift observations not agreeing?

In this talk we will see how string theory gives a resolution of the paradox.

The picture that emerges is called the fuzzball paradigm

The results also indicate a path to understanding the above questions in cosmology

Our conventional picture is that quantum gravity effects are relevant only at distances smaller than the planck length



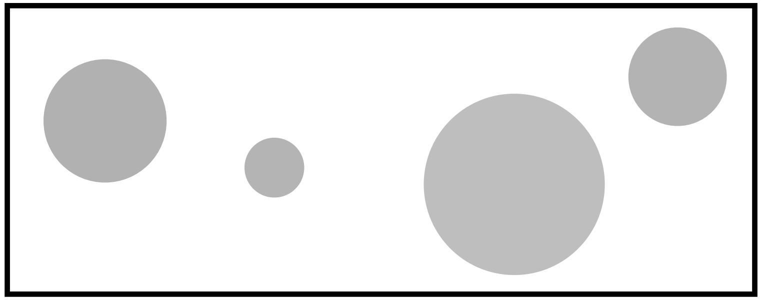
Quantum gravity

Quantum fields, Classical gravity

$$l_p \sim \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-33}\, cm$$

But we will argue that the quantum gravity vacuum has a very different structure

The vacuum has Virtual Fluctuations of black holes of all sizes



One might think that the fluctuations of large black holes will be suppressed, but there are a large number of possible states for large holes

$$S_{bek} = \frac{A}{4G}$$

so such fluctuations are NOT suppressed

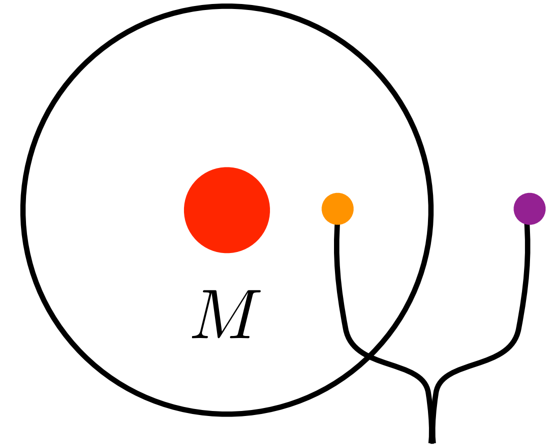
The only input is that the full quantum gravity theory should satisfy causality to leading order

This means that in any region where the curvature is much smaller than planck scale, any effects that violate causality of low energy physics are small

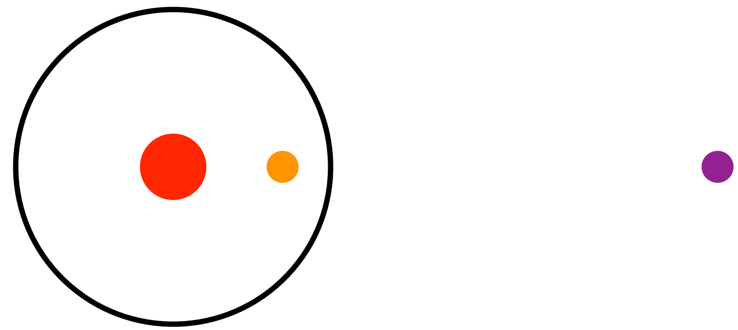
The black hole information paradox
and the Fuzzball paradigm for its resolution

Afshordi, Avery, Bah, Balasubramanian, Bena, Bianchi, Bobev, de Boer, Bossard, Carson, Ceplak, Chowdhury, Craps, Gimon, Giusto, Guo, Hampton, Heidmann, Houppe, Jejjala, Katmadas, Kanitscheider, Keski-Vakkuri, Kraus, Levi, Li, Lunin, Madden, Maldacena, Maoz, Martinec, Massai, Mayerson, Morales, Niehoff, Pani, Park, Peet, Potvin, Puhm, Ross, Ruef, Russo, Rychkov, Saxena, Shigemori, Simon, Skenderis, Srivastava, Taylor, Titchner, Turton, Tyukov, Vasilakis, Walker, Wang, Warner ... and many others

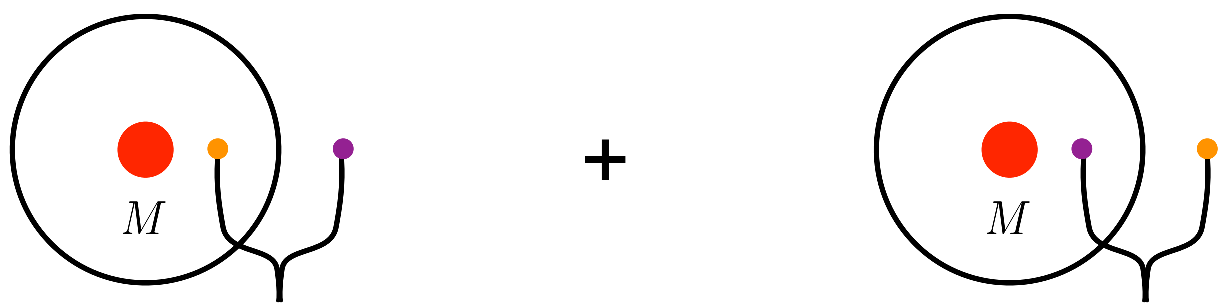
The vacuum around the horizon is unstable
to the creation of particle pairs
with total energy zero



The outer member drifts off to
infinity as Hawking radiation



The two members of the pair are in an entangled state

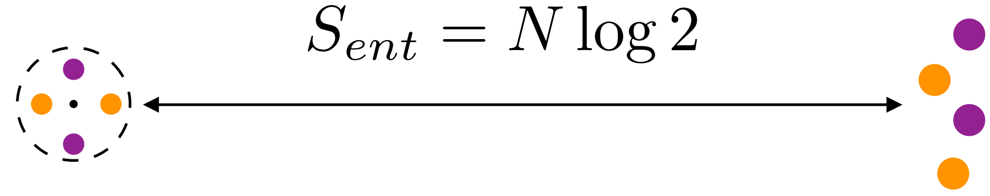


The entanglement can come from different charges, or spins, or occupation numbers ...

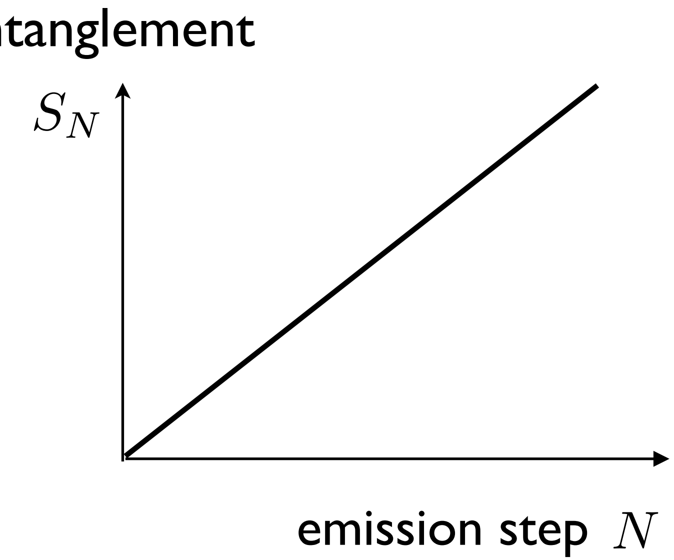
$$|\Psi\rangle = \sum_{i=1}^{n} \psi_i \otimes \chi_i \qquad \Rightarrow \qquad S_{ent} = \ln n$$

After N pairs,

$$S_{ent} = N \log 2$$



Thus the entanglement between the radiation and the hole keeps rising

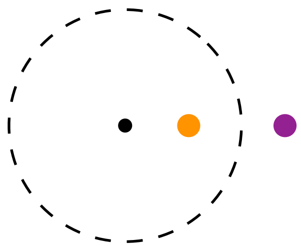


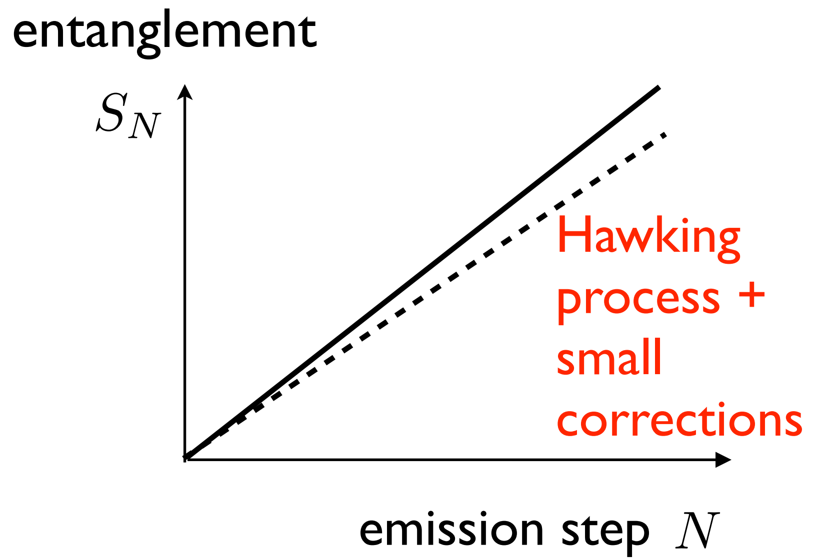
This creates a sharp problem at the endpoint of evaporation

Some people wondered if small, subtle quantum gravity effects could resolve the problem.

Perhaps a small change to the state of each created pair can change the entanglement graph because a large number of pairs are involved?

The small corrections theorem says that this cannot happen





Small corrections theorem: The entanglement must keep growing as

$$S_{N+1} > S_N + \log 2 - 2\epsilon$$

(SDM arxiv: 0909.1038)

No small correction to the semiclassical picture of the hole (i.e., using quantum fields on curved space) can change Hawking's conclusion

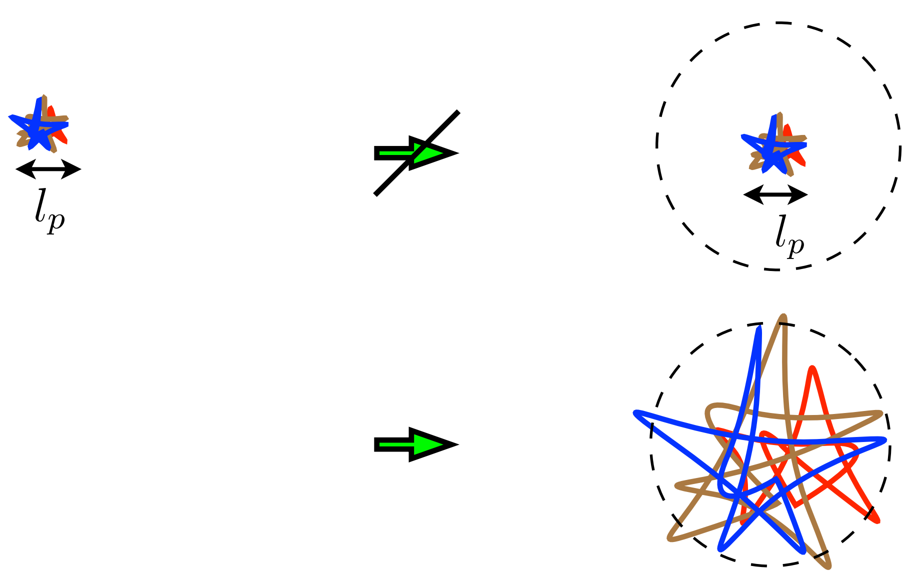
(Unless we add some large long distance nonlocal effects to the gravity theory)

So if small corrections at the horizon will not solve the problem, we need big corrections at the horizon

In string theory we have to make black holes by taking bound states of strings and branes

We find that the size of the bound state is never smaller that the horizon radius

weak coupling strong coupling

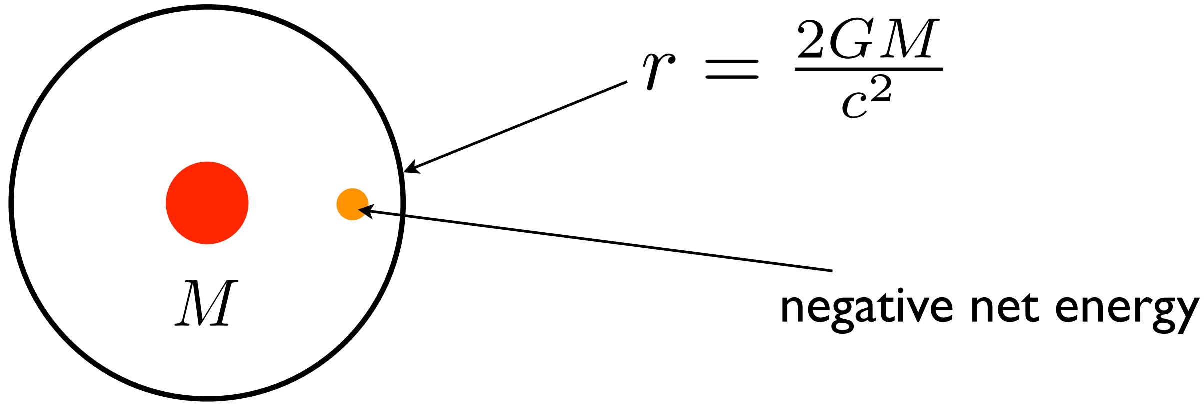


The size of the bound state grows with the number of branes, and a horizon never forms

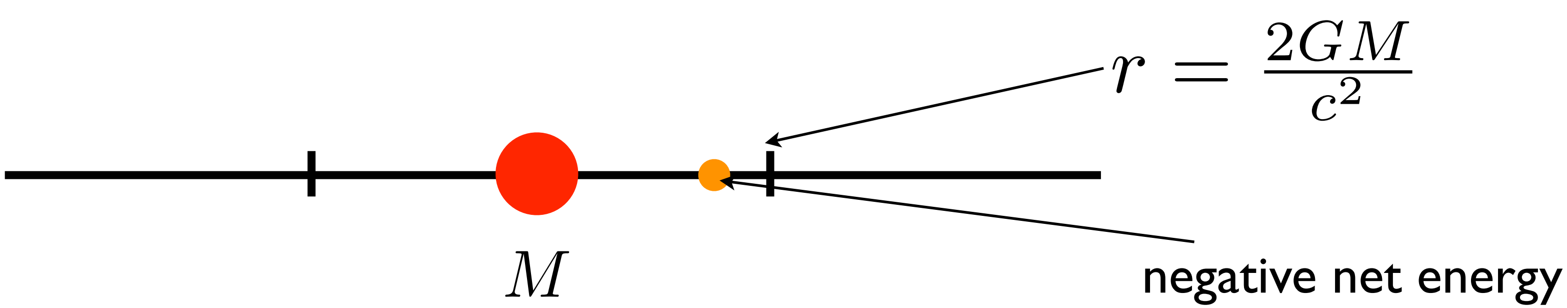
$\longrightarrow$ *fuzzball*

What is the structure of a fuzzball ?

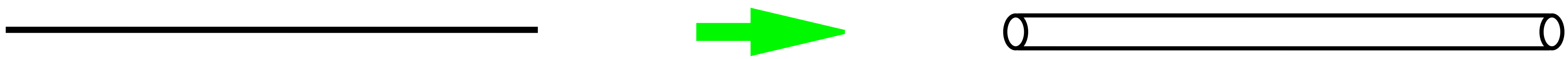
We live in 3 space and 1 time dimension. Recall the black hole ...



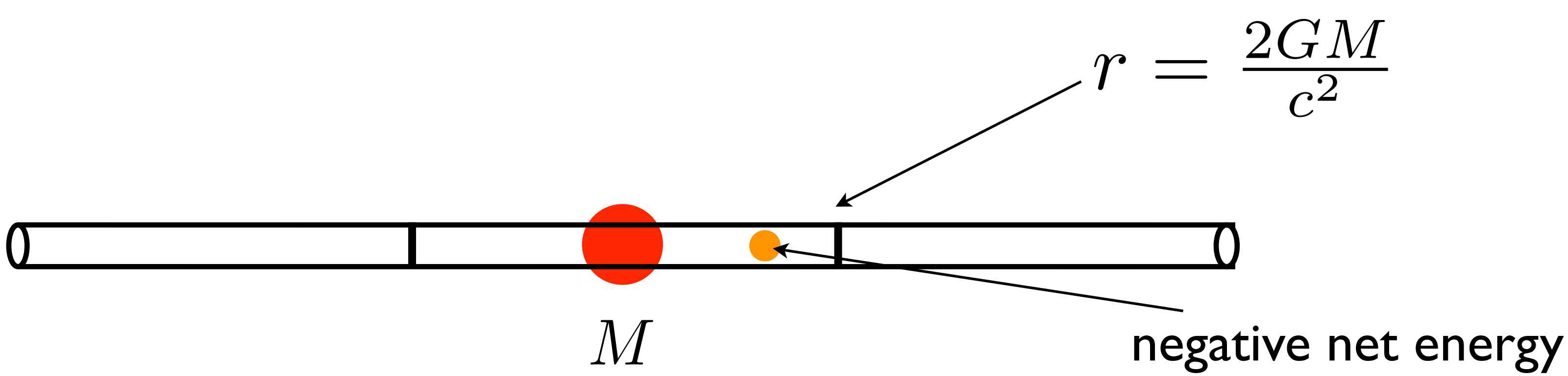
Let us draw just one space direction for simplicity



Now suppose there was an extra dimension (e.g., string theory has 6 extra dimensions)



People have thought of extra dimensions for a long time, but they seemed to have no particular significance for the black hole problem



But there is a completely different structure possible with compact dimensions ...

No place to put particles with net negative energy

The mass M is captured by the energy in the curved manifold

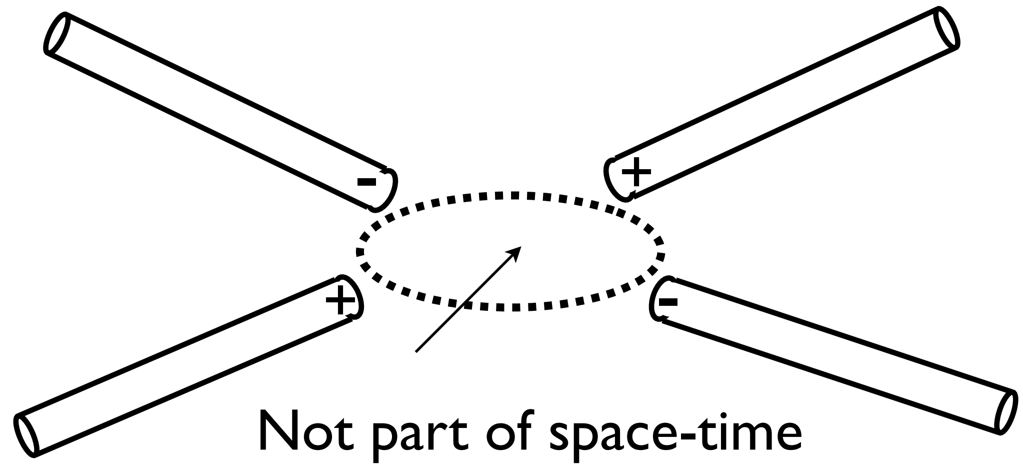
$$r = \frac{2GM}{c^2}$$

There is an extra 'twist' in the space-time which makes it consistent to have both boson and fermion wave functions

(Kaluza Klein monopoles and anti-monopoles)

1-dimension

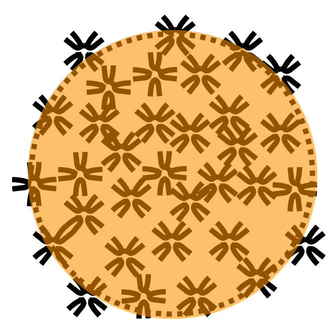
In more dimensions :



Not part of space-time
(no horizon forms)

We will draw only the structure near the horizon :

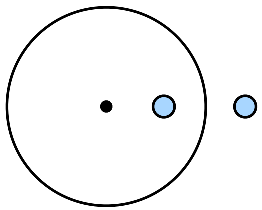
no horizon



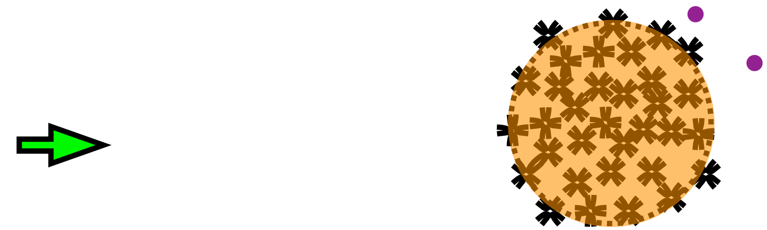
“Fuzzball”

Nothing can fall 'into the hole' because it is like a normal body with no horizon

Fuzzball conjecture: No microstate in string theory will have a traditional horizon which has a vacuum region in its vicinity



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) + O(\epsilon)$$



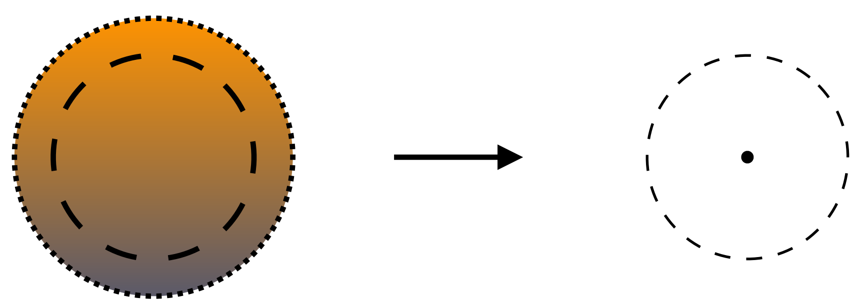
state of radiated quantum depends on details at the surface

How did we bypass the various no-hair theorems?

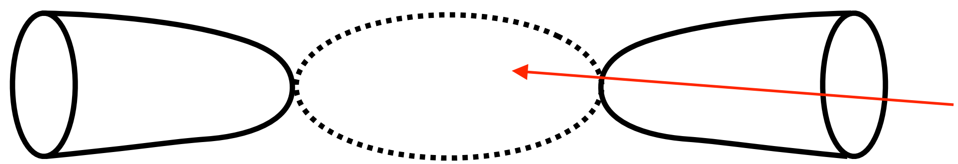
There are special features of a theory like string theory which has extra dimensions/ extended objects/Chern-Simmons terms etc. (Gibbons+Warner 13)

What about Buchdahl theorem? Fluid sphere with pressure decreasing outwards must collapse if

$$R < \frac{9}{4}M$$



Toy model: Euclidean Schwarzschild plus time ('neutral fuzzball')



$$ds^2 = -dt^2 + (1 - \frac{r_0}{r})d\tau^2 + \frac{dr^2}{1 - \frac{r_0}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ , \qquad 0 \leq \tau < 4\pi r_0$$

We can reduce on the direction τ to get a scalar field in 3+1 gravity.

The stress tensor is the standard one for a scalar field $T_{\mu\nu} = \Phi_{,\mu}\Phi_{,\nu} - \frac{1}{2}g^E_{\mu\nu}\Phi_{,\lambda}\Phi^{,\lambda}$

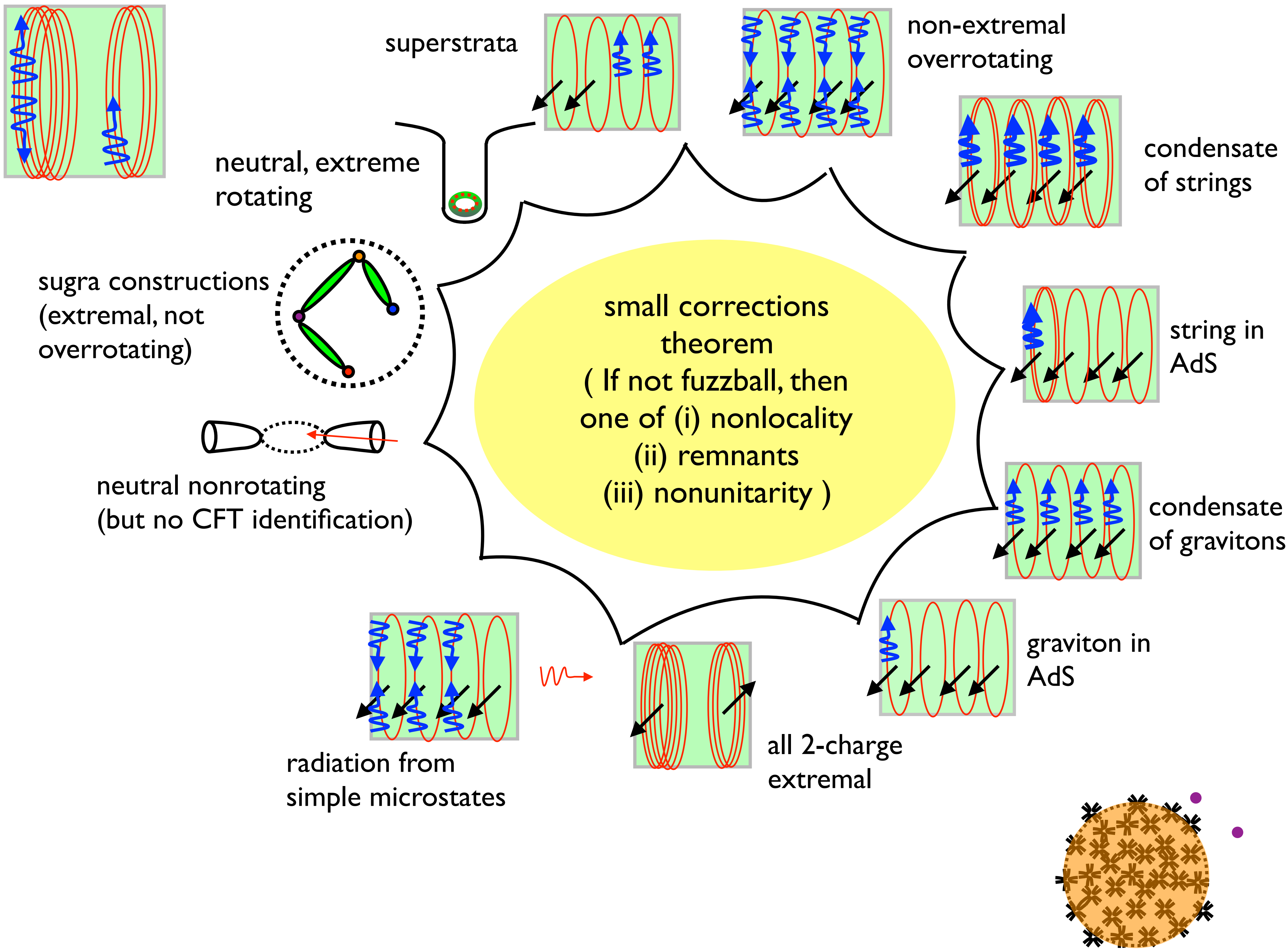
Why does this shell of scalar field not collapse inwards ?

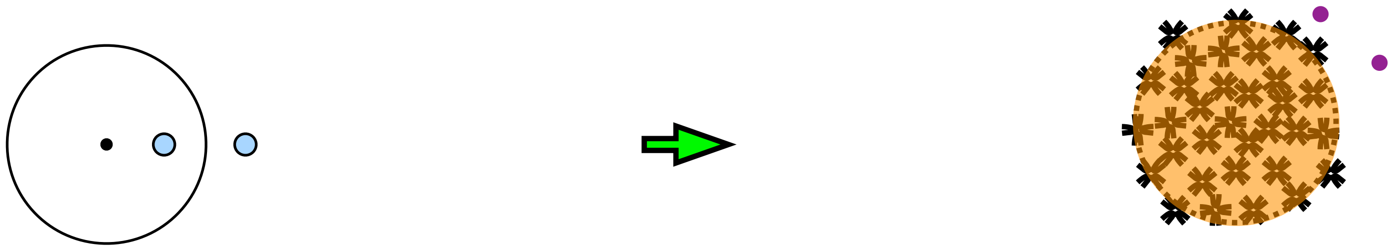
$$T^\mu{}_\nu = \mathrm{diag}\{-\rho, p_r, p_\theta, p_\phi\} = \mathrm{diag}\{-f, f, -f, -f\} \, , \qquad f = \frac{3r_0^2}{8r^4(1-\frac{r_0}{r})^{\frac{3}{2}}}$$

Pressure diverges at tip of cigar so a Buchdahl type analysis would call this a singularity

But the 4+1 dimensional solution is completely regular

(g_{tt} never changes sign, so there is no horizon)


superstrata
non-extremal
overrotating
condensate
of strings
neutral, extreme
rotating
sugra constructions
(extremal, not
overrotating)
small corrections
theorem
(If not fuzzball, then
one of (i) nonlocality
(ii) remnants
(iii) nonunitarity)
string in
AdS
neutral nonrotating
(but no CFT identification)
condensate
of gravitons
graviton in
AdS
all 2-charge
extremal
radiation from
simple microstates



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) + O(\epsilon)$$

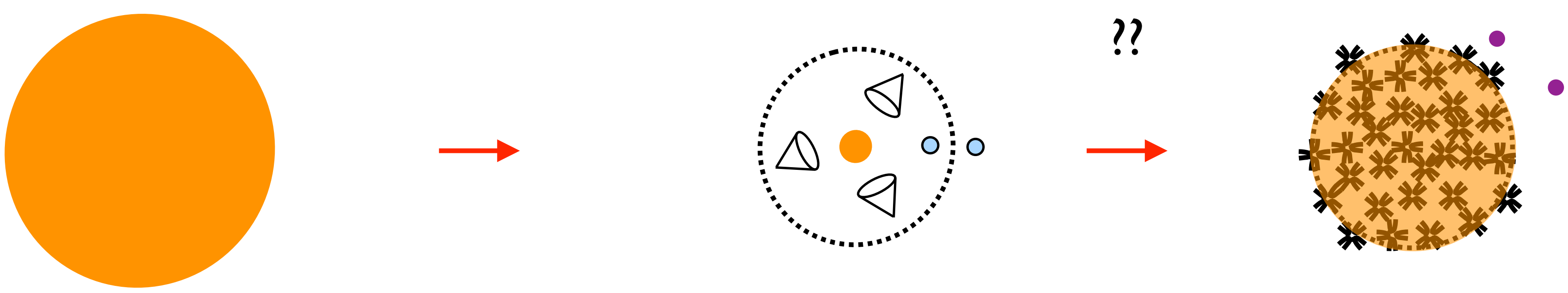
state of radiated quantum depends on details at the surface

This resolves the information paradox ...

Virtual fluctuations of fuzzballs

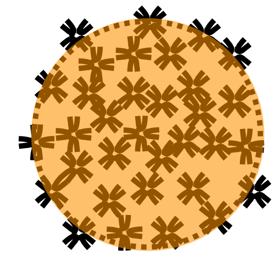
OR

What goes wrong with the semiclassical approximation?



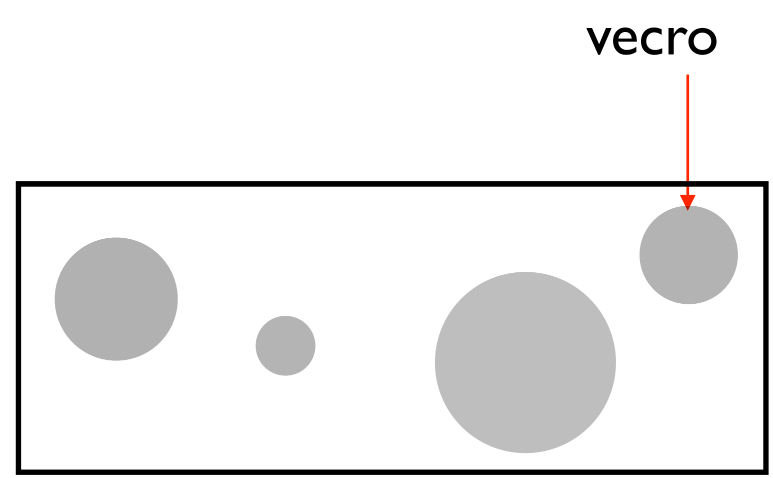
The argument

(A) If fuzzballs exist as on-shell configurations describing the microstates of black holes, then the gravitational vacuum must contain virtual fluctuations corresponding to fuzzball type configurations



(B) Fluctuations corresponding to large mass configurations are expected to be suppressed. But the degeneracy of such configurations rises quickly with the energy, similar to the large degeneracy of on-shell states

$$\mathcal{N} \sim e^{S_{bek}}$$



(C) Thus the gravitational vacuum has these extended-size fluctuations everywhere

VECRO: Virtual Extended Compression-Resistant Objects

Why are large vecro fluctuations $R_v \gg l_p$ not suppressed?

The fluctuation to any large fuzzball type configuration is indeed highly suppressed:

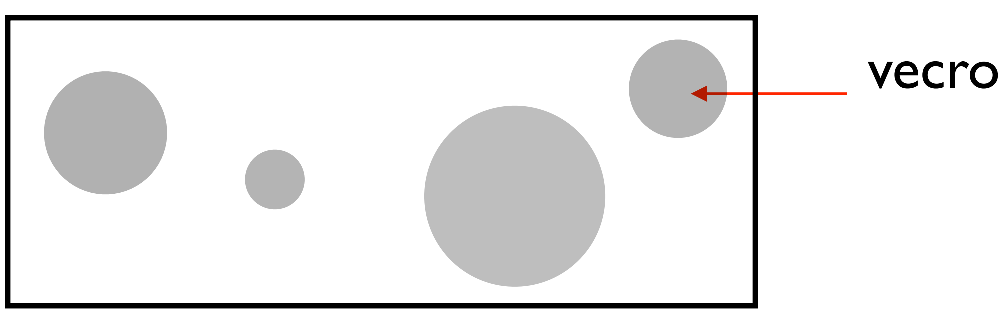
$$P \sim e^{-S} \sim e^{-ET}$$

$$E \sim M \sim \frac{R^{d-2}}{l_p^{d-1}}, \quad T \sim R \quad \longrightarrow \quad S \sim \left(\frac{R}{l_p}\right)^{d-1}$$

But there are a very large number of such fuzzball type configurations:

$$\mathcal{N} \sim e^{S_{bek}} \quad , \quad S_{bek} \sim \frac{A}{G} \sim \left(\frac{R}{l_p}\right)^{d-1}$$

Thus we can have $P\mathcal{N} \sim 1$; i.e., the suppression is offset by the large degeneracy



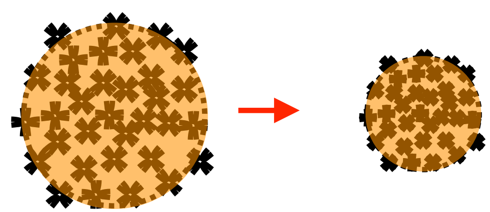
Compression-Resistance: Fuzzball configurations are resistant to compression or stretching

Suppose we have a fuzzball of radius R

Suppose we compress or expand this by a factor of order unity $R \to (1 \pm \delta) R$

Several indirect arguments about fuzzballs suggest that the energy cost if of the order of the mass of a black hole of radius R

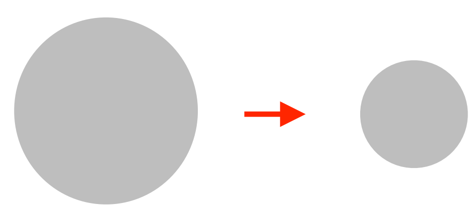
$$R \to (1 \pm \delta) R$$



$$\Delta E \sim M(R)$$

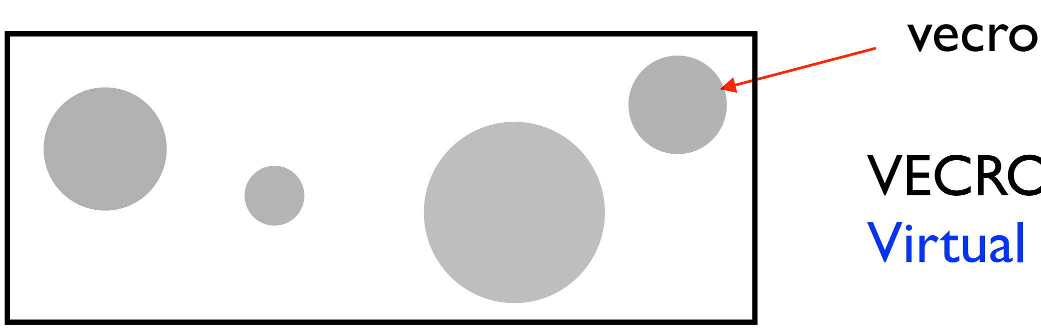
We assume that a similar property holds for virtual fuzzballs (vecros)

$$R_v \to (1 \pm \delta) R_v$$



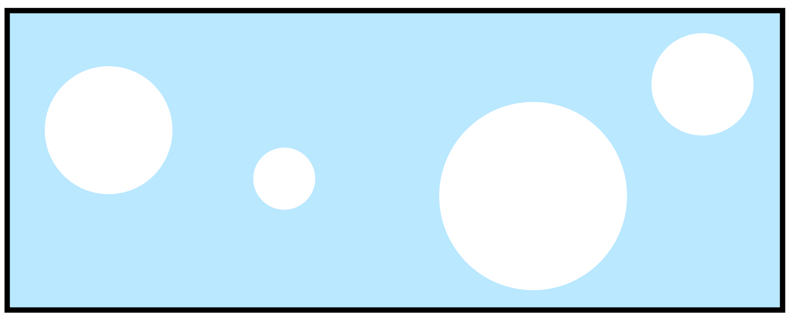
$$\Delta E \sim M(R_v)$$

We get the following picture of the vacuum of quantum gravity

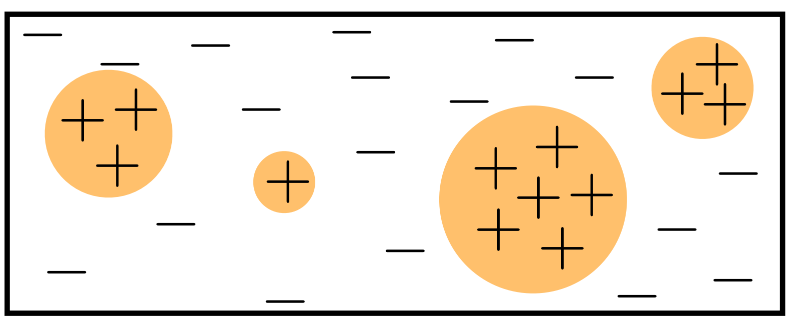


VECRO:
Virtual Extended Compression-Resistant Objects

So in the fuzzball paradigm the vacuum has a structure similar to that in phase transitions ...



Steam bubbles form in water near boiling point

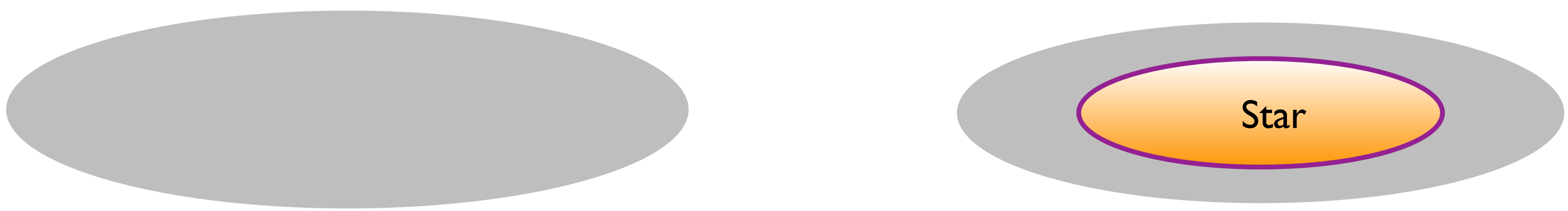


Ising model near criticality

(We cannot get this in a theory with no fuzzballs; e.g. 3+1 canonically quantized gravity)

What is the effect of this vecro structure of the vacuum ?

First consider a low curvature object like a star



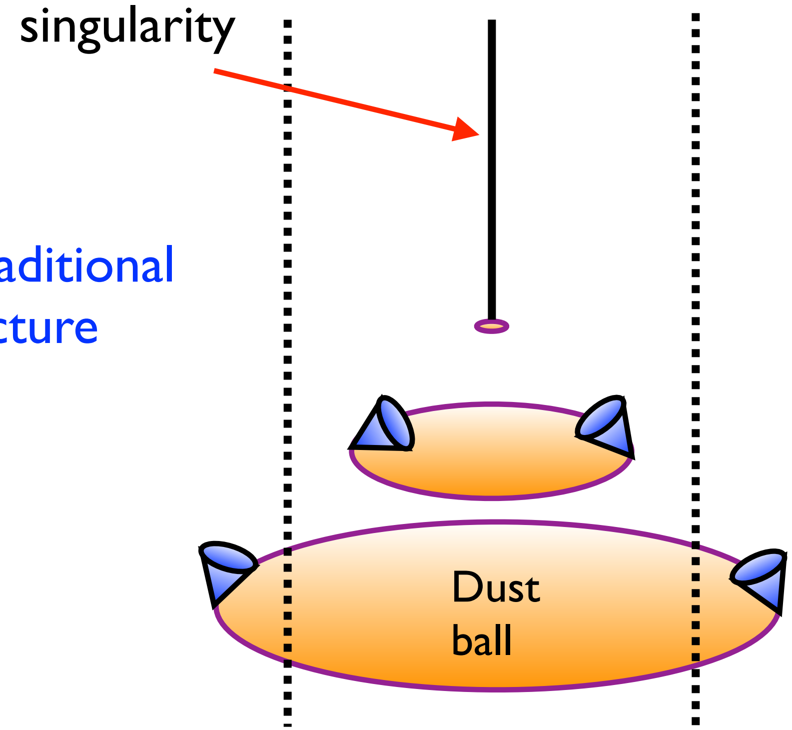
A star has a weak gravitational pull

So the vecro compresses slightly and stabilizes. This distortion of the wavefunctional is included in the Einstein action dynamics

Let us now ask what happens in a black hole ....

This is the semiclassical picture of black hole formation

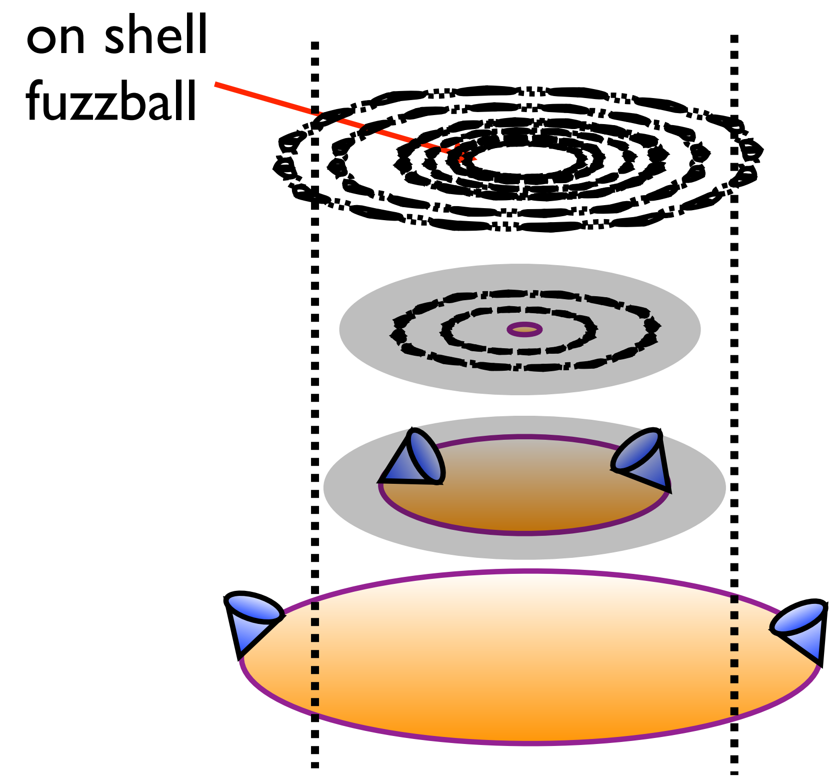
Traditional picture



But now we have to consider what the vecro fluctuations are doing in this spacetime ....

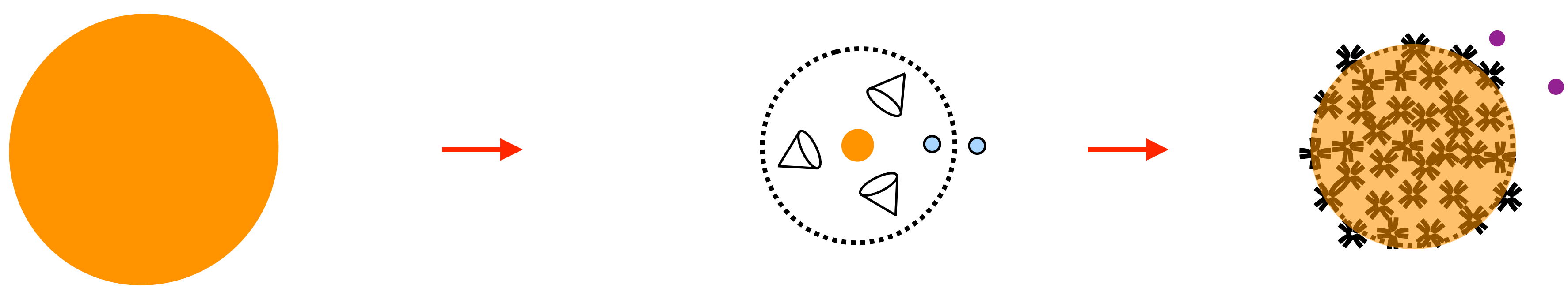
Inside the horizon, the light cones point inwards, so a vecro must keep on compressing (i.e., it cannot stabilize in size) ....

Inside a closed trapped surface, the vecros are forced to keep compressing



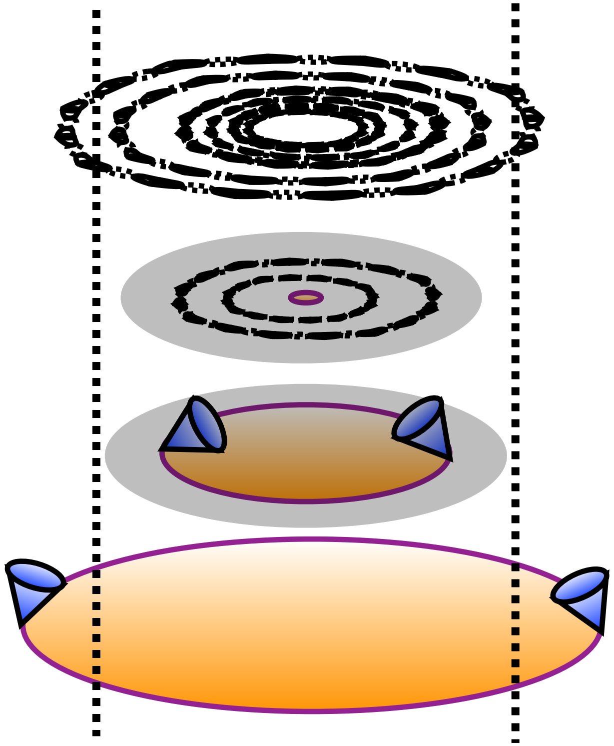
The resulting distortion of the vacuum wavefunctional turns the vecros to on-shell fuzzballs

This is analogous to how a large distortion of a vacuum mode of scalar field gives pair creation



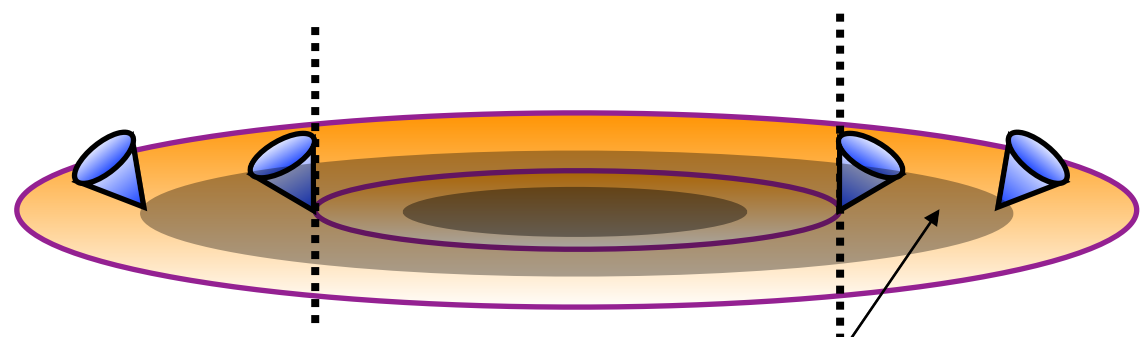
The extended nature of the vecro fluctuation allows it to detect the formation of a closed trapped surface, violating the equivalence principle

Applying this to cosmology



With a black hole, vecros inside a closed trapped surface were forced to compress

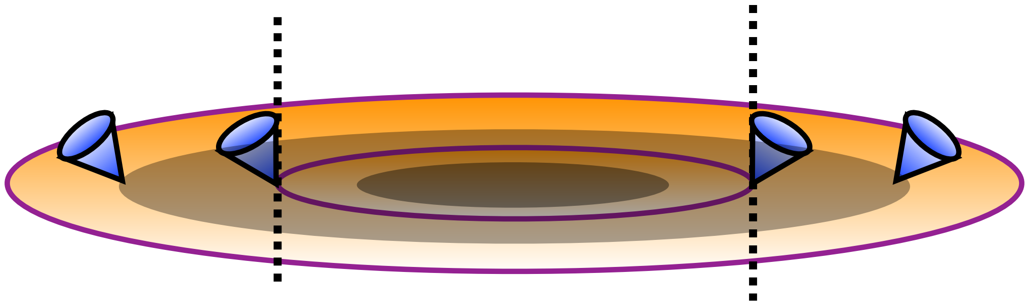
In a cosmology, vecros larger than the cosmological horizon will be forced to stretch



Vecros with radii lager than the horizon

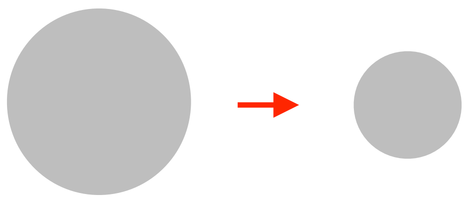
$$R_v > H^{-1}$$

forced to stretch



Vecros with radii lager than the horizon

$R_v > H^{-1}$ stretch



R_v $(1 \pm \delta) R_v$

(Order unity compression or stretching)

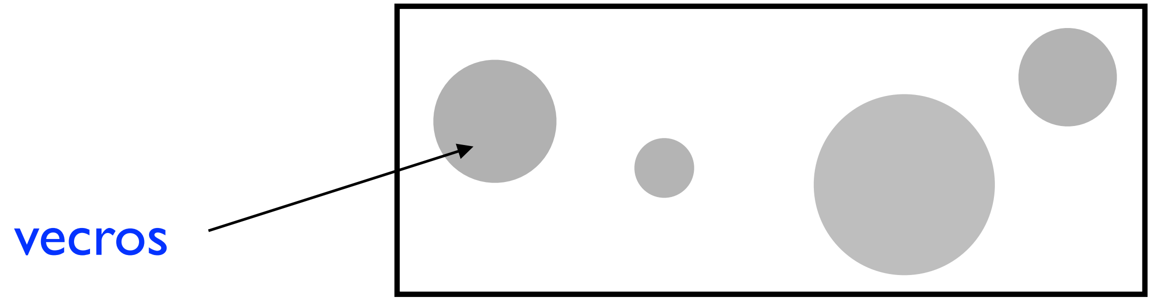
$\Delta E \sim M(R_v)$

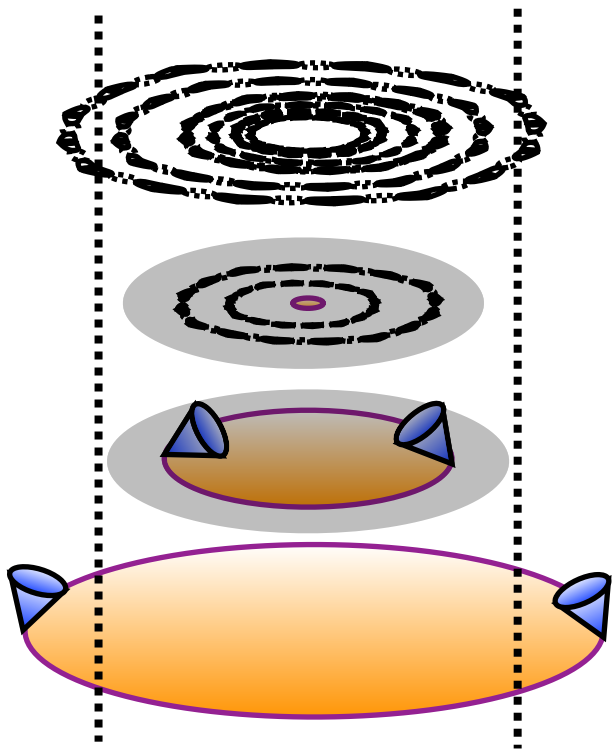
The energy from this stretching will be the source of energy for the effects we seek

(Energy for inflation, dark energy ...)

At this point we note an important difference between Minkowski spacetime and the cosmological spacetime ...

In Minkowski space, we have vecros with all radii $0 < R_v < \infty$





This is necessary in order that we can resolve the Hawking puzzle for black holes of all sizes

But in an expanding cosmology, extended structures cannot have a radius larger than the distance that light has been able to travel since the big bang

We take a flat cosmology

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2 d\Omega^2]$$

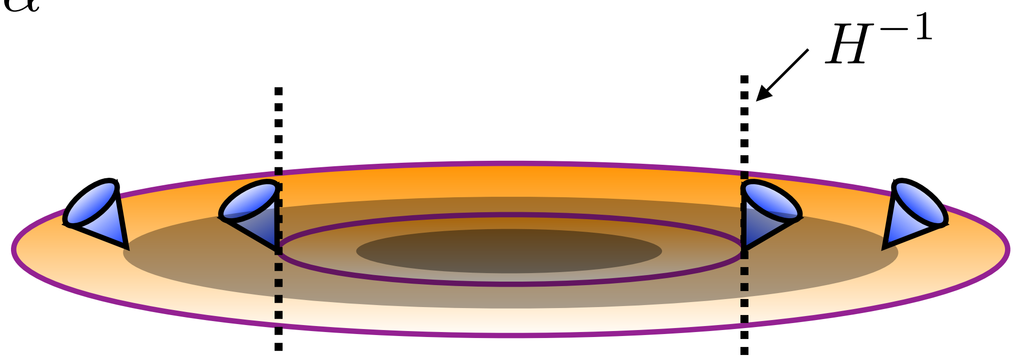
$$a(t) = a_0 t^\alpha, \quad H = \frac{\dot{a}}{a} = \frac{\alpha}{t}, \quad H^{-1} = \frac{t}{\alpha}$$

The distance that light has travelled since the big bang is

$$R_{max}(t) = a(t)\int_{t'=0}^{t} \frac{dt'}{a(t')} = \frac{t}{1-\alpha}$$

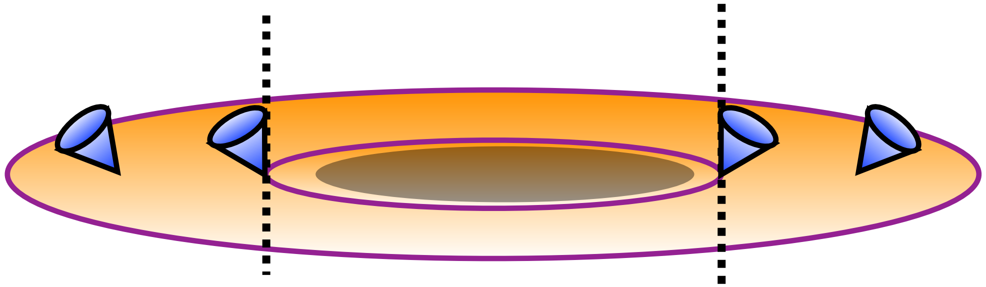
$$R_v \leq R_{max} \longrightarrow \frac{R_v}{H^{-1}} \leq \frac{\alpha}{1-\alpha}$$

So we can form vecros with
all radii upto this value



(A) $$\frac{R_v}{H^{-1}} \leq \frac{\alpha}{1-\alpha}$$

In the radiation phase, $\alpha = \frac{1}{2}$ so $\frac{R_v}{H^{-1}} \leq 1$

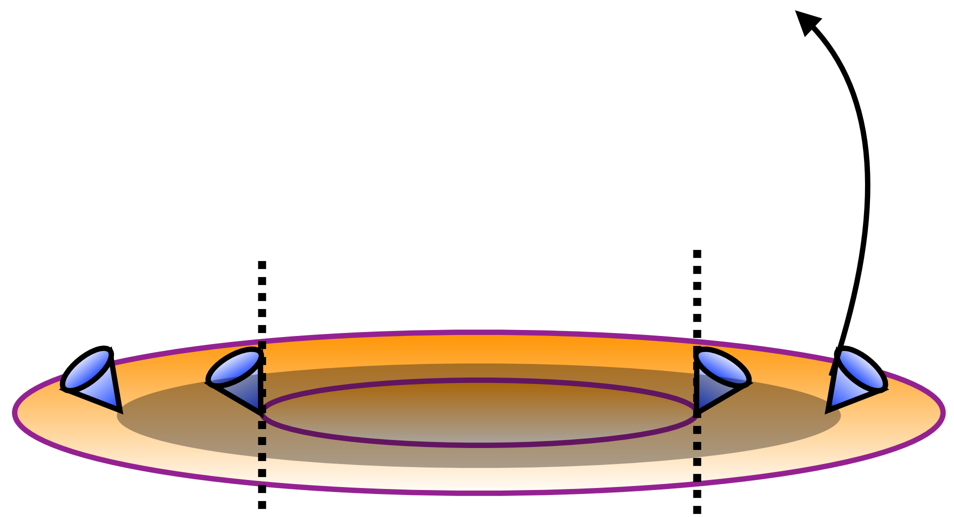


So there cannot be any energy from the forcible stretching of vecros in this phase ..

This is important, since observations do not allow much freedom in the amount of energy in this phase of the universe

(Extra energy would lead to faster expansion, which would alter the predictions of Big Bang Nucleosynthesis (BBN))

(B) $$\frac{R_v}{H^{-1}} \leq \frac{\alpha}{1-\alpha}$$



For a dust cosmology, $\alpha = \frac{2}{3}$

So vecros in the range

$$1 \leq \frac{R_v}{H^{-1}} \leq 2$$

will be forced to stretch

Suppose that at time $t = t_0$ the radius of the cosmological horizon is $H_0^{-1} \equiv R_{h,0}$

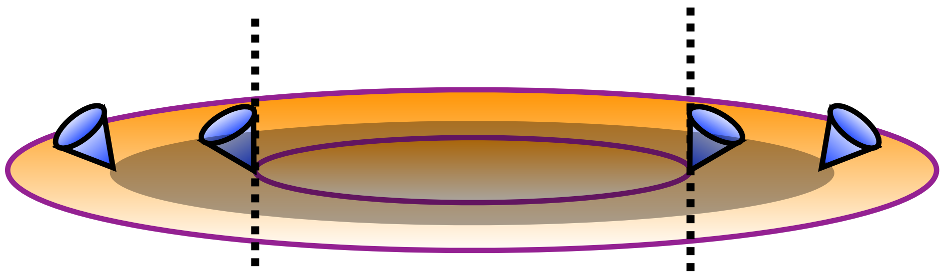
Let the vecro start with a radius $R_0 > R_{h,0}$

Then it must stretch to at least the radius

$$R_{max} = R_{h,0} \left((1-\alpha) \frac{R_0}{R_{h,0}} + \alpha \right)^{\frac{1}{1-\alpha}}$$

What happens at the time when the radiation phase turns into the dust phase?

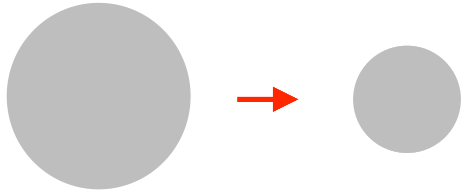
When the radiation phase turns to the dust phase, there will be a stretching of vecros



This will lead to an extra energy in the universe which is not part of our usual semiclassical stress tensor

How much is this energy ?

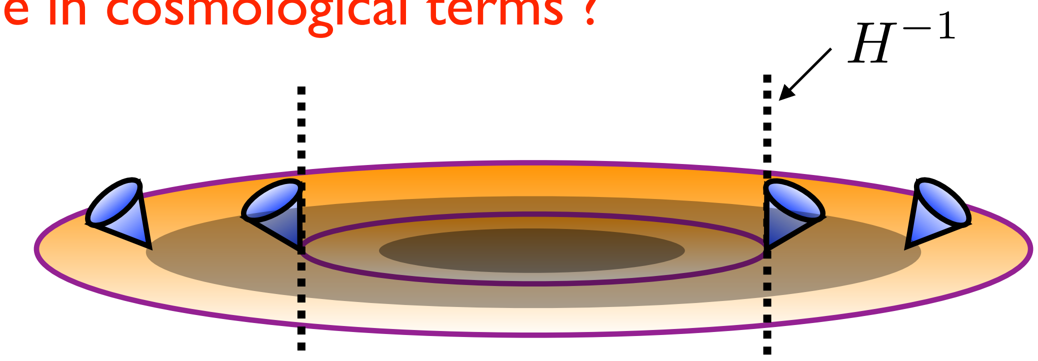
We had noted the scale of energies associated with an order unity distortion of the vecro distribution function



R_v $\quad (1-\delta)R_v$

$$\Delta E \sim M(R_v)$$

What is the significance of this energy scale in cosmological terms ?



Setting $R_v \sim R_h = H^{-1}$

we get $$\Delta E \sim M(R_h) \sim \frac{R_h}{G}$$

This corresponds to an energy density $$\Delta\rho \sim \frac{1}{R_h^3}\frac{R_h}{G} \sim \frac{H^2}{G}$$

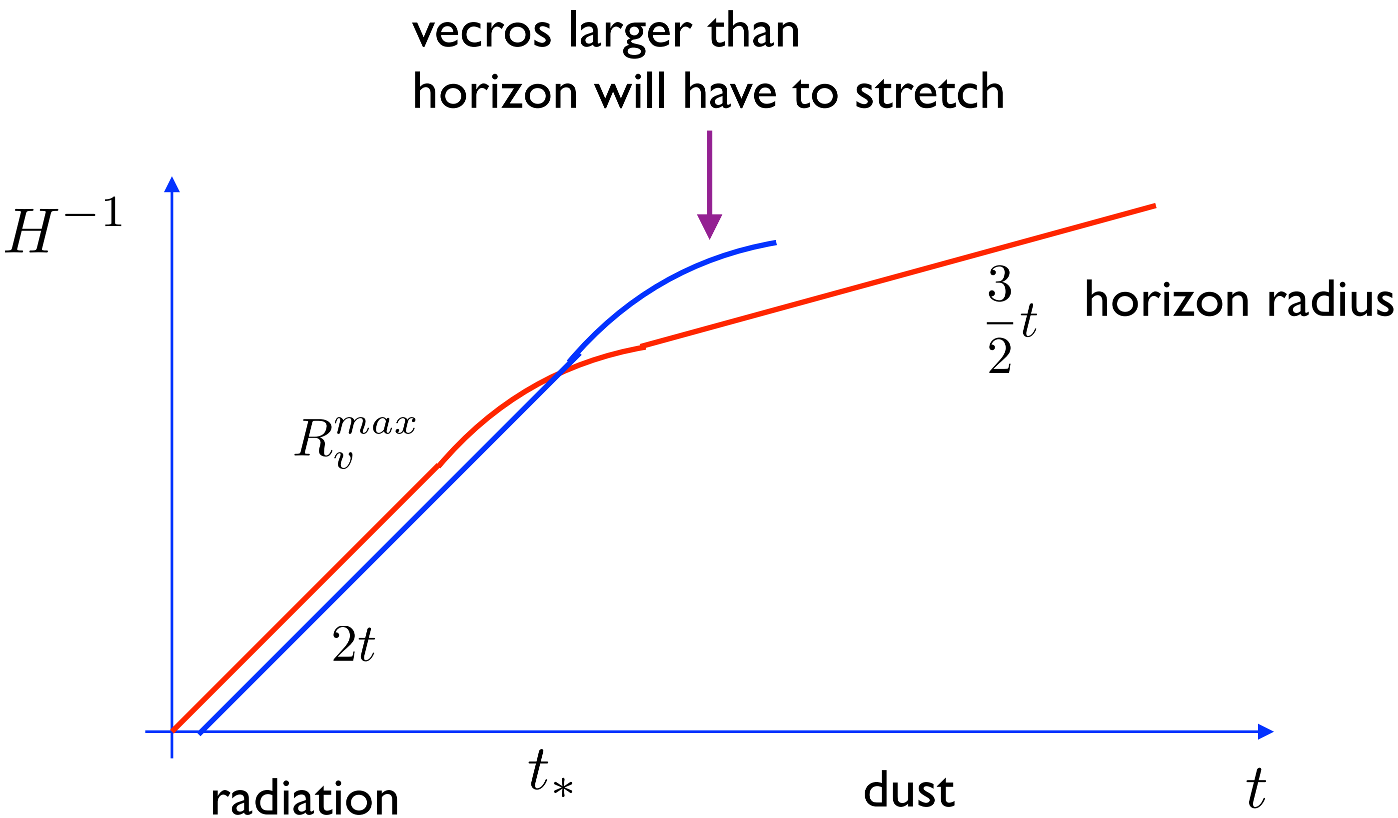
The Hubble expansion law gives $H^2 \sim G\rho_{closure}$

Then we find that $\Delta\rho = \mu\,\rho_{closure}$

Here μ is order unity and $\rho_{closure}$ is the closure energy density of the universe at the given epoch

We have seen that there is no stretching energy from vecros in the radiation phase, while there can be such an energy in the dust phase

How much is this energy?



If the changeover were adiabatic, then the vecro distribution will adjust to a minimum energy one, and there will be no extra energy.

If the changeover were sudden, we will get a stretching of horizon scale vecros by a factor of order unity

The actual changeover in the power law is somewhere in between, happening over a few Hubble timescales

Thus we expect to get an energy from the vecro distribution that is some small fraction of the closure density at the epoch of matter radiation equality $z \approx 2500$

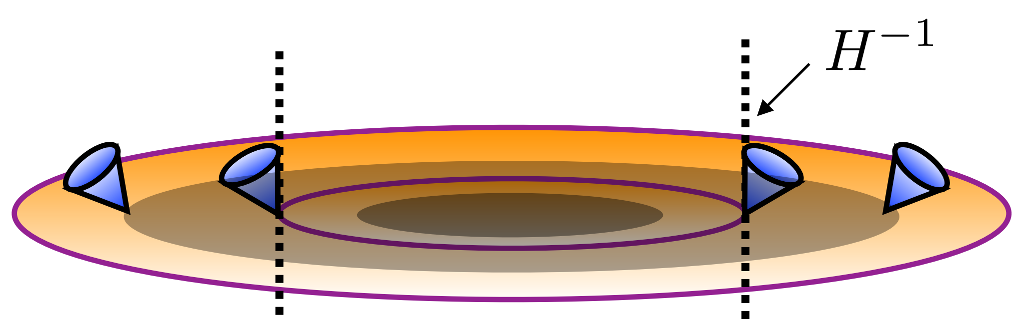
Interestingly, just this scale of energy is required at around just this epoch to resolve a tension in two different observations of the Hubble constant

Measurements on local objects suggest $H_0 \approx 74\, Km/s/Mpc$

The $\Lambda\mathrm{CDM}$ model applied to cosmic microwave background measurements suggests $H_0 \approx 67\, Km/s/Mpc$

An extra energy density of order $\sim 10\%$ of the closure density at t_* can explain this tension ... such energy is called Early Dark Energy (EDE)

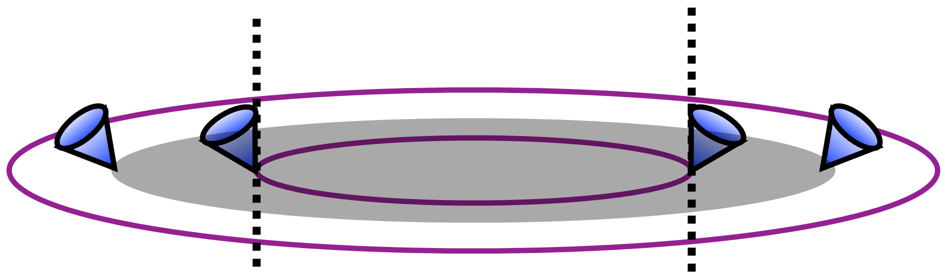
We see that the energy required may arise naturally from the dynamics of vecros ...



(C) Dark energy ?

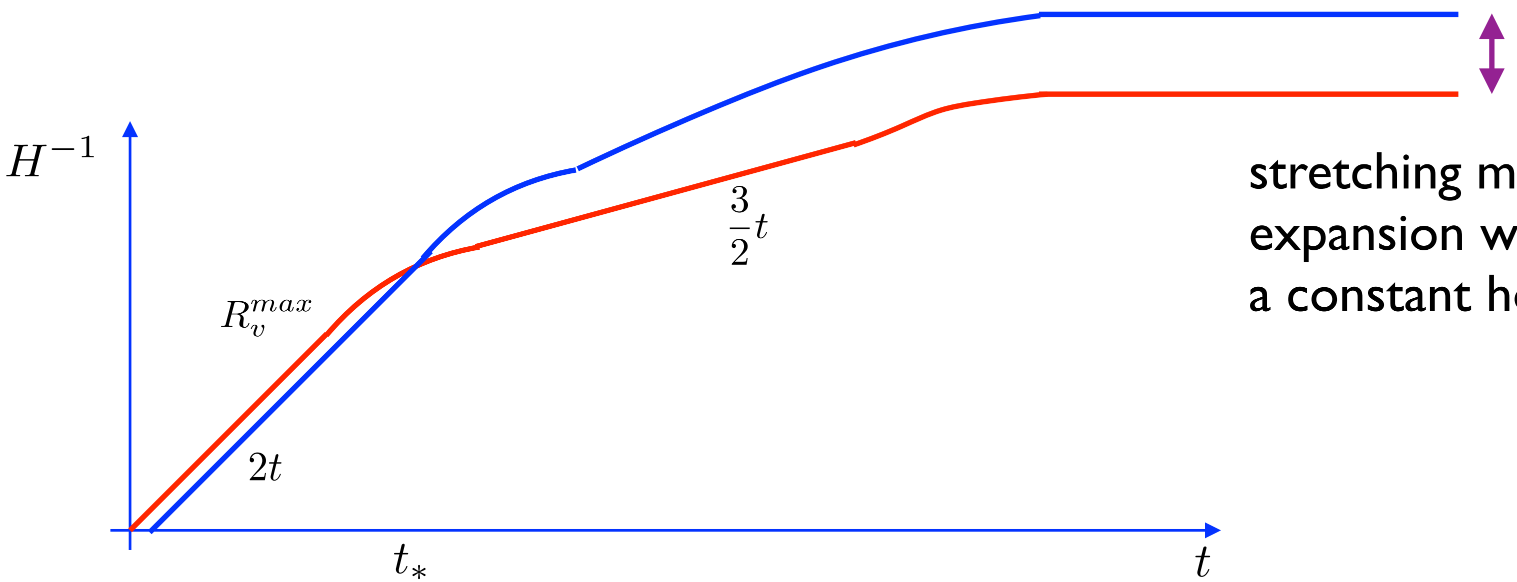
We have noted that $\Delta\rho = \mu\,\rho_{closure}$

from the stretching of vecros



Suppose the stretching of vecros is such that $\Delta\rho = \rho_{closure}$

Then we do not need any matter to support the expansion: the energy of stretching vecros maintains the expansion



stretching maintains the expansion with a constant horizon radius

Then we get a universe with expansion dominated by dark energy ....

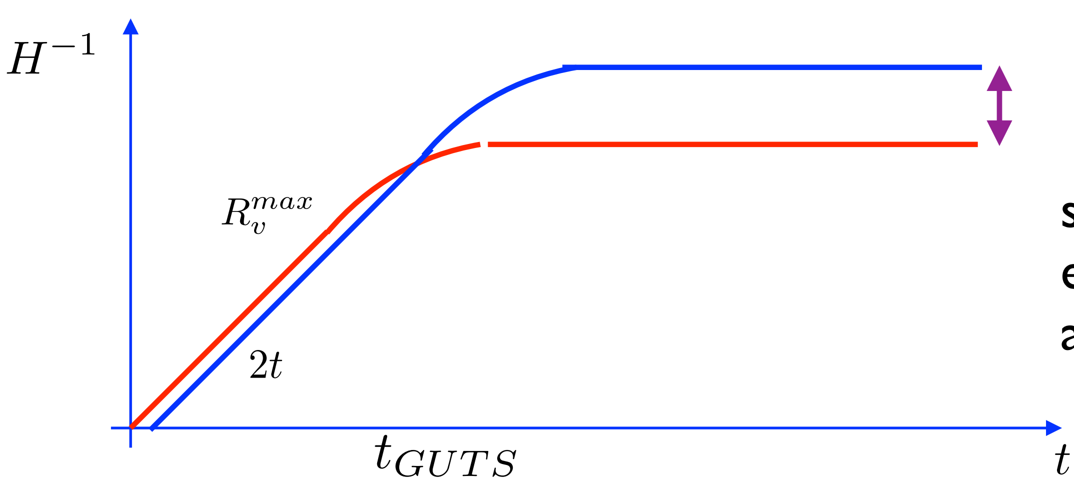
(D) Inflation ?

We have seen that we get a stretching of vecros when the slope of the Hubble expansion decreases suddenly

This happens when the pressure drops suddenly

At the end of the GUTS epoch, the heavy GUTS particles become nonrelativistic,, leaving only the light standard model particles to provide pressure

We again have the possibility of getting stuck in a phase where the energy of vecro stretching maintains the expansion, with a fixed Hubble radius



$$\Delta\rho = \rho_{closure}$$

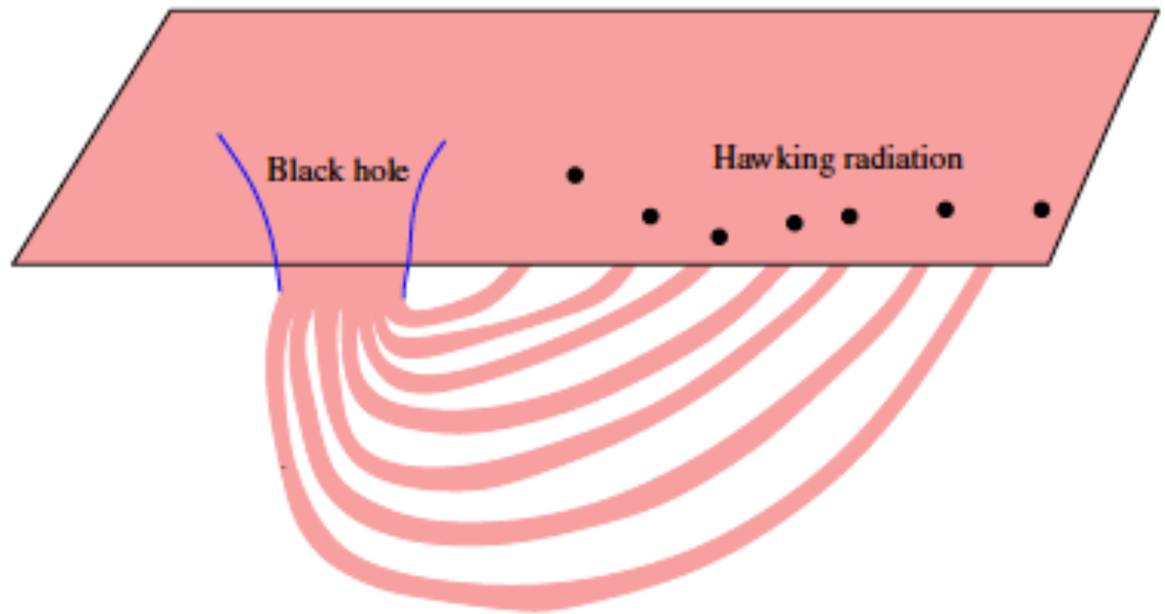
stretching maintains the expansion with a constant horizon radius

Are there any alternatives?

Some people looked for an alternative resolution of the paradox

→ The wormhole paradigm

Nonlocal effects between
the hole and infinity
are responsible for resolving
the information paradox

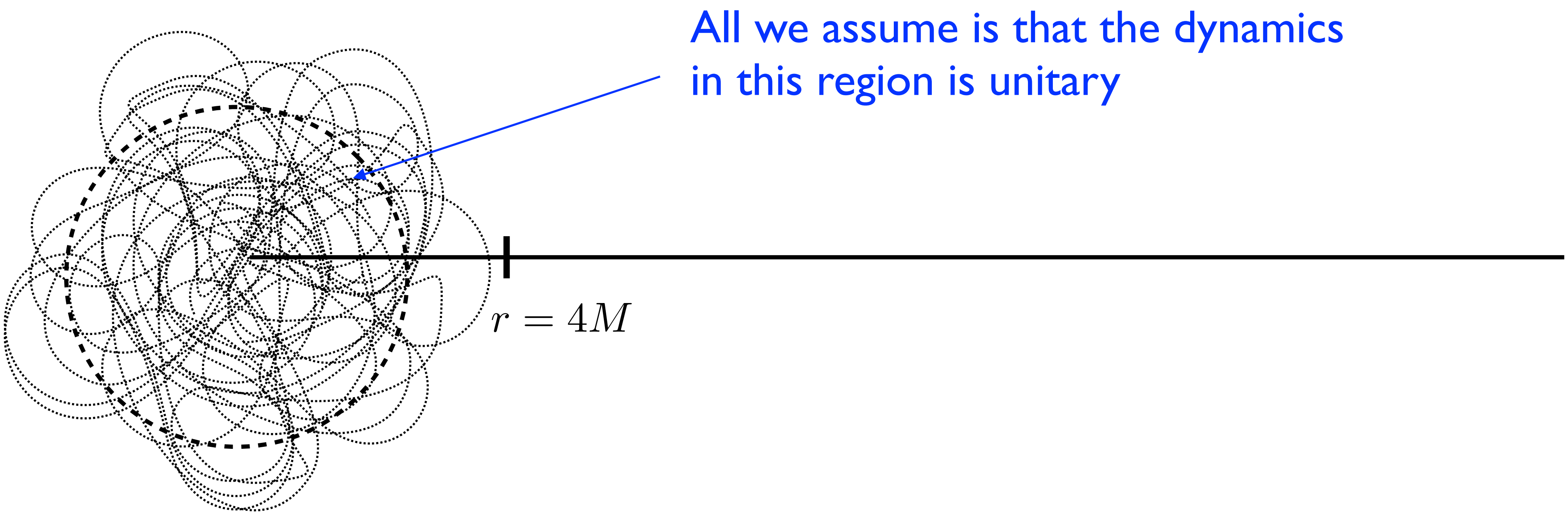


The goal of this approach is to somehow maintain a smooth horizon, where low energy semiclassical dynamics holds just like in this room

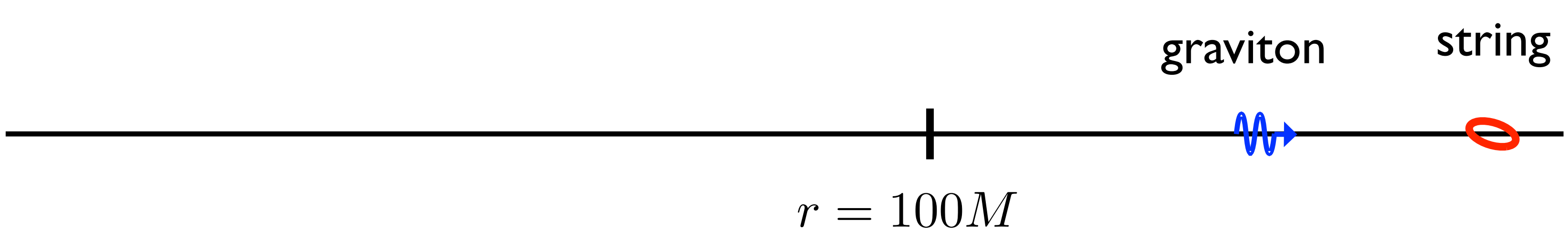
It is important to understand that having such semiclassical dynamics FORCES one to require nonlocal effects in the exact quantum gravity theory

Consider a scenario that is tempting but which does NOT work:

(A) The EXACT description of the black hole is some very complicated string theory dynamics in the region of the hole (say $r \lesssim 4M$)

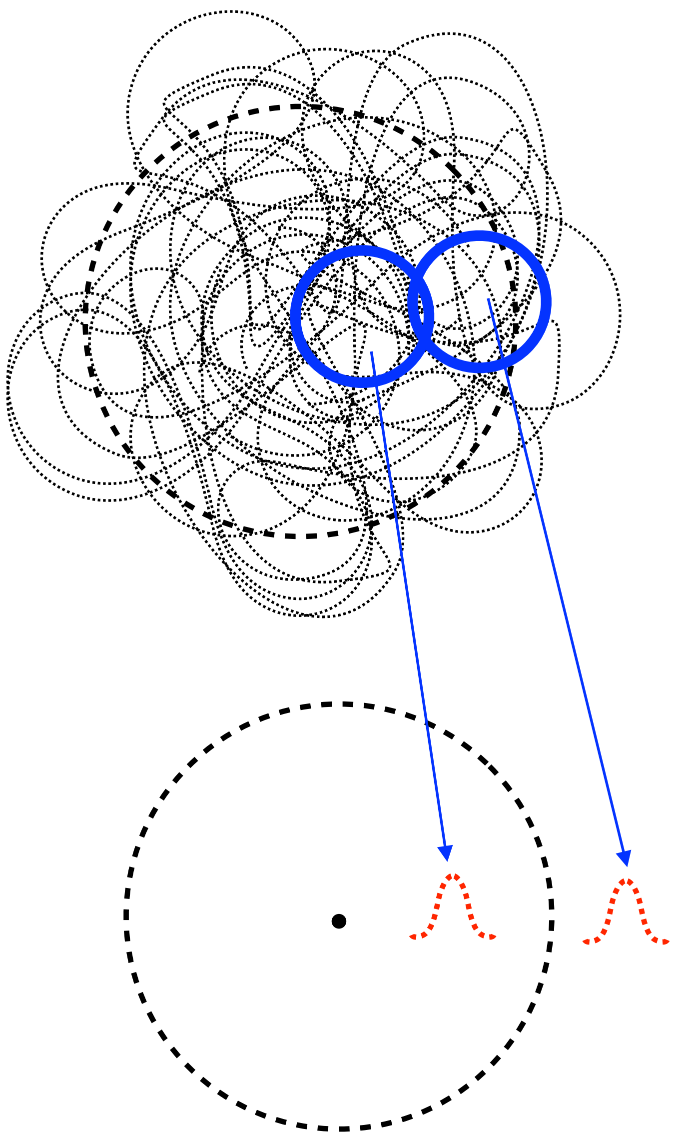


(B) Far from the hole (say $r \gtrsim 100M$) the EXACT dynamics is given by standard quantization of string theory around flat spacetime (no novel nonlocal effects)



(C) We can extract an EFFECTIVE SEMICLASSICAL DYNAMICS from these EXACT degrees of freedom.

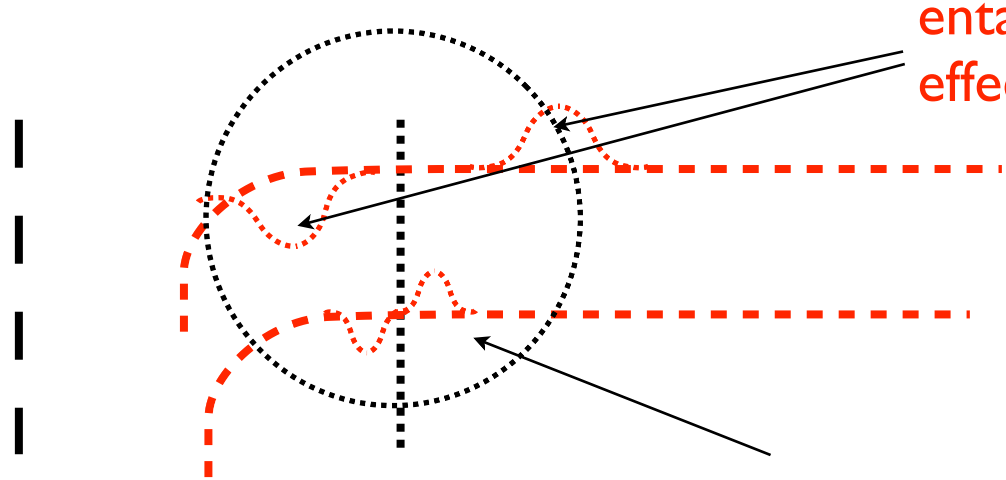
Let these semiclassical degrees of freedom describe a scalar field



Some complicated combinations of these EXACT degrees of freedom yield the effective semiclassical quanta

$$\hat{\phi} = \sum_k \left(\hat{a}_k f_k(x) + \hat{a}_k^\dagger f_k^*(x) \right)$$

$$\Box \hat{\phi} = 0$$



$$\frac{1}{\sqrt{2}}(|0\rangle|0\rangle + |1\rangle|1\rangle) + O(\epsilon)$$

We will be forced to get the creation of entangled pairs in the effective description

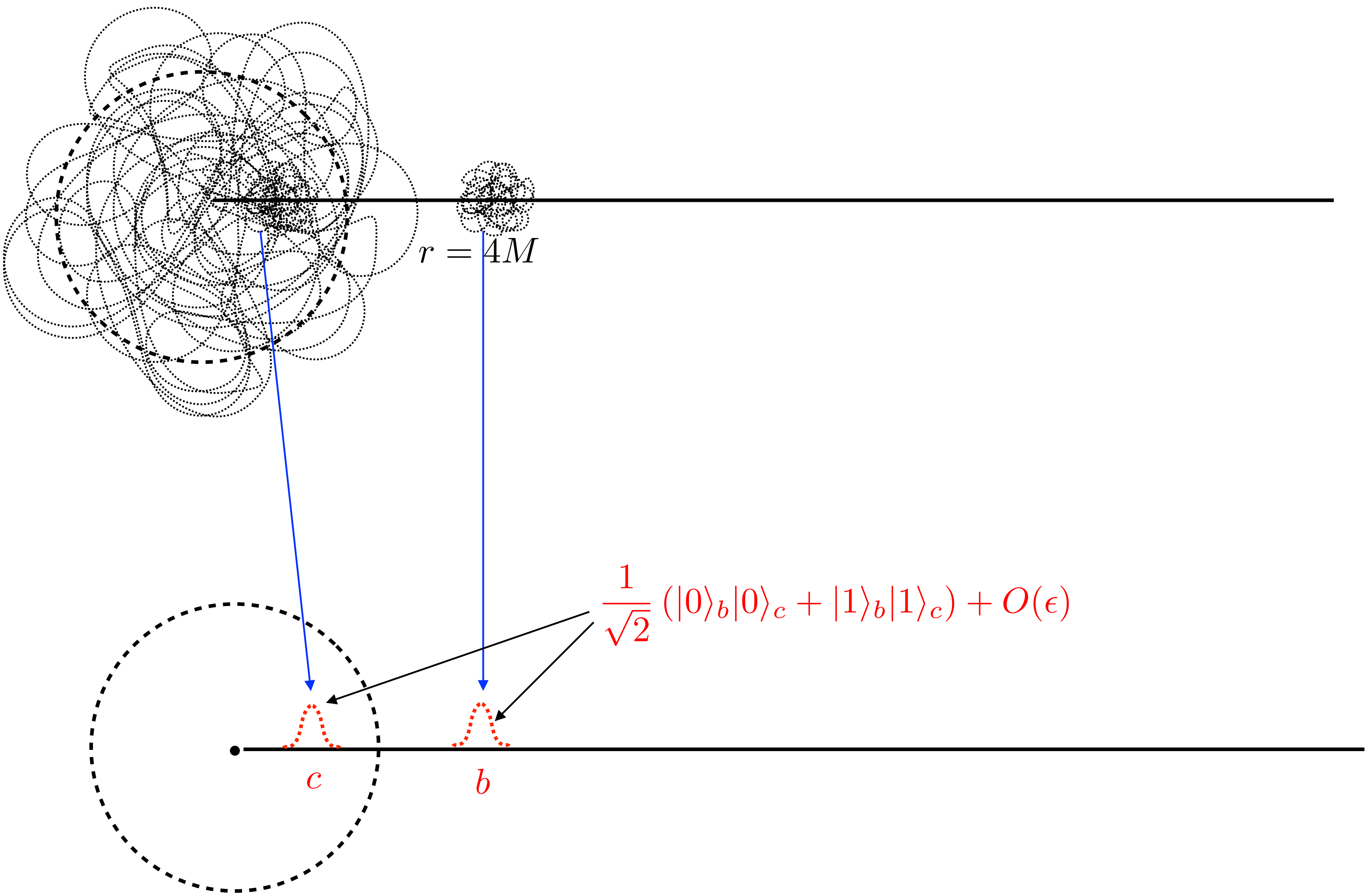
(i) We only ask that this effective field work for low energy modes, say wavelengths

$\lambda \gtrsim 1\,\mathrm{fermi}$

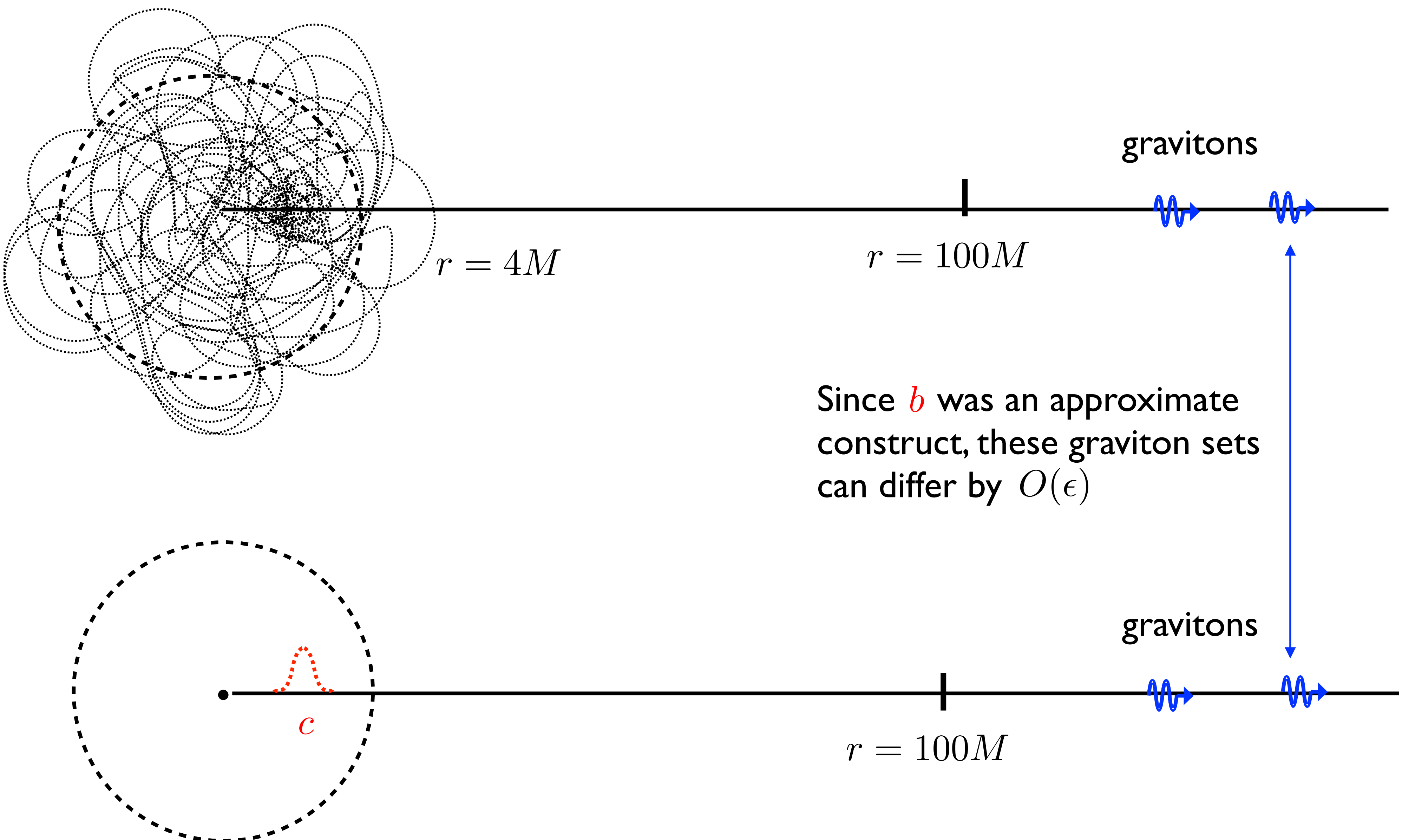
(ii) The dynamics of $\Box\hat{\phi} = 0$ needs to be reproduced only upto some approximation, specified by some a parameter $\epsilon \ll 1$

(iii) We do not even assume that this map work for all times: it just works over the spacetime region required to emit a few Hawking modes, so that we can use it for the information puzzle

(D) The following depicts what will happen with the above assumptions:



This situation will evolve as follows:



b travels to far region and must be written as normal excitations of the exact string theory

The effective small corrections theorem: (Guo, Hughes, SDM, Mehta, to appear)

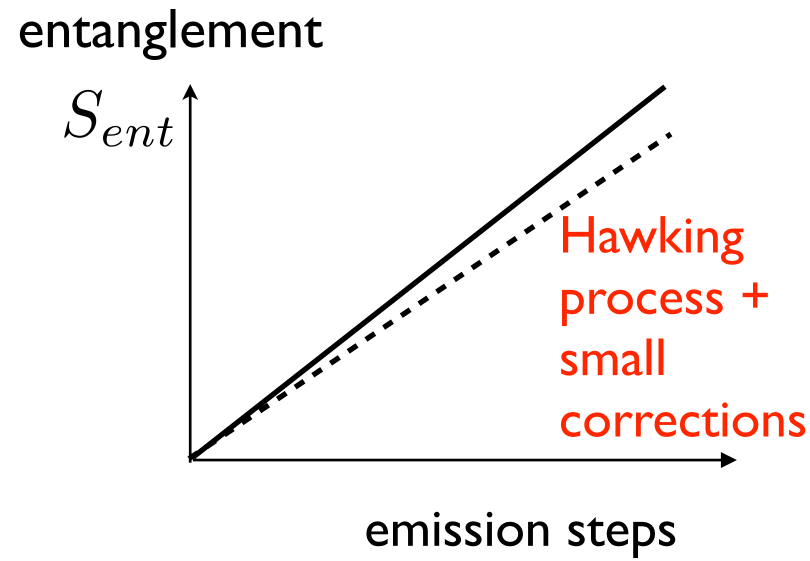
Under these assumptions, the entanglement graph in the EXACT quantum gravity theory will keep rising

(It is also rising in the approximate semiclassical theory)

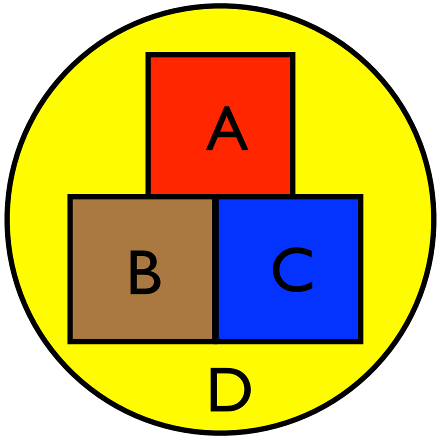
The entanglement at step N must keep growing as

$$S_{N+1} > S_N + \log 2 - 2\epsilon$$

(SDM 2009)

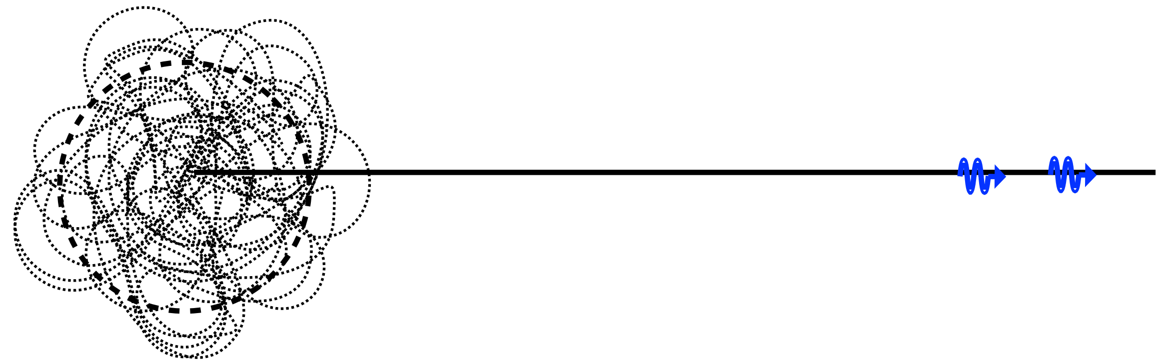


The result is not an obvious one since it uses the power of the strong subadditivity inequality of quantum entanglement entropy, which is nontrivial to prove



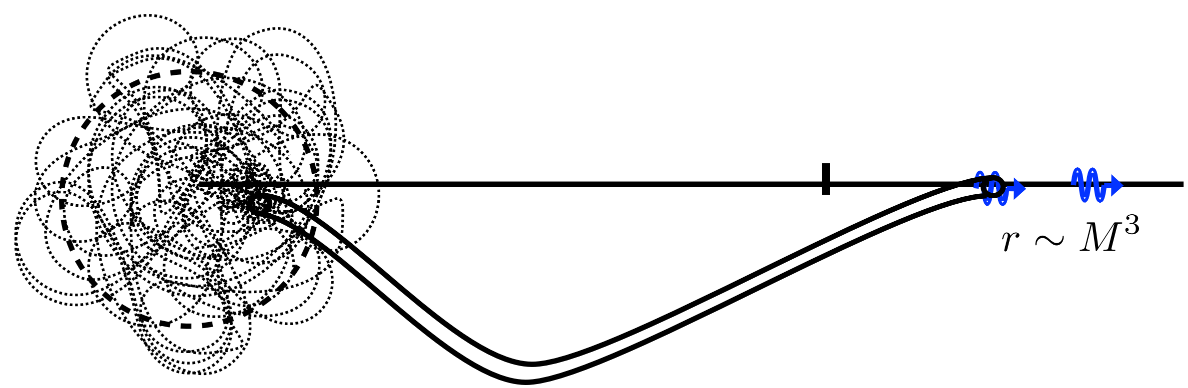
Thus the small corrections theorem leaves us with two sharply different possibilities

(1) The fuzzball paradigm:

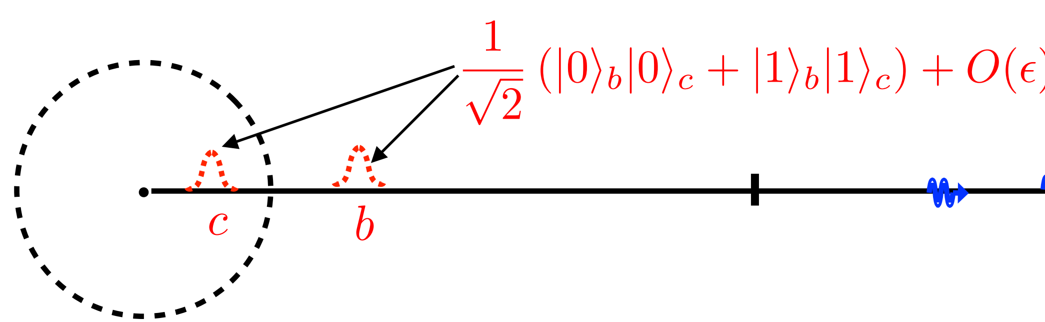


Just like a piece of coal; no effective description with semiclassical physics at horizon

(2) The wormhole paradigm:



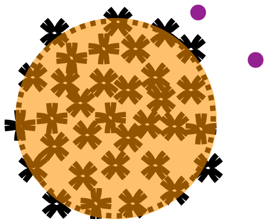
An effective semiclassical description can be obtained, at least for describing the emission of a few quanta



Then the EXACT theory MUST have nonlocal effects

Fuzzball paradigm:

Many exciting things to do !!



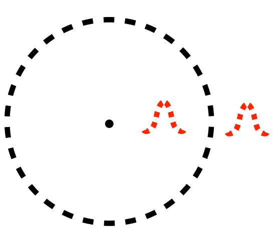
Good agreements of emission rates, scrambling, energy gaps, 3 and 4 point functions, observational signatures …

Approximate universality in $E \gg T$ limit ? (Fuzzball complementarity)

Constraints from causality: The VECRO hypothesis for the nature of the gravitational vacuum

Cosmology: Source of energy in inflation, Dark energy ...

Wormhole paradigm: I find that different approaches have one or more of the following features, which I find myself not willing to reconcile to:



Non-unitarity evolution in black hole interior in EXACT theory

Long-distance nonlocality of Hamiltonian interactions in EXACT theory

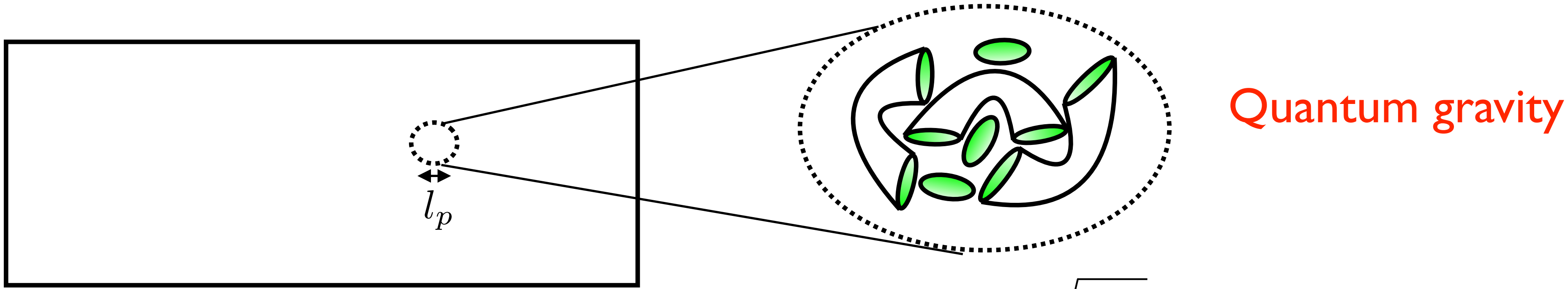
Remnants, baby universes in EXACT theory

Altered dynamics at infinity in EXACT theory: non-standard laboratory dynamics, nonlocal identification of bits, ensemble averaged theory ...

No map between EXACT theory and semiclassical approximation

SUMMARY

Traditional picture of the quantum gravity vacuum

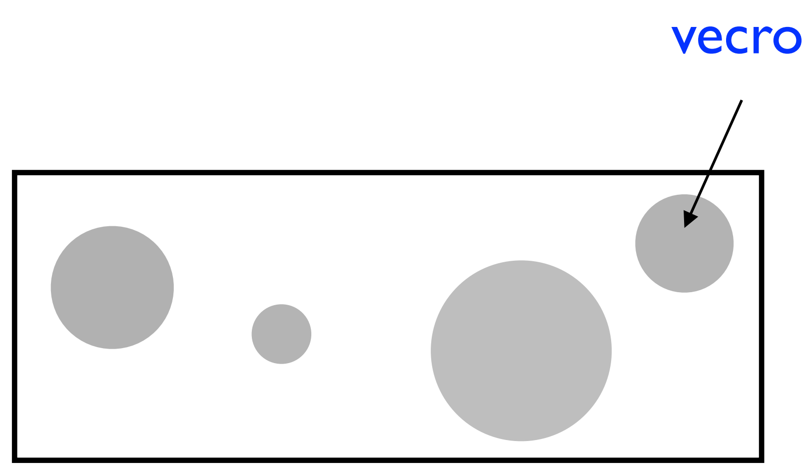


$$l_p \sim \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-33}\, cm$$

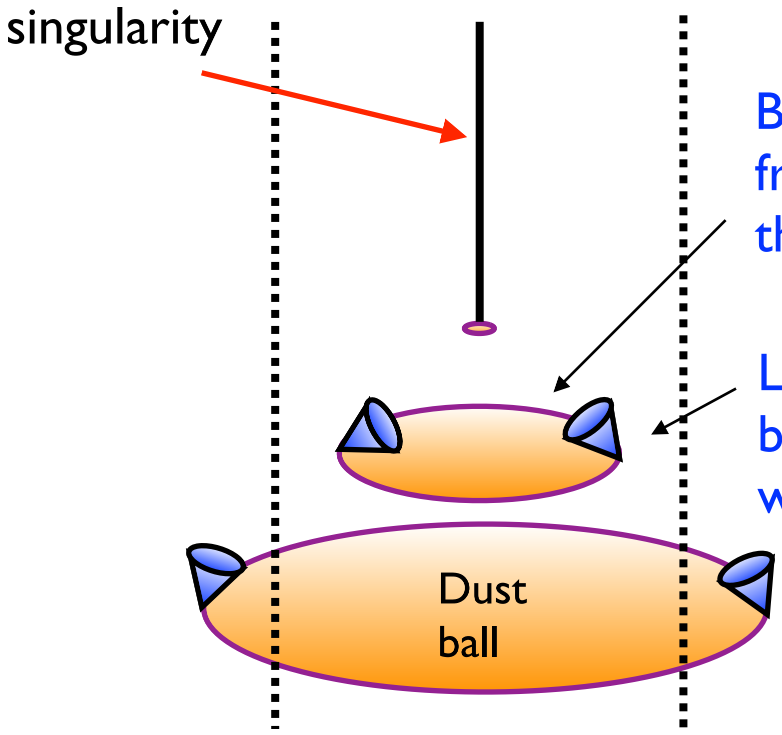
Quantum fields, Classical gravity

But this picture leads to the black hole information paradox

Resolving the paradox leads to a new picture of the gravitational vacuum



This new picture is forced, if we assume that causality holds to leading order for low energy dynamics in gently curved spacetime



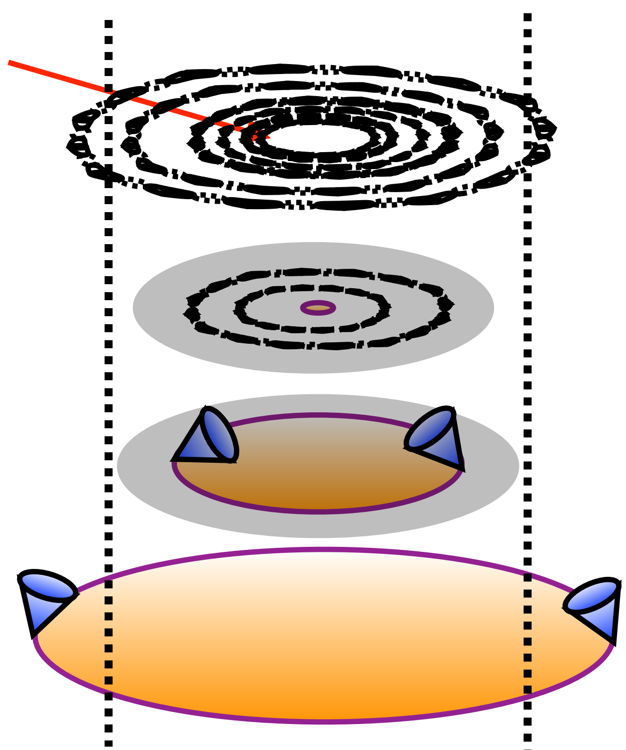
But no new physics from here can affect the horiozn

Low curvature, dust ball passes through without new physics

Small corrections cannot help

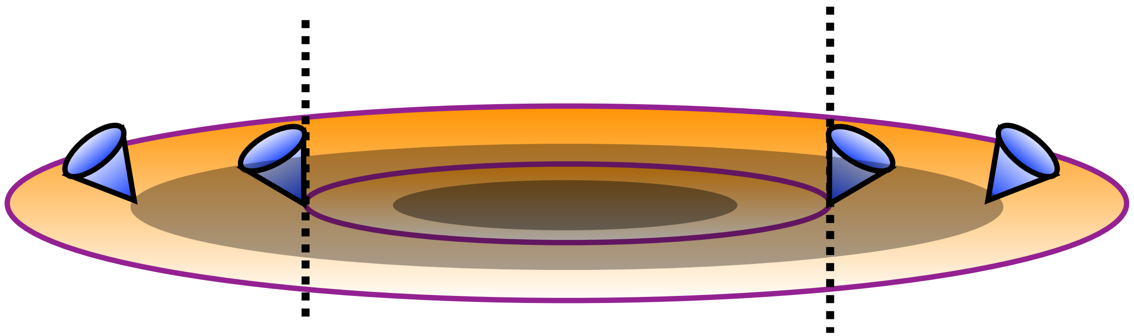
$$S_{N+1} > S_N + \log 2 - 2\epsilon$$

on shell fuzzball



The extended nature of the vecro fluctuation allows it to detect the formation of a closed trapped surface, violating the equivalence principle

In a cosmology, the vecro fluctuations cannot have arbitrary size since they cannot be larger than the particle horizon (the distance that light can travel since the big bang)



The stretching of vecros generates an extra energy at the scale of the cosmological horizon ...

But this is the scale at which we need new physics to explain many features of the universe : inflation, dark energy, Hubble constant tensions ...

Thus there is a beautiful interplay between the black hole puzzles and the puzzles of cosmology ..

THANK YOU !!