Symmetries at Null Boundaries: 3-dimensional gravity

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Based on: arXiv: 2007.12759 & 2104.03992

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IRCHP1400 (IPM, Iran)

November 2021

Outline

- Solution phase space near a null boundary in 3-dimensional Einstein gravity
- Null boundary symmetries, surface charges and algebra
- Change of slicing on solution phase space
- Topologically Massive Gravity

Gauge Theories in Presence of Boundaries

Gauge transformations in presence of **boundaries** fall in two classes

- Trivial gauge transformations $\rightarrow$ vanishing charge
- Physical or Large gauge transformations $\rightarrow$ non-vanishing charge

Motivations:

- Black hole horizons are null.
- Boundaries of asymptotically flat spacetimes are null.
- Null hypersurfaces are one-way membranes.

Solution Phase Space

Einstein gravity in 3-dimensions:

$$S=\frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\,\sqrt{-g}(R-2\Lambda)$$

The line-element:

$$ds^2=-V dv^2+2\eta dv dr+g\,(d\phi+U dv)^2$$

Null boundary: $r=0$.

Expansion around $r=0$:

$$\begin{aligned}
V&=2\left(\eta\kappa-\partial_v\eta+\mathcal{U}\partial_\phi\eta\right)r+\mathcal{O}(r^2)\,,\\
U&=\mathcal{U}-\frac{\eta}{\Omega^2}\Upsilon r+\mathcal{O}(r^2)\,,\\
g&=\Omega-2\eta\lambda\, r+\mathcal{O}(r^2)\,.
\end{aligned}$$

Solution phase space: $\{\eta,\Omega,\Upsilon\}$.

Null Boundary Symmetry

Null Boundary Symmetries:

$$\xi = T\,\partial_v + r\,(\partial_v T - \mathcal{U}\partial_\phi T - W)\,\partial_r + \left(Y - r\frac{\eta}{\Omega}\partial_\phi T\right)\partial_\phi + \mathcal{O}(r^2)$$

where $T = T(v,\phi)$, $W = W(v,\phi)$ and $Y = Y(v,\phi)$.

Variation of Fields:

$$\begin{aligned}
\delta_\xi \Omega &= T\partial_v\Omega + \mathcal{U}\Omega\partial_\phi T + \partial_\phi(Y\Omega)\,,\\
\delta_\xi \eta &= 2\eta\partial_v T + T\partial_v\eta - 2\eta\mathcal{U}\partial_\phi T - W\eta + Y\partial_\phi\eta\,,\\
\delta_\xi \Upsilon &= T\partial_v\Upsilon + 2\Upsilon\mathcal{U}\partial_\phi T + 2\Upsilon\partial_\phi Y + Y\partial_\phi\Upsilon + \Omega(\partial_\phi W - \Gamma\partial_\phi T)\,.
\end{aligned}$$

Algebra of Null Boundary Symmetries (NBS):

$$\boxed{\text{Diff}(\mathcal{N}) \in\!\!\!\!+ \ \text{Weyl}(\mathcal{N})}$$

Surface Charges

Using the covariant phase space method (CPSM), surface charges become

$$\not{\delta} Q_\xi = \frac{1}{16\pi G}\int_{\mathcal{N}_v} d\phi\left(W\delta\Omega + Y\delta\Upsilon + T\not{\delta}\mathcal{A}\right)\,,$$

with

$$\not{\delta}\mathcal{A} = -2\Omega\delta\Theta + \Omega\Theta\frac{\delta\eta}{\eta} - \Gamma\delta\Omega + \mathcal{U}\delta\Upsilon\,.$$

where

$$\Theta := \frac{1}{\Omega}\left(\partial_v\Omega - \partial_\phi(\mathcal{U}\Omega)\right), \qquad \Gamma := -2\kappa + 2\Theta + \frac{\partial_v\eta}{\eta} - \frac{\mathcal{U}\partial_\phi\eta}{\eta}$$

The charge variation is obviously **non-integrable**.

Algebra of the Surface Charges

- **Representation theorem**: Charge algebra is isomorphic to symmetry algebra up to central extension terms.

- **Criterion**: We require the central extension term to be field independent.

  Integrable:

$$Q_\xi = \frac{1}{16\pi G}\int_{\mathcal{N}_v} d\phi \left\{W\ \Omega + Y\ \Upsilon + T\ \left(-\Gamma\Omega + \mathcal{U}\Upsilon\right)\right\}\,,$$

  Non-integrable (flux):

$$F_\xi(\delta g; g) = \frac{1}{16\pi G}\int_{\mathcal{N}_v} d\phi\, T\left[-2\Omega\delta\Theta + \Omega\Theta\frac{\delta\eta}{\eta} + \Omega\delta\Gamma - \Upsilon\delta\mathcal{U}\right].$$

- **Charge algebra**: The surface charge algebra is exactly the same as NBS algebra without any central extension term.

Change of Slicing and Heisenberg algebra

Change of slicing:

$$\tilde{W} = W - \Gamma T - (Y + T\mathcal{U})\,\partial_\phi \mathcal{P}\,,$$
$$\tilde{T} = \Omega\Theta T + \partial_\phi[\Omega(Y + T\mathcal{U})]\,,$$
$$\tilde{Y} = Y + T\mathcal{U}\,.$$

Charge expression in new slicing:

$$\delta\tilde{Q} = \frac{1}{16\pi G}\int_{\mathcal{N}_v} d\phi\left(\tilde{W}\delta\Omega + \tilde{Y}\delta\mathcal{J} + \tilde{T}\delta\mathcal{P}\right)$$

where

$$\mathcal{J} = \Upsilon + \Omega\partial_\phi\mathcal{P}, \qquad \mathcal{P} = \ln\frac{\eta}{\Theta^2}\,.$$

Surface Charges:

- Ω: **entropy aspect charge**
- $\mathcal{J}$: **angular momentum aspect charge**
- $\mathcal{P}$: **expansion aspect charge**

Charge Algebra

Transformation laws are as

$$
\begin{aligned}
\delta_\xi \Omega &= \tilde{T}\,, \\
\delta_\xi \mathcal{P} &\approx -\tilde{W}\,, \\
\delta_\xi \mathcal{J} &\approx 2\mathcal{J}\partial_\phi \tilde{Y} + \tilde{Y}\partial_\phi \mathcal{J}\,.
\end{aligned}
$$

The charge algebra is the **Heisenberg $\oplus$ Witt algebra**

$$
\begin{aligned}
&\{\Omega(v,\phi), \Omega(v,\phi')\} = \{\mathcal{P}(v,\phi), \mathcal{P}(v,\phi')\} = 0, \\
&\{\Omega(v,\phi), \mathcal{P}(v,\phi')\} = 16\pi G \delta\left(\phi - \phi'\right), \\
&\{\mathcal{J}(v,\phi), \Omega(v,\phi')\} = \{\mathcal{J}(v,\phi), \mathcal{P}(v,\phi')\} = 0, \\
&\{\mathcal{J}(v,\phi), \mathcal{J}(v,\phi')\} = 16\pi G \left(\mathcal{J}(v,\phi')\partial_\phi - \mathcal{J}(v,\phi)\partial'_\phi\right)\delta\left(\phi - \phi'\right).
\end{aligned}
$$

Non-uniqueness and Existence of Genuine Slicings!

Non-uniqueness:

The genuine (integrable) slicings are not ***unique****.*

Existence:

Integrability Conjecture*: In the absence of genuine flux passing through the boundary, there are specific slicings,* **genuine slicings***, such that the charge variation becomes integrable.*

3**-dimensional Einstein gravity**:

No bulk propagating d.o.f $\rightarrow$ *Integrable charge expression.*

Topologically Massive Gravity as a Nontrivial Check

Solution Phase Space: It involves **four** independent functions over the two dimensional null boundary.

- **boundary phase space**:

  entropy aspect charge, angular momentum aspect charge and expansion aspect charge.

- **bulk phase space**:

  the massive chiral propagating graviton mode of TMG.

Surface charge algebra: **Heisenberg $\oplus$ Virasoro**

Questions

1. What happens in Einstein's theory in higher dimensions?
2. How universal is the following algebra

$$\textbf{Heisenberg} \oplus \textbf{Diff}(\mathcal{N}_v)$$

3. Physical realization of these surface charges through the **memory effects**.
4. How do our results tackle the BH microstate problem?

Thank You!