What Really the Decoherence Theory is About?

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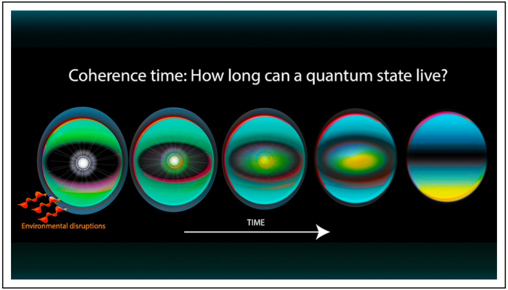
Decoherence Theory



- Decoherence theory based on a system-environment interaction.
- Quantum characteristics decay as $e^{-\Gamma t}$ because of coupling with environment and the system shows classical behaviour. (does it?!)

[1] H. P. Breuer and F. Petruccione, The Theory of Open Quantum Systems Oxford University Press (2002).

Decoherence Theory

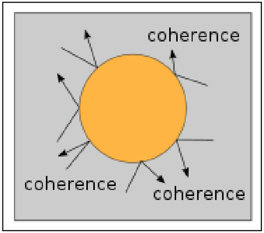
$$\hat{\rho}_{\text{particle}} = \begin{pmatrix} \rho_{1,1} & \rho_{1,2} \\ \rho_{2,1} & \rho_{2,2} \end{pmatrix}$$

$\Downarrow$ Environment monitoring

$$\hat{\rho}_{\text{particle}} = \begin{pmatrix} \rho'_{1,1}(t) & \rho'_{1,2} e^{-\Gamma t} \\ \rho'_{2,1} e^{-\Gamma t} & \rho'_{2,2}(t) \end{pmatrix}$$

$\Downarrow$ Decoherence process is complete

$$\hat{\rho}_{\text{particle}} = \begin{pmatrix} \rho'_{1,1}(t \to \infty) & 0 \\ 0 & \rho'_{2,2}(t \to \infty) \end{pmatrix}$$



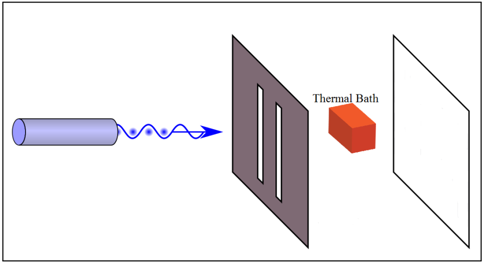
Quantum Brownian Motion Lindblad Master Equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}_s(t) = \hat{\mathcal{L}}[\hat{\rho}_s(t)] = \underbrace{-\mathrm{i}[\hat{H}'_s, \hat{\rho}_s(t)]}_{\text{unitary evolution}} + \underbrace{\hat{\mathcal{D}}[\hat{\rho}_s(t)]}_{\text{decoherence}}$$

- Born Approximation: The density operator remains at all times in a product form of system and environment and the changes in the environment is negligible.
- Markov Approximation: The environment memory effect is negligible.
- Secular Approximation: Discarding fast oscillating terms.
- Lindblad Master Equation: Ensure the positivity of reduced density matrix.

[1] S. Maniscalco, J. Piilo, F. Intravaia, F. Petruccione and A. Messina, Phys. Rev. A 70, 032113 (2004).

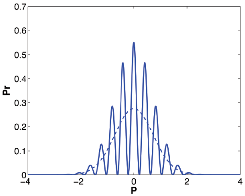
A Simple Example

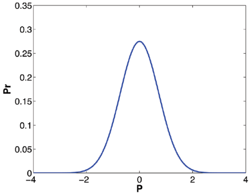


$$\hat{\rho}_s = \begin{pmatrix} 1/2 & -\mathrm{i}/2 \\ \mathrm{i}/2 & 1/2 \end{pmatrix} \overset{\text{Thermal Bath}}{\Longrightarrow} \hat{\rho}_s = \begin{pmatrix} \frac{2(\bar{n}+1)-\eta^2}{2(2\bar{n}+1)} & \frac{-i}{2}\eta \\ \frac{i}{2}\eta & \frac{2\bar{n}+\eta^2}{2(2\bar{n}+1)} \end{pmatrix}$$

A Simple Example

$$\hat{\rho}_s = \begin{pmatrix} \frac{2(\bar{n}+1)-\eta^2}{2(2\bar{n}+1)} & \frac{-i}{2}\eta \\ \frac{i}{2}\eta & \frac{2\bar{n}+\eta^2}{2(2\bar{n}+1)} \end{pmatrix} \xrightarrow{\eta\to 0} \hat{\rho}_s = \begin{pmatrix} \frac{2(\bar{n}+1)}{2(2\bar{n}+1)} & 0 \\ 0 & \frac{2\bar{n}}{2(2\bar{n}+1)} \end{pmatrix}$$

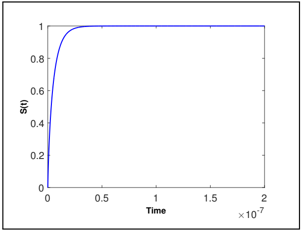




$$S(t\to\infty) = -\frac{\bar{n}+1}{2\bar{n}+1}\log_2\frac{\bar{n}+1}{2\bar{n}+1} - \frac{\bar{n}}{2\bar{n}+1}\log_2\frac{\bar{n}}{2\bar{n}+1}.$$

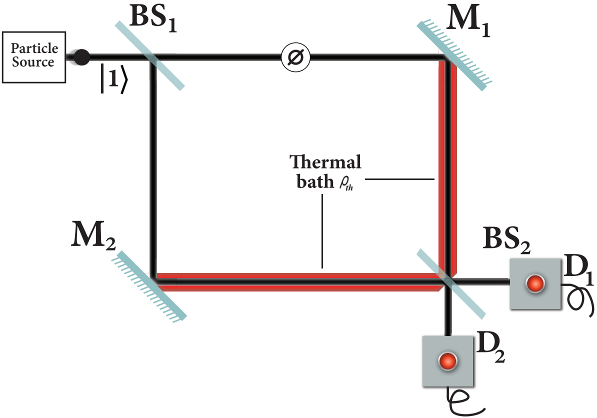
Quantum Thermodynamic Program

- Hatano-Sasa inequality (for CPTP map): $\Delta S \geq -\mathrm{tr}[\{M(\rho) - \rho\} \log(\xi)]$
  For Lindblad master equation[1] :$\Delta S \geq 0$
- $\delta W = 0$ and δQ depends on the changes of $\langle X^2 \rangle$ and $\langle P^2 \rangle$ through time[2,3], which is zero.
  Clausius inequality: $\Delta S \geq 0$



[1] F. Binder, S. Vinjanampathy, K. Modi and J. Goold, Phys. Rev. E 91, 032119 (2015).
[2] A. E. Allahverdyan and T. M. Nieuwenhuizen, Phys. Rev. Lett. 85, 1799 (2000).
[3] T. M. Nieuwenhuizen and A. E. Allahverdyan, Phys. Rev. E 66, 036102 (2002).

Decoherence in System-Bath Interferometery

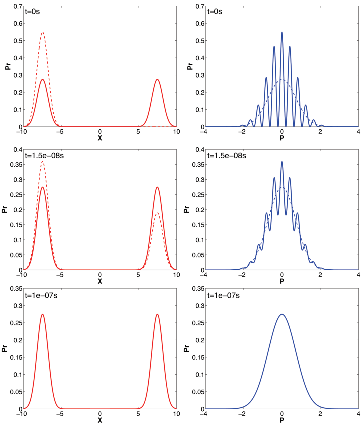


Decoherence in System-Bath Interferometery

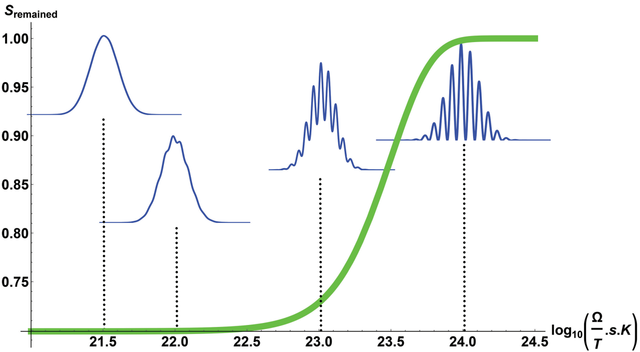
$$\Pr(P) = \sqrt{\frac{1}{\Omega\pi}} e^{-\frac{P^2}{\Omega}} [1 + \frac{\eta^2 - 1}{2\bar{n} + 1} \sin(Pd) + \eta \sin\phi \cos(Pd)],$$

- $\eta \sin\phi \cos(Pd)$
- $\frac{\eta^2-1}{2\bar{n}+1} \sin(Pd)$

$$\bar{n} = (e^{\Omega/kT} - 1)^{-1}$$



Decoherence in System-Bath Interferometery

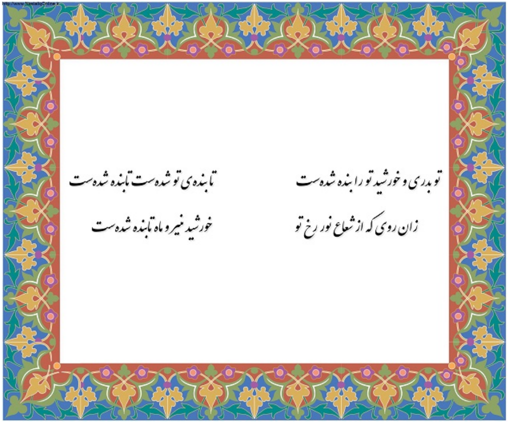


$$S_{rem} := S_{cm} - S(t \to \infty)$$

Conclusion

- Oops!... The decoherence theory is no longer an acceptable explanation for quantum to classical transition. Had it been so?
- The decoherence theory is about that a quantum system reaches to the maximum possible entropy. Does it necessarily mean that a classical situation is attained?
- A measure introduced. The greater the amount of the remained entropy is the more quantum properties the system shows.
- A quantum system, whenever created, it never completely lose its quantum properties. However, they are not easy to observe.

THANK YOU


تو بدری و خورشید تو را بنده شده ست
تا بنده ی تو شده ست تابنده شده ست
زان روی که از شعاع نور رخ تو
خورشید نیر و ماه تابنده شده ست