Energy Cost of Getting Information

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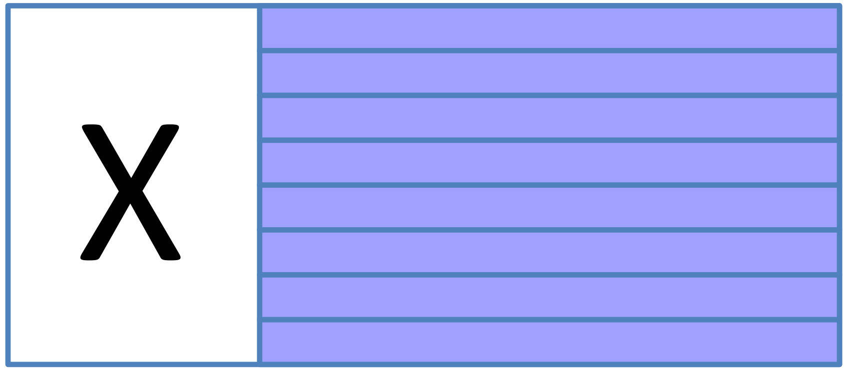
Information

Information

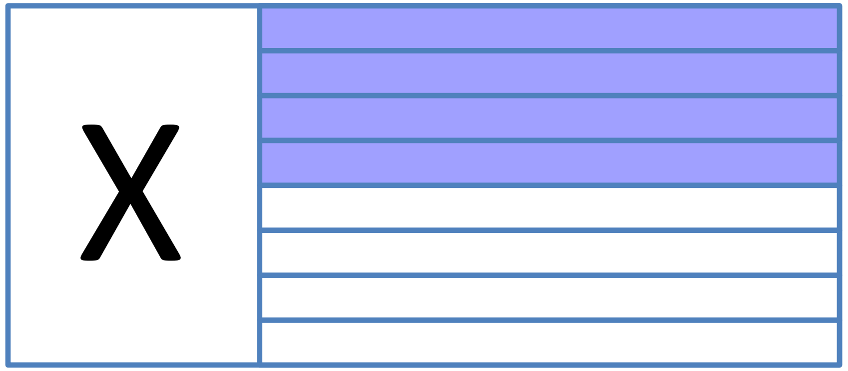
- **Communication Theory**
  - Shannon Information
- **Estimation Theory**
  - Fisher Information

Information : Shannon



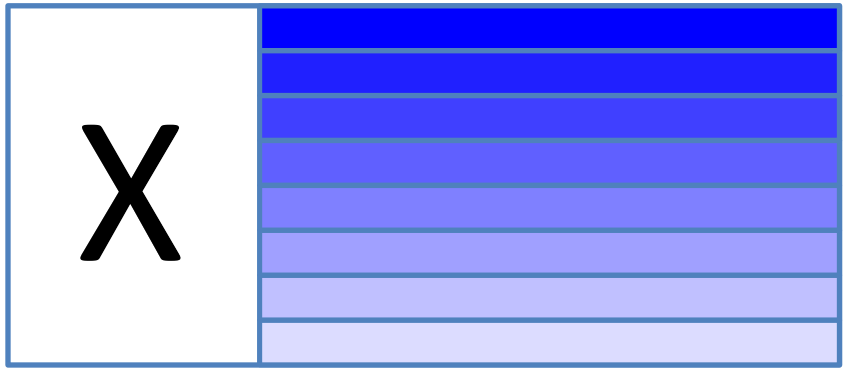
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

3



0	0
0	1
1	0
1	1

2



1/2	1							
1/4	0	1						
1/8	0	0	1					
1/16	0	0	0	1				
1/32	0	0	0	0	1			
1/64	0	0	0	0	0	1		
1/128	0	0	0	0	0	0	1	
2/256	0	0	0	0	0	0	0	1

1.9

$$H[\rho] = -\sum \rho(x_i) \log \rho(x_i)$$

Information : Fisher

Random Variable $X = \{x_1, x_2, .., x_m\}$

Probability Distribution $\rho(x_i, \theta)$

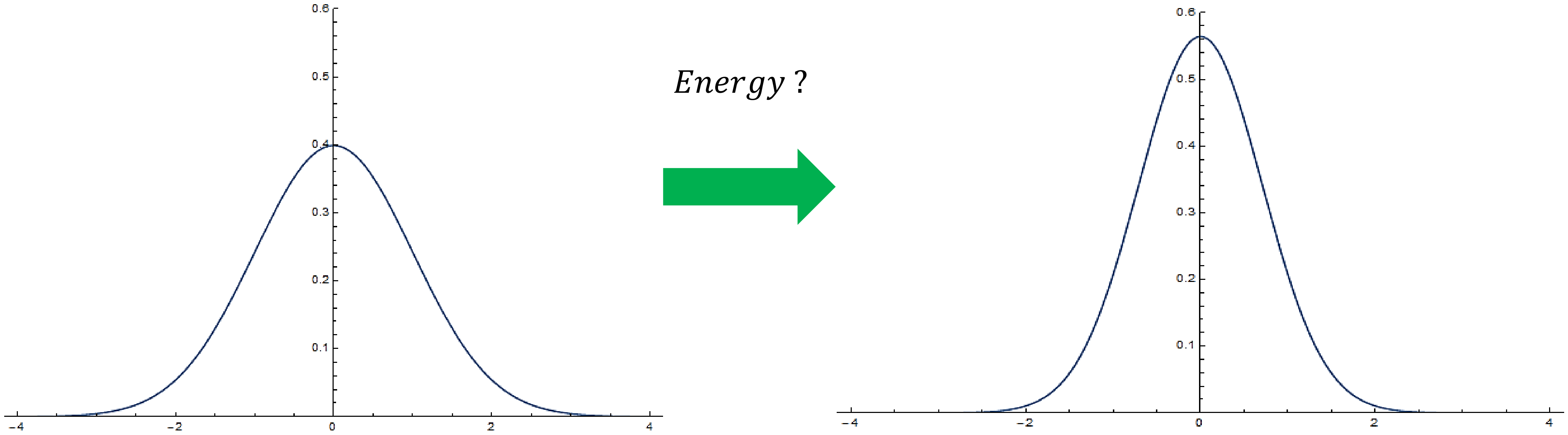
Results of N-Trial Experiment $\mathbf{x} = \{x_{i_1}, x_{i_2}, .., x_{i_N}\}$

Estimator $\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}(\mathbf{x})$

$$\mathrm{var}(\hat{\boldsymbol{\theta}}(\mathbf{x})) \geq \frac{1}{N\, F[\rho]}$$

$$F[\rho] = \sum \frac{(\nabla_\theta \rho(x_i, \theta))^2}{\rho(x_i, \theta)}$$

Energy Cost of Getting Information



Proposal

$$\rho \longrightarrow \rho + \delta\rho$$

$$\nu \longrightarrow \nu + \delta\nu$$

$$\delta E \propto v\, \delta v$$

$$E \propto v^2$$

$$E = \frac{1}{2} m\, v^2$$

$$\delta E \propto \nu\, \delta \nu$$

$$E \propto \nu^2$$

$$E = \frac{\hbar^2}{m}\, v^2$$

Proposal

$$v = \lim_{dt \to 0} \frac{\mathbf{d}[x(t+dt), x(t)]}{dt}$$

$$\nu = \lim_{dx \to 0} \frac{\mathbf{d}[\rho(x+dx), \rho(x)]}{dx}$$

$$\mathbf{d}[x(t+dt), x(t)] = x(t+dt) - x(t)$$

$$\mathbf{d}[\rho(x+dx), \rho(x)] = ?$$

Distance Properties

- $\mathbf{d}[\rho_1, \rho_2] \geq 0$
- If $\mathbf{d}[\rho_1, \rho_2] = 0$ then $\rho_1 = \rho_2$
- $\mathbf{d}[\rho_1, \rho_2] = \mathbf{d}[\rho_2, \rho_2]$ Symmetry
- $\mathbf{d}[\rho_1, \rho_2] + \mathbf{d}[\rho_2, \rho_3] \geq \mathbf{d}[\rho_1, \rho_3]$ Triangle Inequality

Jenson-Shannon distance

$$\mathbf{d}_{JS}\left[\rho_1, \rho_2\right] \equiv \sqrt{H\left(\frac{\rho_1 + \rho_2}{2}\right) - \frac{1}{2}H(\rho_1) - \frac{1}{2}H(\rho_2)}$$

$$H(\rho) = -\sum \rho(x_i) \ln \rho(x_i)$$

Jenson-Shannon distance

$$\nu = \sqrt{\frac{1}{8}\int \frac{(\nabla_x \rho(x))^2}{\rho(x)} dx}$$

$$E = \frac{\hbar^2}{8m}\int \frac{(\nabla_x \rho(x))^2}{\rho(x)} dx$$

Example : Particle in a Box

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \rightarrow \rho_n = |\psi_n|^2 = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)$$

$$E = \frac{n^2 \hbar^2 \pi^2}{2mL^2}$$

$$E = \frac{\hbar^2}{8m} \int_0^L \frac{(\nabla_x \rho_n(x))^2}{\rho_n(x)} dx = \frac{n^2 \hbar^2 \pi^2}{2mL^2}$$

Equations of Motion (Classical Mechanics)

Deterministic (q, p) $\dot{p} = -\frac{dU}{dq}$ $\dot{q} = \frac{p}{m}$

Probabilistic (ρ, s) $\dot{\rho} = -\nabla\left(\rho \frac{\nabla s}{m}\right)$ $\dot{s} = -\left(\frac{(\nabla s)^2}{2m} + U\right)$

$$H = H_{kinetic} + H_{potential}$$

$$\frac{dH}{dt} = 0$$

$$H_{kinetic} = \int \frac{(\nabla s)^2}{2m} \rho \, dx \qquad H_{potential} = \int U \rho dx$$

Equations of Motion (Quantum Mechanics)

$$H = H_{kinetic} + H_{potential} + H_{information}$$

$$H_{kinetic} = \int \frac{(\nabla s)^2}{2m} \rho \, dx$$

$$H_{potential} = \int U \rho dx$$

$$H_{information} = \frac{\hbar^2}{8m} \int \frac{(\nabla \rho)^2}{\rho} dx$$

$$\frac{dH}{dt} = 0$$

$$\dot{\rho} = -\nabla \left(\frac{\nabla s}{m} \rho \right)$$

$$\dot{s} = -\left(\frac{(\nabla s)^2}{2m} + U + \frac{\hbar^2}{8m} \frac{(\nabla \rho)^2}{\rho^2} - \frac{\hbar^2}{4m} \frac{\nabla^2 \rho}{\rho} \right)$$

Equations of Motion (Quantum Mechanics)

$$\dot{\rho} = -\nabla\left(\frac{\nabla s}{m}\rho\right)$$

$$\dot{s} = -\left(\frac{(\nabla s)^2}{2m} + U + \frac{\hbar^2}{8m}\frac{(\nabla\rho)^2}{\rho^2} - \frac{\hbar^2}{4m}\frac{\nabla^2\rho}{\rho}\right)$$

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + U\psi$$

$$\psi = \sqrt{\rho}\exp\left(\frac{is}{\hbar}\right)$$

Thanks to Your Attention