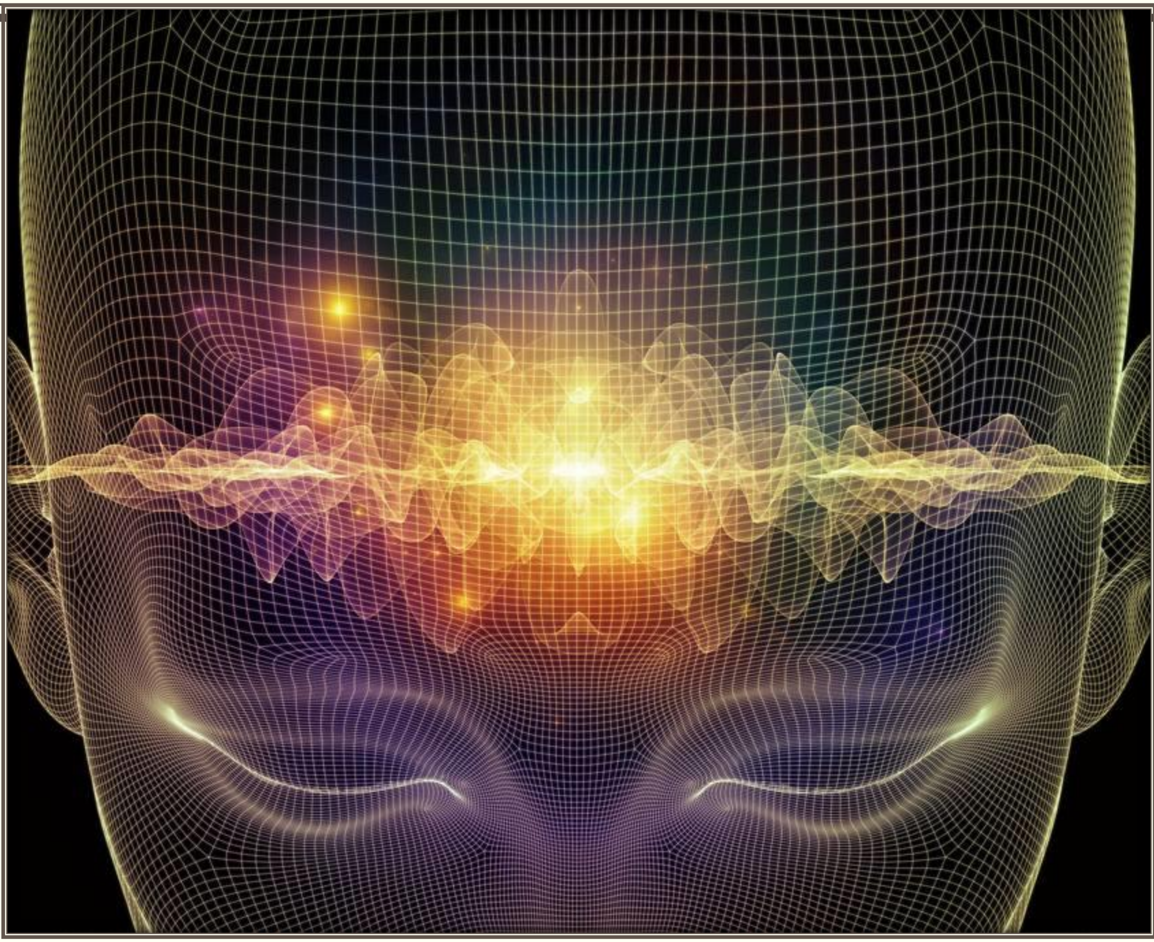
A Quantum Cognition Analysis of Human Behavior by Hardy’s Non-locality Argument



Introduction

Human cognition and behavior is both probabilistic and dynamic. There are two ways to build such systems:

- ✓ Classical probabilistic dynamics
- ✓ Quantum probabilistic dynamics

Previously, all cognitive researches relied on classical probability theory and principles of classical mechanics. But recently it was found some experimental data on human cognition that cannot be explained by classical theory.

In the recent years, many researches have done on using quantum theory in cognitive science. Quantum probability seems to be a useful framework to describe a variety of subjects' behaviour.

- Some reasons for quantum approach to cognition

Some reasons for quantum approach to cognition

Why Quantum Theory?

- **Violation of sure thing principle:**

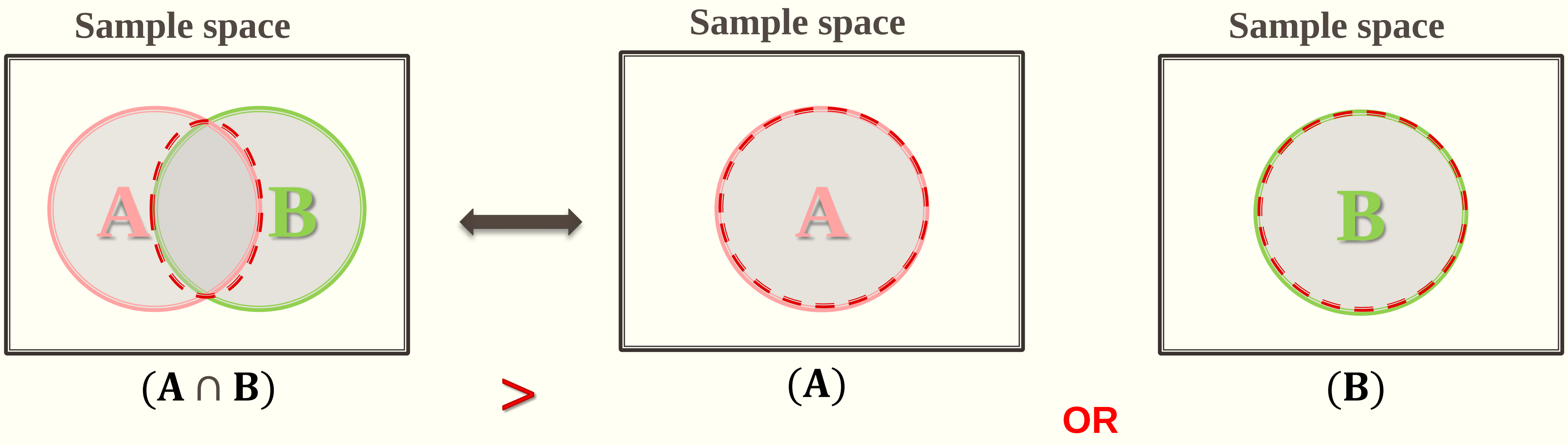
A consequence of classical probability theory, is violated by human decision-makers.

In decision theory, the sure-thing principle states that a decision maker who would take a certain action if she/he knew that event *X* obtained, and also if he knew that the negation of *X* obtained, should also take that same action if he knows nothing about *X*.

$$\begin{cases} X : A \to B \\ X^{\sim} : A \to B \end{cases} \quad \longrightarrow \quad X \text{ or } X^{\sim} : A \to B \quad \longrightarrow \quad ???: B \to A$$

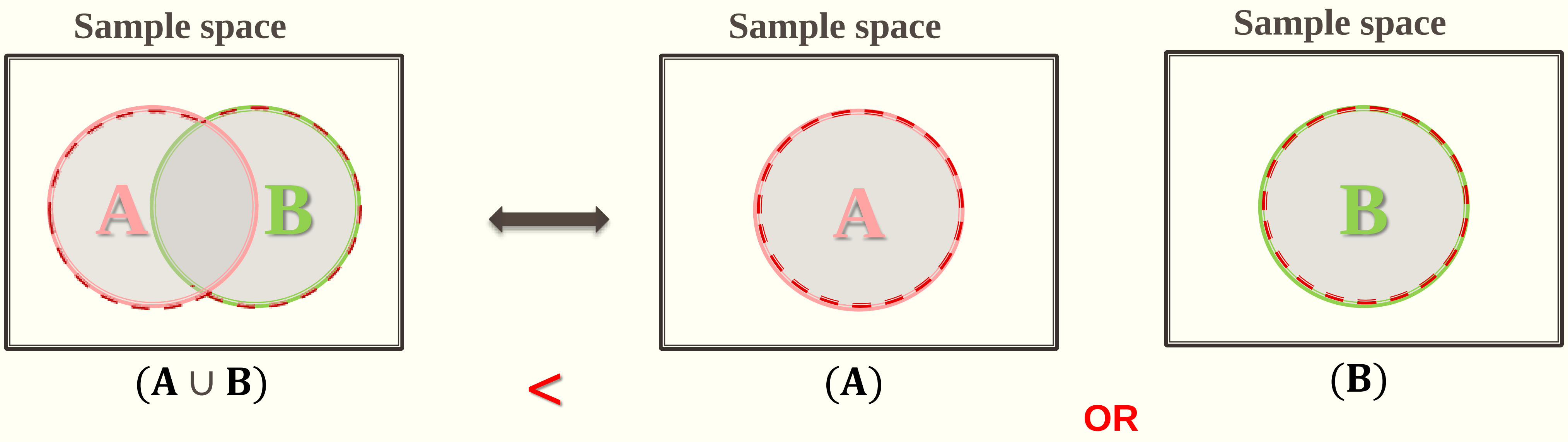
- **Conjunction fallacy:**

Under some conditions, people judge the probability of event A and B to be greater than the probability of event B.



- **Disjunction fallacy:**

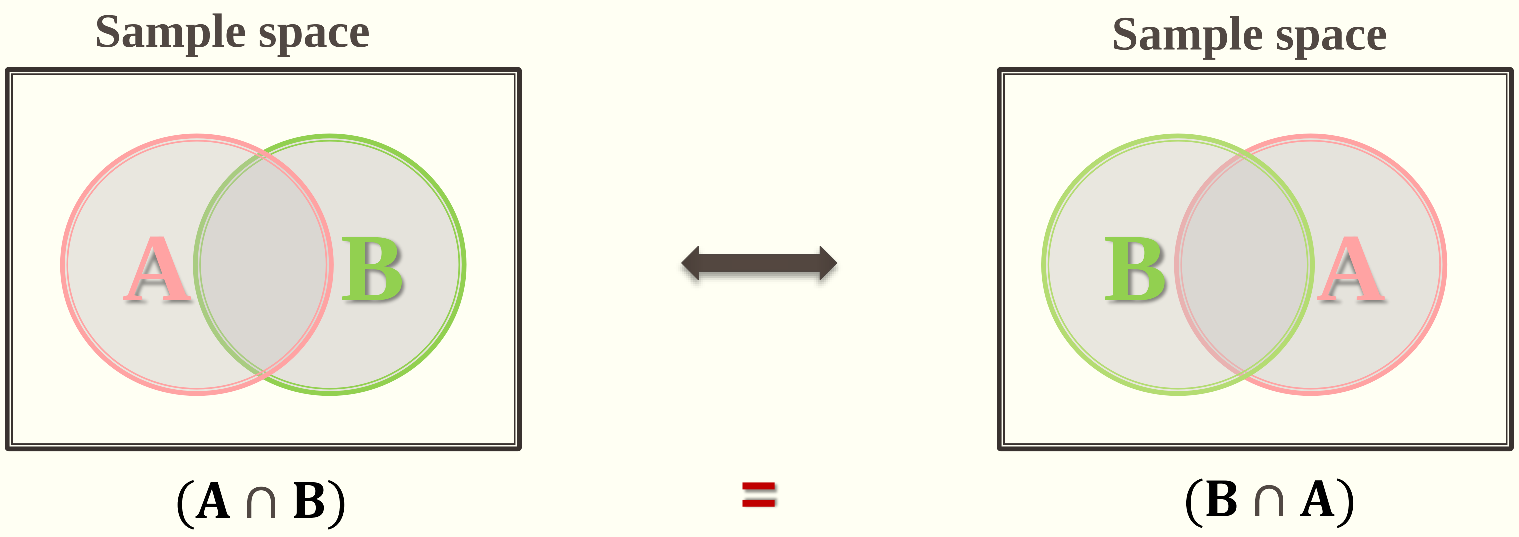
Under the same conditions, they judge the probability of A or B to be less than the probability of event A.



- **Order effects:**

An order effect occur when the subspace used to describe one question is incompatible with the subspace used to describe the other.

✓ Note that **classical** probability theory cannot explain order effects because events are represented as **sets** and are **commutative**.



Some reasons for quantum approach to cognition

- Quantum theory is not easy for researchers in cognition and decision making to accept. In fact, quantum mechanics was not easy for physicists to accept either, but it was forced on them by several paradoxical findings that could not be explained using classical physics.

- Principles from quantum theory resonate with deeply rooted psychological intuitions and conceptions about human cognition and decision.

  - ***e.g., superposition* : ambiguity, uncertainty**
  - ***complementarity*: constructive view of judgment, attitude, etc.**
  - ***contextuality:* mental background**

- **Quantum entanglement in concept combinations** (Diederik Aerts & Sandro Sozzo)

- **Bell's inequality:**

The seminal work by John Stewart Bell(1964), it is known that the results obtained when measuring a quantum state in space separated regions can display some counter-intuitive form of correlations, often named as quantum nonlocality. We consider the simplest case where Alice and Bob have to perform two different Stern Gerlach measurements on two spin-one-half particles.

The measurements are defined by two directions, corresponding to the directions of the Stern–Gerlach measurement apparatuses for each party, namely a_1 and a_2 for Alice and b_1 and b_2 for Bob, while the outcomes of these measurements are $\mathrm{a}_1, \mathrm{a}_2, \mathrm{b}_1$ and $\mathrm{b}_2 \in \{-1, +1\}$.

$$-2 \leq BL = |\langle a_1 b_1 \rangle + \langle a_1 b_2 \rangle + \langle a_2 b_1 \rangle - \langle a_2 b_2 \rangle| \leq 2$$

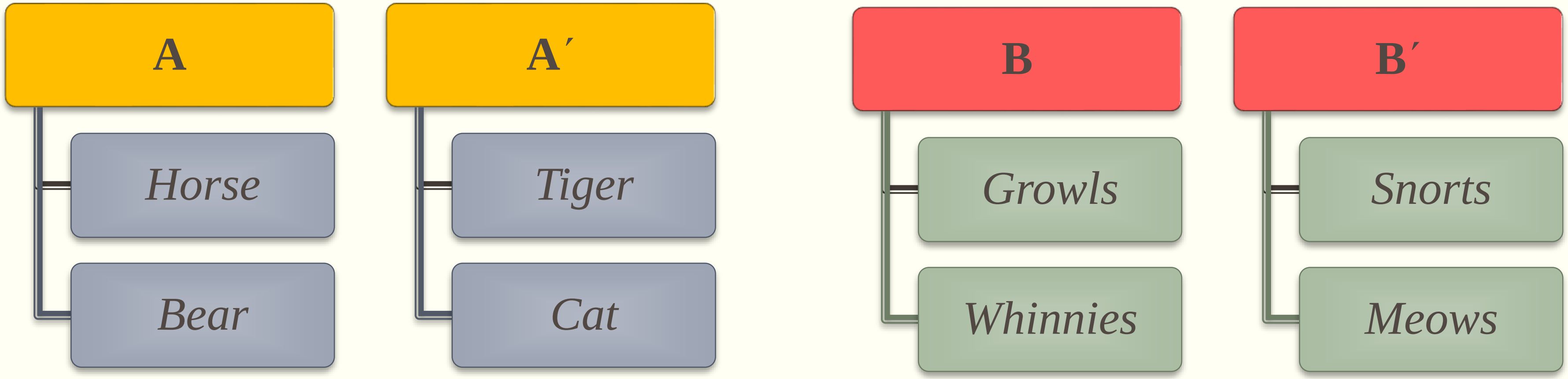
The mathematical formalism of quantum mechanics predicts a maximum value $2\sqrt{2}$, which is greater than 2.

Quantum entanglement in concept combinations (Diederik Aerts & Sandro Sozzo)

D Aerts and S Sozzo (2013) Published the study of the structure of the combination of two concepts, the concept Animal, and the concept Acts, in the sentence The Animal Acts, by means of experiments with human subjects to the identification of entanglement in such compound systems.

Quantum entanglement in concept combinations (Diederik Aerts & Sandro Sozzo)

- Number of participants: 81 people
- They consider two pairs of states, of Animal, and two pairs of states, of Acts (by Acts they mean the specific action of Making A Sound).
- Theyintroduce the single experiments A and A′ for the concept Animal, andBandB′for the concept Acts.



Experiment performed on groups A and A ′:

- Experiments A and A′ consist in participants choosing between Animalsanswering the question ‘**Which animal do you like?’**

$$\text{Out comes (Experiments A)}\begin{cases}\lambda_H = +1 \ \text{ if Horse is chosen}\\ \lambda_B = -1 \ \text{ if Bear is chosen}\end{cases}$$

$$\text{Out comes (Experiments A}'\text{)}\begin{cases}\lambda_T = +1 \ \text{ if Tiger is chosen}\\ \lambda_C = -1 \ \text{ if Cat is chosen}\end{cases}$$

Results:

Animal	Probability
Horse	$p(A_1) = 0.5309$
Bear	$p(A_2) = 0.4691$

Animal	Probability
Tiger	$p(A'_1) = 0.7284$
Cat	$p(A'_2) = 0.2716$

Experiment performed on groups B and B ′:

- Experiments B and B′ consist in participants choosing between Actsanswering the question ‘**Which act for an animal do you like?’**

Out comes (Experiments B) $\begin{cases} \lambda_G = +1 & \text{if Growls is chosen} \\ \lambda_W = -1 & \text{if Whinnies is chosen} \end{cases}$

Out comes (Experiments B′) $\begin{cases} \lambda_S = +1 & \text{if Snorts is chosen} \\ \lambda_M = -1 & \text{if Meows is chosen} \end{cases}$

Results:

Act	Probability
Growls	$p(B_1) = 0.4815$
Whinnies	$p(B_2) = 0.5185$

Act	Probability
Snorts	$p(B'_1) = 0.3210$
Meows	$p(B'_2) = 0.6790$

Quantum entanglement in concept combinations (Diederik Aerts & Sandro Sozzo)

Let us now come to the coincidence experiments $AB, A'B, AB'$ and $A'B'$ for the conceptual combination The Animal Acts.

Results:

The Animal Acts	Probability
Horse Growls	$p(A_1, B_1) = 0.049$
Horse Whinnies	$p(A_1, B_2) = 0.630$
Bear Growls	$p(A_2, B_1) = 0.259$
Bear Whinnies	$p(A_2, B_2) = 0.062$

The Animal Acts	Probability
Horse Snorts	$p(A_1, B'_1) =$
Horse Meows	$p(A_1, B'_2) = 0.025$
Bear Snorts	$p(A_2, B'_1) = 0.296$
Bear Meows	$p(A_2, B'_2) = 0.086$

The Animal Acts	Probability
Tiger Growls	$p(A'_1, B_1) = 0.778$
Tiger Whinnies	$p(A'_1, B_2) = 0.086$
Cat Growls	$p(A'_2, B_1) = 0.086$
Cat Whinnies	$p(A'_2, B_2) = 0.049$

The Animal Acts	Probability
Tiger Snorts	$p(A'_1, B'_1) = 0148$
Tiger Meows	$p(A'_1, B'_2) = 0.086$
Cat Snorts	$p(A'_2, B'_1) = 0.099$
Cat Meows	$p(A'_2, B'_2) = 0.667$

Now evaluate the expectation values $E(A,B)$ ‘$E(A',B)$ ‘$E(A,B')$and$E(A',B')$associated with the experiments $AB, A'B, AB'$ and $A'B'$respectively, and insert the values into Bell’s inequality.

$$E(A,B) = P(A_1,B_1) + P(A_2,B_2) - P(A_2,B_1) - P(A_1,B_2) = \mathbf{-0.7778}$$

$$E(A',B) = P(A'_1,B_1) + P(A'_2,B_2) - P(A'_2,B_1) - P(A'_1,B_2) = \mathbf{0.6543}$$

$$E(A,B') = P(A_1,B'_1) + P(A_2,B'_2) - P(A_2,B'_1) - P(A_1,B'_2) = \mathbf{0.3580}$$

$$E(A',B') = P(A'_1,B'_1) + P(A'_2,B'_2) - P(A'_2,B'_1) - P(A'_1,B'_2) = \mathbf{0.6296}$$

$$-2 \le E(A',B') + E(A',B) + E(A,B') - E(A,B) \le 2$$

And according to the collected dad

$$E(A',B') + E(A',B) + E(A,B') - E(A,B) =$$
$$0.6296 + 0.3580 + 0.6543 - (-0.7778) = \mathbf{2.4197}$$

Which is greater than 2. s close the maximal possible violation in quantum theory, $2\sqrt{2} \approx 2.8284$.

- **A quantum cognition analysis of human behaviour by Hardy's non-locality argument**

- **Hardy's Non-locality:**

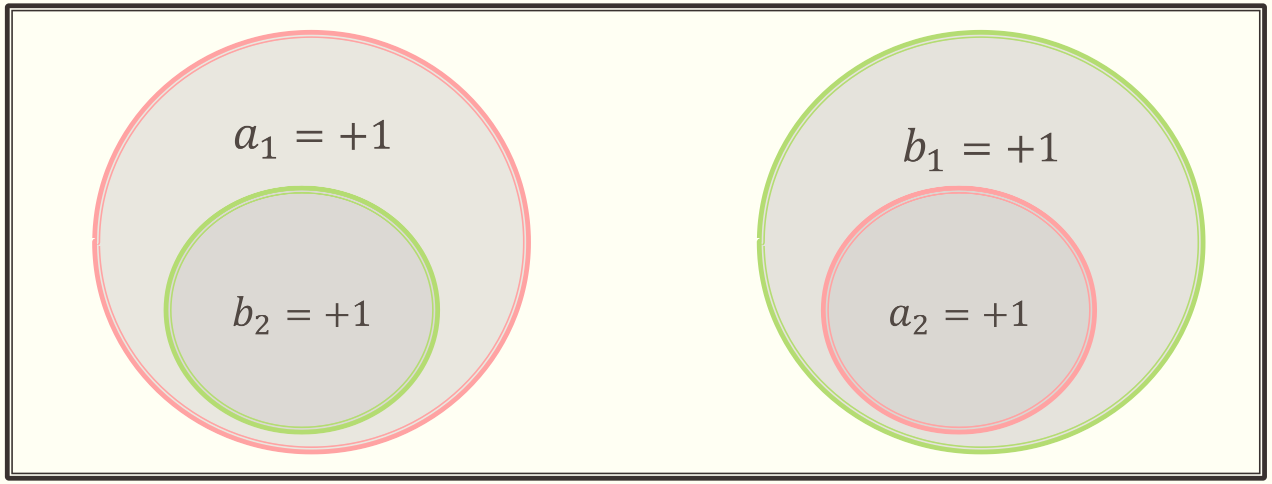
Bell's inequality is not the only way to express non-locality in quantum mechanics. Lucien Hardy (1992-93), describes equations which show the quantum contradiction directly with local realism not with inequalities such as Bell's inequality.

Hardy's logical structure is as follows: consider four events a_1, a_2, b_1 and $b_2 \in \{+1, -1\}$ where the positive sign means that an event occurs and the negative sign means that the event does not take place. Also, we consider two observers, Alice and Bob, where a_1 and a_2 may happen on Alice's side and b_1 and b_2 happen on Bob's side which is far apart from Alice.

A quantum cognition analysis of human behaviour by Hardy's non-locality argument

We can represent Hardy's non-locality argument as:

$$Prob(a_1 = +1, b_1 = +1) = 0$$
$$Prob(a_1 = -1, b_2 = +1) = 0$$
$$Prob(a_2 = +1, b_1 = -1) = 0$$
$$Prob(a_2 = +1, b_2 = +1) = q$$



A quantum cognition analysis of human behaviour by Hardy's non-locality argument

In some papers, it is asserted that cognitive experiments are not as direct as experiments in quantum mechanics. In fact, in almost all cognitive experiments subjects are required to give an answer to a question and they usually express a response they think is correct, not the first answer that comes into their mind.

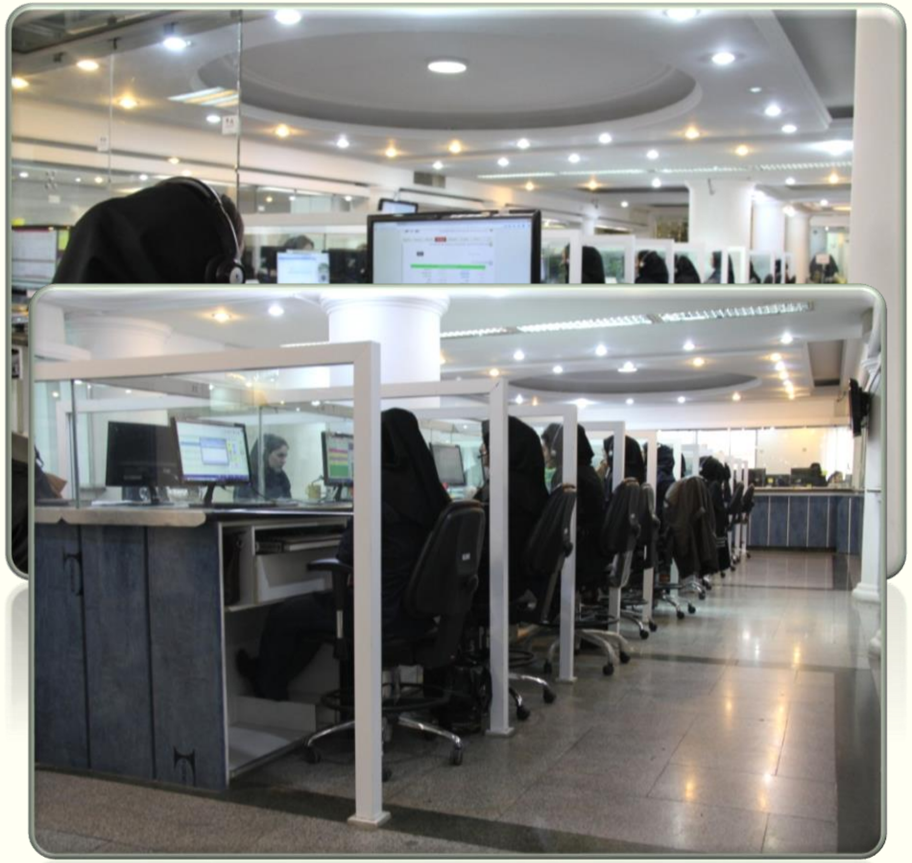
So we just observe the subjects’ behaviour and the question and answer method was not applied to prevent any mental background on participants’ minds.

A quantum cognition analysis of human behaviour by Hardy's non-locality argument

In the call centre that we have done our experiment, sales operators were divided into two groups; the inbound group that was responsible to answer incoming customers’ calls and the outbound group that was responsible to call customers to inform them about the new services or check out the subscription period.

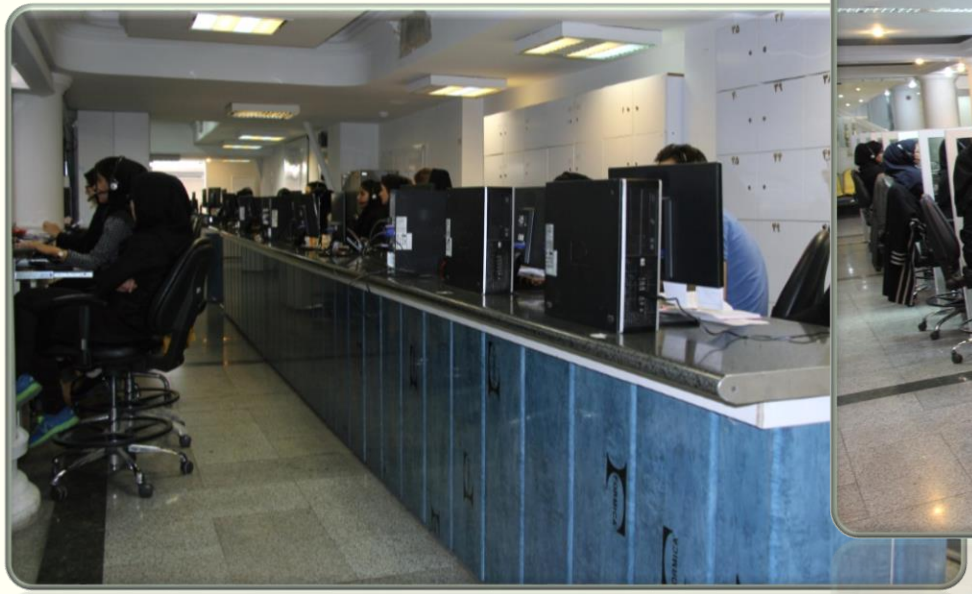
❖ In this research, incoming customers’ calls were the considered.

❖ On average, the number of inbound calls was more than 2000 calls per working day and more than 1200 calls per holiday.



A quantum cognition analysis of human behaviour by Hardy's non-locality argument

- ❖ The average number of inbound operators was 19 people per working day and 13 people per holiday.
- ❖ Operators had different work time depending on a number of incoming calls in each time period.



1) Sales Amount
2) Purchase Time
3) Operators’ Working Time
4) Number of Inbound Calls
5) Number of Abandoned Calls
6) Operators’ Talk Time
7) Operators DND Time

We considered the parameter $a_1$abandonedcalls, a_2 as buy, b_1 as an operator was responding to incoming calls and b_2 as it was not an operator's working time.

A customer disconnected her/his call before an operator's responding → $a_1 = +1$
A customer did not disconnect the call → $a_1 = -1$
A customer has bought a service → $a_2 = +1$
A customer has not bought any services → $a_2 = -1$

An operator was responding to incoming calls → **An**
An operator was not responding to incoming calls → **An o**
It was not an operator's working time →
It was an operator's working time →

- As we see, events a_1 and a_2 are related to the customers' behaviour and events b_1 and b_2 are related to the operators' behaviour.
- We had study all incoming calls from August 22, 2016, to October 21, 2016. In total, over 115000 calls have been reviewed in this study.

A quantum cognition analysis of human behaviour by Hardy's non-locality argument

According to the Hardy's equations:

$$Prob(a_1 = +1, b_1 = +1) = 0$$

- The probability that an operator was responding to a disconnected call.

$$Prob(a_1 = -1, b_2 = +1) = 0$$

- The probability that a customer's call was being responded by an operator who was not at work.

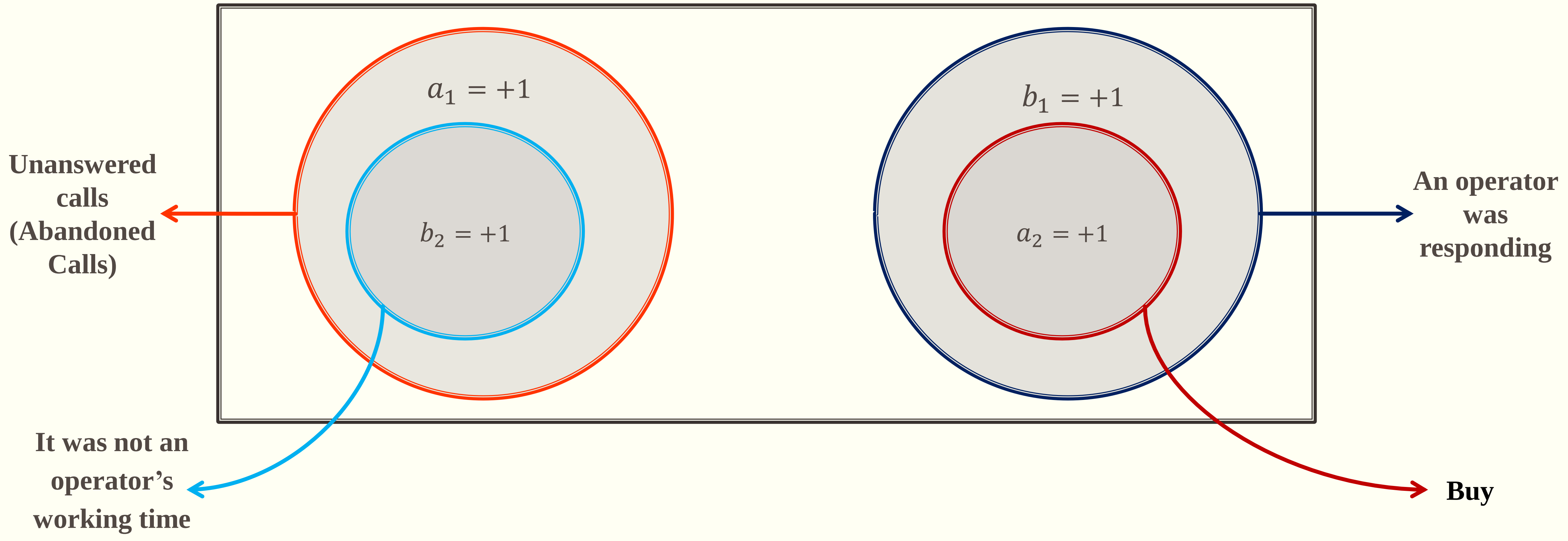
$$Prob(a_2 = +1, b_1 = -1) = 0$$

- The probability that a customer has bought a service, and this sale was registered for an operator who did not respond to the customer's call.

$$Prob(a_2 = +1, b_2 = +1) = q$$

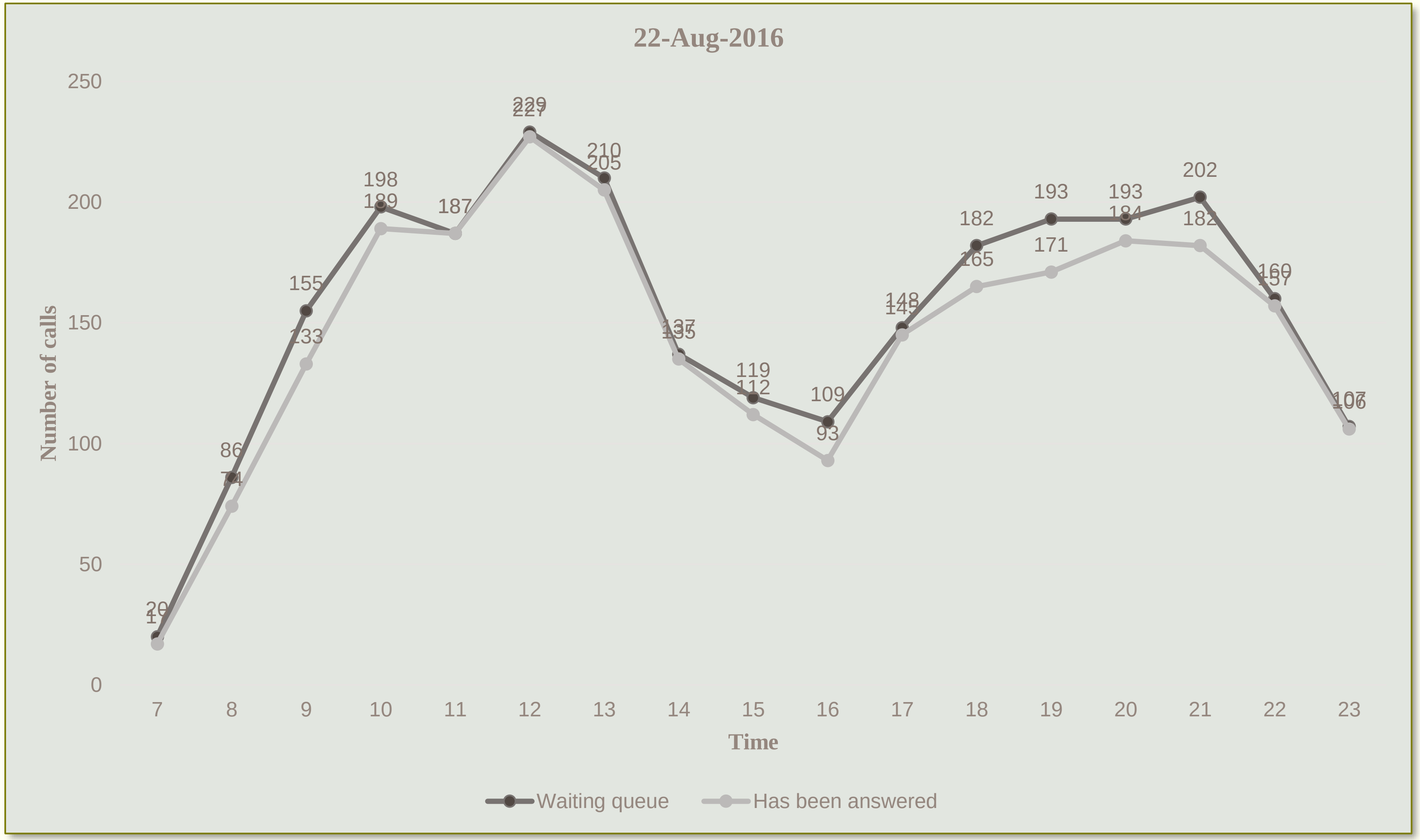
- The probability that a customer has bought a service, and this sale was registered for an operator who was not at work at the purchase time.

A quantum cognition analysis of human behaviour by Hardy's non-locality argument



A quantum cognition analysis of human behaviour by Hardy's non-locality argument

Calls Pick Graph Sample



A quantum cognition analysis of human behaviour by Hardy's non-locality argument

Part of collected data

Date	The number of responded calls	Call drop rate (Call)	Sales amount for operators' who were not at work (Toman)	Sales amount for operators' who were at work (Toman)
22-Aug-16	2473	164	874,180	18,654,118
23-Aug-16	2288	151	886,860	25,890,741
24-Aug-16	2112	54	751,010	23,879,117
25-Aug-16	1919	67	717,220	23,697,522
26-Aug-16	1300	23	476,330	14,345,285
27-Aug-16*	1146	44	725,286	26,122,732
28-Aug-16	2072	63	478,728	23,064,912
29-Aug-16	2056	51	1,054,030	22,061,473
30-Aug-16	1984	41	725,286	24,230,032
31-Aug-16	1929	48	998,974	20,948,317
1-Sep-16	1815	90	825,784	24,046,560
2-Sep-16*	581	38	961,380	14,216,797
3-Sep-16	2282	77	634,660	26,390,214
4-Sep-16	2022	104	542,820	24,551,341
5-Sep-16	2030	53	877,050	23,821,728
6-Sep-16	1888	70	574,430	22,233,120
7-Sep-16	1964	66	518,460	21,551,614
8-Sep-16	1723	122	913,585	21,202,381
9-Sep-16	1046	17	609,310	12,161,949

- A customer after hearing an operator's presentation must decide whether to buy or not, so the state vector in this case is:

$$|Decision\ Making\rangle = \alpha_d|Buy\rangle + \beta_d|Do\ Not\ Buy\rangle$$

So q is equal to the total sales amount for the operators who were not at work divided to the total sales for all operators;

$$Pr(a_2 = +1, b_2 = +1) = {}^{50373989}/_{1273102156 + 50373989} = \mathbf{0.038062}$$

❑Conclusions

Conclusions

- As we know, one of the purposes of statistical analysis is predicting accurately the factors that can help authorities to make the right decisions commensurate with the predicted values, and it is obvious that as much as the predicted values are accurate authorities can adopt better necessary policies and applying appropriate solutions. Thus, given the results of the experiment, in cases that are related to the analysis of individuals' behaviour, we should also check the results in quantum mechanics to provide more accurate data.

- Quantum mechanics formulas may not give the entire mathematical requirements framework for cognition and decision research; however, by continuing to study and research in this field in addition to the development of cognitive science, it may create new approaches to quantum theory.

Thank You