Chronology Protection Conjecture in String Theory

M.S.Costa, C.H., J.Penedones, N.Sousa

Nucl. Phys. B728 (2005) 148-178 [hep-th/0504102]

Carlos Herdeiro

Physics Department, University of Porto, Portugal

@ IPM String School & Workshop, Tehran, 16/04/2006

Topics

- Introduction to Closed Causal Curves (CCCs)

Topics

- Introduction to Closed Causal Curves (CCCs)
- The O-plane orbifold

Topics

- Introduction to Closed Causal Curves (CCCs)
- The O-plane orbifold
  - *Geometry*

Topics

- Introduction to Closed Causal Curves (CCCs)
- The O-plane orbifold
  - *Geometry*
  - *Classical and Quantum Particle Dynamics*

Topics

- Introduction to Closed Causal Curves (CCCs)
- The O-plane orbifold
  - *Geometry*
  - *Classical and Quantum Particle Dynamics*
  - *Classical and Quantum String Dynamics*

Topics

- Introduction to Closed Causal Curves (CCCs)
- The O-plane orbifold
  - *Geometry*
  - *Classical and Quantum Particle Dynamics*
  - *Classical and Quantum String Dynamics*
  - *Partition function and Hagedorn divergence*

Topics

- Introduction to Closed Causal Curves (CCCs)
- The O-plane orbifold
  - *Geometry*
  - *Classical and Quantum Particle Dynamics*
  - *Classical and Quantum String Dynamics*
  - *Partition function and Hagedorn divergence*
- A stringy Chronology Protection Conjecture

Classical General Relativity possesses solutions with CCC

Whenever there are CCCs through any point, the Cauchy problem is ill-defined.

Classical General Relativity possesses solutions with CCC

Whenever there are CCCs through any point, the Cauchy problem is ill-defined.

⋆ We cannot get rid of them just by requiring “good matter” (energy conditions);

Classical General Relativity possesses solutions with CCC

Whenever there are CCCs through any point, the Cauchy problem is ill-defined.

⋆ We cannot get rid of them just by requiring “good matter” (energy conditions);

⋆ Maybe we cannot create them:

Classical General Relativity possesses solutions with CCC

Whenever there are CCCs through any point, the Cauchy problem is ill-defined.

⋆ We cannot get rid of them just by requiring “good matter” (energy conditions);

⋆ Maybe we cannot create them:

Hawking (1992) considers quantum fields driving evolution in “quasi-static” process; shows 2-point correlators and $\langle \hat{T}_{\mu\nu} \rangle$ blow up along Closed Null Curve;

Hawking’s proposal

Hawking’s Chronology Protection Conjecture is based on a toy model (Misner space) and relies on High Energy = Ultra Violet effects of a quantum field;

Hawking’s proposal

Hawking’s Chronology Protection Conjecture is based on a toy model (Misner space) and relies on High Energy = Ultra Violet effects of a quantum field;

But String Theory provides a UV completion of GR with very different behaviour from ordinary QFT;

Hawking's proposal

Hawking's Chronology Protection Conjecture is based on a toy model (Misner space) and relies on High Energy = Ultra Violet effects of a quantum field;

But String Theory provides a UV completion of GR with very different behaviour from ordinary QFT;

We will build a toy model with CCCs (O-plane orbifold), and suggest that string theory will not allow CCCs... but for a very different reason from Hawking's proposal:

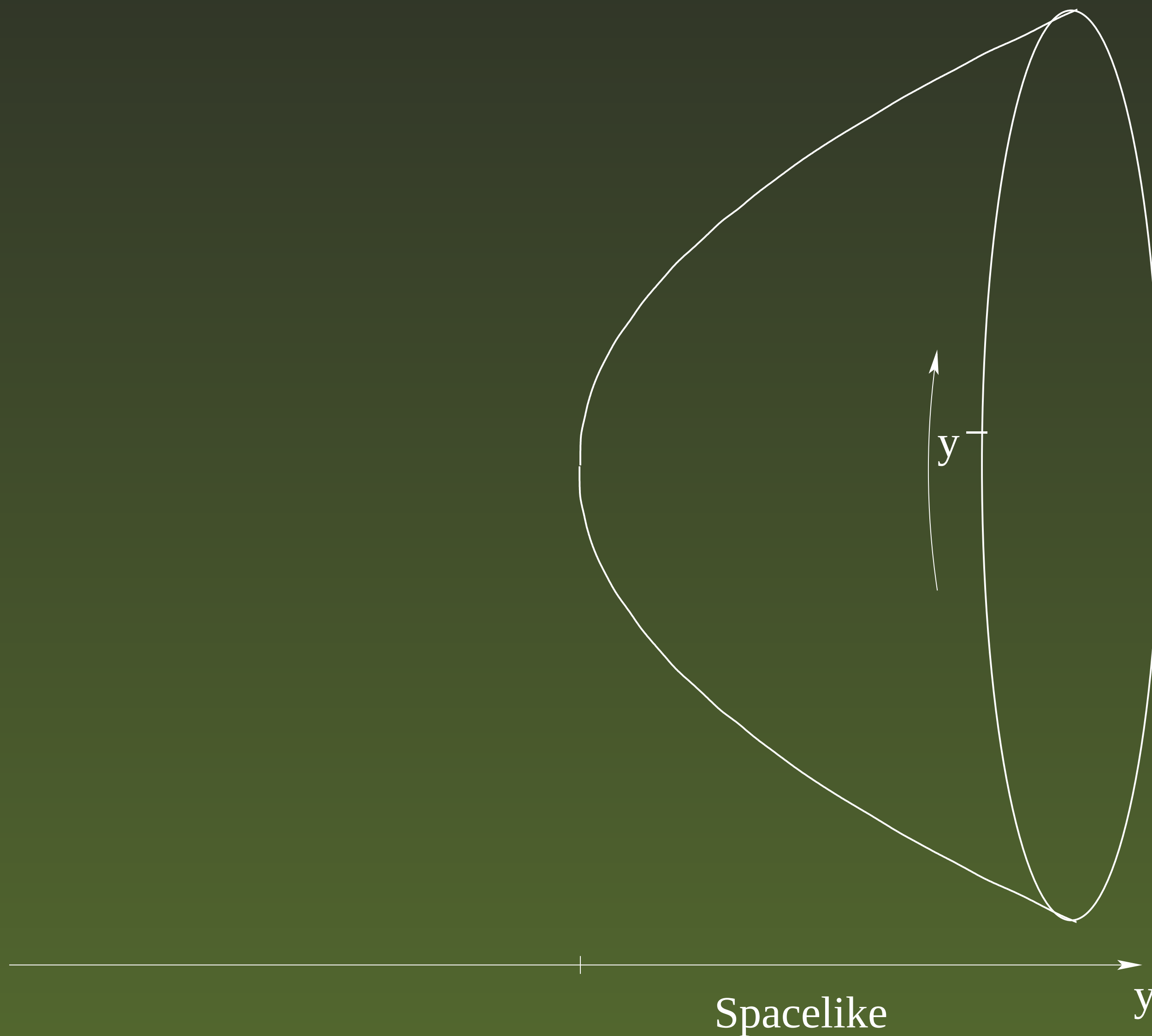
Hawking's proposal

Hawking's Chronology Protection Conjecture is based on a toy model (Misner space) and relies on High Energy = Ultra Violet effects of a quantum field;

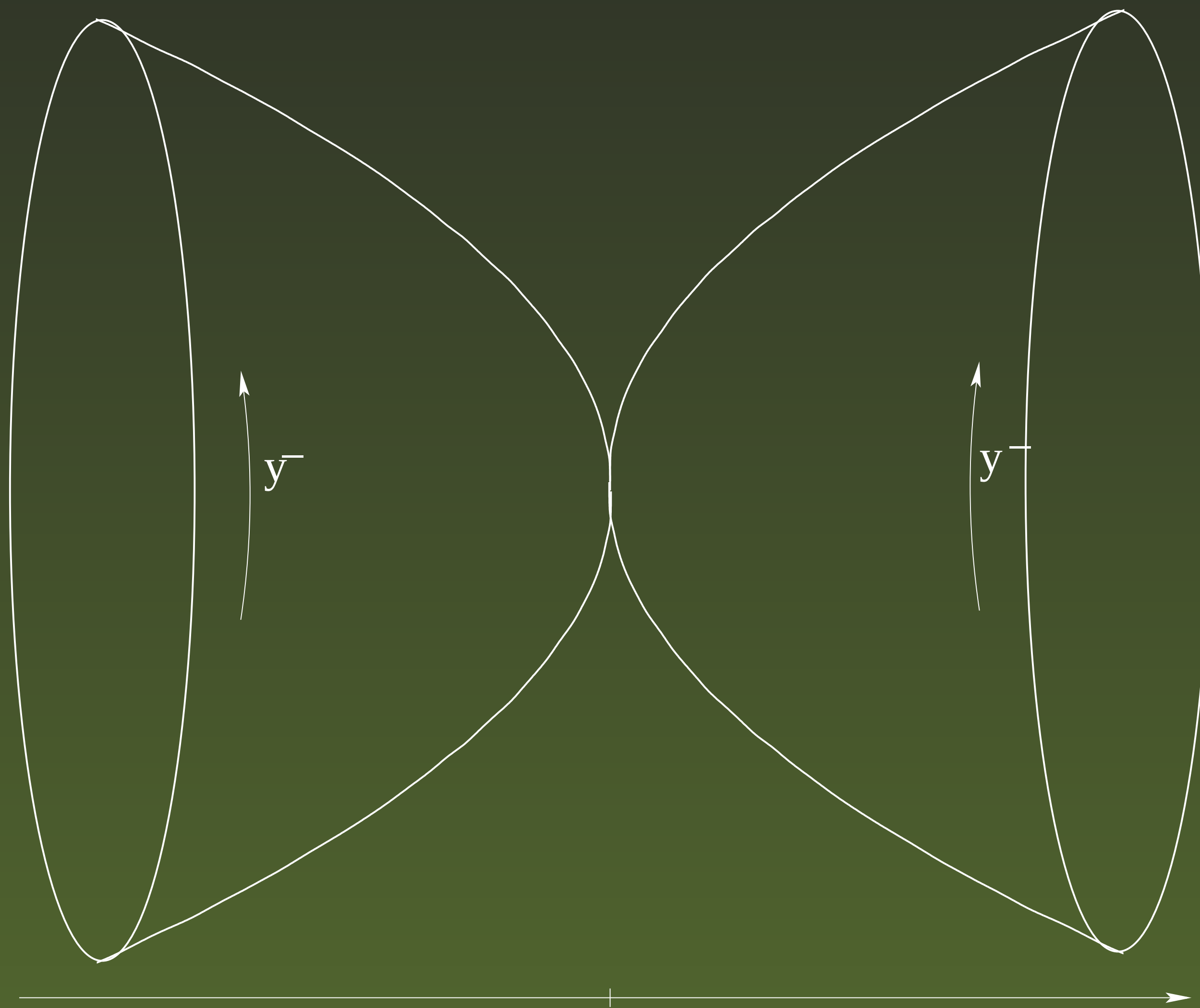
But String Theory provides a UV completion of GR with very different behaviour from ordinary QFT;

We will build a toy model with CCCs (O-plane orbifold), and suggest that string theory will not allow CCCs... but for a very different reason from Hawking's proposal:
it is an *Infrared effect*.

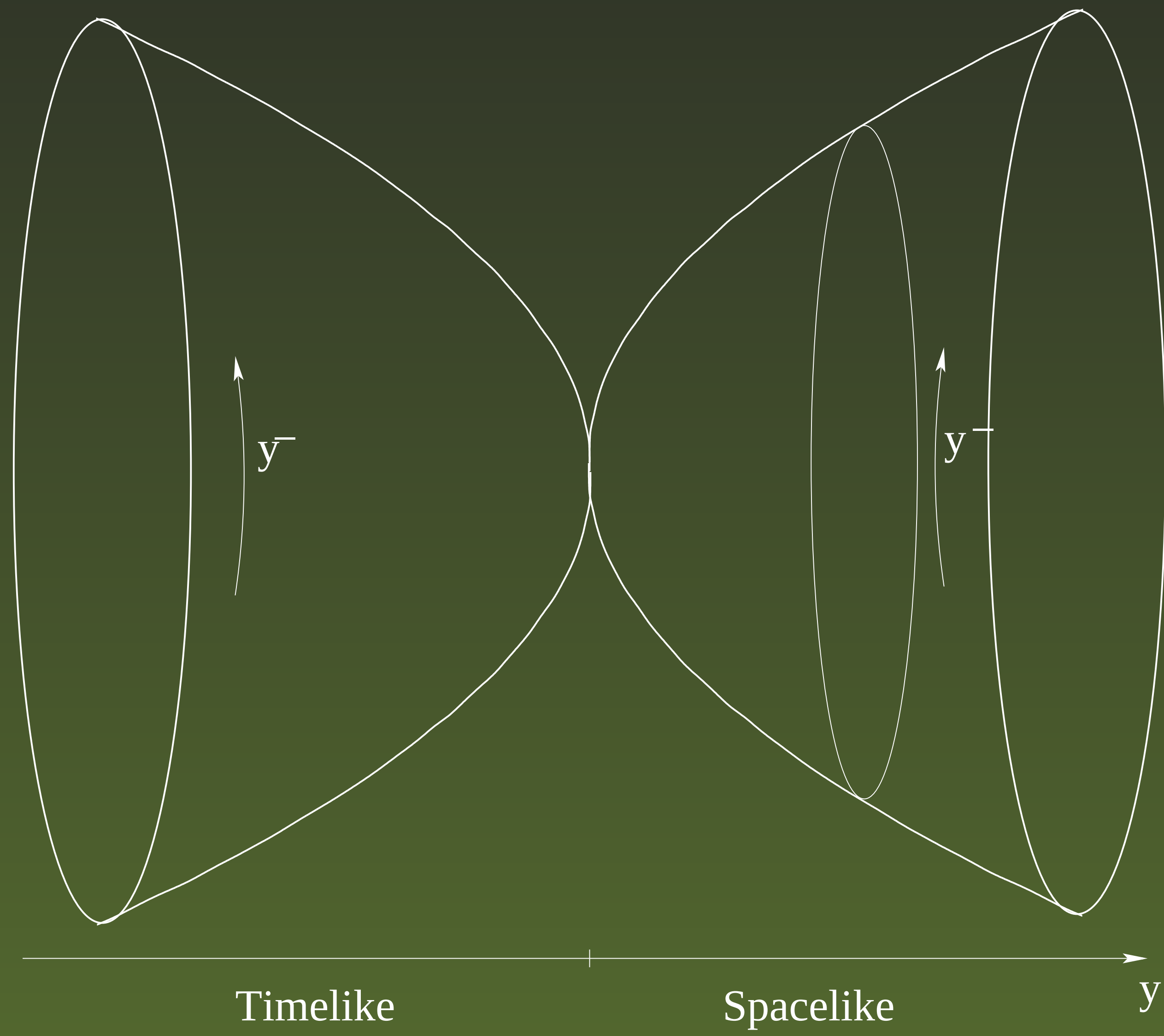
Our Toy Model:



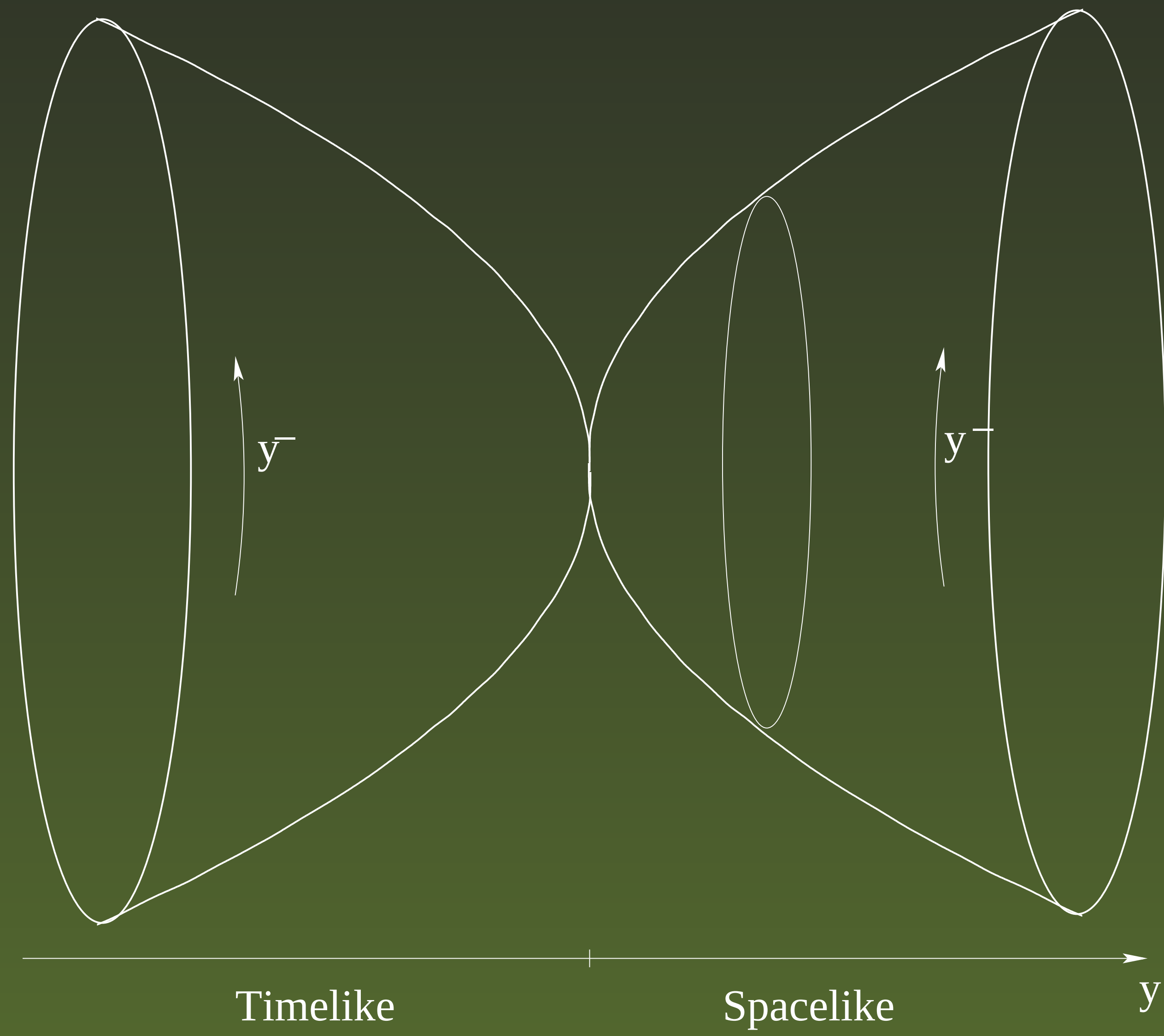
Our Toy Model:



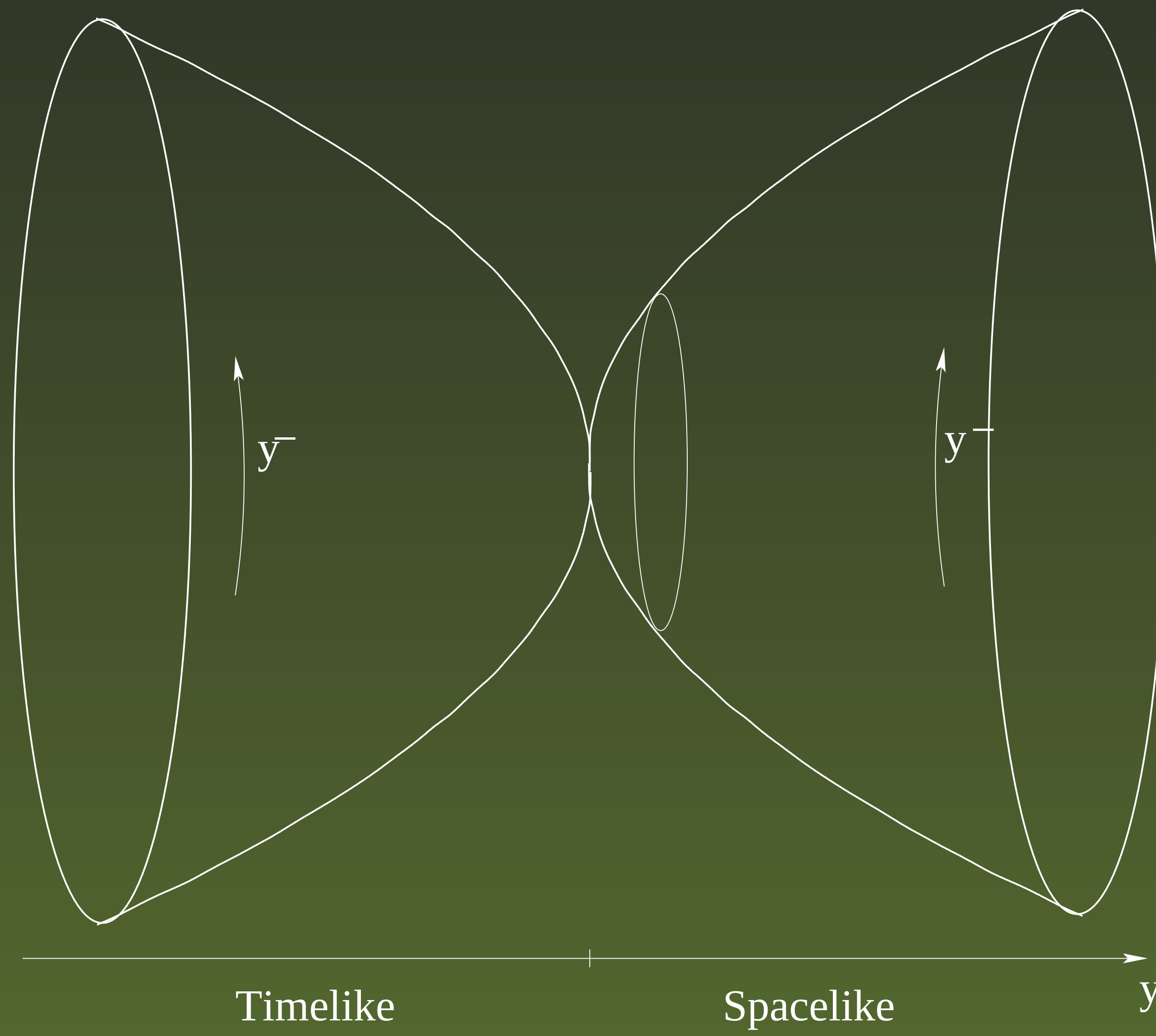
Our Toy Model:



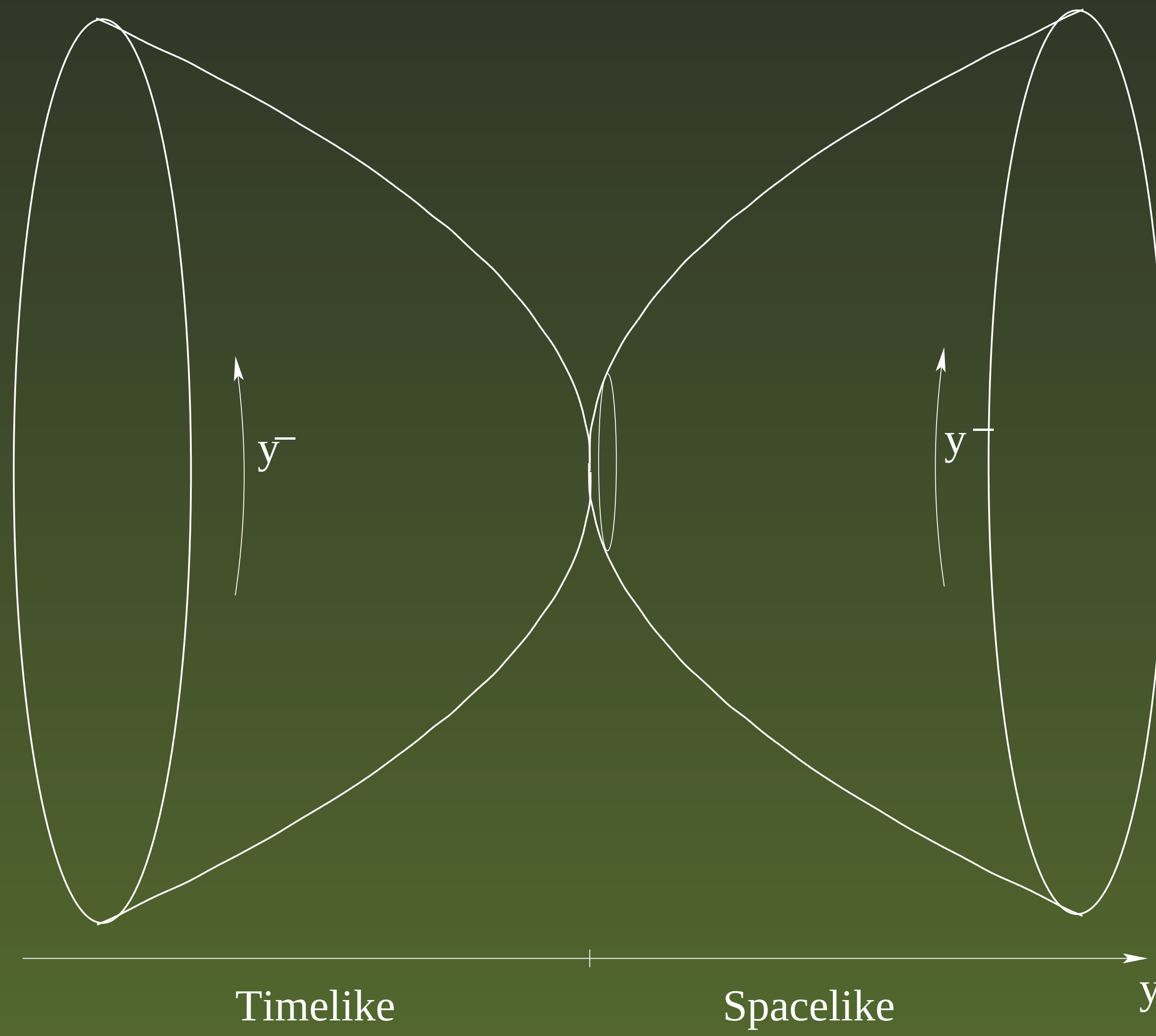
Our Toy Model:



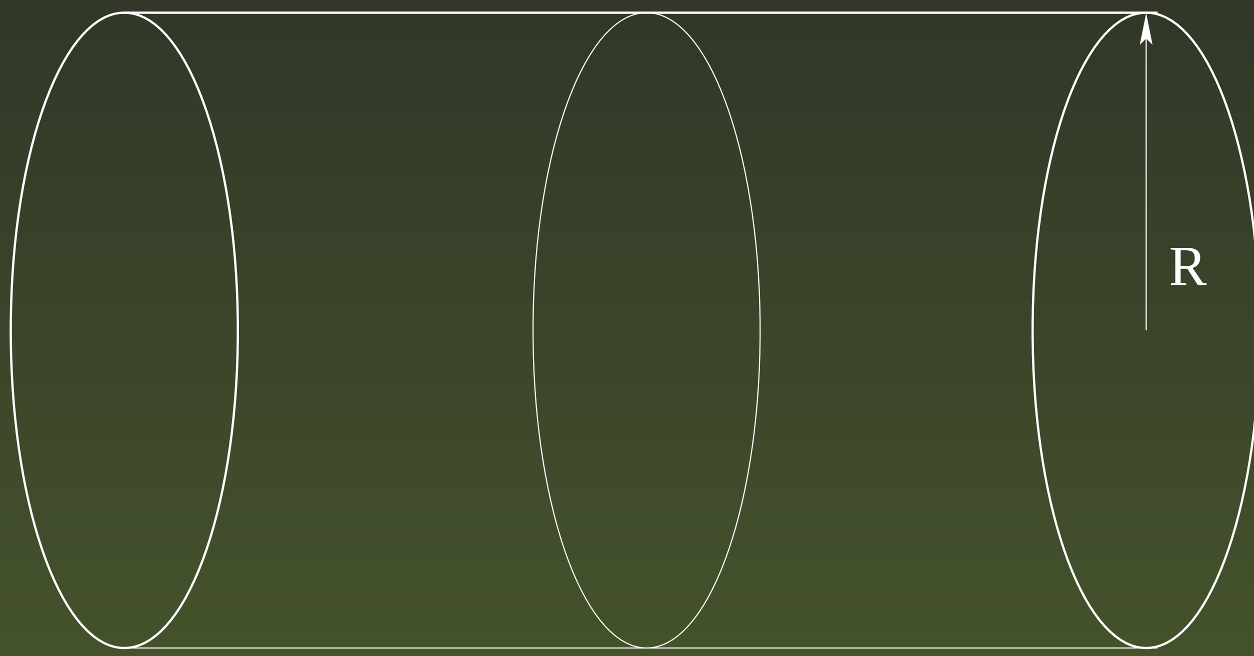
Our Toy Model:



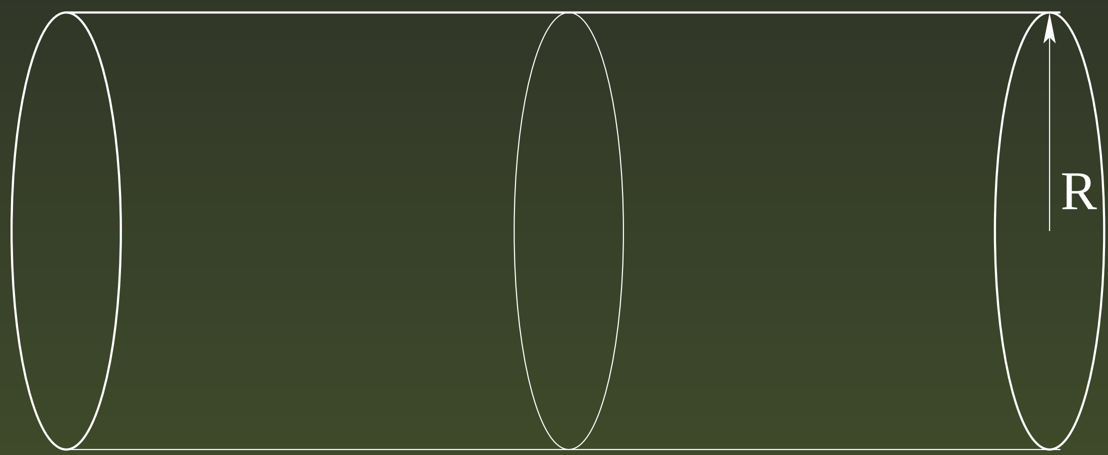
Our Toy Model:



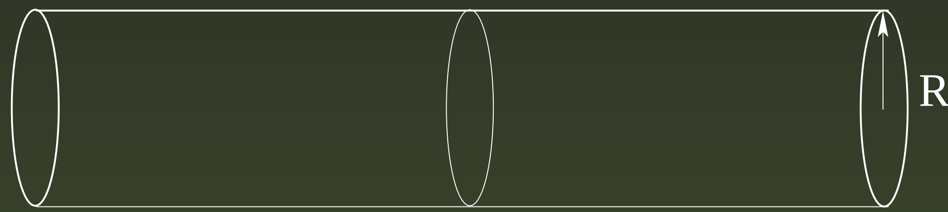
Circle compactification:



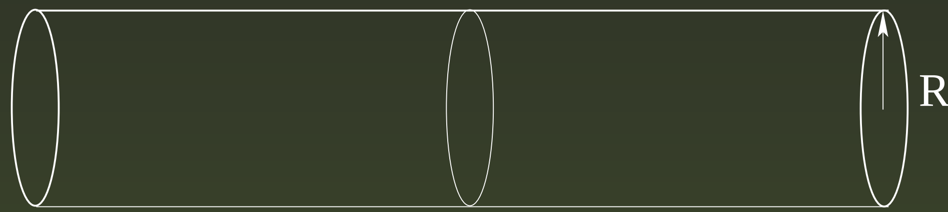
Circle compactification:



Circle compactification:

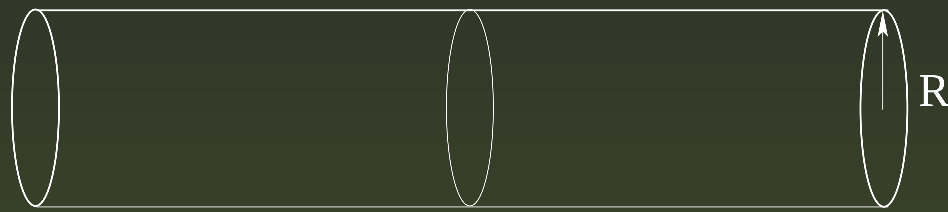


Circle compactification:



When $R \sim \ell_s$ new massless modes appear in the string spectrum and the oscillator sum in the partition functions diverges - *Hagedorn behaviour*

Circle compactification:



When $R \sim \ell_s$ new massless modes appear in the string spectrum and the oscillator sum in the partition functions diverges - *Hagedorn behaviour*

It has been suggested that such winding tachyons could lead to topology changes (E.Silverstein and collaborators).

The O-plane orbifold

It is an orbifold of $\mathbb{M}^{2,1}$. Consider light cone coordinates (x^-, x, x^+) and the flat metric $ds^2 = -2dx^+dx^- + dx^2$.

The O-plane orbifold

It is an orbifold of $\mathbb{M}^{2,1}$. Consider light cone coordinates (x^-, x, x^+) and the flat metric $ds^2 = -2dx^+dx^- + dx^2$.

The orbifold group element $\Omega = e^\kappa$ is generated by the Killing vector

$$\kappa = 2\pi i\left(RP_- + \Delta J\right)$$

where

$$iJ = x_+\frac{\partial}{\partial x} - x\frac{\partial}{\partial x^+}\,, \qquad iP_- = \frac{\partial}{\partial x^-}\,.$$

The O-plane orbifold

It is an orbifold of $\mathbb{M}^{2,1}$. Consider light cone coordinates (x^-, x, x^+) and the flat metric $ds^2 = -2dx^+dx^- + dx^2$.

The orbifold group element $\Omega = e^\kappa$ is generated by the Killing vector

$$\kappa = 2\pi i \left(RP_- + \Delta J \right)$$

where

$$iJ = x_+ \frac{\partial}{\partial x} - x \frac{\partial}{\partial x^+} \,, \qquad iP_- = \frac{\partial}{\partial x^-} \,.$$

$$\vec{x} \equiv \begin{pmatrix} x^- \\ x \\ x^+ \end{pmatrix} \sim \begin{pmatrix} x^- + 2\pi R \\ x - 2\pi\Delta x^- - 2\pi^2 R\Delta \\ x^+ - 2\pi\Delta x + 2\pi^2\Delta^2 x^- + \frac{4}{3}\pi^3 R\Delta^2 \end{pmatrix}$$

y-coordinates

We can introduce coordinates (y^-, y, y^+) in which the orbifold action becomes trivial:

y-coordinates

We can introduce coordinates (y^-, y, y^+) in which the orbifold action becomes trivial:

$$
\begin{aligned}
x^- &= y^- \\
x &= y - \frac{E}{2}(y^-)^2 \\
x^+ &= y^+ - Eyy^- + \frac{E^2}{6}(y^-)^3 \,,
\end{aligned}
$$

where $E = \Delta/R$.

y-coordinates

We can introduce coordinates (y^-, y, y^+) in which the orbifold action becomes trivial:

$$\begin{aligned} x^- &= y^- \\ x &= y - \frac{E}{2}(y^-)^2 \\ x^+ &= y^+ - Eyy^- + \frac{E^2}{6}(y^-)^3 \,, \end{aligned}$$

where $E = \Delta/R$. The orbifold identification and metric become

$$y^- \sim y^- + 2\pi R \,, \qquad ds^2 = -2dy^- dy^+ + 2Ey(dy^-)^2 + dy^2$$

y-coordinates

We can introduce coordinates (y^-, y, y^+) in which the orbifold action becomes trivial:

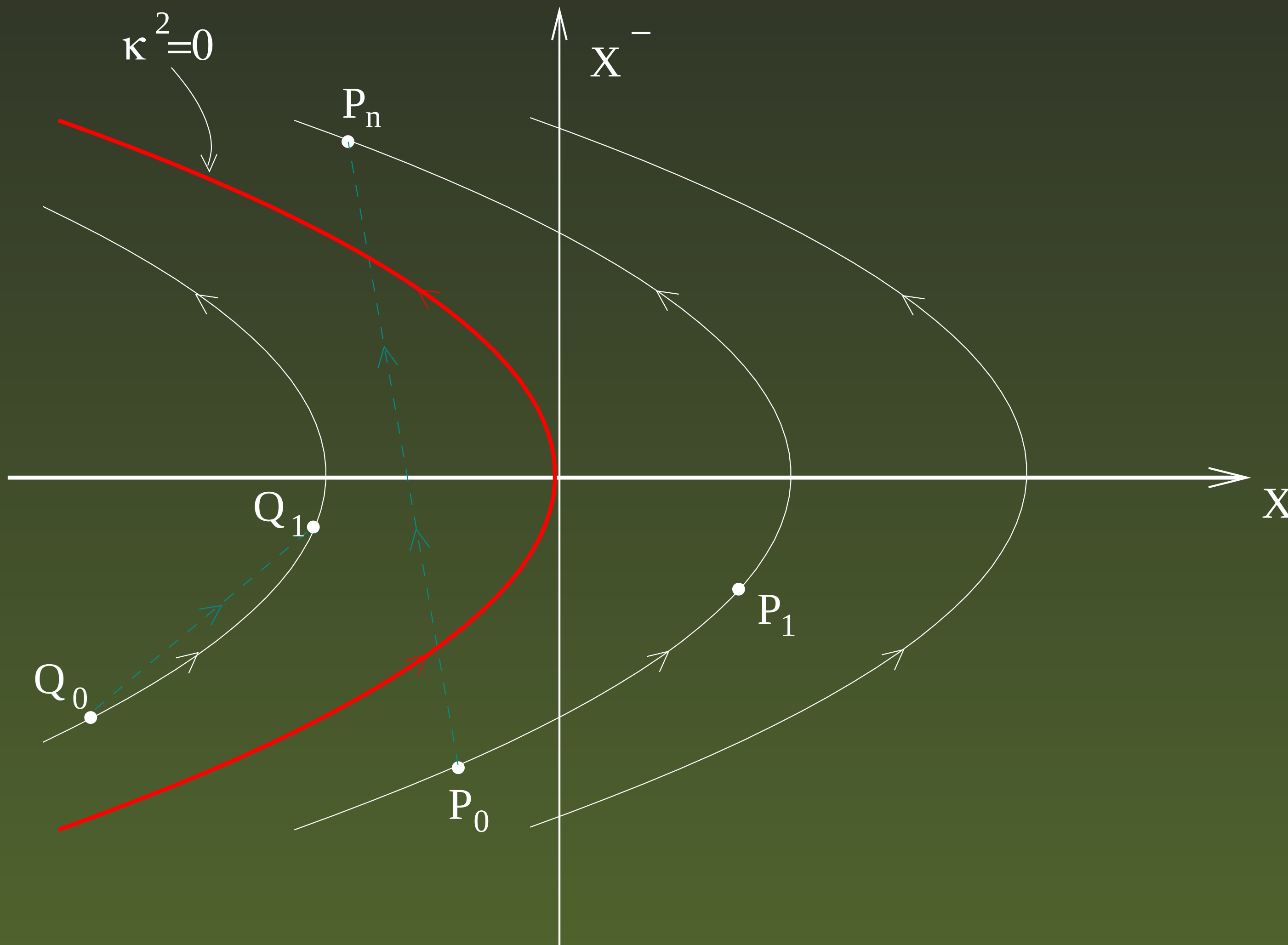
$$
\begin{aligned}
x^- &= y^- \\
x &= y - \frac{E}{2}(y^-)^2 \\
x^+ &= y^+ - Eyy^- + \frac{E^2}{6}(y^-)^3 \,,
\end{aligned}
$$

where $E = \Delta/R$. The orbifold identification and metric become

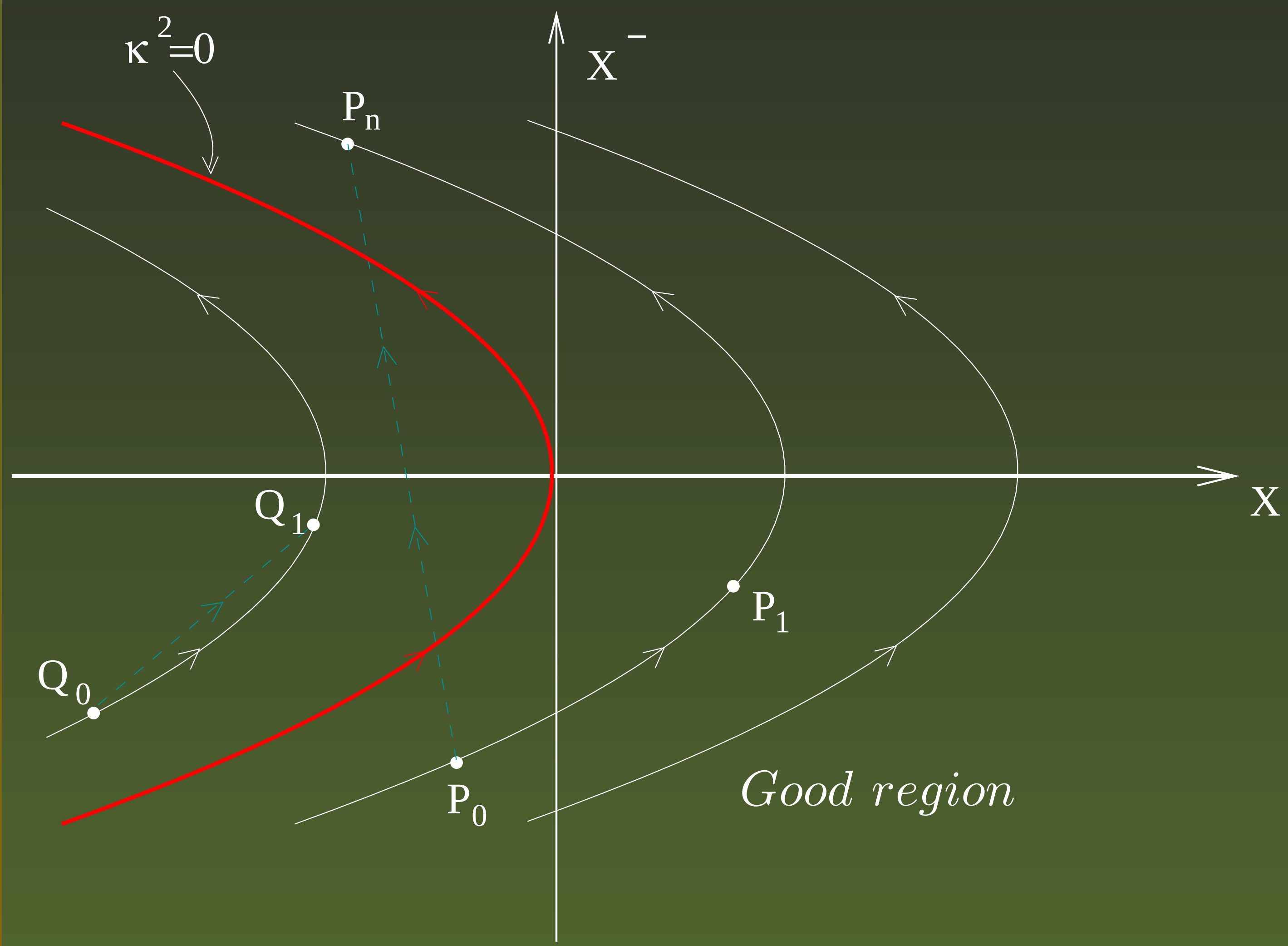
$$
y^- \sim y^- + 2\pi R \,, \qquad ds^2 = -2dy^-dy^+ + 2Ey(dy^-)^2 + dy^2
$$

$$
\kappa \text{ is } \begin{cases} \textit{spacelike} \text{ for } y > 0 \\ \textit{null} \text{ for } y = 0 \\ \textit{timelike} \text{ for } y < 0 \end{cases}
$$

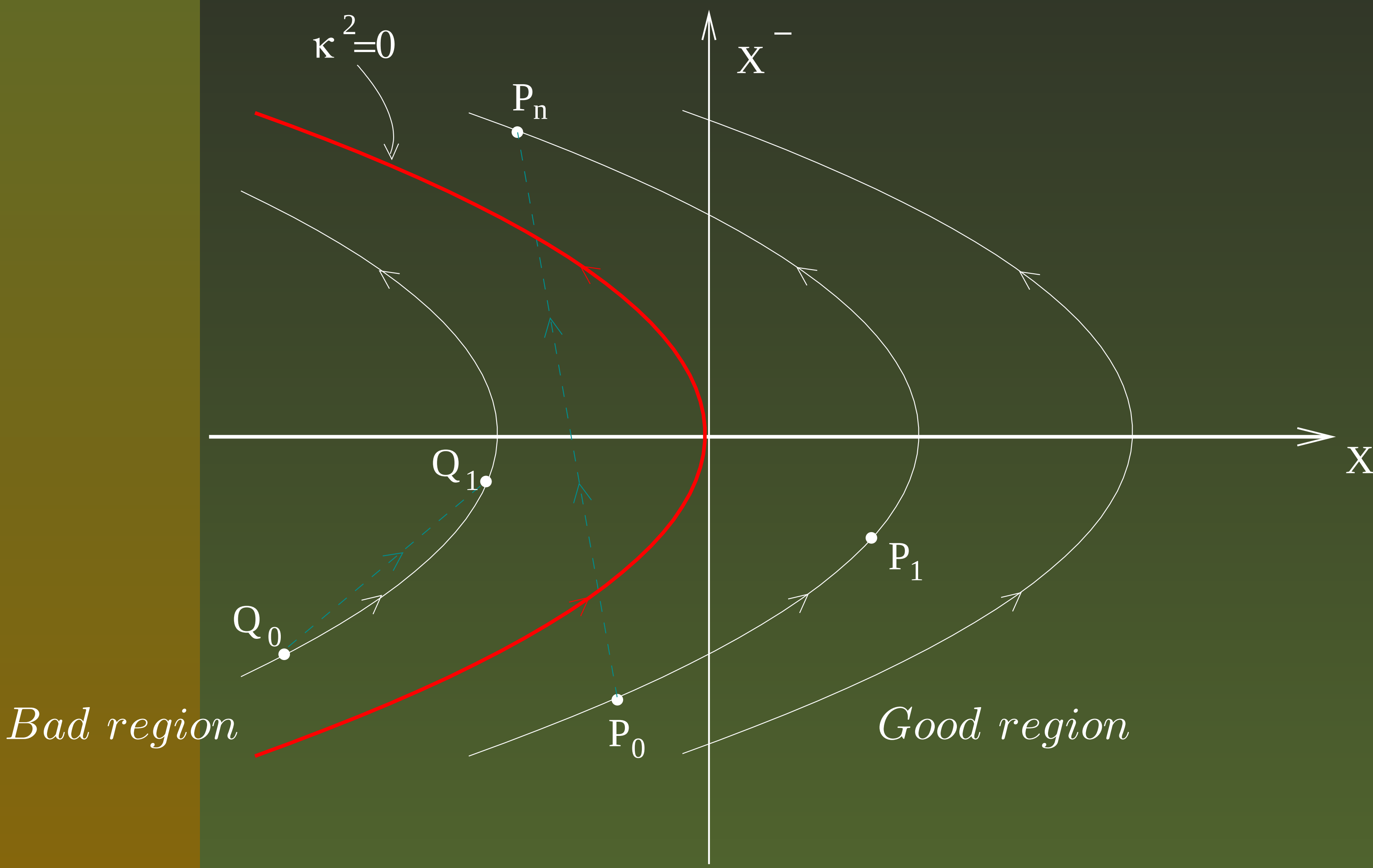
O-plane as a Parabolic orbifold



O-plane as a Parabolic orbifold



O-plane as a Parabolic orbifold



Classical particle dynamics I

Hamiltonian for particle dynamics in y-coordinates

$$\mathcal{H} = q^2 + V(y)\,, \qquad V(y) \equiv -2Eq_+^2 y - 2q_+ q_-$$

Classical particle dynamics I

Hamiltonian for particle dynamics in y-coordinates

$$\mathcal{H} = q^2 + V(y) \, , \qquad V(y) \equiv -2Eq_+^2 y - 2q_+ q_-$$

- There is a linear potential $\Leftrightarrow$ constant (inertial) force because y-coordinates are accelerating.

Classical particle dynamics I

Hamiltonian for particle dynamics in y-coordinates

$$\mathcal{H} = q^2 + V(y)\,, \qquad V(y) \equiv -2Eq_+^2 y - 2q_+q_-$$

- There is a linear potential $\Leftrightarrow$ constant (inertial) force because y-coordinates are accelerating.

- The Hamiltonian is a constant of motion $\quad \mathcal{H} = q_\mu q^\mu = -M^2$.

Classical particle dynamics I

Hamiltonian for particle dynamics in y-coordinates

$$\mathcal{H} = q^2 + V(y)\ , \qquad V(y) \equiv -2Eq_+^2 y - 2q_+ q_-$$

• There is a linear potential ⇔ constant (inertial) force because y-coordinates are accelerating.

• The Hamiltonian is a constant of motion $\mathcal{H} = q_\mu q^\mu = -M^2$.

• Mass-shell condition in terms of: the classical turning point y_0, light-cone energy q_+ and Kaluza-Klein momentum q_-

$$M^2 = 2q_+ q_- + 2Eq_+^2 y_0$$

Classical particle dynamics II

$$V(y) \equiv -2Eq_+^2 y - 2q_+q_- \,, \qquad M^2 = -V(y_0)$$

Classical particle dynamics II

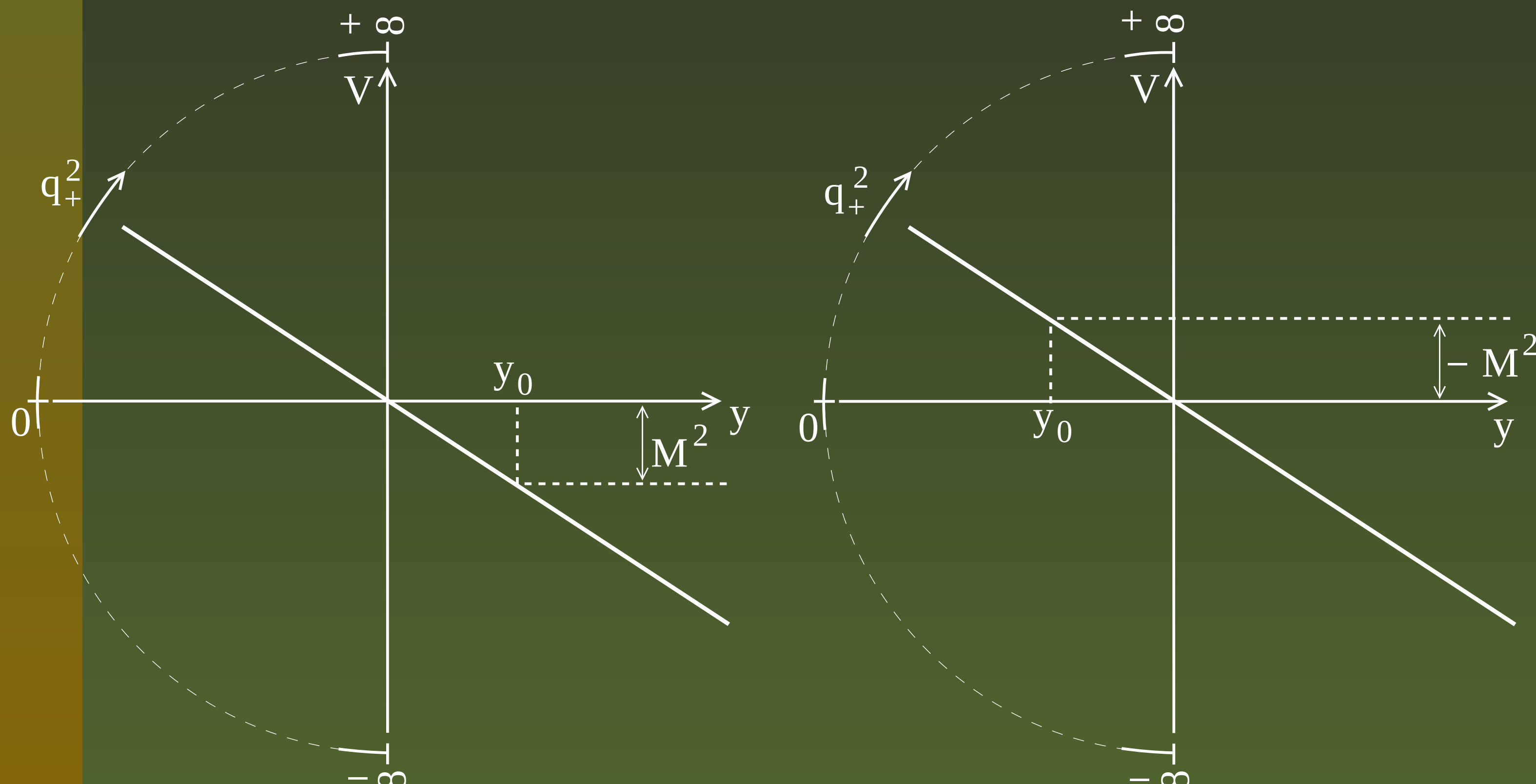
$$V(y) \equiv -2Eq_+^2 y - 2q_+q_- \ , \qquad M^2 = -V(y_0)$$

For $q_- = 0$:

Classical particle dynamics II

$$V(y) \equiv -2Eq_+^2 y - 2q_+q_- \ , \qquad M^2 = -V(y_0)$$

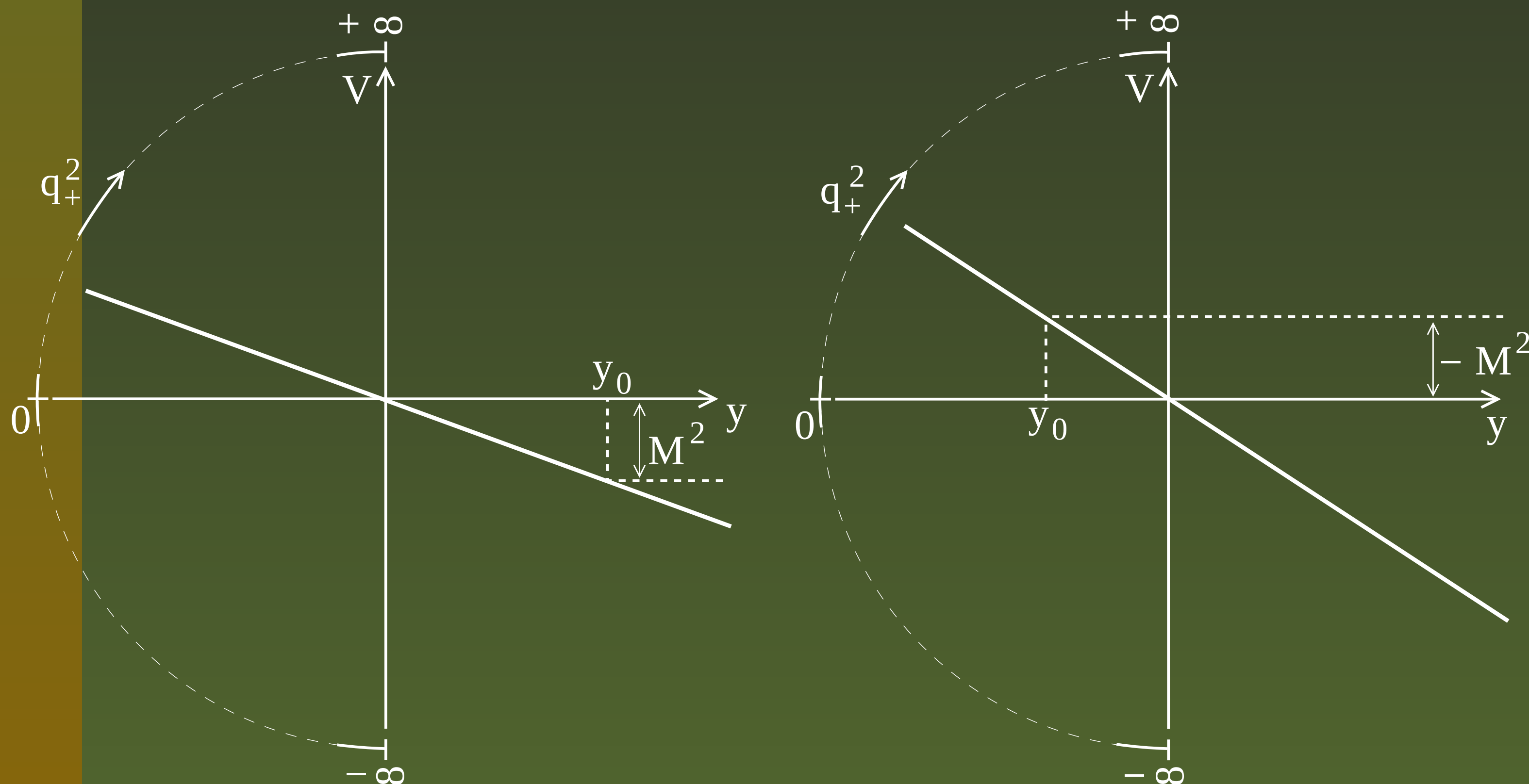
For $q_- = 0$:



Classical particle dynamics II

$$V(y) \equiv -2Eq_+^2 y - 2q_+q_- \ , \qquad M^2 = -V(y_0)$$

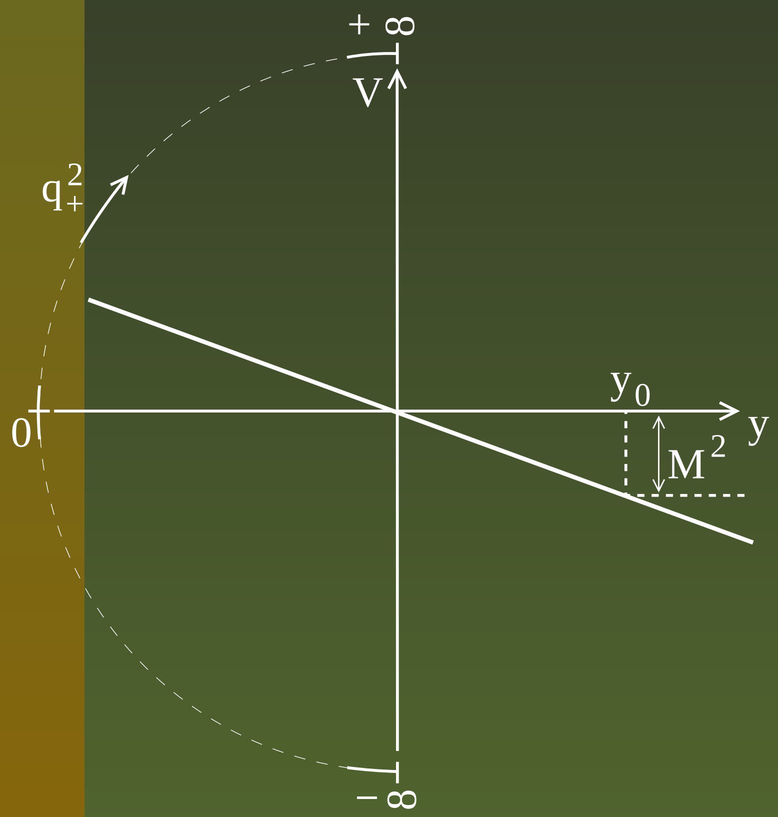
For $q_- = 0$:

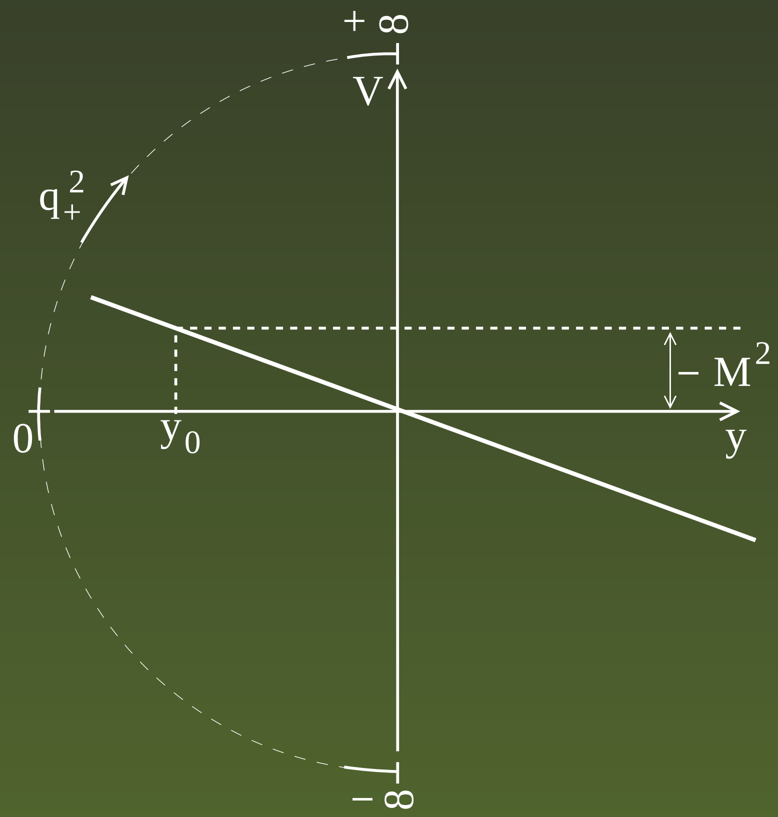


Classical particle dynamics II

$$V(y) \equiv -2Eq_+^2 y - 2q_+ q_- \ , \qquad\qquad M^2 = -V(y_0)$$

For $q_- = 0$:

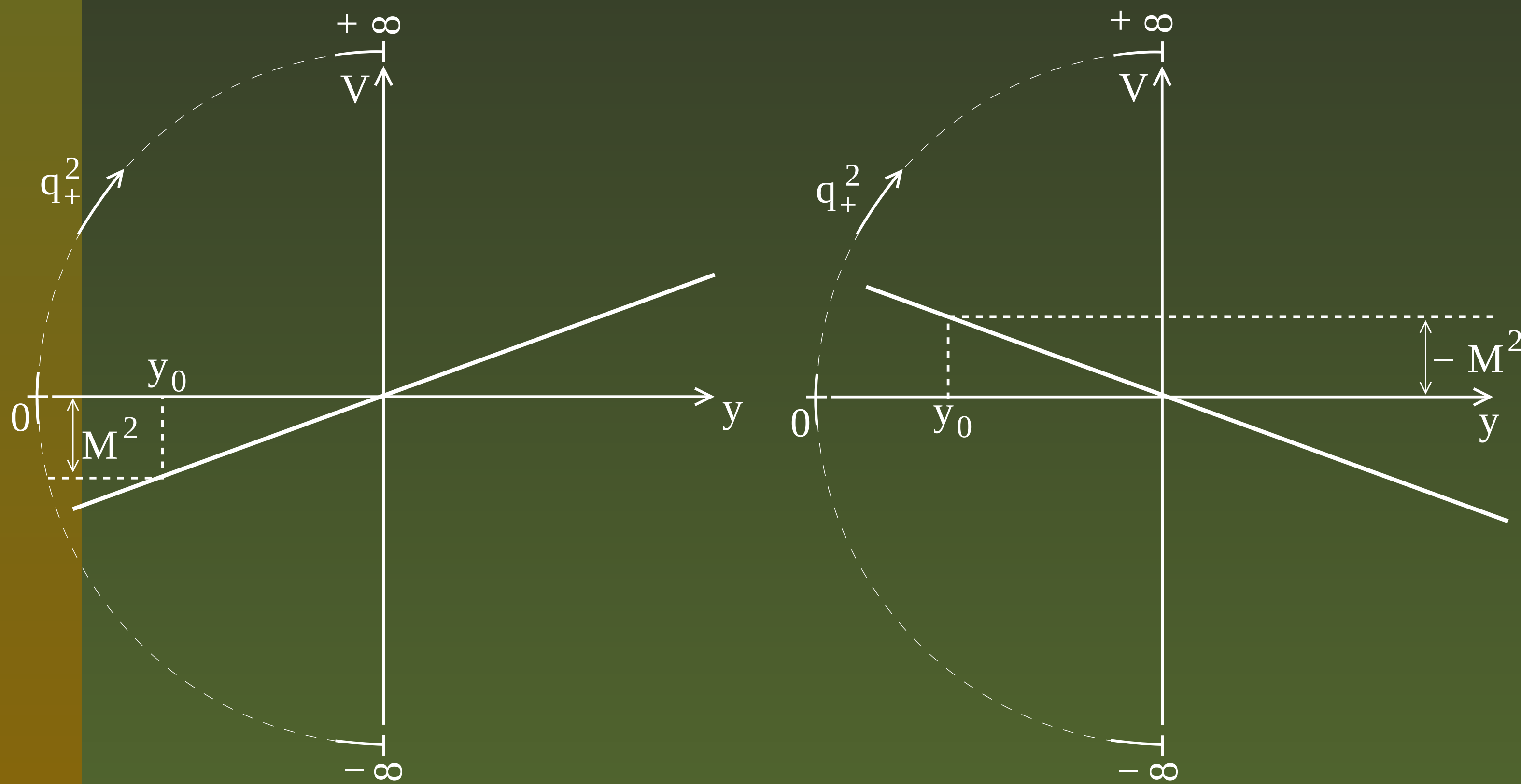




Classical particle dynamics II

$$V(y) \equiv -2Eq_+^2 y - 2q_+q_- \ , \qquad M^2 = -V(y_0)$$

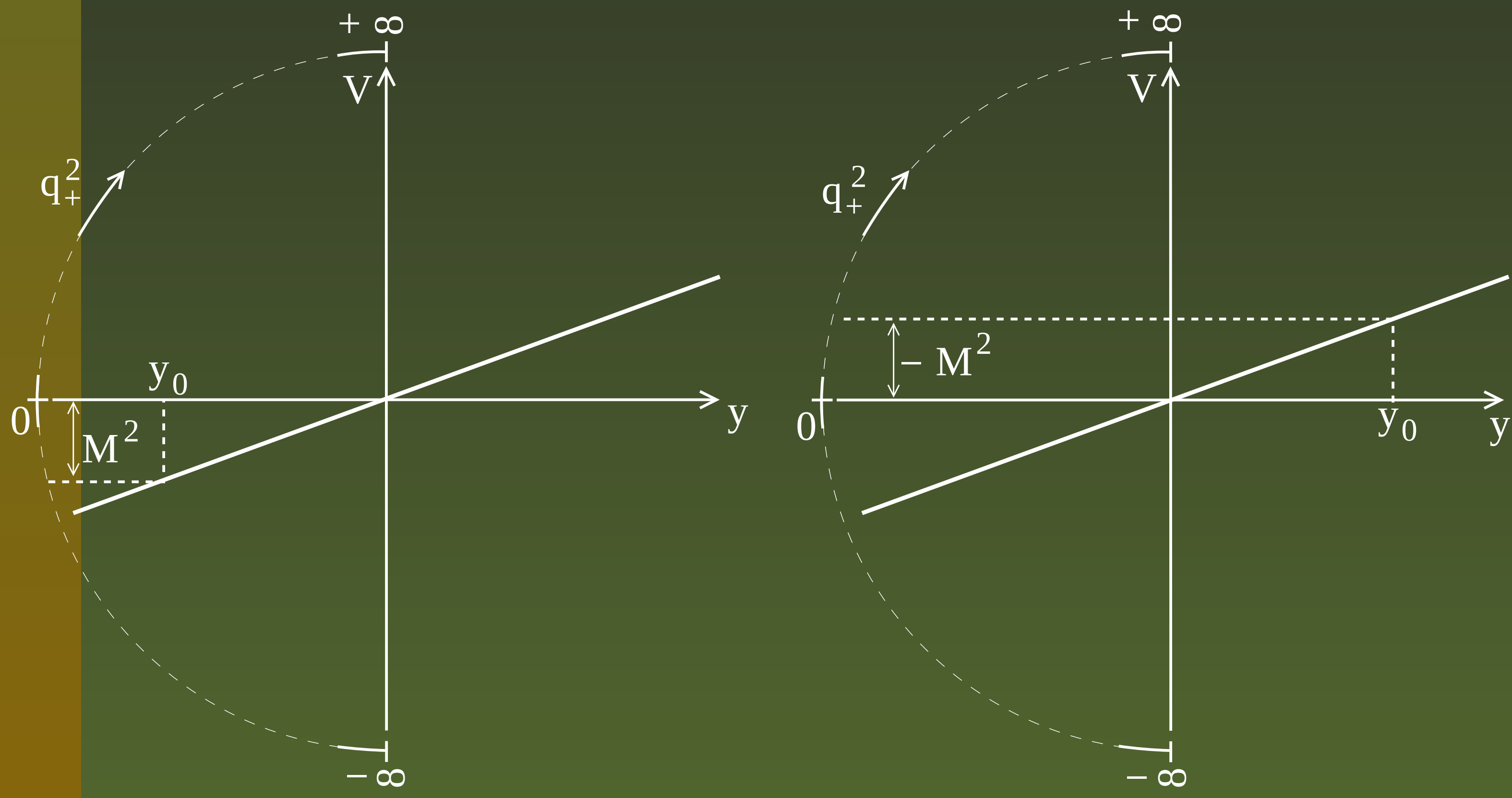
For $q_- = 0$:



Classical particle dynamics II

$$V(y) \equiv -2Eq_+^2 y - 2q_+q_- \ , \qquad M^2 = -V(y_0)$$

For $q_- = 0$:



Single particle wave functions I

The Klein-Gordon equation

$$\left(-2\partial_+\partial_- - 2Ey\,\partial_+^2 + \partial_y^2 \right)\Phi = M^2\,\Phi$$

Single particle wave functions I

The Klein-Gordon equation

$$\Big(-2\partial_+\partial_- - 2Ey\,\partial_+^2 + \partial_y^2 \Big)\,\Phi = M^2\,\Phi$$

is solved by

$$\Phi_{q_+,y_0,m}(\vec{y}) = \frac{|K(q_+)|^{1/3}}{\sqrt{2\pi R L_+ \rho(y_0,q_+)}}\,Ai(z)\,e^{\,i\left(q_+y^+ + \frac{m}{R}y^-\right)}$$

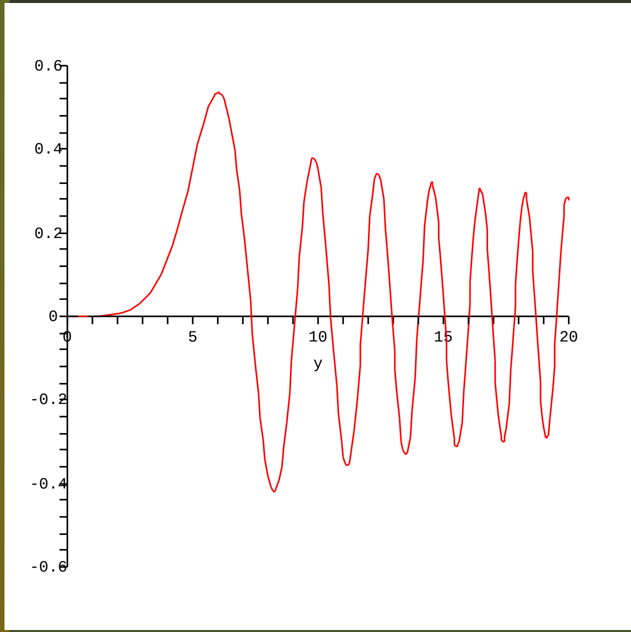
• $\mathrm{Ai}(z)$ are Airy functons;

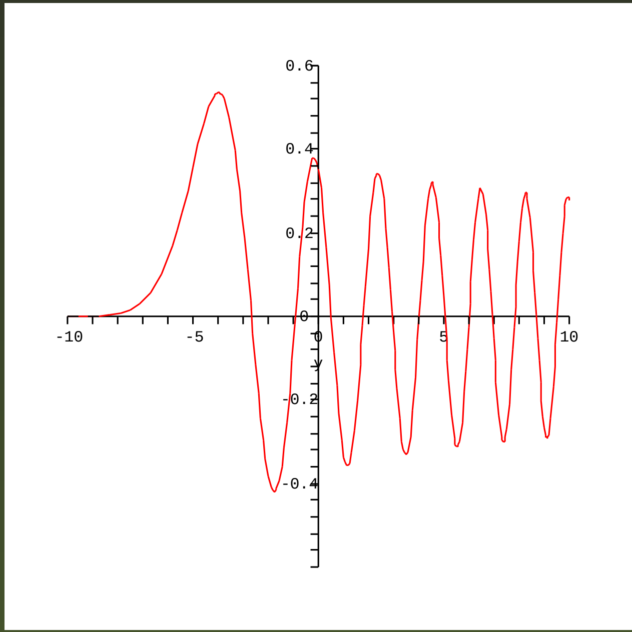
• For normalisation we considered a “box” defined by $0 \leq y^+ \leq L_+$ and $-L/2 \leq y \leq L/2$;

• $q_- = m/R$ is the Kaluza-Klein momentum;

• $z^3 = K(y_0 - y)^3$ and $K(q_+) = 2Eq_+^2$.

Single particle wave functions II





$M^2 > 0,\ q_+^2 > 0$ $M^2 < 0,\ q_+^2 > 0$

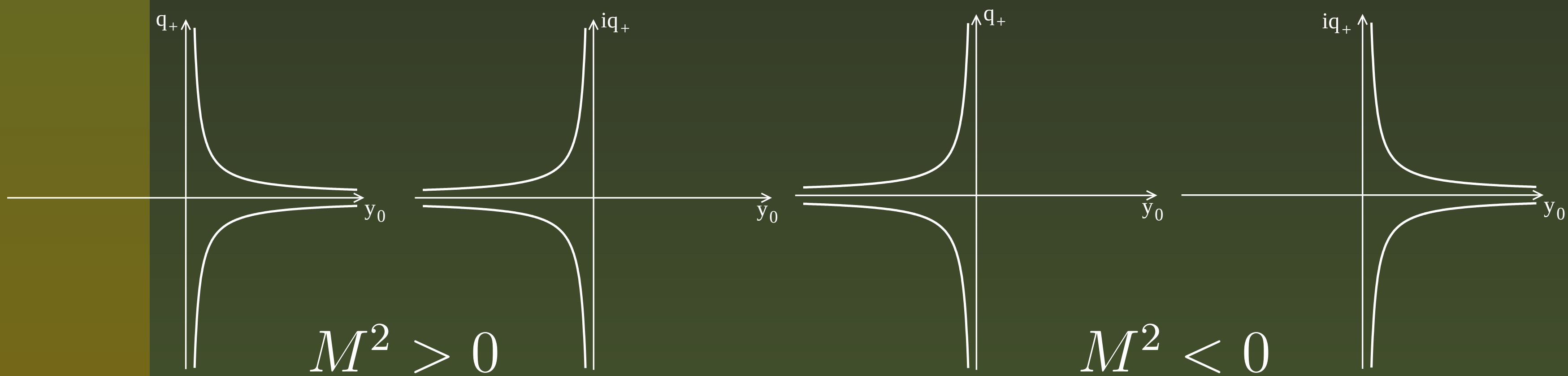
y_0 is where wave functions turn from oscillatory to evanescent.

Single particle wave functions: summary

Consider q_+ as a function of M^2 and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{M^2}{2Ey_0}}$

Single particle wave functions: summary

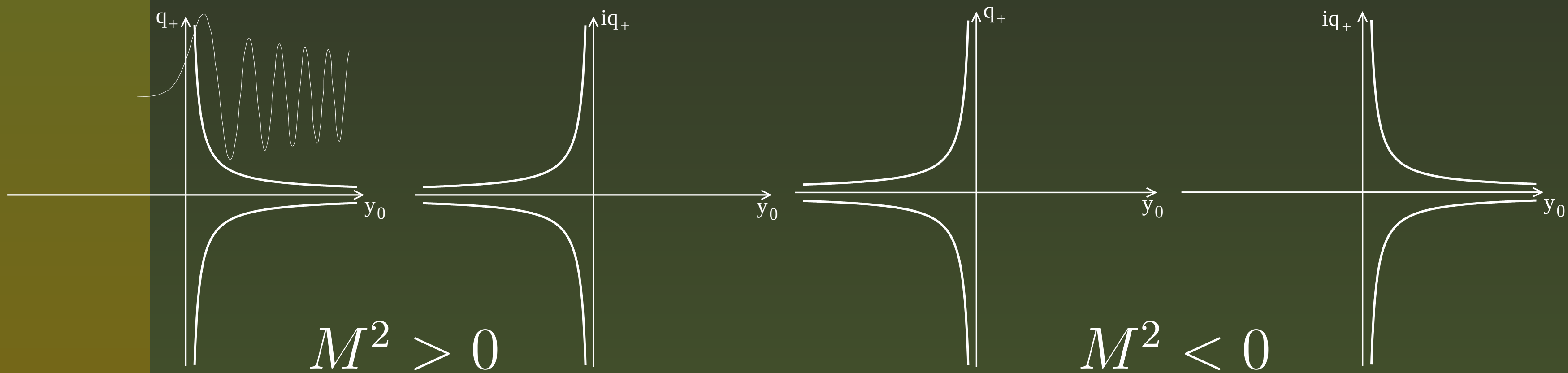
Consider q_+ as a function of M^2 and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{M^2}{2Ey_0}}$



$M^2 > 0$ $\qquad$ $M^2 < 0$

Single particle wave functions: summary

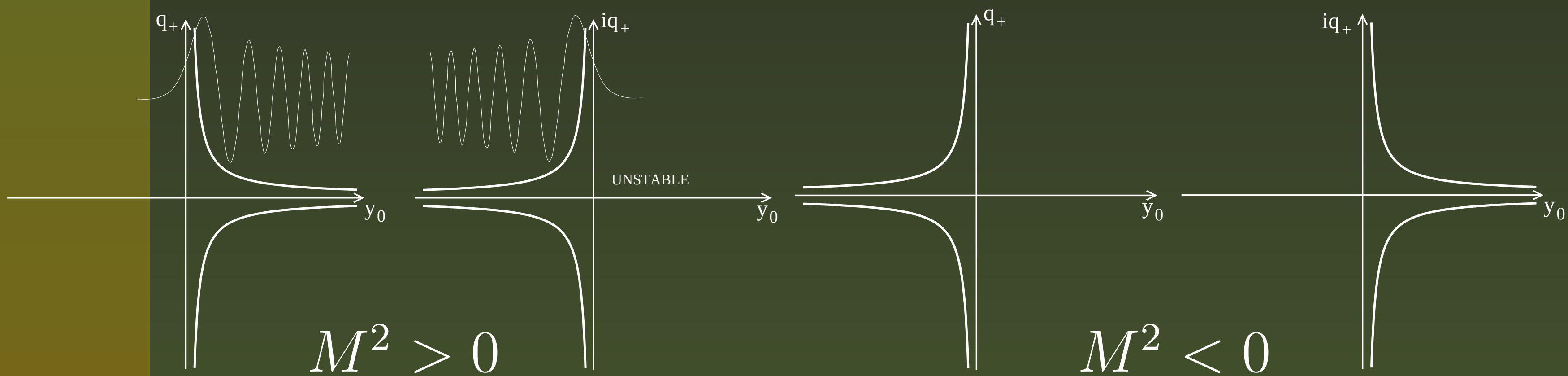
Consider q_+ as a function of M^2 and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{M^2}{2Ey_0}}$



$M^2 > 0$ $\qquad\qquad$ $M^2 < 0$

Single particle wave functions: summary

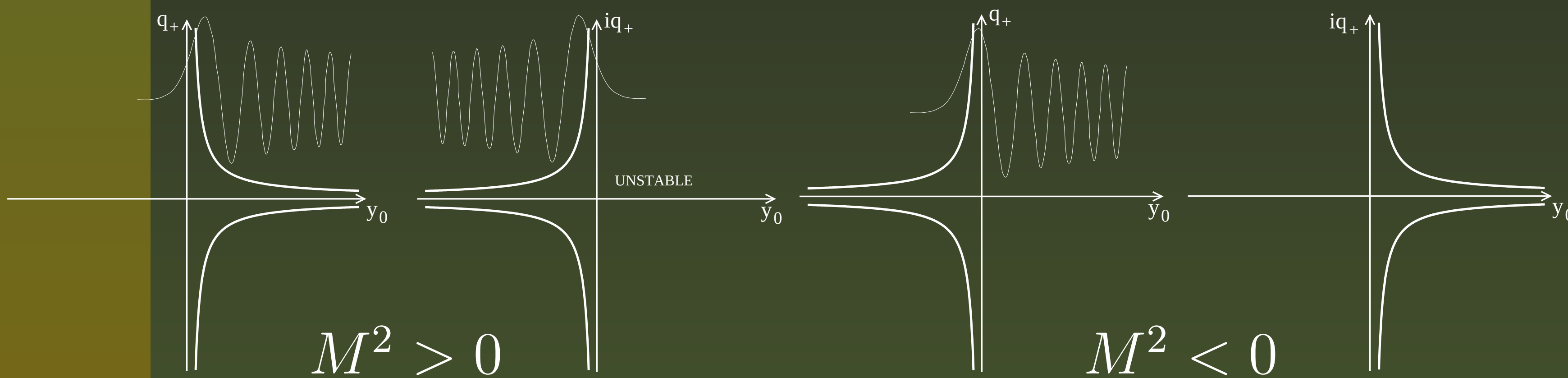
Consider q_+ as a function of M^2 and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{M^2}{2Ey_0}}$



$M^2 > 0$ $\qquad$ $M^2 < 0$

Single particle wave functions: summary

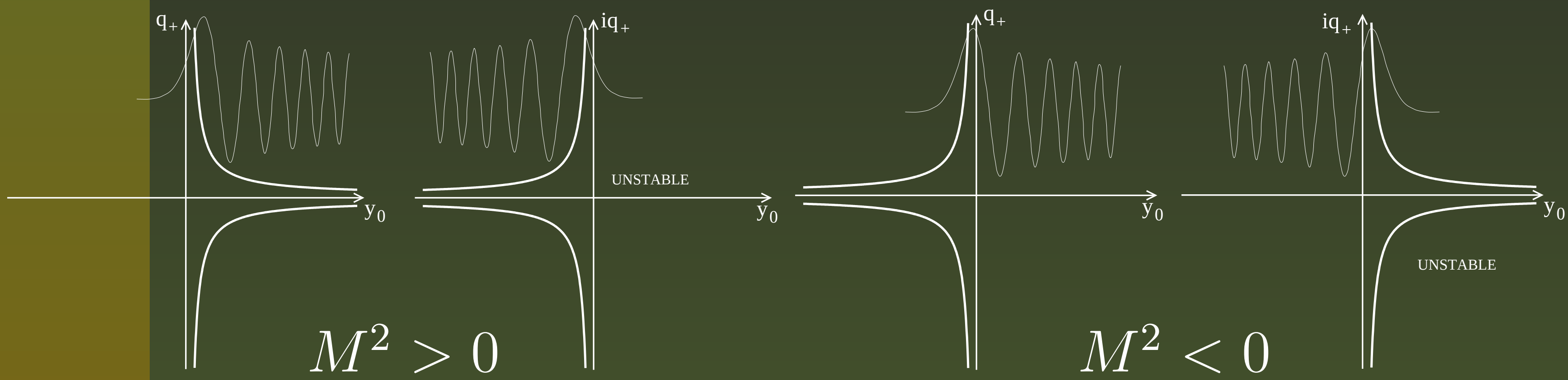
Consider q_+ as a function of M^2 and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{M^2}{2Ey_0}}$



$M^2 > 0$ $M^2 < 0$

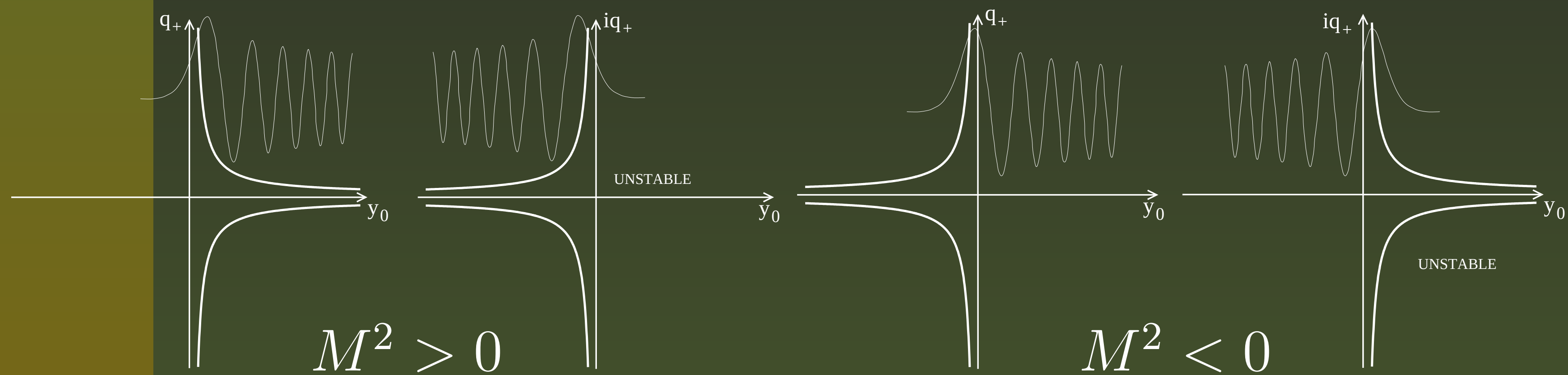
Single particle wave functions: summary

Consider q_+ as a function of M^2 and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{M^2}{2Ey_0}}$



Single particle wave functions: summary

Consider q_+ as a function of M^2 and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{M^2}{2Ey_0}}$



$M^2 > 0$ $M^2 < 0$

Note: Tachyon ($M^2 < 0$) $\neq$ Instability ($Im(q_+) \neq 0$); well known example Freedman-Breitenlohner bound in AdS.

Classical string dynamics I

The novelty is the winding;

Classical string dynamics I

The novelty is the winding; Embedding functions $(0 \leq \sigma \leq \pi)$

$$Y^{-}(\tau,\sigma) = 2R\omega\sigma + y^{-}(\tau)\ , \qquad Y(\tau,\sigma) = y(\tau)\ , \qquad Y^{+}(\tau,\sigma) = y^{+}(\tau)$$

Classical string dynamics I

The novelty is the winding; Embedding functions ($0 \leq \sigma \leq \pi$)

$$Y^-(\tau,\sigma) = 2R\omega\sigma + y^-(\tau)\ , \qquad Y(\tau,\sigma) = y(\tau)\ , \qquad Y^+(\tau,\sigma) = y^+(\tau)$$

Hamiltonian for string dynamics

$$\mathcal{H} = 2\Big(q^2 + V(y)\Big)\ , \qquad V(y) \equiv -2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right) - 2q_+q_-$$

Classical string dynamics I

The novelty is the winding; Embedding functions ($0 \leq \sigma \leq \pi$)

$$Y^-(\tau,\sigma) = 2R\omega\sigma + y^-(\tau)\ , \qquad Y(\tau,\sigma) = y(\tau)\ , \qquad Y^+(\tau,\sigma) = y^+(\tau)$$

Hamiltonian for string dynamics

$$\mathcal{H} = 2\Big(q^2 + V(y)\Big)\ , \qquad V(y) \equiv -2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right) - 2q_+q_-$$

Conserved quantity is: $\lambda = -\frac{\mathcal{H}}{2} = L_0 + \tilde{L}_0$

Classical string dynamics I

The novelty is the winding; Embedding functions ($0 \leq \sigma \leq \pi$)

$$Y^-(\tau,\sigma) = 2R\omega\sigma + y^-(\tau)\ , \qquad Y(\tau,\sigma) = y(\tau)\ , \qquad Y^+(\tau,\sigma) = y^+(\tau)$$

Hamiltonian for string dynamics

$$\mathcal{H} = 2\Big(q^2 + V(y)\Big)\ , \qquad V(y) \equiv -2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right) - 2q_+q_-$$

Conserved quantity is: $\lambda = -\frac{\mathcal{H}}{2} = L_0 + \tilde{L}_0$ and not the mass

$$M^2(y) \equiv -q_\mu q^\mu = \lambda + 2Ey\left(\frac{\omega R}{2}\right)^2$$

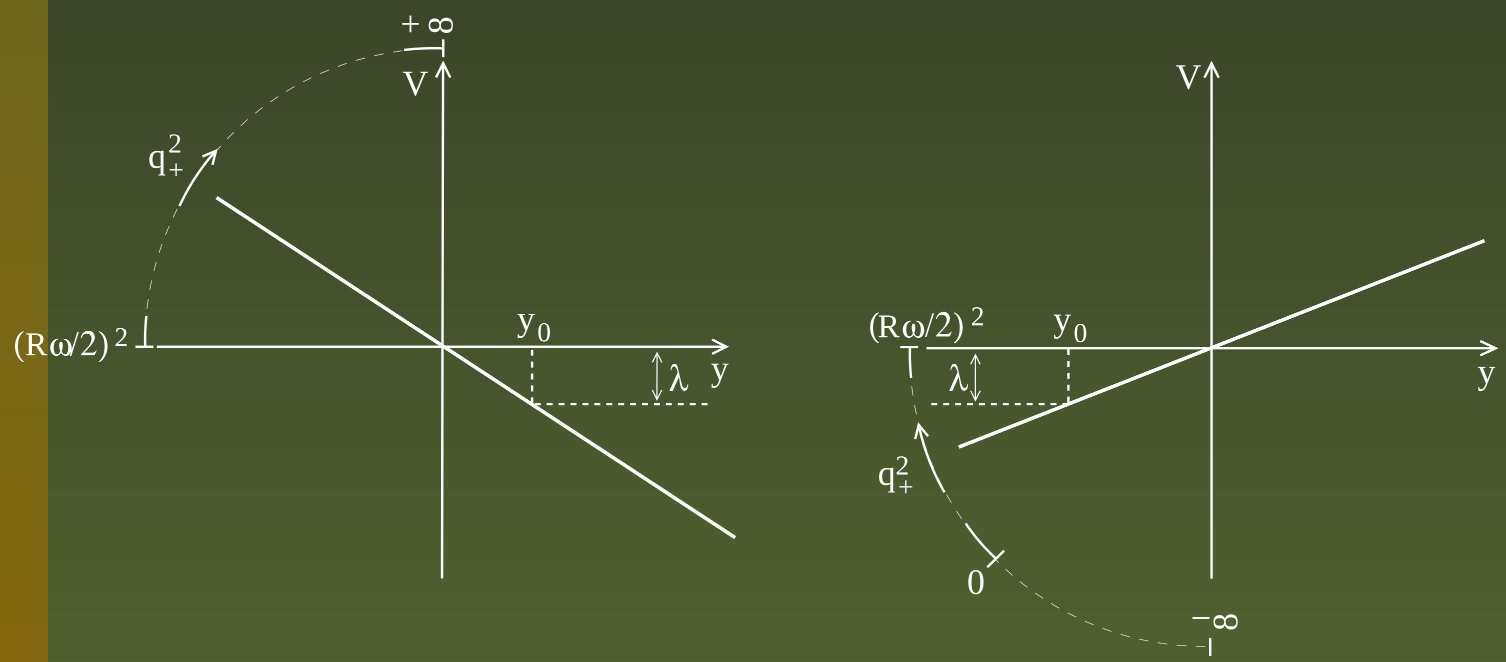
Classical string dynamics II

$$V(y) \equiv -2q_+q_- - 2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right) \;, \quad \lambda = -V(y_0)$$

Classical string dynamics II

$$V(y) \equiv -2q_+q_- - 2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right), \quad \lambda = -V(y_0)$$

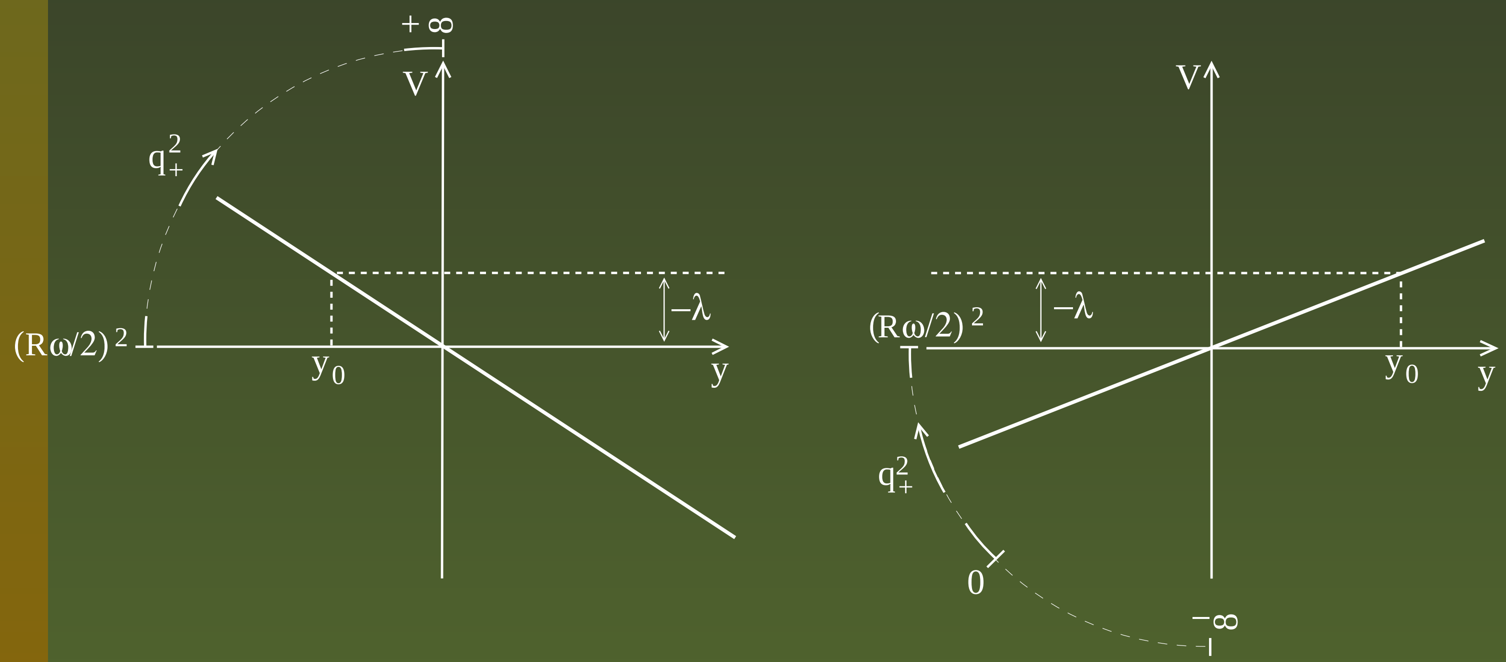
$q_- = 0, \lambda > 0$:



Classical string dynamics II

$$V(y) \equiv -2q_+q_- - 2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right), \quad \lambda = -V(y_0)$$

$q_- = 0, \lambda < 0$:

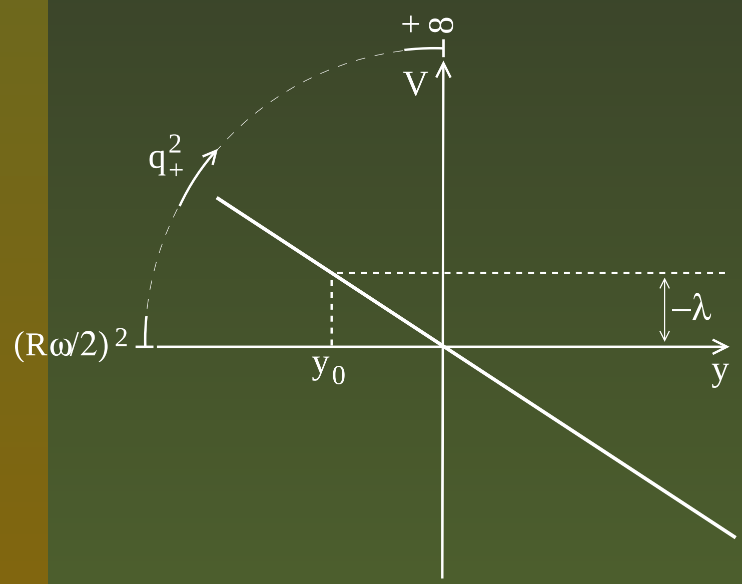


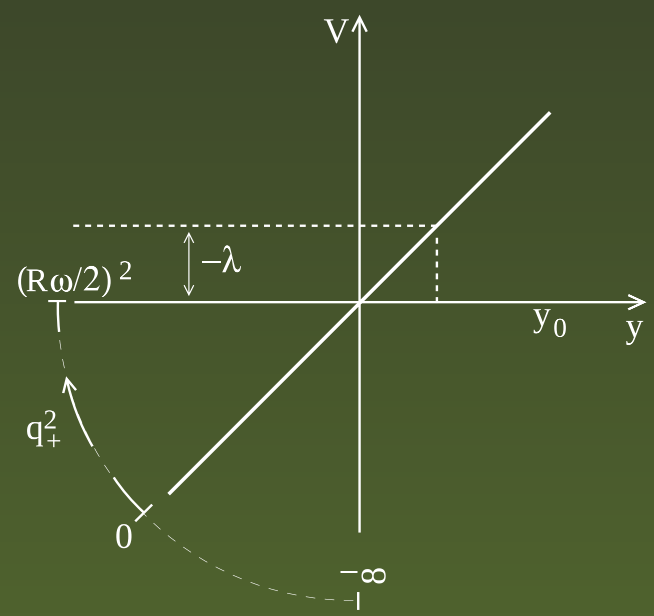
Classical string dynamics II

$$V(y) \equiv -2q_+q_- - 2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right) \;, \quad \lambda = -V(y_0)$$

$q_- = 0, \lambda < 0$:

Unstable tachyons have limited penetration



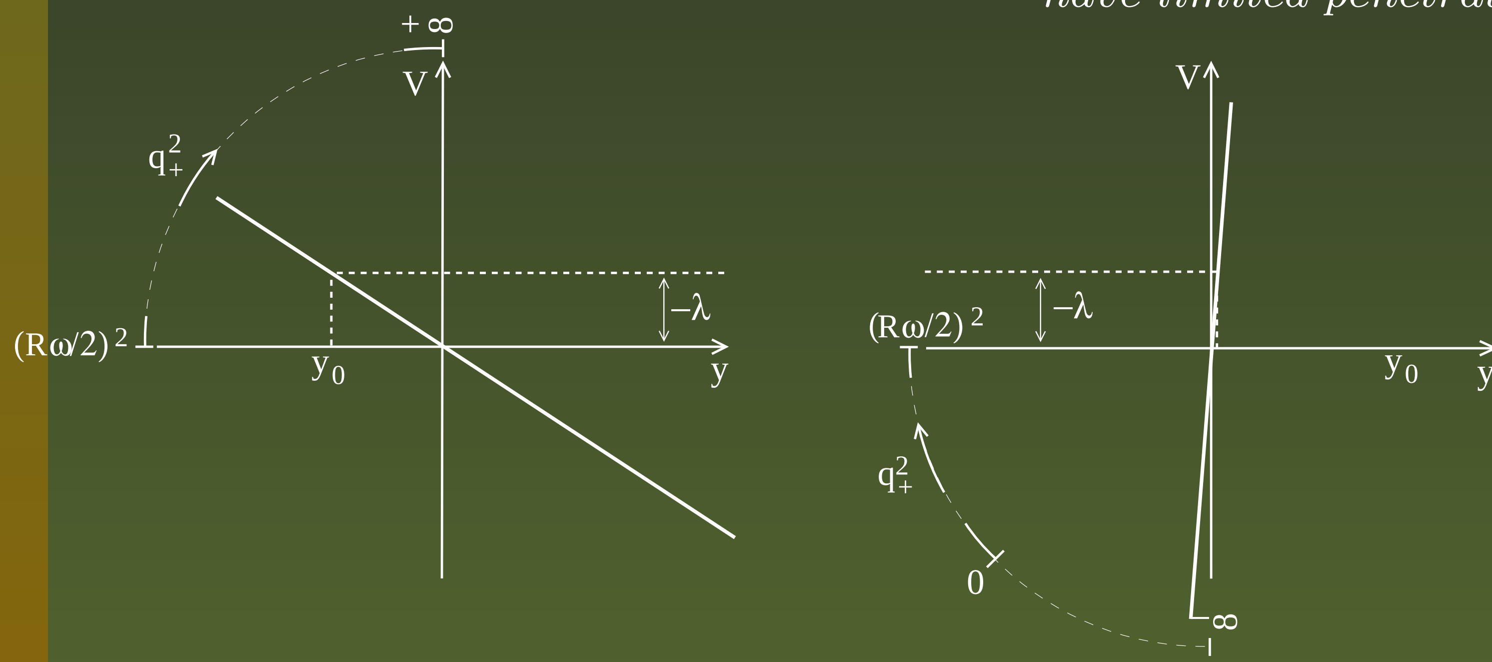


Classical string dynamics II

$$V(y) \equiv -2q_+q_- - 2Ey\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right)\ , \quad \lambda = -V(y_0)$$

$q_- = 0, \lambda < 0$:

Unstable tachyons have limited penetration

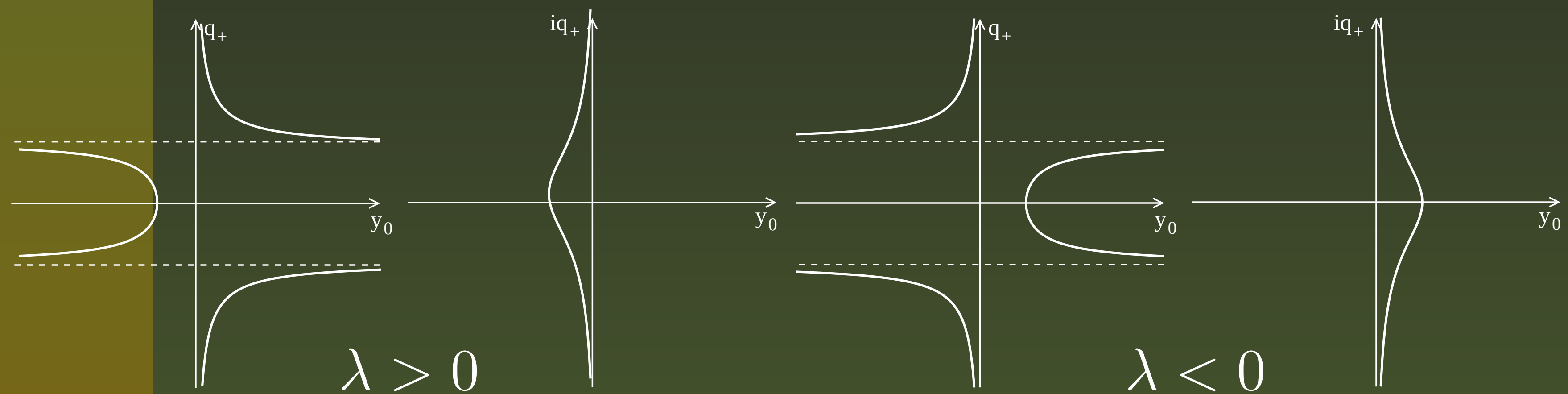


String wave functions

q_+ as function of λ and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{\lambda}{2Ey_0} + \left(\frac{\omega R}{2}\right)^2}$

String wave functions

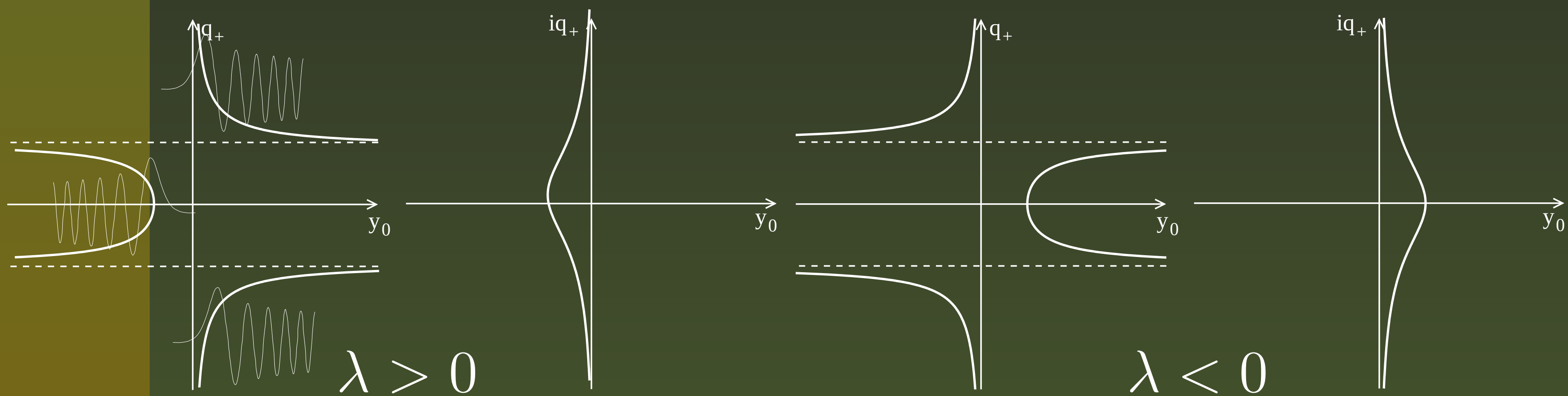
q_+ as function of λ and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{\lambda}{2Ey_0} + \left(\frac{\omega R}{2}\right)^2}$



$\lambda > 0$ $\lambda < 0$

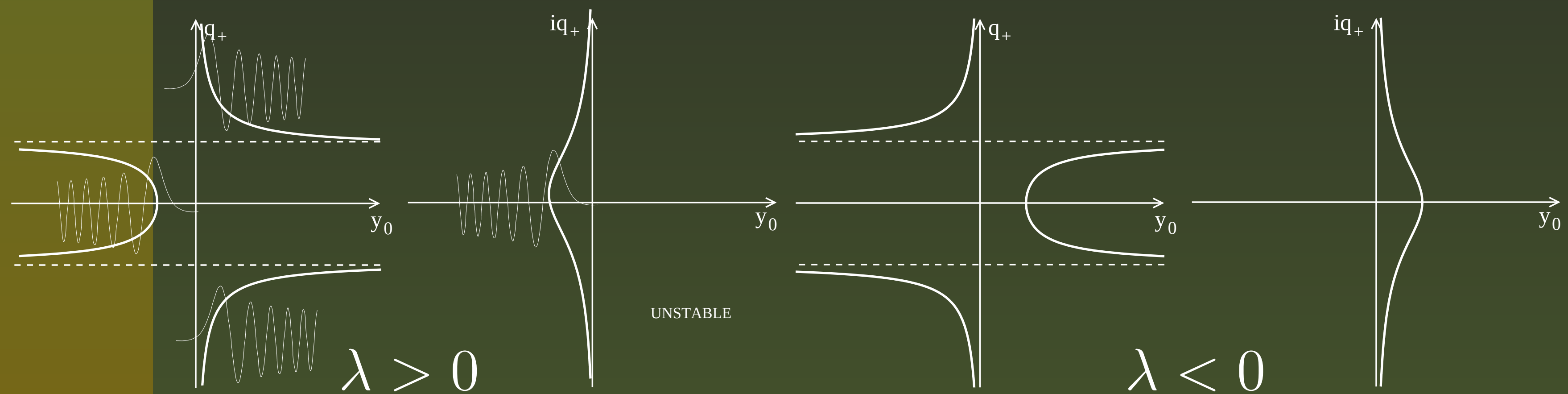
String wave functions

q_+ as function of λ and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{\lambda}{2Ey_0} + \left(\frac{\omega R}{2}\right)^2}$



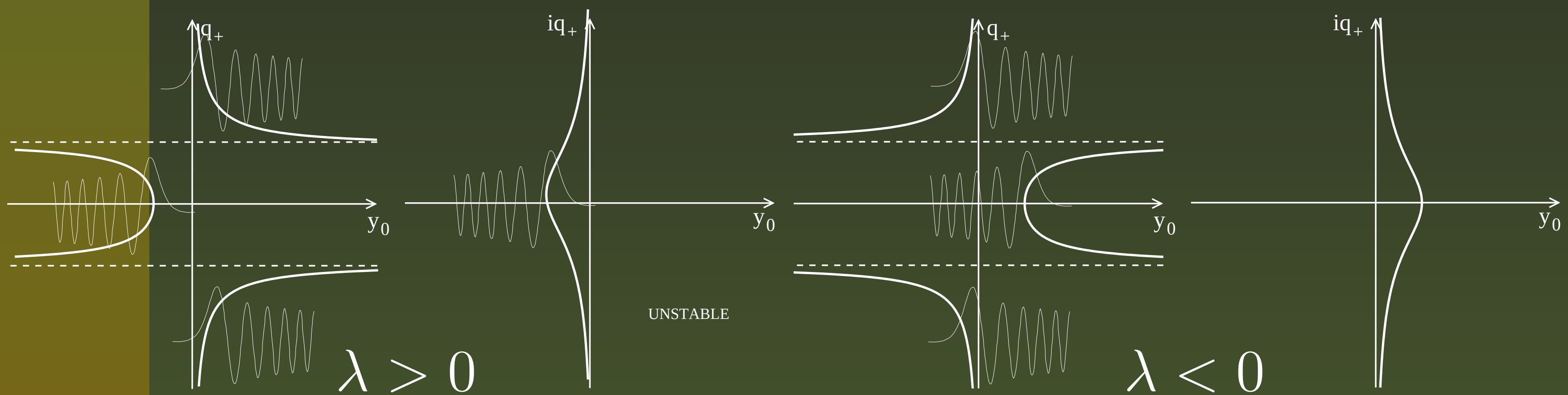
String wave functions

q_+ as function of λ and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{\lambda}{2Ey_0} + \left(\frac{\omega R}{2}\right)^2}$



String wave functions

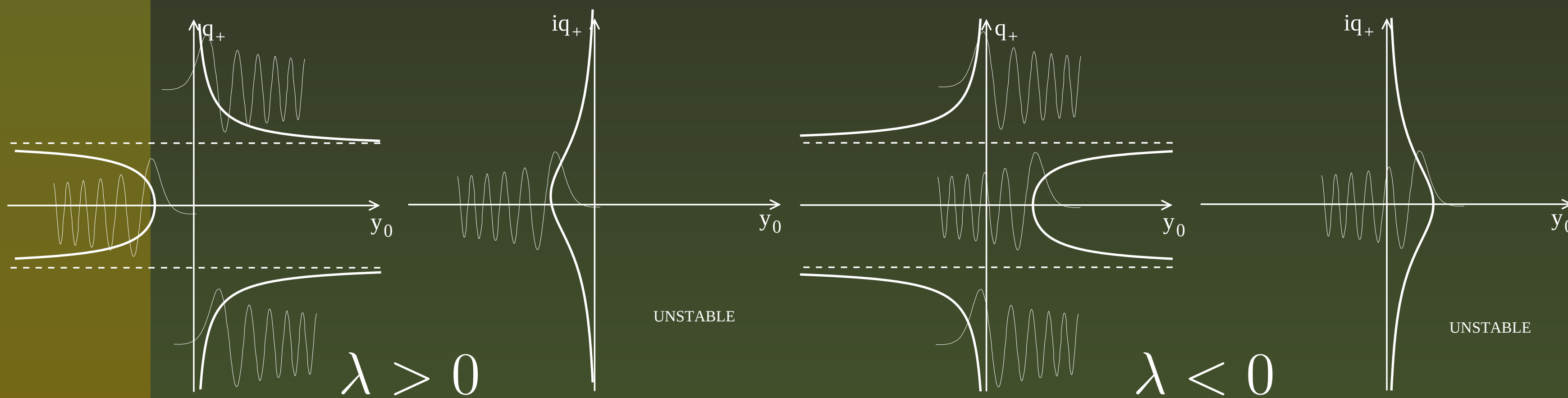
q_+ as function of λ and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{\lambda}{2Ey_0} + \left(\frac{\omega R}{2}\right)^2}$



$\lambda > 0$ $\lambda < 0$

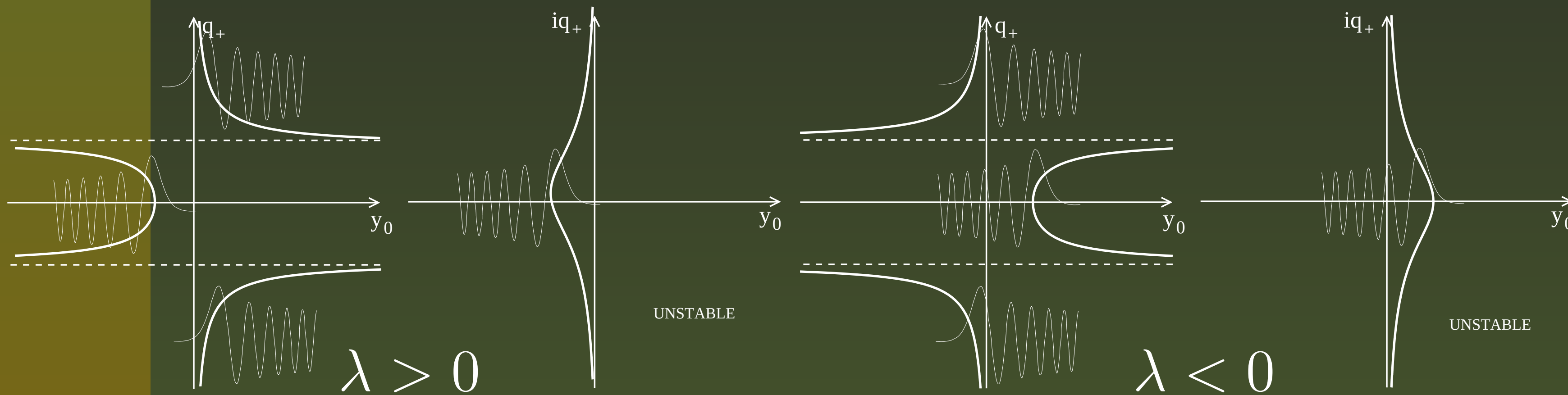
String wave functions

q_+ as function of λ and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{\lambda}{2Ey_0} + \left(\frac{\omega R}{2}\right)^2}$



String wave functions

q_+ as function of λ and y_0, for $q_- = 0$: $q_+ = \pm\sqrt{\frac{\lambda}{2Ey_0} + \left(\frac{\omega R}{2}\right)^2}$



Unstable tachyonic modes cannot penetrate the good region beyond a certain critical y.

Canonical Quantisation

Oscillators and zero modes decouple; so normal ordering is standard. For bosonic string $L_0 = \tilde{L}_0 = 1$ on physical states; these satisfy

$$M^2(y) \equiv -q_\mu q^\mu = 2Ey \left(\frac{\omega R}{2}\right)^2 + N + \tilde{N} - 2$$

Canonical Quantisation

Oscillators and zero modes decouple; so normal ordering is standard. For bosonic string $L_0 = \tilde{L}_0 = 1$ on physical states; these satisfy

$$M^2(y) \equiv -q_\mu q^\mu = 2Ey\left(\frac{\omega R}{2}\right)^2 + N + \tilde{N} - 2$$

The quantum numbers for the string centre of mass wave functions $\psi_{q_+,y_0,m}$ obey the on-shell relation

$$\lambda = -2 + N + \tilde{N} = 2q_+q_- + 2Ey_0\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right)$$

Level matching condition: $N - \tilde{N} = m\omega.$

Canonical Quantisation

Oscillators and zero modes decouple; so normal ordering is standard. For bosonic string $L_0 = \tilde{L}_0 = 1$ on physical states; these satisfy

$$M^2(y) \equiv -q_\mu q^\mu = 2Ey\left(\frac{\omega R}{2}\right)^2 + N + \tilde{N} - 2$$

The quantum numbers for the string centre of mass wave functions $\psi_{q_+,y_0,m}$ obey the on-shell relation

$$\lambda = -2 + N + \tilde{N} = 2q_+q_- + 2Ey_0\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right)$$

Level matching condition: $N - \tilde{N} = m\omega$.

Due to zero point energy (-2), both $\lambda > 0$ and $\lambda < 0$ are allowed.

Partition Function I

In the canonical formalism one takes the trace over the Hilbert space

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \, \mathbf{Tr} \left(q^{\,L_0 - 1} \bar{q}^{\,\tilde{L}_0 - 1} \right)$$

where $q = e^{2\pi i \tau}$ and $\tau = \tau_1 + i\tau_2$.

Partition Function I

In the canonical formalism one takes the trace over the Hilbert space

$$Z = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2}\, \mathbf{Tr}\left(q^{\,L_0-1}\bar{q}^{\,\tilde{L}_0-1}\right)$$

where $q = e^{2\pi i\tau}$ and $\tau = \tau_1 + i\tau_2$.

- Choose the basis $|q_+, y_0, m\rangle$ to perform trace;
- Do analytic continuation: $q_+ \longrightarrow iq_+$;
- Perform q_+ integral (well defined for $y_0 > 0$) to get in the integrand

$$\exp\left[-2\pi\tau_2\left(-2 + n + \tilde{n} + \frac{m^2}{R^2(y_0)} + \frac{\omega^2 R^2(y_0)}{4}\right)\right]$$

where $R^2(y_0) = 2Ey_0R^2$ is the proper radius of compact direction.

Partition Function II - Infrared Divergences

For $y_0 > 0$, the condition

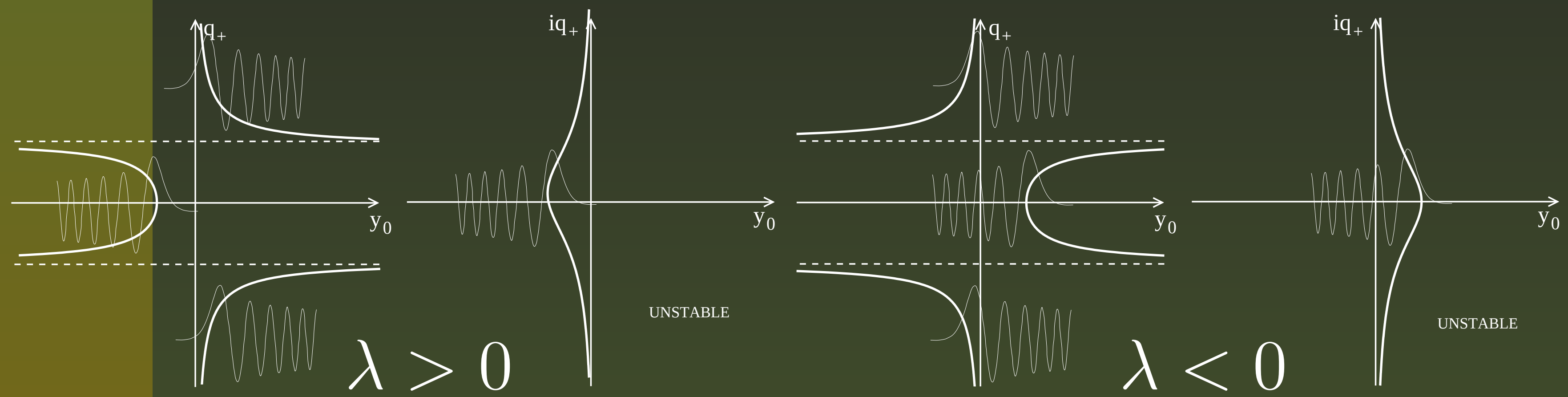
$$\lambda + \frac{m^2}{R^2(y_0)} + \frac{\omega^2 R^2(y_0)}{4} < 0$$

is the condition for solutions of the quadratic equation in q_+

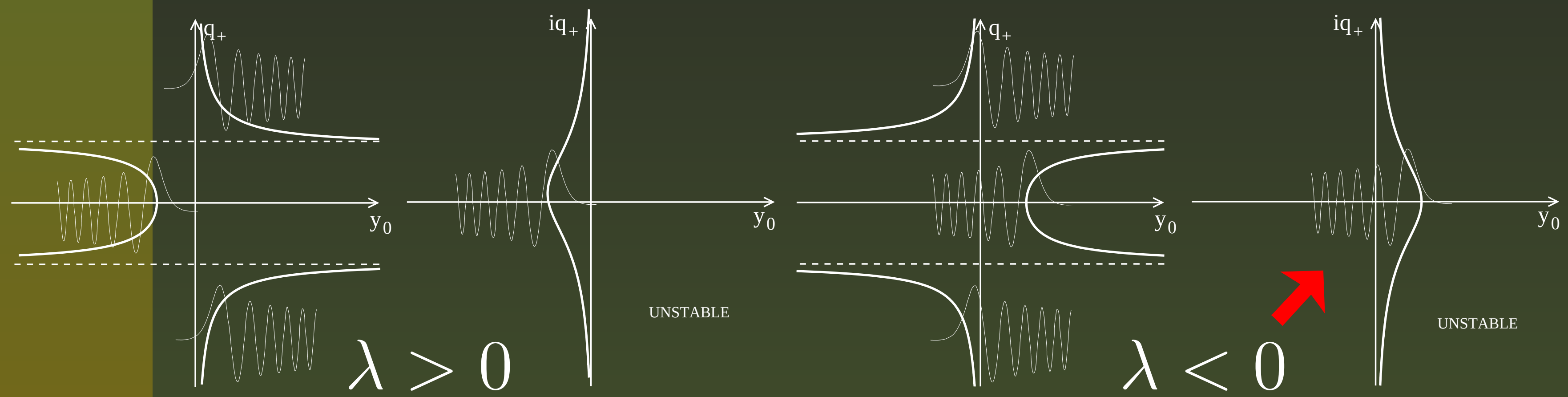
$$\lambda = 2q_+q_- + 2Ey_0\left(q_+^2 - \left(\frac{\omega R}{2}\right)^2\right)$$

to have an imaginary part, when $\lambda < 0$.

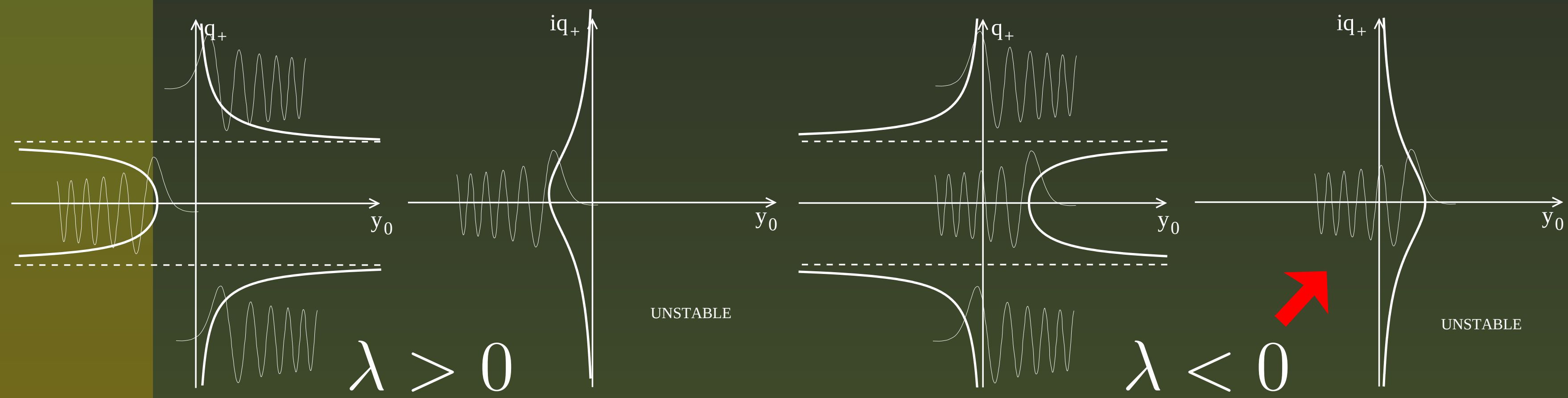
Partition Function III - States causing Infrared Divergences



Partition Function III - States causing Infrared Divergences



Partition Function III - States causing Infrared Divergences



The state with $n = \tilde{n} = 0$ and $\omega = 1$ renders the partition function divergent for

$$y_0 < 4/(ER^2)$$

This is the state that condenses the furthest into the $y > 0$ region.

Partition Function IV - Hagedorn Divergence

The same behaviour can be seen from the large n behaviour of Z.

Partition Function IV - Hagedorn Divergence

The same behaviour can be seen from the large n behaviour of Z.

- Perform Wick rotation and Poisson re-sum the KK momentum.
- Perform the q_+ integral;
- Use integral representation of the measure;
- Replace the fundamental region $\mathcal{F}$ by the strip;
- Expand the Dedekind eta function $\eta(\tau)$ in a Taylor series;
- Perform the τ_1 integration;

Partition Function V - Hagedorn Divergence

$$Z \propto \sum_{\omega'=1, n=0}^{\infty} d_n^2 \int_{-\infty}^{\infty} dy \int_0^{\infty} \frac{d\tau_2}{\tau_2^{14}} \exp\left(-4\pi(n-1)\tau_2 - \frac{2\pi}{\tau_2} \frac{\omega'^2 R^2(y)}{4}\right)$$

Partition Function V - Hagedorn Divergence

$$Z \propto \sum_{\omega'=1,n=0}^{\infty} d_n^2 \int_{-\infty}^{\infty} dy \int_0^{\infty} \frac{d\tau_2}{\tau_2^{14}} \exp\left(-4\pi(n-1)\tau_2 - \frac{2\pi}{\tau_2}\frac{\omega'^2 R^2(y)}{4}\right)$$

- $n = 0$: IR divergence ($\tau_2 \to \infty$) from tachyon;

Partition Function V - Hagedorn Divergence

$$Z \propto \sum_{\omega'=1,n=0}^{\infty} d_n^2 \int_{-\infty}^{\infty} dy \int_0^{\infty} \frac{d\tau_2}{\tau_2^{14}} \exp\left(-4\pi(n-1)\tau_2 - \frac{2\pi}{\tau_2}\frac{\omega'^2 R^2(y)}{4}\right)$$

• $n = 0$: IR divergence ($\tau_2 \to \infty$) from tachyon;
• $n \geq 1$: Perform τ_2 integral for $y > 0$; The large n expansion of the integrand is dominated by

$$e^{2\pi\sqrt{n}\left(4-\sqrt{2\omega^2 R^2(y)}\right)}$$

We conclude that the sum in n diverges when $y < y_c = \frac{4}{ER^2}$.

Partition Function V - Hagedorn Divergence

$$Z \propto \sum_{\omega'=1,n=0}^{\infty} d_n^2 \int_{-\infty}^{\infty} dy \int_0^{\infty} \frac{d\tau_2}{\tau_2^{14}} \exp\left(-4\pi(n-1)\tau_2 - \frac{2\pi}{\tau_2}\frac{\omega'^2 R^2(y)}{4}\right)$$

• $n = 0$: IR divergence ($\tau_2 \to \infty$) from tachyon;
• $n \geq 1$: Perform τ_2 integral for $y > 0$; The large n expansion of the integrand is dominated by

$$e^{2\pi\sqrt{n}\left(4-\sqrt{2\omega^2 R^2(y)}\right)}$$

We conclude that the sum in n diverges when $y < y_c = \frac{4}{ER^2}$.

$\star$ Hagedorn behaviour!

Superstring

NS⊗NS sector: $\lambda = -1 + N + \tilde{N}$.

Then one needs to specify the orbifold spin structure:

Superstring

NS$\otimes$NS sector: $\lambda = -1 + N + \tilde{N}$.

Then one needs to specify the orbifold spin structure:

- Supersymmetry breaking case (anti-periodic):

Superstring

NS$\otimes$NS sector: $\lambda = -1 + N + \tilde{N}$.

Then one needs to specify the orbifold spin structure:

• Supersymmetry breaking case (anti-periodic):

$\star$ Even ω have the usual supersymmetric GSO projection; odd ω have reversed GSO projection;

Superstring

NS⊗NS sector: $\lambda = -1 + N + \tilde{N}$.

Then one needs to specify the orbifold spin structure:

- Supersymmetry breaking case (anti-periodic):

$\star$ Even ω have the usual supersymmetric GSO projection; odd ω have reversed GSO projection;

$\star$ Usual bosonic string tachyon absent;

Superstring

NS⊗NS sector: $\lambda = -1 + N + \tilde{N}$.

Then one needs to specify the orbifold spin structure:

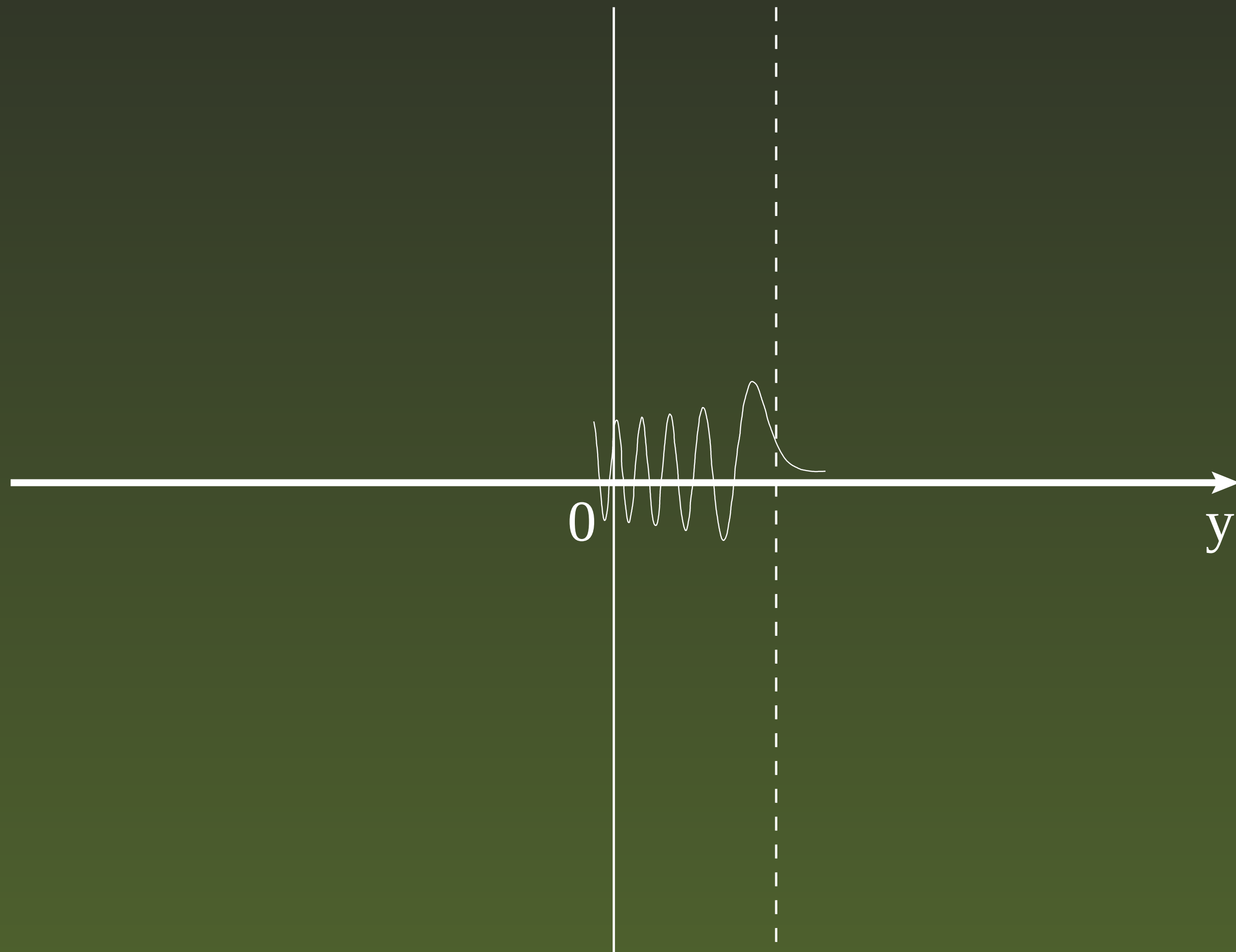
- Supersymmetry breaking case (anti-periodic):

$\star$ Even ω have the usual supersymmetric GSO projection; odd ω have reversed GSO projection;

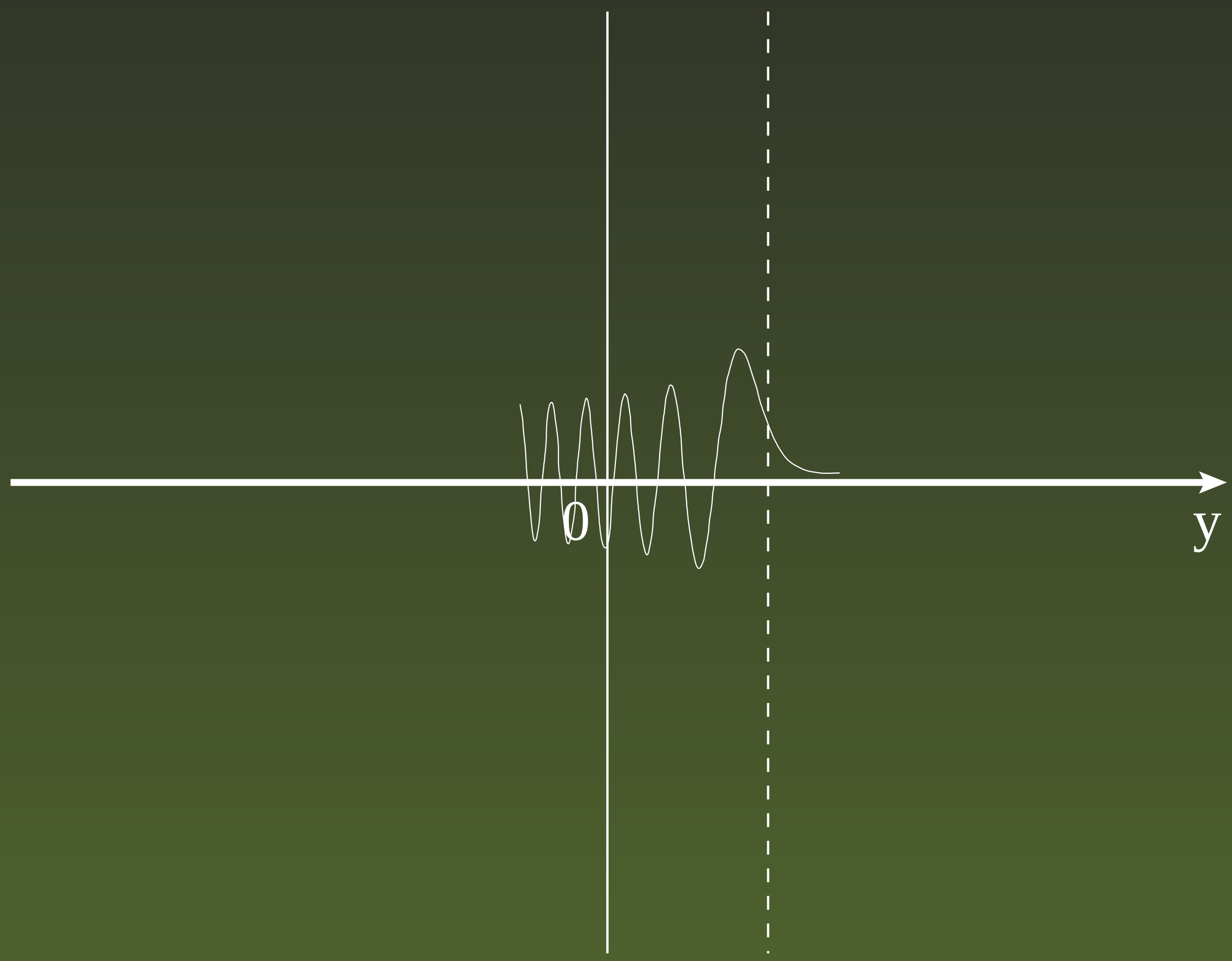
$\star$ Usual bosonic string tachyon absent;

$\star$ States with $\lambda = -1$ and odd ω penetrate good region to a maximum of $y = y_c = \frac{2}{ER^2}$ (for $\omega = \pm 1$).

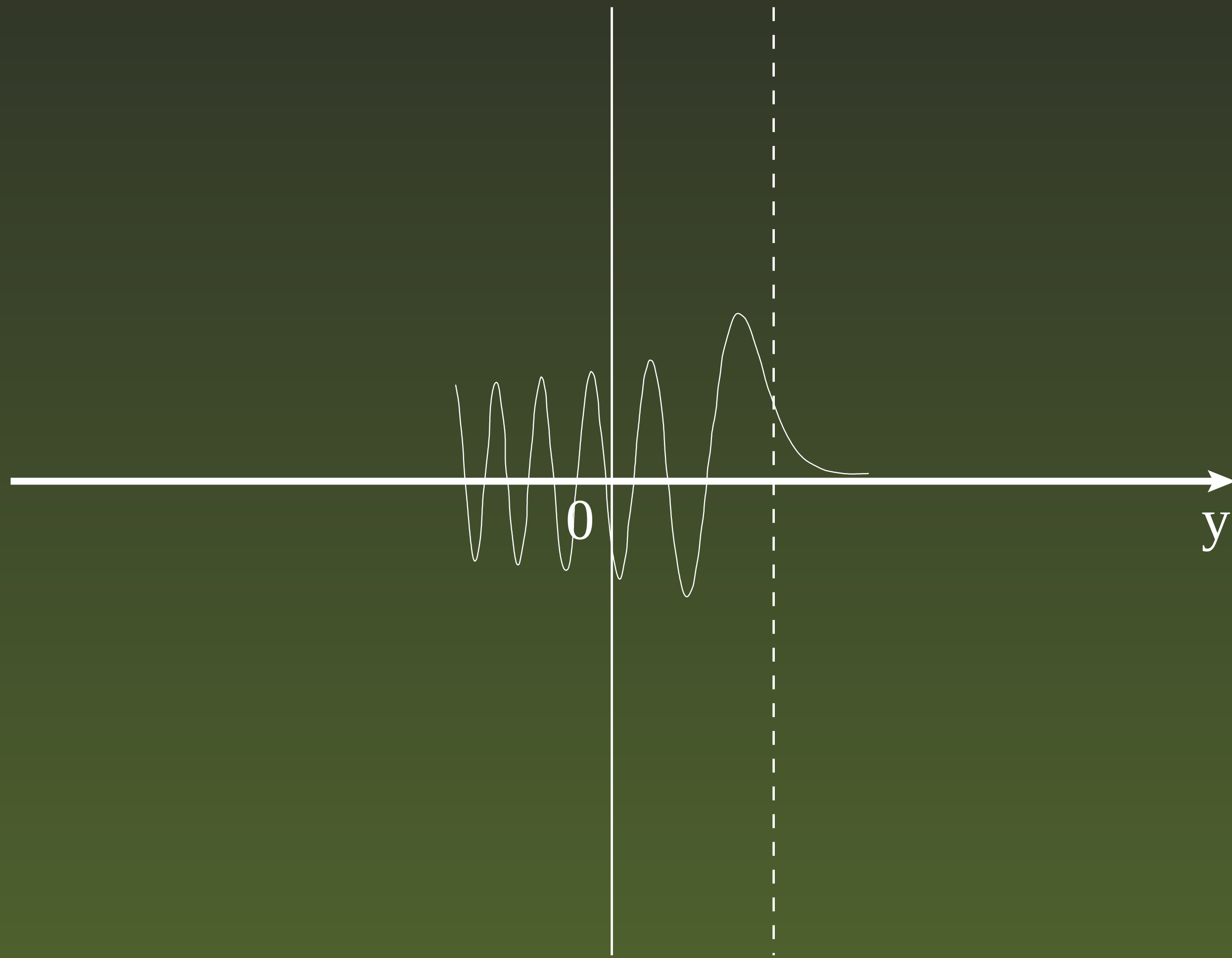
Growth of unstable modes



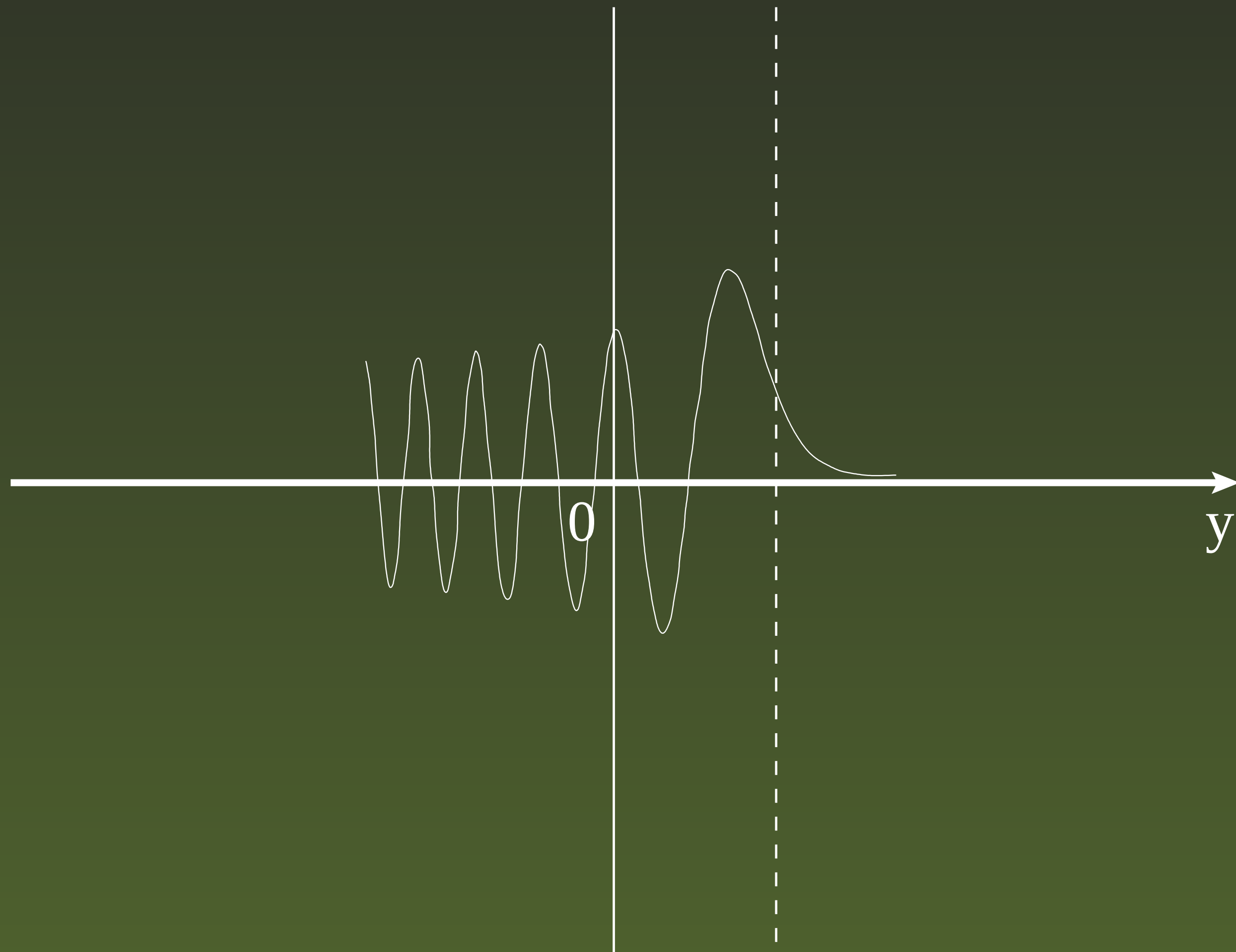
Growth of unstable modes



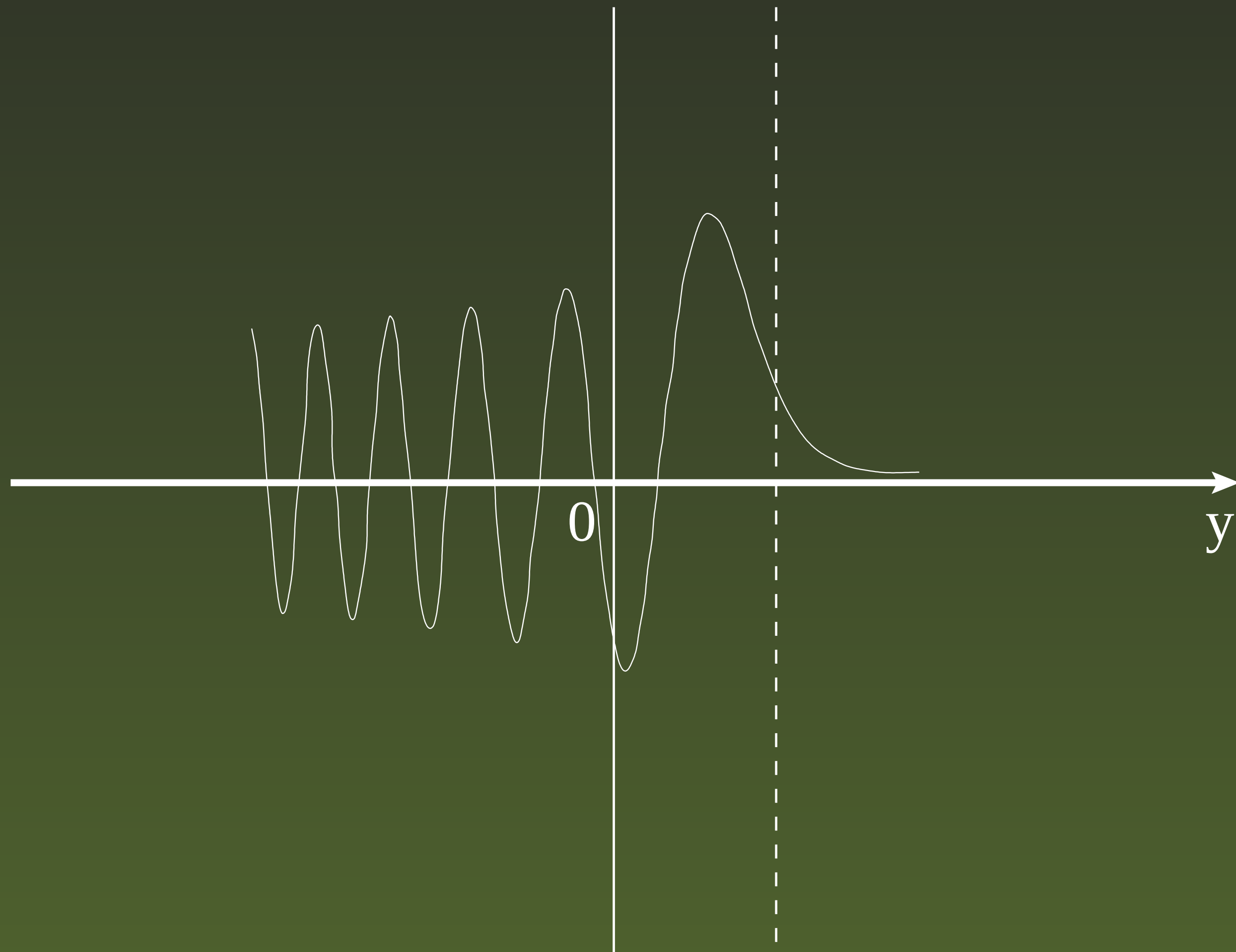
Growth of unstable modes



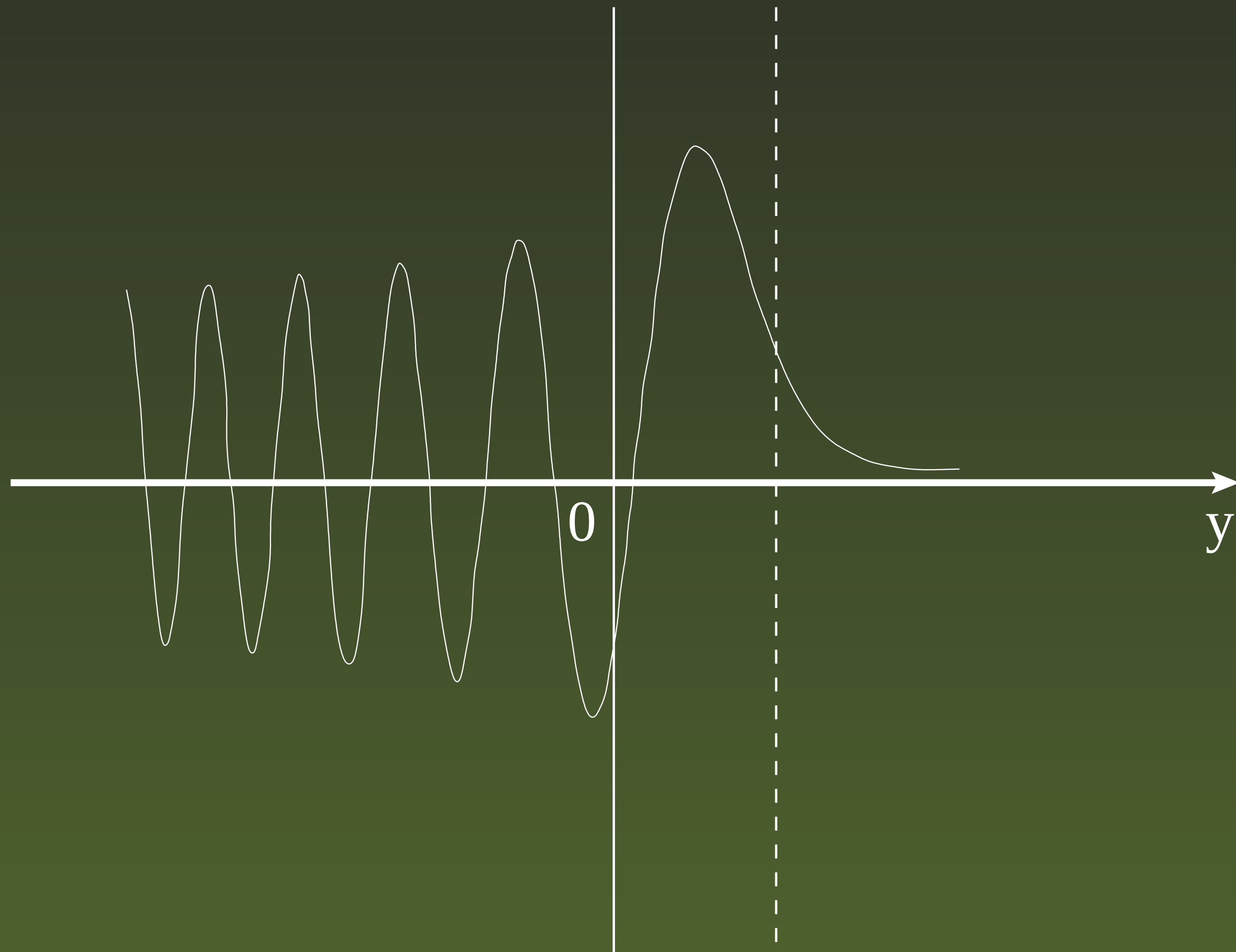
Growth of unstable modes



Growth of unstable modes



Growth of unstable modes



Chronology Protection Conjecture in String Theory

Chronology Protection Conjecture in String Theory

Closed Null curves do not form in string theory because light winding states condense, causing a phase transition whose end point target space geometry is chronological.

Chronology Protection Conjecture in String Theory

Closed Null curves do not form in string theory because light winding states condense, causing a phase transition whose end point target space geometry is chronological.

Nice toy model but... How general?...End point?