Small Black Strings/Holes

Based on

M. A., F. Ardalan, H. Ebrahim and S. Mukhopadhyay, arXiv:0712.4070,

Our aim is to study the symmetry of the near horizon geometry of the extremal black holes in $\mathcal{N} = 2$ and 4 supergravities in five dimensions.

The corresponding black holes could be either small or large depending on whether the corresponding classical horizon area is zero or non-zero, respectively.

In the $\mathcal{N} = 2$ case both small and large black holes are $\frac{1}{2}$ BPS.

In the $\mathcal{N} = 4$ case the large black hole is $\frac{1}{4}$ BPS whereas the small one is $\frac{1}{2}$ BPS.

An interesting feature of these extremal black holes is that in the near horizon limit they exhibit supersymmetry enhancement (A. Chamseddine, S. Ferrara, G. W. Gibbons and R. Kallosh, hep-th/9610155).

At the leading order, in the near horizon limit large black holes undergo supersymmetry doubling.

The near horizon of large black holes in both $\mathcal{N} = 4$ and $\mathcal{N} = 2$ preserves eight supercharges.

On the other hand the small black holes in both cases are singular at the leading order and as a result going near horizon we will not led to supersymmetry doubling, i.e. for $\mathcal{N} = 2$ small black holes at near core limit there are just four supercharges while for $\mathcal{N} = 4$ case the number of supercharges are eight.

We will show that taking into account the R^2 corrections the small black holes will also exhibit supersymmetry doubling.

That means in $\mathcal{N} = 4$ case the small black holes preserve all the sixteen supercharges while for the $\mathcal{N} = 2$ theory the near horizon geometry of small black holes preserve eight supercharges.

The main property underlying the supersymmetry enhancement is the appearance of AdS_2 geometry in the near horizon limit due to the extremality. This is the case both for large and small black holes when the higher order corrections are taken into account.

Having established the supersymmetry enhancement for extremal small black holes, it may be possible to study the near horizon symmetry.

• To Understand the holographic dual of string theory/gravity on the AdS_2 background.

• Gravity on AdS_2 geometry is important on its own as it carries entropy unlike the higher dimensional cases. The AdS_2 space is a background which naturally appears in the general near horizon geometry of the extremal black holes, and therefore its holographic dual could be used to understand the entropy of the black holes better.

The extremal black holes in five dimensional $\mathcal{N} = 2$ supergravity in the presence of supersymmetrized R^2 corrections (K. Hanaki, K. Ohashi and Y. Tachikawa, hep-th/0611329) were studied in

A. Castro, J. L. Davis, P. Kraus and F. Larsen, hep-th/0702072, hep-th/0703087,arXiv:0705.1847

M. Alishahiha., hep-th/0703099

M. Cvitan, P. D. Prester, S. Pallua and I. Smolic, arXiv:0706.1167

It was shown that these corrections stretch the horizon leading to $AdS_2 \times S^3$ near horizon geometry.

Our main results

- In $\mathcal{N} = 4$ theory the near horizon geometry of small black holes preserve sixteen supercharges and the corresponding global near horizon symmetry is $OSp(4^*|4) \times SU(2)$.

- In $\mathcal{N} = 2$ case the near horizon geometry of the small black hole preserves eight supercharges with global near horizon symmetry of $OSp(4^*|2) \times SU(2)$.

This is to be compared with the symmetry of the near horizon geometry of the large black holes $SU(1,1|2) \times SU(2)$ (J. P. Gauntlett, R. C. Myers and P. K. Townsend, hep-th/9810204)

$\mathcal{N}=2$ Five dimensional Black hole

Consider five dimensional $\mathcal{N}=2$ supergravity. This can be obtained from M-theory compactified on Calabi-Yau threefold.

- The five dimensional theory contains the gravity multiple coupled to $h_{(1,1)}-1$ vector multiplets.

- The $(h_{(1,1)}-1)$ dimensional space of the scalars can be regarded as a hypersurface of a $h_{(1,1)}$ dimensional manifold. The hypersurface is defined by

$$\frac{1}{6}C_{IJK}X^I X^J X^K = X^I X_I = 1, \quad I,J,K = 1,\cdots,h_{(1,1)},$$

where

- C_{IJK}: intersection number of C.Y.

- X^I: the volumes of 2-cycles.

- X_I: the volumes of 4-cycles.

- The bosonic part of the Lagrangian is

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{4}G_{IJ}F^{I}_{\mu\nu}F^{\mu\nu\ J} - \frac{1}{2}\mathcal{G}_{ij}\partial_{\mu}\phi^{i}\partial^{\mu}\phi^{j}$$
$$+ \frac{g^{-1}}{48}c^{\mu\nu\rho\sigma\lambda}C_{IJK}F^{I}_{\mu\nu}F^{J}_{\rho\sigma}A^{K}_{\lambda}$$

where $(X = X(\phi))$

$$G_{IJ} = -\frac{1}{2}\partial_I\partial_J \log \nu|_{\nu=1}\ , \qquad \mathcal{G}_{ij} = G_{IJ}\ \partial_i X^I \partial_j X^J|_{\nu=1}$$

There are several known supersymmetric solutions for this model including

Black hole, Black string, Black ring, Rotating black hole

The near horizon geometry could be either $AdS_2 \times S^3$ or $AdS_3 \times S^2$ or some deformation of them.

Although in the leading order we use very special geometry, sometime it is useful to work with a more general context, namely the superconformal approach. This approach, in particular, is useful when we want to write the explicit form of the action. In this approach we start with a five dimensional theory which is invariant under a larger group that is superconformal group and therefore we construct a conformal supergravity. Then by imposing a gauge fixing condition, one breaks the conformal supergravity to standard supergravity model.

The representation of superconformal group includes Weyl, vector and hyper multiples.

1. Weyl multiplet

$$e_\mu^a,\ \psi_\mu,\ V_\mu,\ v^{ab},\ \chi,\ D$$

2. Vector multiplet

$$A_\mu^I,\ X^I,\ \Omega^I, Y^I, \quad I = 1, \cdots, n_v$$

The hypermultiplet contains scalar fields $\mathcal{A}_\alpha^i$ where $i = 1, 2$ is $SU(2)$ doublet index and $\alpha = 1, \cdots, 2r$ refers to $USp(2r)$ group.

Although we won't couple the theory to matters, we shall consider the hyper multiplet to gauge fix the dilatational symmetry reducing the action to the standard $\mathcal{N} = 2$ supergravity action.

Since the Weyl and vector multiplets are irr. rep. the variations of the fields under SUSY transformations are independent of the action. This is the point of retaining the auxiliary fields

The supersymmetry variations of the fermions in Weyl, vector and hyper multiplets are

$$\begin{aligned}
\delta\psi^i_\mu &= \mathcal{D}_\mu\varepsilon^i + \frac{1}{2}v^{ab}\gamma_{\mu ab}\varepsilon^i - \gamma_\mu\eta^i, \\
\delta\chi^i &= D\varepsilon^i - 2\gamma^c\gamma^{ab}\varepsilon^i\mathcal{D}_a v_{bc} - 2\gamma^a\varepsilon^i\epsilon_{abcde}v^{bc}v^{de} + 4\gamma\cdot v\eta^i, \\
\delta\Omega^{Ii} &= -\frac{1}{4}\gamma\cdot F^I\varepsilon^i - \frac{1}{2}\gamma^a\partial_a X^I\varepsilon^i - X^I\eta^i, \\
\delta\zeta^\alpha &= \gamma^a\partial_a\mathcal{A}^\alpha_i\varepsilon^i - \gamma\cdot v\mathcal{A}^\alpha_i\varepsilon^i + 3\mathcal{A}^\alpha_i\eta^i
\end{aligned}$$

In this notation at leading order the bosonic part of the action is

$$I = \frac{1}{16\pi G_5}\int d^5x\, \mathcal{L}_0$$

with

$$\begin{aligned}\mathcal{L}_0 &= \partial_a \mathcal{A}^i_\alpha \partial^a \mathcal{A}^\alpha_i + (2\nu + \mathcal{A}^2)\frac{D}{4} + (2\nu - 3\mathcal{A}^2)\frac{R}{8} \\ &+ (6\nu - \mathcal{A}^2)\frac{v^2}{2} + 2\nu_I F^I_{ab} v^{ab} \\ &+ \frac{1}{4}\nu_{IJ}(F^I_{ab}F^{J\,ab} + 2\partial_a X^I \partial^a X^J) \\ &+ \frac{g^{-1}}{24} C_{IJK}\epsilon^{abcde} A^I_a F^J_{bc} F^K_{de}\end{aligned}$$

where $\mathcal{A}^2 = \mathcal{A}^i_{\alpha\ ab}\mathcal{A}^{\alpha\ ab}_i$, $v^2 = v_{ab}v^{ab}$ and

$$\nu = \frac{1}{6}C_{IJK}X^I X^J X^K,\ \nu_I = \frac{1}{2}C_{IJK}X^J X^K,\ \nu_{IJ} = C_{IJK}X^K$$

To fix the gauge it is convenient to set $\mathcal{A}^2 = -2$. Then integrating out the auxiliary fields by making use of their equations of motion one finds

$$\mathcal{L}_0 = R - \frac{1}{2} G_{IJ} F^I_{ab} F^{Jab} - \mathcal{G}_{ij} \partial_a \phi^i \partial^a \phi^j + \frac{g^{-1}}{24} \epsilon^{abcde} C_{IJK} F^I_{ab} F^J_{cd} A^K_e$$

The black hole solution of the above action has the following form

$$\begin{aligned} ds^2 &= e^{-4U} dt^2 - e^{2U}(dr^2 + r^2 d\Omega_3^2), \qquad & e^{2U} X_I = \frac{1}{3} H_I, \\ F^I_{tr} &= -\partial_r (e^{-2U} X^I), & e^{6U} = H_1 H_2 H_3, \end{aligned}$$

where $H_I = h_I + \frac{q_I}{r^2}$.

Using the supersymmetry variation one can see that the above solution preserves four supercharges constrained by

$$\gamma^{\hat{t}} \varepsilon^i = -\varepsilon^i.$$

Now the aim is to study the supersymmetry properties of the near horizon limit of the black hole solution.

• When $q_I \neq 0$ for all I where we get a large black hole with non-vanishing horizon and therefore non-zero macroscopic entropy is given by $S_{BH} = 2\pi\sqrt{q_1 q_2 q_3}$.

• When one of the charges is zero, say $q_1 = 0$ we get a small black hole with vanishing horizon and macroscopic entropy to this order.

For the large black hole the near horizon geometry is given by

$$ds^2 = l^2 \left[(\rho^2 dt^2 - \frac{d\rho^2}{\rho^2}) - 4 d\Omega_3^2 \right],$$

where $l = (q_1 q_2 q_3)^{1/6}/2$ and $r^2 = (q_1 q_2 q_2)^{1/2} \rho/2$.

Using the supersymmetry variation expressions at leading order it was shown that the above near horizon solution preserves all the eight supercharges. Thus supersymmetry enhancement occurs in the near horizon limit of five dimensional large black holes.

A. Chamseddine, S. Ferrara, G. W. Gibbons and R. Kallosh, hep-th/9610155

A. H. Chamseddine and W. A. Sabra, hep-th/9903046

For the small black hole where $q_1 = 0$ the near horizon geometry can be recast in the following form

$$ds^2 = l^2 \left[(\rho^4 dt^2 - \frac{d\rho^2}{\rho}) - \frac{16}{9} \rho d\Omega_3^2 \right].$$

This solution is singular and unlike the large black hole preserves only four supercharges, which can easily be verified using the above near horizon geometry and the supersymmetry variations.

The corrected near horizon geometry of the small black hole is given by

$$ds^2 = l^2 \left[(r^2 dt^2 - \frac{dr^2}{r^2}) - 4 d\Omega_3^2 \right], \qquad v_{\hat{t}\hat{r}} = \frac{3}{4} l.$$

where $l = \frac{1}{6}\sqrt{\frac{c}{8} q_2 q_3}$. The scalars in the vector multiplet are constant and in what follows we do not need their explicit form. $d\Omega_3^2$ is the round metric on a three-sphere of unit radius and can be written in Hopf coordinates

$$d\Omega_3^2 = d\theta^2 + \sin^2\theta \; d\phi^2 + \cos^2\theta \; d\psi^2.$$

The gravitino variation is

$$\delta\psi_\mu^i = (\partial_\mu + \frac{1}{4}(\omega_\mu)^{ab}\gamma_{ab} + v_{\widehat{r}\widehat{t}}\gamma_\mu{}^{\widehat{r}\widehat{t}} - \frac{2}{3}v_{\widehat{r}\widehat{t}}\gamma_\mu\gamma^{\widehat{r}\widehat{t}})\varepsilon^i.$$

Setting the variations of components of gravitino equal to zero leads to the following set of equations:

$$\begin{aligned}
\left(\partial_t - \frac{r}{2}\gamma^{\widehat{r}}(\gamma^{\widehat{t}} + 1)\right)\varepsilon^i &= 0,\\
\left(\partial_r + \frac{1}{2r}\gamma^{\widehat{t}}\right)\varepsilon^i &= 0,\\
\left(\partial_\theta + \frac{1}{2}\gamma^{\widehat{\theta}\widehat{r}\widehat{t}}\right)\varepsilon^i &= 0,\\
\left(\partial_\phi - \frac{1}{2}\cos\theta\gamma^{\widehat{\phi}\widehat{\theta}} + \frac{1}{2}\sin\theta\gamma^{\widehat{\phi}\widehat{r}\widehat{t}}\right)\varepsilon^i &= 0,\\
\left(\partial_\psi + \frac{1}{2}\sin\theta\gamma^{\widehat{\psi}\widehat{\theta}} + \frac{1}{2}\cos\theta\gamma^{\widehat{\psi}\widehat{r}\widehat{t}}\right)\varepsilon^i &= 0.
\end{aligned}$$

The solurion is

$$\varepsilon^i = \sqrt{\frac{r}{l}}\Omega\, \varepsilon^i_0, \qquad \lambda^i = \frac{l}{2}(t - \frac{\gamma^{\hat{r}}}{r})\, \varepsilon^i, \qquad \text{with} \quad \Omega = e^{\frac{1}{2}\gamma^{\hat{t}\hat{r}\hat{\theta}}\theta}\, e^{-\frac{1}{2}\gamma^{\hat{\theta}\hat{\phi}}\phi}\, e^{\frac{1}{2}\gamma^{\hat{t}\hat{r}\hat{\psi}}\psi}$$

where ε^i_0 is a constant spinor such that $\gamma^{\hat{t}}\varepsilon^i_0 = -\varepsilon^i_0$.

Two different chiralities are related by $\gamma^{\hat{r}}$, i.e. $\varepsilon^i_- = \gamma^{\hat{r}}\varepsilon^i_+$.

Since the five dimensional theory is non-chiral, one may choose $\gamma^{\hat{t}\hat{r}\hat{\theta}\hat{\phi}\hat{\psi}} = 1$, therefore the angular dependence of the spinors may be simplified as follows

$$\Omega = \sqrt{\frac{r}{l}}\, e^{\frac{1}{2}\gamma^{\hat{\psi}\hat{\phi}}\theta}\, e^{-\frac{1}{2}\gamma^{\hat{\theta}\hat{\phi}}(\psi+\phi)}.$$

- Altogether there are eight supercharges in the near horizon limit for the small black hole when R^2 correction is taken into account, where the solution is non-singular with $AdS_2 \times S^3$ near horizon geometry.

- These Killing spinors are exactly the same as leading order. Of course here we have R^2 corrections.

The only parameter which appears in the expressions of the supersymmetry variation is the value of the central charge evaluated at near horizon. This is also the parameter which fixes the radii of AdS and sphere factors. Therefore one may always rescale the coordinates such that central charge can be dropped from the supersymmetry variations. On the other hand, when taking into account the higher order corrections (R^2 correction in our case) with the assumption of $AdS_2 \times S^3$ near horizon geometry, the corrections will only change the radii of the AdS and sphere factors. As a result we would not expect to get any corrections to the supersymmetry variation at the near horizon limit. The only new feature is that for small black holes we get supersymmetry enhancement since higher order corrections stretch the horizon leading to $AdS_2 \times S^3$ near horizon geometry.

Small black holes in 5D $\mathcal{N} = 4$ supergravity.

- These solutions are $\frac{1}{2}$ BPS preserving eight supercharges and are singular at the tree level.

- We would not expect to see supersymmetry doubling and the near horizon geometry still preserves eight supercharges.

- The higher order corrections will stretch the horizon in such a way as to make the near horizon geometry $AdS_2 \times S^3$, the supersymmetry enhancement emerges again.

To make the issue precise one first needs to show that there is a small black hole solution in $\mathcal{N} = 4$ supergravity in five dimensions in the presence of higher order corrections. In other words although the near horizon information is useful, it is not enough to prove whether or not there is a solution interpolating between the near horizon geometry and asymptotically flat space times and a priori it is not obvious whether the solution exists. So far, such a solution has not been found. Nevertheless there is an indirect evidence for the existence of such a solution.

An indication of the existence of a small black hole solution in the $\mathcal{N} = 4$ case in the presence of higher order corrections would be if the five dimensional small back hole solution of the $\mathcal{N} = 2$ (A. Castro, J. L. Davis, P. Kraus and F. Larsen, hep-th/0703087) could indeed be embedded in the $\mathcal{N} = 4$ theory. (J. M. Lapan, A. Simons and A. Strominger, arXiv:0708.0016 [hep-th].)

The reason that the embedding is possible is the fact that if we regard the 5D small black hole solution to be the result of the reduction to 10 dimensional supergravity, because of the particular form of the charges and fields, the reduced background will not break the $Sp(4)$ R-symmetry of the $\mathcal{N} = 4$ model.

Therefore the supersymmetry variation expressions are exactly the same as those in the previous section for $\mathcal{N} = 2$ case.

The only difference is that the index of the spinors ε^i, $i = 1, 2$ of the $\mathcal{N} = 2$ model will now run from 1 to 4, $i = 1, 2, 3, 4$ for $\mathcal{N} = 4$ case. In other words for the former case the spinors are in the **2** of $Sp(2)$ while for latter case it is the **4** of $Sp(4)$.

In the near horizon geometry of the small back hole of the $\mathcal{N} = 4$ model where the geometry is $AdS_2 \times S^3$, we get sixteen supercharges corresponding to

$$\varepsilon^i = \sqrt{\frac{r}{l}}\Omega\, \varepsilon_0^i, \qquad \lambda^i = \frac{l}{2}(t - \frac{\gamma^{\hat{r}}}{r})\, \varepsilon^i, \qquad \text{with} \quad \Omega = \sqrt{\frac{r}{l}}\, e^{\frac{1}{2}\gamma^{\hat{\psi}\hat{\phi}}\theta}\, e^{-\frac{1}{2}\gamma^{\hat{\theta}\hat{\phi}}(\psi+\phi)},$$

for $i = 1, 2, 3, 4$.

Near Horizon superalgebra

we would like to construct the near horizon superalgebra for the $\mathcal{N} = 4$ five dimensional small black holes.

We have a factor of AdS_2 one would expect to get a factor of $SO(2,1)$ in the near horizon superalgebra.

$$L_1 = \frac{2}{l}\partial_t \ , \qquad L_0 = t\partial_t - r\partial_r \ , \qquad L_{-1} = \frac{l}{2}(r^{-2} + t^2)\partial_t - lrt\partial_r \ ,$$

satisfying

$$[L_m, L_n] = (m-n)L_{m+n},$$

for $m, n = \pm 1, 0$.

In order to find the near horizon superalgebra, one first needs to see how the AdS isometry acts on the supercharges.

Using the explicit representation of the generators one finds

$$L_0\lambda^i = \frac{1}{2}\lambda^i, \qquad L_0\varepsilon^i = -\frac{1}{2}\varepsilon^i, \qquad L_1\lambda^i = \varepsilon^i, \qquad L_{-1}\varepsilon^i = -\lambda^i.$$

Here the action of generators is defined by the Lie derivative

$$\mathcal{L}_K\varepsilon^i = (K^\mu \mathcal{D}_\mu + \frac{1}{4}\partial_\mu K_\nu \gamma^{\mu\nu})\varepsilon^i.$$

One may consider a correspondence between λ^i and ε^i and the $G_{-\frac{1}{2}}$ and $G_{\frac{1}{2}}$ modes of supercurrent G, respectively.

$$[L_m, G_r] = (\frac{m}{2} - r)G_{m+r}.$$

The next step is to do the same for S^3 part. In other words we will be looking for the action of $SO(4)$ generators on the spinors. To study the action of the generators we use the fact that locally $SO(4) \approx SU(2) \times SU(2)$. In our notation the generators of the two $SU(2)$'s are

$$J^3 = -\frac{i}{2}(\partial_\phi + \partial_\psi), \qquad J^\pm = \frac{1}{2}e^{\pm i(\psi+\phi)}(-i\partial_\theta \pm \cot\theta\,\partial_\phi \mp \tan\theta\,\partial_\psi),$$

$$K^3 = -\frac{i}{2}(\partial_\phi - \partial_\psi), \qquad K^\pm = \frac{1}{2}e^{\mp i(\psi-\phi)}(-i\partial_\theta \pm \cot\theta\,\partial_\phi \pm \tan\theta\,\partial_\psi).$$

On the other hand since $\gamma^{\hat{t}}$ and $\gamma^{\hat{\theta}\hat{\phi}}$ commute, we can always choose ε_0^i such that

$$\gamma^{\hat{\theta}\hat{\phi}}\varepsilon_0^i = \pm i\varepsilon_0^i, \qquad \gamma^{\hat{t}\hat{r}\hat{\psi}}\varepsilon_0^i = \mp i\varepsilon_0^i.$$

With this definition we have

$$J^3\varepsilon^i = \mp\frac{1}{2}\varepsilon^i, \qquad J^3\lambda^i = \mp\frac{1}{2}\lambda^i, \qquad K^3\varepsilon^i = 0, \qquad K^3\lambda^i = 0.$$

The Killing spinor ε^i and λ^i are in the **2** representation of the first $SU(2)$ group generated by J and are neutral under the second $SU(2)$ group generated by K.

Let us start with a constant spinor ε_0 such that $\gamma^{\widehat{\theta}\widehat{\phi}}\varepsilon_0 = -i\varepsilon_0$. Then we can define

$$\xi_+ = \sqrt{\frac{r}{l}}\, e^{\frac{\theta}{2}\gamma^{\widehat{\psi}\widehat{\phi}}} e^{\frac{i}{2}(\psi+\phi)}\varepsilon_0, \qquad \xi_- = \sqrt{\frac{r}{l}}\, e^{\frac{\theta}{2}\gamma^{\widehat{\psi}\widehat{\phi}}} e^{-\frac{i}{2}(\psi+\phi)}\gamma^{\widehat{\psi}\widehat{\theta}}\varepsilon_0,$$

and normalize to $\varepsilon_0^\dagger\varepsilon_0 = 1$. It is easy to verify that color

$$J^3\xi_\pm = \pm\xi_\pm, \qquad J^\pm\xi_\pm = 0, \qquad J^\pm\xi_\mp = \xi_\pm,$$

and therefore ξ is in the **2** of the first $SU(2)$ group.

Using this notation one may express the Killing spinors, ε^I, corresponding to the supercharges as follows

$$\varepsilon^1 = \begin{pmatrix} \xi_+ \\ i\xi_- \\ 0 \\ 0 \end{pmatrix}, \quad \varepsilon^2 = \begin{pmatrix} -i\xi_+ \\ -\xi_- \\ 0 \\ 0 \end{pmatrix}, \quad \varepsilon^3 = \begin{pmatrix} \xi_- \\ -i\xi_+ \\ 0 \\ 0 \end{pmatrix}, \quad \varepsilon^4 = \begin{pmatrix} i\xi_- \\ -\xi_+ \\ 0 \\ 0 \end{pmatrix},$$
$$\varepsilon^5 = \begin{pmatrix} 0 \\ 0 \\ \xi_+ \\ i\xi_- \end{pmatrix}, \quad \varepsilon^6 = \begin{pmatrix} 0 \\ 0 \\ -i\xi_+ \\ -\xi_- \end{pmatrix}, \quad \varepsilon^7 = \begin{pmatrix} 0 \\ 0 \\ \xi_- \\ -i\xi_+ \end{pmatrix}, \quad \varepsilon^8 = \begin{pmatrix} 0 \\ 0 \\ i\xi_- \\ -\xi_+ \end{pmatrix},$$

which correspond to $G^I_{\frac{1}{2}}$, $I = 1, \cdots, 8$. Similarly $G^I_{-\frac{1}{2}}$ corresponds to λ^I given by

$$\lambda^I = \frac{l}{2}(t - \frac{\gamma^{\hat{r}}}{r})\varepsilon^I, \qquad \text{for } I = 1, \cdots, 8.$$

Here each λ^I or ε^I transforms as the **4** of $Sp(4)$.

To complete the near horizon superalgebra we need to compute the anti-commutators of supercharges. To do this we use the supersymmetry transformations of the five dimensional supergravity given by

$$\begin{aligned} \{G_r^I, G_s^J\} &= l\Omega_{ij}\left[(\bar{\varepsilon}_r^I)^i\gamma^\mu(\varepsilon_s^J)^j + (\bar{\varepsilon}_s^J)^i\gamma^\mu(\varepsilon_r^I)^j\right]\partial_\mu \\ &+ \left[(\bar{\varepsilon}_r^I)_i\gamma^{\hat{r}\hat{t}}(\varepsilon_s^J)^j + (\bar{\varepsilon}_s^J)_i\gamma^{\hat{r}\hat{t}}(\varepsilon_r^I)^j\right], \end{aligned}$$

Ω_{ij} is a symplectic matrix which raises and lowers indices as $\varepsilon_i = \Omega_{ij}\varepsilon^j$. We choose a basis in which $\Omega_{12} = \Omega_{34} = 1$.

We get the anticommutators of the supercharges, e.g.,

$$\{G^I_{\pm\frac{1}{2}}, G^J_{\pm\frac{1}{2}}\} = 2\delta^{IJ} L_{\pm 1},$$

$$\{G^I_{\frac{1}{2}}, G^J_{-\frac{1}{2}}\} = \begin{pmatrix} 2L_0 & 2iJ^3 + iA_3 & 2iJ^2 + iA_1 & -2iJ^1 + iA_2 \\ -2iJ^3 - iA_3 & 2L_0 & -2iJ^1 - iA_2 & -2iJ^2 + iA_1 \\ -2iJ^2 - iA_1 & 2iJ^2 + iA_1 & 2L_0 & -2iJ^3 - iA_3 \\ 2iJ^1 - iA_2 & 2iJ^2 - iA_1 & 2iJ^3 + iA_3 & 2L_0 \end{pmatrix}.$$

$I, J = 1, 2, 3, 4.$

One can summarize the entire superalgebra as follows

$$\{G_r^I, G_s^J\} = 2\delta^{IJ} L_{r+s} + (r-s)(M_a)^{IJ} J^a + (r-s)(N_A)^{IJ} T^A,$$

$$[L_m, G_r^I] = (\frac{m}{2} - r) G_{m+r}^I, \qquad [L_m, L_n] = (m-n) L_{m+n}$$

$$[T^A, G_r^I] = (N^A)^{IJ} G_r^J, \qquad [J^a, G_r^I] = (M^a)^{IJ} G_r^J,$$

T^A are the generators of $Sp(4)$ parameterized by $T^A = \{A_\alpha, B_\alpha, C_\alpha, C_0\}$

$$A_\alpha = \begin{pmatrix} \sigma^\alpha & 0 \\ 0 & 0 \end{pmatrix}, \quad B_\alpha = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^\alpha \end{pmatrix}, \quad C_\alpha = \begin{pmatrix} 0 & \delta^\alpha \\ \delta^{\alpha\dagger} & 0 \end{pmatrix}, \quad C_0 = \begin{pmatrix} 0 & \frac{i}{2} \\ \frac{-i}{2} & 0 \end{pmatrix}.$$

Here $\delta^\alpha = \frac{1}{2}(\sigma^1, i\sigma^2, \sigma^3)$ with σ^α being the Pauli matrices. M_a and N_A are the representation matrices for $SU(2)$ and $Sp(4)$, respectively.

This is, indeed, the commutation relations of the supergroup $OSp(4^*|4)$ which also appeared for the near horizon of $\mathcal{N}=4$ five dimensional small black string.

In our case we have another $SU(2)$ coming from the $SO(4)$ isometry of the S^3 factor.

$$OSp(4^*|4) \times SU(2).$$

The bosonic part of the global supergroup $OSp(4^*|4) \times SU(2)$ is $SL(2) \times SU(2) \times SU(2) \times Sp(4)$ while the isometry of the near horizon geometry is $SL(2) \times SO(4)$.

There is an extra $Sp(4)$ symmetry which can not be geometrically realized.

We may identify this symmetry with R-symmetry of $\mathcal{N}=4$ supergravity in five dimensions. (J. M. Lapan, A. Simons and A. Strominger, arXiv:0708.0016 [hep-th].)

$\mathcal{N}=2$ revisited; From $\mathcal{N}=4$ to $\mathcal{N}=2$

• We have seen the supersymmetry enhancement depends crucially on the geometry of the near horizon limit. The appearance of AdS_2 factor is essential in getting supersymmetry doubling.

• It is not clear how the supersymmetry distinguishes between the large and the small black holes or black strings in $\mathcal{N}=2$ theory considering that in both cases eight supercharges are preserved in the presence of higher order corrections.

This question does not arise for $\mathcal{N} = 4$ case, as the large black holes/strings are $\frac{1}{4}$ BPS while the small ones are $\frac{1}{2}$ BPS. Thus taking the near horizon limit they lead to AdS geometries with different number of supercharges; eight and sixteen supercharges, respectively.

Global supergroup of near horizon geometry of small (S) and large (L) black holes and strings in the $\mathcal{N}=4$ five dimensional supergravity.

Object	**Bosonic Symmetry**	**supercharges**	**Supergroup**
L-black string	$SL(2)\times SU(2)$	8	$SU(1,1\|2)$
L-black bole	$SL(2)\times SO(4)$	8	$SU(1,1\|2)\times SU(2)$
S-black string	$SL(2)\times SU(2)\times Sp(4)$	16	$OSp(4^*\|4)$
S-black hole	$SL(2)\times SO(4)\times Sp(4)$	16	$OSp(4^*\|4)\times SU(2)$

For $\mathcal{N} = 2$ model the situation is quite different. As we already mentioned the problem appears because both the small and large black holes (strings) are $\frac{1}{2}$ BPS and in the near horizon, when higher order corrections are taken into account, preserve the same number of eight supercharges.

We would like to pose the question of how to distinguish between the small and large black holes for the higher order corrected action.

To answer to this question we will resort to the $\mathcal{N} = 4$ model by a process of reduction of the number of supersymmetries.

To get the $\mathcal{N} = 2$ model from the $\mathcal{N} = 4$ theory one may follow two different routes:

1. Adding some matter fields.

2. Truncateing some supercharges.

Depending on which route we choose we get either a large or a small black hole (string).

Of course since we do not have an explicit solution for small back holes (strings) in the presence of R^2 corrections in general it is difficult to do this reduction explicitly. Nevertheless one may proceed for the near horizon geometry. To be specific consider the small black string in $\mathcal{N} = 4$ theory and try to find the near horizon superalgebra when the reduction to the $\mathcal{N} = 2$ case is carried out.

The small black string in five dimensions from the Heterotic string theory point of view, corresponds to a fundamental string living on $R^{1,4} \times T^5$.

Adding matter fields from string theory point of view corresponds to turning on some other charges. In particular we can add a set of NS5-branes wrapped on the T^4, with its fifth direction along the fundamental string.

In this case the background in the near horizon geometry preserves just eight supercharges and indeed this will turn out to be a large black string with near horizon supergroup $SU(1,1|2)$.

Now truncating the solution to $\mathcal{N} = 2$ supergravity we will end up with near horizon geometry of large black strings in the $\mathcal{N} = 2$ theory. The near horizon global symmetry will still remain the same, i.e. $SU(1,1|2)$.

J. P. Gauntlett, R. C. Myers and P. K. Townsend, hep-th/9810204

One could start from a small black string in $\mathcal{N}=4$ theory and just throw away half of the supercharges.

Doing so we end up with a small black string in $\mathcal{N}=2$ theory. It will preserve eight supercharges corresponding to

$$\varepsilon^1 = \begin{pmatrix} \xi_+ \\ i\xi_- \end{pmatrix}, \quad \varepsilon^2 = \begin{pmatrix} -i\xi_+ \\ -\xi_- \end{pmatrix}, \quad \varepsilon^3 = \begin{pmatrix} \xi_- \\ -i\xi_+ \end{pmatrix}, \quad \varepsilon^4 = \begin{pmatrix} i\xi_- \\ -\xi_+ \end{pmatrix},$$

and

$$\lambda^I = \frac{l}{2}(t - \frac{\gamma^{\hat{r}}}{r})\varepsilon^I, \qquad \text{for } I = 1,2,3,4.$$

In this case the $Sp(4)$ R-symmetry will break to $Sp(2)$ and the bosonic part of the symmetry will be $SL(2) \times SU(2) \times Sp(2)$.

Searching in the literature we find that there is, indeed, a supergroup with this bosonic part and supporting eight supercharges which is

$$D(2,1;\alpha)$$

.

The parameter $0 < \alpha \leq 1$ is a relative weight of $SU(2)$ and $Sp(2)$.

What is α?

The procedure is the same for small black holes in $\mathcal{N} = = 4$. The global algebra is

$$\{G_r^I, G_s^J\} = 2\delta^{IJ} L_{r+s} + (r-s)(M_a)^{IJ} J^a + (r-s)(N_A)^{IJ} T^A,$$

$$[L_m, G_r^I] = (\frac{m}{2} - r) G_{m+r}^I, \qquad [L_m, L_n] = (m-n) L_{m+n}$$

$$[T^A, G_r^I] = (N^A)^{IJ} G_r^J, \qquad [J^a, G_r^I] = (M^a)^{IJ} G_r^J.$$

Here T^A are the generators of $Sp(2)$ given by the Pauli matrices and N_A is representation matrix for $Sp(2)$.

This is, indeed, the commutation relations of $Osp(4^*|2) = D(2,1;1)$, i.e. $\alpha = 1$.

With an extra $SU(2)$ coming from $SO(4)$ generated by K we get

$$Osp(4^*|2) \times SU(2)$$

as the global near horizon supergroup of the small black hole in $\mathcal{N} = 2$ supergravity in five dimensions.

Therefore as we see the near horizon supergroup of small and large black strings/holes in $\mathcal{N} = 2$ does distinguish between being small or large.

Global supergroup of near horizon geometry of small (S) and large (L) black holes and strings in the $\mathcal{N} = 2$ five dimensional supergravity.

Object	Bosonic Symmetry	supercharges	Supergroup
L-black string	$SL(2) \times SU(2)$	8	$SU(1,1\|2)$
L-black bole	$SL(2) \times SO(4)$	8	$SU(1,1\|2) \times SU(2)$
S-black string	$SL(2) \times SU(2) \times Sp(2)$	8	$OSp(4^*\|2)$
S-black hole	$SL(2) \times SO(4) \times Sp(2)$	8	$OSp(4^*\|2) \times SU(2)$

Unlike the large one, there is no one to one correspondence between the isometry of the near horizon geometry and the bosonic part of the supergroup. In particular there is an extra $Sp(2)$ factor in the corresponding supergroup. Nevertheless we note that there is a novel way to interpret this extra symmetry: It can be interpreted as the R-symmetry of $\mathcal{N} = 2$ five dimensional supergravity.

If this interpretation is correct it is not clear to us why for the large black holes this factor is absent?

Whenever we have AdS_2 or AdS_3 factor the superisometry must have an affine extension containing a Virasoro algebr. Therefore the supergroup we have obtained for small black hole, $OSp(4^*|2) \times SU(2)$, is expected to be the zero mode algebra of a corresponding unknown affine algebra.

The above conclusions can also be made for small and large black strings in $\mathcal{N} = 2$ five dimensional supergravity. The corresponding near horizon supergroup is $OSp(4^*|2)$ for small and $SU(1,1|2)$ for large black strings.

Since the near horizon supergroup for small and large black strings are different, it would be interesting to understand how this will affect the properties of the corresponding 2D conformal field theory holographically dual to these backgrounds.

Small black string of $\mathcal{N} = 4$ supergravity in five dimensions and its holographic dual have recently been studied.

• A. Dabholkar and S. Murthy, "Fundamental Superstrings as Holograms," arXiv:0707.3818 [hep-th].

• C. V. Johnson, "Heterotic Coset Models of Microscopic Strings and Black Holes," arXiv:0707.4303 [hep-th].

• J. M. Lapan, A. Simons and A. Strominger, "Nearing the Horizon of a Heterotic String," arXiv:0708.0016 [hep-th].

• P. Kraus, F. Larsen and A. Shah, "Fundamental Strings, Holography, and Nonlinear Superconformal Algebras," arXiv:0708.1001 [hep-th].

The next step would be to look for an affine extension of the supergroup.

There are no linear superconformal algebras with more than eight supercharges.

If we relax the linearity condition for the algebra there is non-linear affine algebra $\widehat{OSp(4^*|4)}$ which contains the $OSp(4^*|4)$ in the large central charge limit .

Even though this affine algebra contains the part we are interested in (in a specific limit), it is not clear if it is physically acceptable, e.g., this algebra does not have any unitary representations. (P. Kraus, F. Larsen and A. Shah, arXiv:0708.1001.)

Moreover it is not clear how to incorporate the extra $Sp(4)$ factor in the affine structure.