CLASSIFYING SUPERGRAVITY SOLUTIONS AND APPLICATIONS TO AdS/CFT

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Introduction

Supersymmetric solutions of SUGRA theories have played a very important tole in many developments in string/M-theory.

What are they? Consider bosonic solutions with $\psi = 0$:

$$\begin{aligned} E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - T_{\mu\nu} &= 0 \\ Matter\ Equations\ of\ motion &= 0 \\ Bianchi\ Identities &= 0 \end{aligned}$$

and $\delta\psi = 0$:

$$\begin{aligned} \hat{\nabla}_\mu \epsilon = [\nabla_\mu + (fluxes \cdot \Gamma)_\mu]\epsilon &= 0, \\ M(fluxes)\epsilon &= 0. \end{aligned}$$

i.e. admitting a "Killing spinor" ϵ

Can we “classify” such susy solutions?

(i) Matter =0

$$
\begin{aligned}
R_{\mu\nu} &= 0 \\
\nabla_\mu \epsilon &= 0
\end{aligned}
$$

$\Rightarrow$ special holonomy.

Euclidean case:
$SU(n)$ in $d = 2n$ - Calabi-Yau
$Sp(n)$ in $d = 4n$ - Hyper-Kahler
G_2 in $d = 7$
$Spin(7)$ in $d = 8$

Lorentzian: more possibilities. Bryant

(ii) Matter$\neq$0

$\hat{\nabla}$ is a connection on the Clifford bundle and not, in general on the spin bundle.

What should we do?

Motivation

1. Compactifications to e.g. $\mathbb{R}^{1,3}$: fluxes tend to stabilise moduli.

2. Black Holes: Can we classify all supersymmetric black holes?

3. Surprises: new kinds of ansatz. eg black rings, Gödel

4. AdS/CFT: new examples; deeper understanding.

5. Mathematics

Want:
(i) Precise characterisation of geometry (and then theorems!)
(ii) Explicit solutions where possible.

Key Tool for classification: G-Structures

JPG, Martelli, Pakis, Waldram

PLAN:

1. Overview of G-structures and classification programme.

2. AdS/CFT Applications - including consistency of Kaluza-Klein truncations.

Jan Gutowski
Chris Hull
Nakwoo Kim
Dario Martelli
Oisin Mac Conamhna
Eoin O' Colgain
Stathis Pakis
Harvey Reall
James Sparks
Oscar Varela
Dan Waldram

G-Structures

Let M be an n-dimensional manifold. $F(M)$ be the frame bundle: a principal $Gl(n)$ bundle. A G-Structure is a principal G sub-bundle.

Equivalent to no-where vanishing tensors. e.g.

$$\begin{aligned} g_{ab} &\Rightarrow O(n) \quad (or\, O(p,q)) \\ g_{ab}, \epsilon_{a_1 \ldots a_n} &\Rightarrow SO(n) \end{aligned}$$

$n = 2m$

$$\begin{aligned} J_a{}^b, J^2 = -1 &\Rightarrow Gl(m, C) \\ g_{ab}, J_a{}^b, J^2 = -1 &\Rightarrow U(m) \\ g_{ab}, J_a{}^b, \Omega_{a_1 \ldots a_m} &\Rightarrow SU(m) \end{aligned}$$

Classify G-structures by "intrinsic torsion"

Intrinsic Torsion - Measures the deviation from special holonomy.
Let η define a $G \subset SO(n)$ structure. Basic idea:

$$\nabla\eta \leftrightarrow \oplus_i W_i$$

W_i are G-modules which specify the type of G-structure.

In more detail:
$\exists\ \nabla'$ such that $\nabla'\eta = 0$. Define

$$\begin{aligned} T \equiv \nabla - \nabla' = \omega - \omega' \quad &\in \quad \Lambda^1 \otimes \Lambda^2 \\ &\cong \quad \Lambda^1 \otimes so(n) \\ &\cong \quad \Lambda^1 \otimes (g \oplus g^\perp) \end{aligned}$$

Then $\nabla\eta = (\nabla - \nabla')\eta \to$ element of $\Lambda^1 \otimes g^\perp$. This is the part of T that is independent of ∇' and is called the intrinsic torsion:

$$T^{(0)} \in \Lambda^1 \otimes g^\perp = \oplus_i \mathcal{W}_i$$

An example: $SU(3)$ structures in $d = 6$.

Specified by a real form J_{ab} and a complex form Ω_{abc}, satisfying

$$\begin{aligned} J \wedge \Omega &= 0 \\ \Omega \wedge \bar{\Omega} &= -i\frac{4}{3} J \wedge J \wedge J \end{aligned}$$

This defines a metric, and orientation and an almost complex structure.

The intrinsic torsion has 5 components. Decompose fundamental and adjoint of $SO(6)$ into $SU(3)$ reps: $\mathbf{6} = \mathbf{3} + \bar{\mathbf{3}}$, $\mathbf{15} = \mathbf{1} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{8}$. Hence $\Lambda^1 \otimes g^{\perp}$ gives the reps:

$$\begin{aligned} (\mathbf{3} + \bar{\mathbf{3}}) \quad \times \quad & (\mathbf{1} + \mathbf{3} + \bar{\mathbf{3}}) = (\mathbf{1} + \mathbf{1}) + (\mathbf{8} + \mathbf{8}) + \\ & (\mathbf{6} + \bar{\mathbf{6}}) + (\mathbf{3} + \bar{\mathbf{3}}) + (\mathbf{3} + \bar{\mathbf{3}}) \end{aligned}$$

corresponding to 5 $\mathcal{W}_i$.

Each $W_i \in \mathcal{W}_i$ can be expressed entirely in terms of dJ and $d\Omega$:

$$dJ \rightarrow W_1, W_3, W_4$$
$$d\Omega \rightarrow W_1, W_2, W_5$$

e.g.

$$\begin{aligned} (W_4)_a &= J^{b_1 b_2}(dJ)_{a b_1 b_2} \\ (W_5)_a &= \Omega^{b_1 b_2 b_3}(d\Omega)_{a b_1 b_2 b_3} \end{aligned}$$

Examples:

$$\begin{aligned} W_1 = W_2 = 0 &\rightarrow complex \\ W_1 = W_2 = W_3 = W_4 = 0 &\rightarrow Kahler \\ W_i = 0 &\rightarrow Calabi-Yau \\ &\Leftrightarrow dJ = d\Omega = 0 \end{aligned}$$

There exists 32 different $SU(3)$ structures.

To classify supergravity solutions

1. Observe that the isotropy group G of the Killing spinor ϵ defines a G-structure. Explicitly, the tensors defining the G-structure can be constructed as bi-linears:

$$T_{i_1 \dots i_k} \sim \bar{\epsilon}\Gamma_{i_1 \dots i_k}\epsilon, \qquad k = 0, 1, \dots$$

The algebraic conditions satisfied by the tensors can be obtained e.g. by using Fierz identities.

2. $\hat{\nabla}_\mu \epsilon = [\nabla_\mu + (fluxes \cdot \Gamma)_\mu]\,\epsilon = 0$ restricts the intrinsic torsion and determines some of the flux. $M\epsilon = 0$ places additional conditions on the flux and intrinsic torsion. Note that some of the flux components can drop out completely. HARD WORK.

3. Equations of motion. Consider $[\hat{\nabla}_\mu, \hat{\nabla}_\nu]\epsilon = 0$. Impose matter equations of motion and Bianchi identities $\Rightarrow$ $E_{\mu\nu}\Gamma^\nu \epsilon = 0$. Need to impose at most one component of $E_{\mu\nu} = 0$, and only in Lorentzian case. JPG, Pakis

An Example: Heterotic compactified on $\mathbb{R}^{1,3} \times M_6$.

Set Yang-Mills fields to zero for simplicity.

After decomposing $D = 10$ spinor, susy $\Rightarrow$

$$\begin{aligned} \left(\nabla_m + \frac{1}{8} H_{mnp} \Gamma^{np}\right) \epsilon &= 0, \\ \left(\Gamma^m \nabla_m \Phi + \frac{1}{12} H_{mnp} \Gamma^{mnp}\right) \epsilon &= 0, \end{aligned}$$

where ϵ is chiral $D = 6$ spinor.

- $\epsilon \longrightarrow SU(3)$ structure:

$$\begin{aligned} J_{mn} &= -i\epsilon^\dagger \Gamma_{mn} \epsilon \\ \Omega_{mnp} &= \epsilon^T \Gamma_{mnp} \epsilon \end{aligned}$$

• Analyse:

$$\begin{aligned} d(\mathrm{e}^{-2\Phi}\Omega) &= 0, \\ d(\mathrm{e}^{-2\Phi} J \wedge J) &= 0, \\ \mathrm{e}^{2\Phi} d(\mathrm{e}^{-2\Phi} J) &= - * H \end{aligned}$$

Have $W_1 = W_2 = 0$, $\Rightarrow$ complex. $W_4 = -(1/2)W_5 = 2d\Phi$. H restricted by structure.

• H e.o.m. is automatically satisfied.
To satisfy all e.o.m. just need to impose $dH = 0$.

This is equivalent to old results of Strominger and Hull. The point is that the method generalises.

Can apply the programme in 3 broad ways:

1. Classify the most general supergravity solutions in D=10/11 supergravity.

D=11: Find that the most general solutions, preserving 1/32 supersymmmetry have either $SU(5)$ or $Spin(7) \ltimes \mathbb{R}^9$ structure.

JPG, Pakis

Type IIB: Either $Spin(7) \ltimes \mathbb{R}^8$, $SU(4) \ltimes \mathbb{R}^8$ or G_2 structure

Gran, Gutowski, Papadopoulos

Very general results.

Refine the classification to different amounts of supersymmetry. Perhaps we can explicitly determine all solutions with say > 16 supersymmetries? e.g. 29, 30 and 31 susies $\Rightarrow$ 32 susies in IIB.

Gutowski et el

2. Lower-Dimensional Supergravities e.g. $D = 4, 5, 6, 7$.

Can consider ungauged and gauged supergravity and also coupled to various matter multiplets.

Can be much more explicit and this gives powerful new ways of constructing D=10 and D=11 solutions.

JPG, Gutowski, Hull, Pakis, Reall, ...

Highlights:

Black rings – black holes with topology $S^1 \times S^2$.

Elvang, Emparan, Mateos, Reall; JPG, Gutowski; Bena, Warner

AdS_5 black holes in $D = 5$ $N = 1$ gauge supergravity

Gutowski, Reall; Chong, Cvetic, Lu, Pope

3. Special classes of Solutions

• Susy black holes - can we classify them all? Reall...

• Compactifications from $D = 11, 10$ to $\mathbb{R}^{1,3}$, $\mathbb{R}^{1,2}$, ...

Largely worked put but still much to be understood about geometry.

Hitchin/Gualtieri: Generalised geometry: particularly useful for global aspects. Focus on $T \oplus T^*$.

- AdS Solutions

Want to classify most general supersymmetric AdS backgrounds of string/M-theory or equivalently the most general SCFTs with supergravity duals

More precisely, want to classify general warped products of the form: $AdS \times_w M$:

$$ds^2 = e^{2A(y)}[ds^2(AdS) + ds^2(M)(y)]$$

with fluxes preserving isometries of AdS.

Can achieve this using G-structure techniques.

Supersymmetric AdS/CFT Solutions

Aim: characterise the most general supersymmetric warped product solutions of D=11 or type IIB supergravity of the form: $AdS_{d+1} \times_w M$:

$$ds^2 = e^{2A(y)}[ds^2(AdS_{d+1}) + ds^2(M)(y)]$$

with fluxes preserving isometries of AdS.

Solutions are invariant under $SO(d,2)$ $\Leftrightarrow$ characterise the most general SCFTs in d spacetimes that have a supergravity description.

Motivation

$\star$ ($ds^2(M)$, A, fluxes) should be an interesting class of geometries. (Theorems)
$\star$ Rich sets of new explicit solutions
e.g. $Y^{p,q}$ Sasaki-Einstein (JPG, Martelli, Sparks, Waldram)
$\star$ M-theory examples give novel SCFTs

$\star$ Can deform CFT to get different dynamics in IR
$\star$ Can analytically continue $\longrightarrow$ "BPS bubbles" dual to smooth BPS states of SCFTs (LLM)
Also: Wilson Lines Lunin, D'Hoker, Gutperle, ...

PLAN:

1. Special case I: Sasaki-Einstein Geometry

2. Special case II: AdS_3 solutions in IIB AdS_2 solutions in $D = 11$

3. General Classification programme

4. Consistent Kaluza-Klein truncations

Special Case I: Sasaki-Einstein

Type IIB sugra:

$$\begin{aligned} ds^2 &= ds^2(AdS_5) + ds^2(X_5) \\ F_5 &= (1 + *)Vol(X_5) \end{aligned}$$

where X_5 is Sasaki-Einstein. Dual to N=1 SCFTs in d=4.

D=11 sugra:

$$\begin{aligned} ds^2 &= ds^2(AdS_4) + ds^2(X_7) \\ G_4 &= Vol(AdS_4) \end{aligned}$$

where X_7 is Sasaki-Einstein. Dual to N=2 SCFTs in d=3.

Clearly not the most general class of solutions.

Focus on type IIB case.

X_{2n+1} is SE iff the cone metric

$$dr^2 + r^2 ds^2(X_{2n+1})$$

is CY i.e. an $SU(n)$ structure J, Ω with $dJ = d\Omega = 0$.

E.g. $X_5 = S^5$ has cone $\mathbb{R}^6$ and IIB solution arises from D3-branes in $\mathbb{R}^{1,3} \times \mathbb{R}^6$.
E.g. $X_7 = S^7$ has cone $\mathbb{R}^8$ and solution arises from M2-branes in $\mathbb{R}^{1,2} \times \mathbb{R}^8$.

More generally, the cone is singular at $r = 0$ and the solutions arise from D3-branes or M2-branes sitting at the singular tip of the cone over the SE space.

Note $AdS_5 \times S^5$ in Poincaré coordinates:

$$ds^2 = \frac{1}{r^2} ds^2(\mathbb{R}^{1,3}) + \frac{1}{r^2}[dr^2 + r^2 ds^2(X_5)]$$

which is the near horizon limit of solution of D3-branes sitting at apex of cone:

$$ds^2 = (1 + \frac{1}{r^4})^{-1/2} ds^2(\mathbb{R}^{1,3}) + (1 + \frac{1}{r^4})^{1/2}[dr^2 + r^2 ds^2(X_5)]$$

Every SE space has a "Reeb-vector" ξ

$$\xi^j \equiv r(\partial_r)^i J_i{}^j$$

and this turns out to be Killing. This is the geometrical dual of the "$U(1)$" R-symmetry of the SCFT.

Locally, the SE metric can be written

$$ds^2(X_5) = (d\psi + \sigma)^2 + ds^2(KE)$$

where $\xi = \partial_\psi$ and $ds^2(KE)$ is four-d Kähler-Einstein metric with positive curvature and $d\sigma = 2J_{KE}$.

Locally, the SE has an $SU(2)$ structure specified by the one form $K = d\psi + \sigma$ and J_{KE}, Ω_{KE}.

Three possibilities:

1. Regular SE: Have a $U(1)$ symmetry and it is free. KE is globally defined.

2. Quasi regular SE: Have a $U(1)$ symmetry with finite isotropy groups. KE is an orbifold.

3. Irregular SE: Have a non-compact $\mathbb{R}$ symmetry. (If compact, must have additional isometry) KE is not a manifold.

For cases 1 and 2 the dual SCFT has a $U(1)$ R-symmetry, for case 3 there is an $\mathbb{R}$ R-symmetry.

Many explicit constructions of SE metrics: $Y^{p,q}$, $L^{a,b,c}$

Good understanding of toric case $U(1)^3$ symmetry and much understood about the dual SCFTS (quivers, dimers) Hanany,....

Many interesting mathematical theorems: General properties of SCFT should have geometrical manifestations.

1. R-symmetry of SCFT $\leftrightarrow$ Reeb vector.
2. a-maximisation: $\leftrightarrow$ $Vol(SE)$ can be obtained by a variational principle. $Vol(SE)$ are given by algebraic numbers.

Martelli, Sparks, Yau

3. Unitarity and RG flow: can be used to identify interesting obstructions to the existence of conical Calabi-Yau metrics on algebraic varieties.

JPG, Martelli, Sparks, Yau

Special Case II

AdS_3 in Type IIB sugra:

$$\begin{aligned} ds^2 &= e^{2A}[ds^2(AdS_3) + ds^2(Y_7)] \\ F_5 &= (1 + *)Vol(AdS_3) \wedge F_2 \end{aligned}$$

Dual to N=(0,2) SCFTs in d=2.

AdS_2 in D=11 sugra:

$$\begin{aligned} ds^2 &= e^{2A}[ds^2(AdS_2) + ds^2(Y_9)] \\ G_4 &= Vol(AdS_2) \wedge F_2 \end{aligned}$$

Dual to N=2 SCQM.

Can classify using G-structures (Kim)

There is always a Killing vector $\leftrightarrow U(1)$ R-symmetry

Locally, there is an $SU(3)$ or $SU(4)$ structure and the metrics can be written

$$\begin{aligned} ds^2(Y_7) &= \frac{1}{4}(dz+P)^2 + e^{-4A}ds^2(B_6) \\ ds^2(Y_9) &= (dz+P)^2 + e^{-3A}ds^2(B_8) \end{aligned}$$

where B_{2n} is Kähler and satisfies

$$\Box R - \tfrac{1}{2}R^2 + R_{ij}R^{ij} = 0 \qquad (*)$$

with $dP = \mathcal{R}$ and e^{-4A} or $e^{-3A} \propto R > 0$.

Global geometry in $2n+2$ dimensions JPG, Kim :
(ds^2_{2n+2}, f_3, ϕ) with a globally defined $SU(n+1)$ structure J, Ω satisfying

$$\begin{aligned} d[e^{n\phi}\Omega] &= 0 \\ d[e^{2(n-1)\phi} *_{2n+2} J] &= 0 \\ d[e^{2\phi} J] &= f_3 \\ d\left[e^{2(n-3)\phi} *_{2n+2} f\right] &= 0 \end{aligned}$$

Geometry in $2n+1$ dimensions ($n \geq 3$): $ds^2(Y_{2n+1}), F_2, A$ is extracted by demanding the cone form:

$$\begin{aligned} ds^2_{2n+2} &= dr^2 + r^2 ds^2_{2n+1} \\ e^{-2\phi} &= r^{\frac{2(n-1)}{n-2}} e^{\frac{2(n-1)}{2-n} A} \\ f_3 &= r^{\frac{n}{2-n}} dr \wedge F_2 \end{aligned}$$

Many similarities to Sasaki-Einstein geometry

Apply constructions of Sasaki-Einstein $Y^{p,q}$ and $L^{a,b,c}$ to present setting: find infinite classes of examples with known central charges. JPG, Kim, Waldram

To do:

$\star$ The geometries should be dual to D3-branes wrapping holomorphic surfaces inside CY_4 folds. Can we make this more precise? Do interpolating solutions exist?

$\star$ What are the SCFTS dual to explicit examples? Are they related to SCFTs dual to SE geometries wrapped on Riemann surfaces?

$\star$ Is there an analogue of toric geometry?

$\star$ Identify geometrical versions of general properties of CFT.

Most General AdS Solutions

D=11 Have classified most general:

AdS_5, $N = 1$ *

JPG, Martelli, Sparks, Waldram

AdS_5, $N = 2$,

Lin, Lunin, Maldacena

AdS_4, $N = 1$

AdS_4, $N = 2$ (A special case is the $AdS_4 \times SE_7$ solutions)

AdS_3, $N = (0,4)$, $(0,2)$ *, $(2,2)$, $(1,2)$, $(1,1)$, $(1,0)$

JPG, Mateos,Mac Conamhna, Waldram; Figueras, Mac Conamhna, O Colgain,

Type IIB Have classified most general:

AdS_5, $N = 1$ (A special case is the $AdS_5 \times SE_5$ solutions)

JPG, Martelli, Sparks, Waldram

Massive type IIA AdS_4, $N = 1$

Behrndt, Cvetic

Technical comment: Noting that any AdS_{d+1} supersymmetric solution is a special case of a $\mathbb{R}^{1,d-1}$ solution, one can first classify the latter and then extract out the conditions for an AdS solution. Turns out that one can construct the most general AdS solutions from a restricted class of $\mathbb{R}^{1,d-1}$ solutions.

Much to be understood:

$\star$ Still more cases to study

$\star$ For many cases the description is in terms of G-structures and it is not clear what are good local coordinates.

$\star$ For all cases one can recover some known special explicit solutions that were first found in gauged supergravity and describe branes wrapping calibrated cycles.
For some special cases infinite rich classes of explicit solutions have been constructed. Why?

Consistent Kaluza-Klein Truncation

Consider a higher dimensional theory of gravity with a vacuum solution of the form $\mathbb{R}^{1,d-1} \times_w M$ or $AdS_{d+1} \times_w M$.
Expand all higher dimensional fields in terms of modes on M and divide them into "Heavy" H and "Light" L modes.
Substitute into the higher dimensional equations of motion to get

$$\nabla^2 H \sim \sum a_k L^k + \sum b_k H^k + \sum c_{kl} H^k L^l$$
$$\nabla^2 L \sim \sum d_k L^k + \sum e_k H^k + \sum f_{kl} H^k L^l$$

Consistent to set all $H = 0$ only if $a_k = 0$ and then one obtains equations of a lower dimensional theory of gravity.
Any solution of the theory in low dimensions gives a solution of the theory in higher dimensions.

For the truncation to be consistent the light modes should not source the heavy modes.

Simple example: pure $D = 5$ gravity, $R_{MN} = 0$
Vacuum state $\mathbb{R}^{1,3} \times S^1$
Metric:

$$ds^2(x,y) = e^{-\phi/3} g_{\mu\nu} dx^\mu dx^\nu + e^{2\phi/3}(dy + A_\mu dx^\mu)^2$$

where we can expand $\phi(x,y), g_{\mu\nu}(x,y), A_\mu(x,y)$ in an infinite set of modes eg

$$\phi(x,y) = \sum_n \phi_n(x) e^{iny}, \ldots$$

Substitute into $R_{MN} = 0$.
Heavy modes: $n \neq 0$ have mass$\sim n$ and charge$\sim n$
Light modes: $n = 0$ have mass=0 and charge=0.

It will clearly be consistent to set heavy modes to zero, since the light, $U(1)$ invariant, modes cannot source the heavy, non-invariant, states.

Keeping just the light modes (and all of them), we find that they must solve the four dimensional equations of motion

$$\begin{aligned} R_{ij} &= \frac{1}{6}\nabla_i\phi\nabla_j\phi + \frac{1}{2}e^{\phi}F^2_{ij} \\ \nabla^2\phi &= \frac{1}{4}e^{\phi}F^2 \\ d(e^{\phi} * F) &= 0 \end{aligned}$$

Any explicit solution to these $D=4$ equations will give an explicit $D=5$ solution with $R_{MN}=0$.

The consistent KK ansatz provides a powerful way of constructing higher dimensional solutions.

This generalises to tori and also to groups manifolds.
However, in general, there will certainly not exist a consistent KK truncation, but some other cases are known.

If we start with a SUGRA theory and a supersymmetric vacuum, when can we consistently truncate to a lower dimensional theory? Sufficient to check at the level of bosonic fields Cvetic, Lu, Pope

Toroidal reductions give sugra theories
e.g. D=11 Sugra on T^7 or type IIB Sugra on $T^6 \rightarrow$ N=8 supergravity in d=4.

Much less obvious and much more involved are the reductions on spheres to gauged supergravities:

D=11

$AdS_4 \times S^7$: can consistently KK truncate on S^7 to get $N = 8$ gauged supergravity in $D = 4$. De Wit, Nicolai
Gauge group $SO(8) \leftrightarrow R$ symmetry in dual CFT

$AdS_7 \times S^4$: can consistently KK truncate on S^7 to get maximal gauged supergravity in $D = 7$. Nastase, Vaman, van Nieuwenhuizen
Gauge group $SO(5) \leftrightarrow R$ symmetry in dual CFT

Type IIB

$AdS_5 \times S^5$: can consistently KK truncate on S^5 to get maximal gauged supergravity in $D = 5$. Cvetic, Duff, Hoxha, Liu, Lu, Lu, Martinez-Acosta, Pope, Sati, Tran; Lu, Pope, Tran; Cvetic, Lu, Pope, Sadrzadeh, Tran
Gauge group $SO(6) \leftrightarrow R$ symmetry in N=4 SYM

There are also further truncations that one can consider. For example type IIB on S^5 can be further truncated to

• $N = 1$ gauge SUGRA coupled to two vector multiplets ($U(1)^3$) and further to minimal $N = 1$ gauged SUGRA ($U(1)$)

• A different truncation leads to Romans $N = 2$ $SU(2) \times U(1)$ gauge supergravity.

Thus, for example, the AdS_5 black holes of minimal $N = 1$ $D = 5$ gauge supergravity can be uplifted on an S^5 to obtain solutions of type IIB supergravity.

Conjecture : for any supersymmetric $AdS \times_w M$ solution of $D = 10$ or $D = 11$ supergravity there is a consistent KK truncation on M to a gauged supergravity whose fields $g_{\mu\nu}, A_\mu ...$ are dual to the superconformal current multiplet: $T_{\mu\nu}, j_\mu, ...$

JPG, Varela (Duff, Pope)

Some general arguments for why this might be true were put forward by Pope, Stelle . It would be nice to make them more precise.

It would also be nice to have an AdS/CFT proof of the conjecture. For d=2 SCFT David, Sahoo, Sen

We can also tackle this directly by considering G-structure classification of AdS solutions and use it to try and construct the appropriate KK ansatz directly (at the level of bosonic fields).

Then for any explicit AdS solution this will allow one to uplift any explicit solution of the lower dimensional sugra to obtain a new explicit solution in D=10, 11.
Note that one needs to check regularity of higher dimensional solution.

Illustrate with simplest example:

Type IIB sugra: $AdS_5 \times SE_5$

$$\begin{aligned} ds^2 &= ds^2(AdS_5) + (d\psi + \sigma)^2 + ds^2(KE_4) \\ F_5 &= Vol(AdS_5) + Vol(SE_5) \end{aligned}$$

$d\sigma = 2J_{KE}$

Dual to $N = 1$ SCFT in d=4. Bosonic superconformal currents are $T_{\mu\nu}$ and $U(1)$ R-symmetry current j_μ. These are dual to $D = 5$ metric $g_{\mu\nu}$ and $U(1)$ gauge-field A_μ, which are the bosonic fields of minimal D=5 SUGRA.

Hence, Conjecture $\Rightarrow$ should be able to KK reduce type IIB on SE_5 to minimal D=5 gauged sugra

Consistent KK ansatz Buchel, Liu :

$$
\begin{aligned}
ds^2 &= ds_5^2 + (d\psi + \sigma + A)^2 + ds^2(KE_4) \\
F_5 &= (1 + *)\,[Vol_5 + J_{KE} \wedge *_5 F]
\end{aligned}
$$

$F = dA$. Substitute into type IIB equations of motion $\rightarrow$

$$
\begin{aligned}
&R_{\mu\nu} = -4g_{\mu\nu} + \frac{1}{6}F_{\mu\lambda}F_{\nu}{}^{\lambda} - \frac{1}{36}g_{\mu\nu}F^2 \\
&d *_5 F - \frac{1}{3}F \wedge F = 0.
\end{aligned}
$$

Which are indeed the equations of motion of minimal $D = 5$ gauged supergravity.

Generalisations: JPG, O' Colgain, Varela

• Type IIB $AdS_5 \times X_5$ and $D = 11$ $AdS_5 \times_w M_6$ solutions dual to $N = 1$ SCFTs
Have constructed consistent KK reduction on X_5 or M_6 to minimal $D = 5$ gauged sugra. (More difficult)

Recall AdS_5 black hole solutions of minimal gauged sugra. Can be uplifted on S^5, but also on any SE_5, X_5 or M_6.

• $D = 11$ $AdS_5 \times M_6$ with $N = 2$ susy LLM
Have constructed consistent KK reduction to $N = 2$ $SU(2) \times U(1)$ $D = 5$ gauged SUGRA. Bosonic fields: Metric, $SU(2) \times U(1)$ gauge fields and a scalar. (Much more difficult!)

Conclusions

1. Much progress in classifying supersymmetric solutions of sugra theories using G-structures.

2. Very rich set of applications to AdS/CFT:

(i) Sasaki-Einstein
(ii) New AdS_3 solutions of type IIB and AdS_2 solutions of D=11 sugra
(iii) General Classification results

3. Consistent Kaluza-Klein Truncations