Nearing Extremal Intersecting Giants

and

New Decoupled Sectors in $\mathcal{N}=4$ SYM

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Based on My Recent Work [arXiv:0801.4457],

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Introduction and Motivations

- AdS/CFT:
  any state/physical process in the asymptotically $AdS_5 \times S^5$ geometry $\leftrightarrow$ a (perturbative) deformation of $\mathcal{N} = 4,\ d = 4$ SYM.
- A class of such deformations are solutions to $\mathcal{N} = 2,\ d = 5\ U(1)^3$ gauged supergravity.
- These solutions are generically black hole (BH) solutions, among them the static (non-rotating) black holes are specified with four parameters, three charges and one mass parameter.

Introduction, Cont'd

- All of the solutions of $5d\ U(1)^3$ gauged SUGRA can be uplifted as *rotating black three-brane* solutions of $10d$ IIB SUGRA.
- In $10d$ these solutions are only specified by metric and the self-dual five-form and constant dilaton.
- As solutions of IIB these solutions they can be 1/2, 1/4, 1/8 BPS or non-SUSY, respectively preserving 16, 8, 4 and zero SUSY.
- The 1/2 BPS solutions correspond to *smeared (delocalized) spherical D3-branes*, the giant gravitons.

Introduction, Cont'd

- The 1/2 BPS giant gravitons are three-branes wrapping a three sphere inside the S^5 part of the background $AdS_5 \times S^5$ geometry while moving on a geodesic along an $S^1 \in S^5$ transverse to the worldvolume S^3 and smeared (delocalized) over the remaining direction.
- The $1/2$ BPS solutions are specified by a single parameter, the value of the charge.
- The 1/2 BPS solutions in our class can be understood as a collection of smooth LLM geometries; they preserve the same supercharges.

Introduction, Cont'd

- In a similar manner the two-charge $1/4$ BPS and three-charge $1/8$ BPS solutions can be understood as geometries corresponding to intersecting giant gravitons.
- The non-supersymmetric cases then correspond to turning on specific open string excitations on the supersymmetric (intersecting) giant gravitons.

Introduction, Cont'd

- In the dual description in the $\mathcal{N}=4$ SYM on $R\times S^3$, the $1/2$ BPS geometries are described by chiral primary operators in the subdeterminant basis.
- In a similar fashion less BPS solutions correspond to operators involving two or three complex scalars in the $\mathcal{N}=4$ vector multiplet.
- The non-supersymmetric configurations when the solution is near-BPS (*i.e.* when $\frac{\Delta-J}{J}\ll 1$, where Δ is the scaling dimension and J is the R-charge of the corresponding operators) then correspond to insertion of "impurities" in the subdeterminant operators.

The Main Question

Here I'll focus on the two-charge $5d$ black hole solutions. Noting that for these solutions we have a simple interpretation in terms of intersecting giants we pose the following question:

Is there a limit in which the (low energy effective) gauge theory residing on the intersecting spherical brane system decouples from the bulk?

As we will argue, by gathering supportive evidence from various sides, that the answer to this question is positive.

Plan of the Talk

- Review of the $5d$ gauged SUGRA charge black holes.
- Appearance of BTZ $\times S^3$ factors in the near-horizon limit of the corresponding two-charge near-extremal $10d$ IIB solutions, the near-BPS and near-extremal, but far from BPS cases.
- Perturbative addition of the third charge, *rotating BTZ$\times S^3$ geometries.*
- The BTZ$\times S^3$ geometries as solutions to $6d$ (gauged supersymmetric) gravities.

Plan of the Talk, Cont'd

- The Dual Field Theory Descriptions:
  - The $\mathcal{N} = 4$, $D = 4$ SYM descriptions,
    - Identifying the decoupled sectors of the near-BPS and near-extremal cases.
  - The $D = 2$ CFT descriptions,
    - Identification of L_0, $\bar{L}_0$ and the central charge of the two $2d$ CFT's corresponding to near-BPS and near-extremal cases.

Review $5d$ Charged Black Holes

- The $10d$ metric:

$$ds_{10}^2 = \sqrt{\Delta}\, ds_5^2 + \frac{1}{\sqrt{\Delta}}\, d\Sigma_5^2$$

where

$$ds_5^2 = -\frac{f}{H_1 H_2 H_3} dt^2 + \frac{dr^2}{f} + r^2\, d\Omega_3^2$$

$$d\Sigma_5^2 = \sum_{i=1}^{3} L^2 H_i \left(d\mu_i^2 + \mu_i^2 \left[d\phi_i + a_i\, dt \right]^2 \right).$$

- Note that the $5d$ Black Hole Metric is

$$ds_{5d\ BH}^2 = (H_1 H_2 H_3)^{1/3} ds_5^2$$

$$= \frac{-f}{(H_1 H_2 H_3)^{2/3}} dt^2 + \frac{(H_1 H_2 H_3)^{1/3}}{f} dr^2 + r^2 (H_1 H_2 H_3)^{1/3} d\Omega_3^2$$

Review $5d$ *Charged Black Holes, Cont'd*

- $d\Sigma_5^2$ is the metric for a deformed S^5 and

$$\mu_1 = \cos\theta_1, \qquad \mu_2 = \sin\theta_1 \cos\theta_2, \qquad \mu_3 = \sin\theta_1 \sin\theta_2.$$

- H_i, f, Δ and a_i:

$$H_i = 1 + \frac{q_i}{r^2}, \quad f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2} H_1\, H_2\, H_3,$$

$$a_i = \frac{\tilde{q}_i}{q_i} \frac{1}{L} \left(\frac{1}{H_i} - 1 \right), \quad \Delta = H_1\, H_2\, H_3 \left[\frac{\mu_1^2}{H_1} + \frac{\mu_2^2}{H_2} + \frac{\mu_3^2}{H_3} \right],$$

- As $10d$ solution, we also have

$$\mathfrak{F}_5 = F_5 + \ast F_5, \qquad F_5 = dB_4 \ \text{ where,}$$

$$B_4 = -\frac{r^4}{L} \Delta\, dt \wedge d^3\Omega - L \sum_{i=1}^{3} \tilde{q}_i\, \mu_i^2 \left(L\, d\phi_i - \frac{q_i}{\tilde{q}_i} dt \right) \wedge d^3\Omega,$$

Review $5d$ *Charged Black Holes, Cont'd*

- The ADM mass M and physical charges $\tilde{q}_i$ of the corresponding $5d$ black holes are

$$\tilde{q}_i = \sqrt{q_i(\mu + q_i)}$$

$$M = \frac{\pi}{4G_N^{(5)}} \left(\frac{3}{2}\mu + q_1 + q_2 + q_3 + \frac{3L^2}{8} \right).$$

- The last term in M is the Casimir energy.
- $G_N^{(5)}$ is the five-dimensional Newton constant and is related to the ten-dimensional one as

$$G_N^{(5)} = G_N^{(10)} \frac{1}{\pi^3 L^5}.$$

$10d$ *rotating brane charges*

- As $10d$ IIB solutions, the black holes correspond to (smeared or delocalized) stack of *rotating* intersecting spherical three-brane giant gravitons, the angular momentum of each stack of branes is

$$J_i = \frac{\pi L}{4G_N^{(5)}} \tilde{q}_i \ .$$

- The number of branes in each stack is then given by

$$N_i = \frac{2J_i}{N} = \frac{\pi^4}{2N} \cdot \frac{L^8}{G_N^{(10)}} \cdot \frac{\tilde{q}_i}{L^2} = N \cdot \frac{\tilde{q}_i}{L^2}$$

note that, being a D3-brane, each giant is carrying one unit of the RR charge in units of three-brane tension $T_3 = 1/(8\pi^3 l_s^4 g_s)$.

Review $5d$ Charged Black Holes, Cont'd

- μ is a parameter measuring deviation from being BPS.
- For $\mu = 0$ case, $\tilde{q}_i = q_i$ and ADM mass up to the Casimir energy and $\pi/4G_N^{(5)}$ factor is equal to the sum of the physical charges; therefore the solution is BPS.
- The BPS configuration with n number of non-vanishing q_i's ($n = 1, 2, 3$) generically preserve $1/2^n$ of the 32 supercharges of the $AdS_5 \times S^5$ background.
- The three-charge case with $q_1 = q_2 = q_3$, $\mu = 0$ is an exception, it is 1/4 BPS and corresponds to a $5d$ extremal AdS-Reissner-Nordstrom black hole.

Review $5d$ Charged Black Holes, Cont'd

- All supersymmetric BPS solutions have naked singularity. In the 1/2 BPS case it is a light-like, naked singularity, while for 1/4 and 1/8 BPS states it is time-like.
- Black holes with regular horizons can only occur when $\mu \neq 0$ and hence are all non-supersymmetric.
- For the $\mu \neq 0$ cases depending on the number of non-zero charges, which can be one, two or three, we have different singularity and horizon structures:

The causal structure of the $5d$ black holes

- **One-charge black hole:**
  - At $\mu = 0$ we have a null nakedly singular solution which preserves 16 supercharges.
  - As soon as we turn on μ the solution develops a horizon with a space-like singularity sitting behind the horizon.
  - As a $10d$ IIB geometry, the one charge case with $\mu = 0$ corresponds to 1/2 BPS three sphere giant configuration wrapping an S^3 inside the S^5 while moving with the angular momentum $J \propto q$.

- **One-charge black hole, *Cont'd*:**
  - This gravity configuration describes a giant smeared over (delocalized in) two directions inside S^5 transverse to the worldvolume of the brane.
  - Turning on μ then corresponds to adding open string excitations to the giant graviton while keeping the spherical shape of the giant.

The causal structure of the $5d$ black holes

- **Two-charge black hole:**
  - For $0 \leq \mu < \mu_c$ we have a time-like but naked singularity where

$$\mu_c = q_2 q_3 / L^2 .$$

  - At $\mu = \mu_c$ we have an *extremal, but non-BPS* black hole solution with a zero size horizon area (horizon is at $r = 0$) and $r = 0$ in this case is a null naked singularity.
  - As we increase μ from μ_c the solution develops a finite size horizon and the space-like singularity hides behind the horizon.

The causal structure of the $5d$ black holes, Cont'd

- **Two-charge black hole, *Cont'd*:**
  - As a $10d$ solution, the two-charge case at $\mu = 0$ corresponds to two sets of delocalized giant gravitons wrapping two S^3's inside S^5 while rotating on two different S^1 directions.
  - The worldvolume of the giants overlap on a circle.
  - If one of the charges is much smaller than the other one a better (perturbative) description of the system is in terms of a rotating single giant where as a result of the rotation the giant is deformed from the spherical shape.

The causal structure of the $5d$ black holes, Cont'd

- **Two-charge black hole, *Cont'd*:**
  - For the extremal case at $\mu = \mu_c$ we are dealing with intersecting giants which are generically far from being BPS and effectively we are dealing with a stack of giants with worldvolume $R \times S^1 \times \Sigma_2$, where Σ_2 is a compact $2d$ surface inside the S^5.
  - Turning on μ, especially when μ is small enough, corresponds to adding open string excitations while keeping the $U(1)$ symmetry of the giant intersection.
  - Out of extremality, measured by $\mu - \mu_c$, then corresponds to excitations/fluctuations above this stack of giants.

- **Three-charge black hole:**
  - For $0 \leq \mu < \mu_c$ we have a time-like naked singularity, the singularity is, however, behind $r = 0$ (one can extend the geometry past $r = 0$).
  - At some critical μ, $\mu = \mu_c$, we have an extremal solution with a finite size horizon (function f has double zeros at some $r_h \neq 0$).
  - For $\mu > \mu_c$ the geometry has two inner and outer horizons.

- **Three-charge black hole, *Cont'd*:**
  - From the $10d$ viewpoint the three-charge case corresponds to a set of three smeared giant gravitons intersecting only on the time direction and the giants in each set moving on either of the three S^1 directions in the S^5.
  - If one of the charges is much smaller than the other two a better description of the system is in terms of two giants intersecting on an S^1, but the third charge appears as a rotation on the S^1.

The near-horizon limit of the two-charge extremal solutions

- For the two charge case, with vanishing q_1:

$$f = \frac{r^2}{L^2} + f_0 - \frac{\mu - \mu_c}{r^2},$$

$$f_0 = 1 + \frac{q_2 + q_3}{L^2}, \qquad \mu_c = \frac{q_2 q_3}{L^2}.$$

- The horizon of the $5d$ black hole is where g^{rr} vanishes, or at the roots of $r^{4/3} f$.

- For $\mu = \mu_c$ we have a double zero at $r = 0$ and hence the solution is extremal. For $\mu < \mu_c$ f is positive definite and for $\mu > \mu_c$ f has a single positive root.

- Radius of the horizon S^3 in the $5d$ metric is $(H_2 H_3)^{\frac{1}{3}} r^2$, hence the extremal case has vanishing horizon area.

The near-horizon limit of the two-charge extremal solutions, Cont'd

- One can distinguish two extremal black holes (which have double horizons at $r = 0$)
  - The BPS case, with $\mu = 0$ and
  - The extremal but non-BPS case with $\mu = \mu_c$.
- Here we study the near-horizon near-BPS as well as near-horizon near-extremal but non-BPS limits of the two-charge $10d$ solutions separately and argue that these lead to decoupled geometries involving $AdS_3 \times S^3$ factors.

The near-horizon near-BPS limit

- $\mu_1 \sim 1$ *case*

$$\mu - \mu_c = \epsilon^2 M, \qquad q_i = \epsilon \hat{q}_i$$

$$\tau = \frac{t}{L}, \quad r = \frac{L}{(\hat{q}_2 \hat{q}_3)^{1/2}} \epsilon \rho, \qquad \mu_i = \epsilon^{1/2} x_i, \; i = 2, 3,$$

while $\epsilon \to 0$ and keeping $\hat{q}_i, \; M; \; \tau, \; \rho, \; x_i, \phi_i, \; L$ fixed. In this limit $\mu_1 = 1 + \mathcal{O}(\epsilon^2)$ or $\theta_1 \sim \epsilon^{1/2}, \theta_2 =$fixed.

- $\mu_1 \sim \mu_1^0 \neq 1$ *case*

$$\mu - \mu_c = \epsilon^2 M, \qquad q_i = \epsilon \hat{q}_i, \qquad \psi_i = \frac{1}{\epsilon^{1/2}} (\phi_i - \tau),$$

$$r = \frac{L}{(\hat{q}_2 \hat{q}_3)^{1/2}} \epsilon \rho, \qquad \theta_i = \theta_i^0 - \epsilon^{1/2} \hat{\theta}_i, \; 0 \leq \theta_i^0 \leq \pi/2, \; i = 2, 3$$

while $\epsilon \to 0$ and keeping $\tilde{\rho}, \; \hat{q}_i, \; M, \; \theta_i^0, \; x_i, \; L$ fixed.

The near-horizon near-BPS limit, Cont'd

- Taking the above limits we arrive at

$$ds^2 = \epsilon \left[R_S^2 \left(ds^2_{BTZ} + d\Omega_3^2 \right) + \frac{L^2}{R_S^2} ds^2_{\mathfrak{C}_4} \right]$$

where

$$ds^2_{BTZ} = -(\rho^2 - \gamma^2) d\tau^2 + \frac{d\rho^2}{\rho^2 - \gamma^2} + \rho^2 d\phi_1^2$$

with

$$\gamma^2 = \frac{\mu - \mu_c}{\mu_c} = \frac{M}{\hat{\mu}_c}, \qquad \hat{\mu}_c = \hat{q}_2 \hat{q}_3 / L^2$$

and the radius of the S^3 being

$$R_S^2 = \sqrt{\hat{q}_2 \hat{q}_3} \qquad \text{for} \qquad \mu \simeq 1$$

$$R_S^2 = \sqrt{\hat{q}_2 \hat{q}_3}\, \mu_1^0 \qquad \text{for} \qquad \mu \simeq \mu_1^0$$

The near-horizon near-BPS limit, Cont'd

- In either case $\mathcal{C}_4$ is (locally) describing a T^4 and hence the solutions are $AdS_3 \times S^3 \times T^4$. $ds^2_{\mathcal{C}_4}$ have different forms for the two cases:
  - $\mu_1 \sim 1$ *case*

$$ds^2_{\mathcal{C}_4} = \sum_{i=2,3} \hat{q}_i (dx_i^2 + x_i^2 d\psi_i^2)$$

where $\psi_i = \phi_i - \tau$.

  - $\mu_1 \sim \mu_1^0 \neq 1$ *case*

$$ds^2_{\mathcal{C}_4} = \sum_{i=2,3} \hat{q}_i (dx_i^2 + (\mu_i^0)^2 d\psi_i^2)$$

where $\mu_2^0 = \sin\theta_1^0 \cos\theta_2^0$, $\mu_3^0 = \sin\theta_1^0 \sin\theta_2^0$,
$dx_2 = \cos\theta_1^0 \cos\theta_2^0 d\hat{\theta}_1$, $dx_3 = \cos\theta_1^0 \sin\theta_2^0 d\hat{\theta}_1 + \cos\theta_2^0 \sin\theta_1^0 d\hat{\theta}_2$.

The near-horizon near-BPS limit, Cont'd

- For the metric

$$ds^2_{BTZ} = -(\rho^2 - \gamma^2)d\tau^2 + \frac{d\rho^2}{\rho^2 - \gamma^2} + \rho^2 d\phi_1^2$$

- $\gamma^2 = -1$ we have a global AdS_3 space,
- for $-1 < \gamma^2 < 0$ it is a conical space,
- for $\gamma^2 = 0$ we have a massless BTZ and
- for $\gamma^2 > 0$ we are dealing with a static BTZ black hole of mass γ^2.
- These geometries are, upon two T-dualities, related to standard the D1-D5 system and the corresponding arguments are applicable to this case.

The Near-horizon limit, the near-extremal, but non-BPS case

- Here we keep μ_c fixed, with the scalings

$$r = \sqrt{\frac{\mu_c}{f_0}}\,\epsilon\rho, \qquad t = \frac{L}{\sqrt{f_0}}\,\frac{\tau}{\epsilon}, \qquad \mu - \mu_c = \epsilon^2 M$$

$$\phi_1 = \frac{\varphi}{\epsilon}, \qquad \phi_i = \psi_i + \frac{\tilde{q}_i}{q_i L}\,\frac{\tilde{\tau}}{\epsilon}, \quad i = 2, 3$$

  and $\epsilon \to 0$ while $\rho,\ \tau,\ \varphi,\ \psi_i,\ M,\ q_i,\ L$ are kept fixed.

- In this limit q_i/L^2 and hence f_0, μ_c/L^2 are fixed.
- In this limit

$$f = f_0\left(1 - \frac{M}{\mu_c\,\rho^2}\right), \quad \Delta = \mu_1^2\,\frac{L^4 f_0^2}{q_2 q_3}\,\frac{1}{\rho^4}\cdot\frac{1}{\epsilon^4}, \quad H_i = \frac{L^2 f_0}{q_2 q_3}\,\frac{q_i}{\rho^2}\cdot\frac{1}{\epsilon^2}.$$

- Taking the limit we obtain

$$ds_{10}^2 = \mu_1 \left(R_{AdS_3}^2 \, ds_3^2 + R_S^2 \, d\Omega_3^2\right) + \frac{1}{\mu_1} ds_{\mathcal{M}_4}^2$$

where

$$ds_3^2 = -(\rho^2 - \rho_0^2) d\tau^2 + \frac{d\rho^2}{\rho^2 - \rho_0^2} + \rho^2 d\varphi^2,$$

- Note that $\varphi \in [0, 2\pi\epsilon]$.
- $d\Omega_3^2$ is the metric for a three-sphere of unit radius and

$$ds_{\mathcal{M}_4}^2 = \frac{L^2}{R_S^2}\left[q_2 \left(d\mu_2^2 + \mu_2^2 \, d\psi_2^2\right) + q_3 \left(d\mu_3^2 + \mu_3^2 \, d\psi_3^2\right)\right].$$

- In the above

$$R_S^2 \equiv \sqrt{q_2 q_3} = \sqrt{L^2 \mu_c}, \quad R_{AdS_3}^2 = \frac{R_S^2}{f_0}, \quad \rho_0^2 = \frac{M}{\mu_c}.$$

The Near-horizon limit, the near-extremal, but non-BPS case, Cont'

- The φ angle in the BTZ is coming from the part which was in the S^5 part of the original $AdS_5 \times S^5$,
- the rest of the six-dimensional part of metric comes from the original AdS_5 geometry;
- the $\mathcal{M}_4$ is coming from the S^5 piece.
- Although $\varphi \in [0, 2\pi\epsilon]$, the causal boundary of the near-horizon decoupled geometry is still $R \times S^1$, because at large, but fixed ρ the AdS_3 part of the metric takes the form

$$ds_3^2 \sim R^2_{AdS_3}\epsilon^2\rho^2(-dt^2 + d\phi_1^2) \ ,$$

t is the (global) time direction in the original AdS_5.

- As the $10d$ IIB solution, we have a constant dilaton field with the four-form

$$B_4 = -L^2 \,\left(\tilde{q}_2\, \mu_2^2\, d\psi_2 + \tilde{q}_3\, \mu_3^2\, d\psi_3\right) \wedge d^3\Omega_3,$$

where in the near-horizon, near-extremal limit

$$\tilde{q}_2^2 = q_2^2(1 + \frac{q_3}{L^2}), \qquad \tilde{q}_3^2 = q_3^2(1 + \frac{q_2}{L^2}).$$

- Note that even when $M = 0$, that is for $\mu = \mu_c$ the near-horizon geometry is not preserving any SUSY.

Addition of the third charge

- We discussed the near-horizon limits of the two-charge black holes, which lead to BTZ$\times S^3$ geometries.
- Here we are going to turn on the third charge q_1.
- Consider generic values for q_1. That is, take all three charges to be of the same order, for some critical value for μ, μ_c, we have an extremal (but non-BPS) black hole. In the near-horizon limit this extremal but non-BPS black hole goes over to $AdS_2 \times S^3$ geometry.
- What we are going to consider here is the non-generic case, when $q_1 \ll q_2,\ q_3$. That is *perturbative addition of the third charge*.

Perturbative Addition of the third charge, the near-BPS case

- Let us turn on the third charge q_1 and scale it as

$$q_1 = \epsilon^2 \hat{q}_1$$

while keeping $\hat{q}_1$ fixed, and scale the rest of parameters the same as before.

- After shifting the ρ coordinate as

$$\rho^2 \to \rho^2 - \frac{\hat{q}_1 \hat{q}_2 \hat{q}_3}{L^2}$$

- After the limit the metric takes the form

$$ds^2 = \epsilon \left[R_S^2 \; \left(ds^2_{rot.BTZ} + d\Omega_3^2 \right) + \frac{L^2}{R_S^2} ds^2_{\mathcal{C}_4} \right]$$

Perturbative Addition of the third charge, Cont'd

- where $R_S^4 = \hat{q}_2\hat{q}_3$ and $ds^2_{rot.BTZ}$ is the metric for a rotating BTZ black hole in the AdS_3 background of unit radius, with mass and angular momentum

$$M_{BTZ} = \frac{M + 2\hat{q}_1}{\hat{\mu}_c} = \frac{\hat{\mu} + 2\hat{q}_1}{\hat{\mu}_c} - 1, \qquad J_{BTZ} = 2\sqrt{\frac{\hat{q}_1(\hat{\mu} + \hat{q}_1)}{\hat{\mu}_c^2}}.$$

- Again there are two $\mu_1 \sim \mu_1^0 \neq 1$ and $\mu_1 \simeq 1$ cases. As in the previous case, for $\mu_1 \simeq \mu_1^0$, $R_S^4 = \hat{q}_2\hat{q}_3(\mu_1^0)^2$.

- The physical angular momentum of the original $10d$ black-brane (or electric charge of the $5d$ black hole) corresponding to q_1 charge, J_1, is related to J_{BTZ} as

$$J_1 = \frac{N^2\epsilon^2}{4}\,\frac{\hat{\mu}_c}{L^2} J_{BTZ}.$$

De tour to rotating BTZ black holes

- All stationary solutions to
$$R_{\mu\nu} = -\frac{2}{R^2} g_{\mu\nu},$$
which are *locally* AdS_3 space-times, are of the form
$$ds^2 = R^2 \left[-\frac{F(r)}{r^2} dt^2 + \frac{r^2}{F(r)} dr^2 + r^2 \left(d\phi + \frac{a_+^2 - a_-^2}{r^2} dt \right)^2 \right],$$
where $\phi \in [0, 2\pi]$ and
$$F(r) = r^4 + 2(a_+^2 + a_-^2) r^2 + (a_+^2 - a_-^2)^2.$$
- It is useful to introduce two other parameters
$$a_+^2 = -\frac{M+J}{4}, \qquad a_-^2 = -\frac{M-J}{4},$$
- We can always assume $a_+^2 \leq a_-^2$, *i.e.* $J \geq 0$ and $J \in \mathbb{Z}$. We are then left with three possibilities.

De tour to rotating BTZ black holes

- *Conical Singularity:* $a_+^2,\ a_-^2 > 0, \quad or \quad M < -J.$
  - $a_+ = a_- = 1/2$ corresponds to a *global* AdS_3.
  - For the generic case $a_+ = a_- = \gamma/2$, corresponding to $J = 0$, the conic space has the same line element as a global AdS_3 but now $\phi \in [0, 2\pi\gamma]$.
  - In string theory for rational values of γ and only when $\gamma < 1$ the conical singularity can be resolved.
  - For the general $a_+ \neq a_-$ case, the conical space can be resolved only when a_-^2 is a rational number and $0 \leq a_-^2 \leq 1/4$. In terms of M, J that is

$$-1 \leq M - J \equiv -\gamma^2 < -2J\ , \quad \gamma \in \mathbb{Q},\ J \in \mathbb{Z}.$$

De tour to rotating BTZ black holes

- $a_+^2 < 0,\ a_-^2 > 0$, corresponding to $-J < M < J$. The geometry is ill-defined and not sensible in string theory.
- *Rotating BTZ Black hole:* $a_+^2,\ a_-^2 \leq 0, \quad or \quad M \geq J \geq 0$
  - This rotating BTZ black hole of mass M and angular momentum J has temperature
$$T_{BTZ} = \frac{\sqrt{M^2 - J^2}}{2\pi\rho_h}, \qquad \rho_h = \frac{1}{2}\left(\sqrt{M+J} + \sqrt{M-J}\right).$$
  - Static BTZ: Special case of $a_- = a_+$ (*i.e.* $J = 0$).
  - extremal rotating BTZ: Special case of $a_- = 0$ ($M = J$), which has zero temperature.
  - Massless BTZ black hole: Very special case of $a_- = a_+ = 0$ ($M = J = 0$).

De tour to rotating BTZ black holes

- *To summarize the above, the cases with integer-valued J and when $M - J \geq -1$ are those which are sensible geometries in string theory. For the $-1 < M - J < 0$ resolution of conical singularity in string theory also demands $\sqrt{J - M}$ to be a rational number.*
- Among the above cases $M \leq -J$ for any M, J and $M = J,\ M \geq 0$ can be supersymmetrized.
  - For the $M \leq -J$ case, the conic spaces, the solution becomes supersymmetric in a $3d$ gauged supergravity which has at least two $U(1)$ gauge fields.

De tour to rotating BTZ black holes

- *Supersymmetry....*
  - To maintain supersymmetry we should turn on the Wilson lines of both of the $U(1)$ (flat-connection) gauge fields.
  - The two gauge fields which make the above metric supersymmetric are

$$A^{(1)} = a_+(dt + d\phi), \qquad A^{(2)} = a_-(dt - d\phi) \ ,$$

$A^{(1)},\ A^{(2)}$ are the flat connections of the two $U(1)$'s.

  - For $M = J,\ M \geq 0$, the extremal rotating BTZ black hole, no gauge fields are needed to keep supersymmetry.

De tour to rotating BTZ black holes

- *Among the supersymmetric configurations*
  - the *global* AdS_3, that is when $a_+ = a_- = 1/2$, keeps the maximum supersymmetry the $3d$ theory has, with *anti-periodic* boundary conditions for fermions on the ϕ direction.
  - The massless BTZ, that is when $a_+ = a_- = 0$, as well as the extremal BTZ, corresponding to $a_+^2 = a_-^2 > 0$, keep half of the maximal supersymmetry but with *periodic* boundary conditions for fermions on the ϕ direction.
  - The conical spaces also keep half of maximal supersymmetry.

Perturbative Addition of the third charge, Cont'd

- This metric is a rotating black hole only when $M_{BTZ} \geq J_{BTZ}$ (extremality bound) and also $\phi \in [0, 2\pi]$.
- In terms of our parameters the extremality bound is

$$M^2 \geq 4\hat{q}_1\hat{q}_2\hat{q}_3/L^2.$$

  Note that M can be positive or negative.
- The (Hawking) temperature of our rotating BTZ is

$$T_{BTZ} = \frac{\sqrt{M^2 - 4\hat{q}_1\hat{q}_2\hat{q}_3/L^4}}{\pi\sqrt{2\hat{\mu}_c\left(M + 2\hat{q}_1 + \sqrt{M^2 - 4\hat{q}_1\hat{q}_2\hat{q}_3/L^4}\right)}}$$

- For the special case of $M^2 = 4\hat{q}_1\hat{q}_2\hat{q}_3/L^2$ we have an extremal rotating BTZ black hole which has $T_{BTZ} = 0$.

Perturbative Addition of the third charge, Cont'd

- When $M_{BTZ} \leq -J_{BTZ} \leq 0$, we have a sensible conical singularity only if
$$M \leq -2\ Max(\hat{q}_1, \sqrt{\hat{q}_1\hat{q}_2\hat{q}_3/L^2}),$$
while $M + 2\hat{q}_1 \leq 0$ and if γ, $\gamma^2 \equiv J_{BTZ} - M_{BTZ}$, is a rational number.

- In sum, to have a sensible string theory description we should have
$$M_{BTZ} - J_{BTZ} + 1 \geq 0,$$
and if $0 \leq J_{BTZ} - M_{BTZ} \equiv \gamma^2 \leq 1$, γ should be rational.

Perturbative Addition of the third charge, the near-extremal case

- We may turn on the third charge q_1 "perturbatively", with the scaling
$$q_1 = \epsilon^4 \hat{q}_1 \ .$$
- After taking the above limit the metric takes the form
$$ds^2 = \mu_1 \left[R_{AdS}^2 \, ds^2_{rot.\ BTZ} + R_S^2 \, d\Omega_3^2\right] + \frac{1}{\mu_1} d\mathcal{M}_4^2$$
where $R_S^4 = q_2 q_3 , \ R_{AdS}^2 = R_S^2 / f_0$ and
$$ds^2_{rot.\ BTZ} = -N(\rho) d\tau^2 + \frac{d\rho^2}{N(\rho)} + \rho^2 (d\varphi - N_\varphi d\tau)^2$$
in which
$$N(\rho) = \rho^2 - M_{BTZ} + \frac{J_{BTZ}^2}{4\rho^2}, \qquad N_\varphi = \frac{J_{BTZ}}{2\rho^2} \ ,$$

- with

$$M_{BTZ} = \frac{M}{\mu_c}, \quad J_{BTZ} = 2\sqrt{\frac{f_0 \hat{q}_1}{\mu_c}}, \ \ \mu_c = q_2 q_3 / L^2, \quad f_0 = 1 + \frac{q_2 + q_3}{L^2} \ .$$

- Note as in the two-charge case, in the above rotating BTZ the angular coordinate $\varphi \in [0, 2\pi\epsilon]$.

- The above geometry has the interpretation of rotating BTZ only when the extremality bound is satisfied

$$M^2 \geq 4 \mu_c f_0 \hat{q}_1 \, .$$

- The horizon radius, where $N(\rho)$ vanishes, is

$$\rho_h = \frac{1}{2}\left(\sqrt{M_{BTZ} + J_{BTZ}} + \sqrt{M_{BTZ} - J_{BTZ}}\right) \, .$$

The Near-horizon Geometries as solutions to $6d$ SUGRAs

- *Questions:*
  - Are the $AdS_3 \times S^3$ geometries solutions to some six-dimensional (super) gravities?
  - Is there a consistent reduction of 10 IIB theory leading to these possible $6d$ (supergravity) theories?
  - If yes, Do these $AdS_3 \times S^3$ near-horizon limit of a $6d$ *black string* solution?

The Near-horizon Geometries as solutions to $6d$ SUGRAs, Cont'd

- *Answers:*
  - As we will see the answer to first question is affirmative and we present the corresponding $6d$ gravity theories.
  - We also give the consistent reduction relating these $6d$ theories to the $10d$ IIB.
  - As for the last question, for the near-BPS case the answer is affirmative, but for the near-extremal it is yet under construction.

The $6d$ SUGRA corresponding to the near-BPS geometry

- It is readily seen that the $AdS_3 \times S^3$ coming as near-horizon limit of the $10d$ near-BPS solution, *which has equal AdS_3 and S^3 radii* is a solution to

$$S = \frac{1}{16\pi G_N^{(6)}} \int d^6x \sqrt{-g_{(6)}} \left[R_{(6)} - (\partial\Phi)^2 - \frac{1}{3} e^{2\Phi} F_{\mu\nu\rho} F^{\mu\nu\rho} \right],$$

- The three-form $F_{\mu\nu\rho} = (dB_2)_{\mu\nu\rho}$. The two-form is not self-dual.
- The above action is made into a consistent $6d$ $\mathcal{N} = (1,1)$ SUGRA if besides the metric, two-form B_2 and the scalar Φ we also add *two* $U(1)$ gauge fields.

The $6d$ SUGRA corresponding to the near-BPS geometry, Cont'd

- The two $U(1)$ fields are not gauged, *i.e.* it is not a *gauged SUGRA*.
- The action for these gauge fields are
$$S_{gauge} = \int e^{2\Phi}(F^1_{\mu\nu})^2 + e^{-2\Phi}(F^2_{\mu\nu})^2 .$$
- It is evident that the above $6d$ theory can be obtained from the reduction of $10d$ IIB theory on T^4, or $\mathcal{C}_4$.
- The $AdS_3 \times S^3$ is a solution to this $6d$ theory with vanishing gauge fields, constant Φ and q_2 units of electric and q_3 units of magnetic three-form flux over the S^3.

The $6d$ SUGRA corresponding to the near-BPS geometry, Cont'd

- The $AdS_3 \times S^3$ also appears in the near-horizon over near-BPS black string, which is a *marginal bound state* of q_2 electric and q_3 magnetic strings.
- This $6d$ strings, both of the electrically and magnetically charged ones, are $10d$ three-brane giants wrapping two different two-cycles on $\mathcal{C}_4$.
- The tension of the $6d$ string, the electric or magnetic ones both, is

$$T_s^{(6)}|_{Near\ BPS} = \pi\epsilon L^2 \cdot T_3 = \frac{N\epsilon}{2\pi L^2}\ .$$

The $6d$ SUGRA corresponding to the near-BPS geometry, Cont'd

- The $6d$ Newton constant is then

$$G_N^{(6)} = \frac{G_N^{(10)}}{Vol_{\mathfrak{C}_4}}, \qquad Vol_{\mathfrak{C}_4} = \begin{cases} (2\pi)^2 L^4 \mu_2^0 \mu_3^0 \, \epsilon^2 & \mu_1 \sim \mu_1^0 \\ (2\pi)^2 L^4 \, \epsilon^2 & \mu_1 \sim 1 \end{cases}$$

- Recalling that

$$G_N^{(10)} = 8\pi^6 g_s^2 l_s^8, \qquad L^4 = 4\pi g_s N l_s^4$$

$$G_N^{(6)} = \frac{\pi^2}{8} \cdot \frac{L^4}{N^2 \epsilon^2} \frac{1}{\mu_2^0 \mu_3^0}.$$

- Note that to obtain the above for the $\mu_1 \sim \mu_1^0$, we have scaled the $6d$ metric by a factor of $\epsilon \mu_1^0$ so that, $R_S^2 = \sqrt{\hat{q}_2 \hat{q}_3}$ for both the $\mu_1^0 = 1$, and $\mu_1^0 \neq 1$ cases .

The $6d$ SUGRA corresponding to the near-extremal geometry

- One can check that that the $AdS_3 \times S^3$ coming as near-horizon limit of the $10d$ near-extremal solution, *which has unequal AdS_3 and S^3 radii* is a solution to

$$S = \frac{1}{16\pi G_N^{(6)}} \int d^6x \sqrt{-g_{(6)}} \left[R_{(6)} - (\partial\Phi)^2 + \frac{8}{L^2} \cosh\Phi - \frac{1}{3} e^{2\Phi} (F_3)^2 \right]$$

- The three-form $F_3 = dB_2$. The two-form is not self-dual.
- Difference of this action with the previous one is in the potential term for scalar Φ.

The $6d$ SUGRA corresponding to the near-extremal geometry, Cont'

- The $AdS_3 \times S^3$ is a solution to this $6d$ theory constant Φ and $\tilde{q}_2$ units of electric and $\tilde{q}_3$ units of magnetic three-form flux over the S^3.
- The value of constant Φ is completely determined in terms of the charges $\tilde{q}_2,\ \tilde{q}_3$.
- The above $6d$ action can be obtained from *consistent reduction* of IIB theory with the metric reduction ansatz

$$ds^2_{10} = \mu_1\ g^{(6)}_{\mu\nu} dx^\mu dx^\nu + \frac{1}{\mu_1} ds^2_{\mathcal{M}_4}$$

where

$$ds^2_{\mathcal{M}_4} = \frac{L^2}{R_S^2}\left[e^{\Phi}(d\mu_2^2 + \mu_2^2 d\psi_2^2) + e^{-\Phi}(d\mu_3^2 + \mu_3^2 d\psi_3^2)\right].$$

The $6d$ SUGRA corresponding to the near-extremal geometry, Cont'

- The two-form B_2 is coming from the reduction of the self-dual five-form:

$$F_5 = \frac{1}{3!} F_{3\,\mu\nu\rho}\, d\mu_2^2 \wedge d\chi_2 \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho$$
$$+ \frac{1}{3!} e^{2\Phi} (*F_3)_{\mu\nu\rho}\, d\mu_3^2 \wedge d\chi_3 \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho,$$

- The five-form equation of motion, $dF_5 = 0$ implies the equations of motion for the three-form:

$$dF_3 = 0, \qquad d\left(e^{2\Phi} * F_3\right) = 0.$$

- The $6d$ Newton constant is then

$$G_N^{(6)} = \frac{G^{(10)}}{\frac{\pi^2}{2} L^4} = \frac{\pi^2 L^4}{N^2}.$$

The $6d$ SUGRA corresponding to the near-extremal geometry, Cont'

- Unlike the ungauged $6d$ SUGRA, electric and magnetic string solutions to this $6d$ gravity are not mutually BPS.
- The electrically and magnetically charged $6d$ strings are both three-brane giants which are wrapping different two-cycles on $\mathcal{M}_4$.
- The tension of the $6d$ strings are

$$T_s^{(6)} = T_3(\pi L^2) = \frac{N}{2\pi L^2} = \frac{1}{2\sqrt{G_N^{(6)}}}.$$

- These strings form a (p,q)-string type *bound states*. The mass of the bound state is the square root of the sum of the squares of mass of individual electric or magnetic strings.

The Black Hole entropy Analyses

To argue that our near-horizon limits are indeed decoupling limits we first compute the Bekenstein-Hawking entropy of the original $5d$ black holes and compare it with the entropy of the $3d$ (or $6d$) black holes.

As we will show these entropies match for both of the near-BPS and near-extremal cases. This matching is a strong evidence in support of the fact that in our decoupling limits we have not lost any degrees of freedom.

The Black Hole entropy, the $5d$ Analysis

- The $5d$ Bekenstein-Hawking entropy is

$$S_{BH} = \frac{A_h^{(5)}}{4G_N^{(5)}} \,.$$

where

$$A_h^{(5)} = 2\pi^2 r_h^3 (H_1 H_2 H_3)^{1/2}|_{r=r_h} \,.$$

- Recalling that

$$G_N^{(10)} = 8\pi^6 g_s^2 l_s^8, \quad G_N^{(5)} = \frac{G_N^{(10)}}{\pi^3 L^5}, \quad L^4 = 4\pi g_s N l_s^4 \,,$$

- we obtain

$$S_{BH} = \frac{1}{2\pi} N^2 \cdot \frac{A_h^{(5)}}{L^3} .$$

The $5d$ black hole entropy analysis, the near-BPS case

- In the near-BPS limit the horizon is located at

$$r_h^2 = \mu - \mu_c$$

and hence

$$S_{BH}^{Near\ BPS} = \pi\gamma \frac{\hat{\mu}_c}{L^2}\ N^2 \epsilon^2\ ,$$

where

$$\gamma^2 = \frac{\mu - \mu_c}{\mu_c}, \qquad \hat{\mu}_c = \mu_c/\epsilon^2$$

- Once the third charge is also added perturbatively, the above is replaced with

$$S_{BH}^{Near\ BPS} = \pi \frac{\hat{\mu}_c}{L^2}\ \rho_h\ N^2 \epsilon^2\ ,$$

where

$$\rho_h^2 = \frac{1}{2\hat{\mu}_c}\left(M + 2\hat{q}_1 + \sqrt{M^2 - 4\hat{q}_1\hat{q}_2\hat{q}_3/L^4}\right).$$

The $5d$ black hole entropy analysis, the near-BPS case

- The validity of classical gravity analysis demands that
  - All curvature components should remain small in string units l_s and
  - the entropy, should be large:

$$S_{BH} \gg 1$$

- All curvature components scale as $1/\epsilon$ (in units of L^{-2}).
- The large entropy condition implies that together with $\epsilon \to 0$, $N \to \infty$, *e.g.* as $N \sim \epsilon^{-\alpha}, \ \alpha \geq 2$.
- This consideration is not strong enough to fix α.

The $5d$ black hole entropy analysis, the near-BPS case

- Noting the form of metric, that it has a factor of ϵ in front and that one expects the string scale to be the shortest physical length leads to

$$\epsilon \sim l_s^2 \quad \Rightarrow \quad N \sim \epsilon^{-2} \ .$$

- Once the above scaling of ϵ and N is considered,

$$S_{BH} \sim N \sim \epsilon^{-2} \to \infty .$$

- In sum, our complete near-horizon, near-BPS limit is defined as an $\alpha' = l_s^2 \sim \epsilon \to 0$ limit together with scaling $q_2,\ q_3 \sim \epsilon;\ \ q_1,\ \mu \sim \epsilon^2$, while keeping $L^4 \sim N l_s^4$ fixed.

The $5d$ black hole entropy analysis, the near-extremal case

- In the near-extremal limit to order ϵ, we have

$$r_h^2 = \frac{\mu - \mu_c}{f_0} + \mathcal{O}(\epsilon^4).$$

- Therefore

$$S_{BH}^{Near\ Extremal} = \pi \frac{\mu_c}{L^2} \cdot \frac{\rho_0}{\sqrt{f_0}} N^2 \epsilon.$$

- With the perturbative addition of the third charge

$$S_{BH} = \pi \rho_h \frac{1}{\sqrt{f_0}} \frac{\mu_c}{L^2} N^2 \epsilon\ ,$$

where

$$\rho_h = \frac{1}{2}\left(\sqrt{M_{BTZ} + J_{BTZ}} + \sqrt{M_{BTZ} - J_{BTZ}}\right)$$

$$M_{BTZ} = \frac{M}{\mu_c}, \qquad J_{BTZ} = 2\sqrt{\frac{f_0 \hat{q}_1}{\mu_c}}$$

The $5d$ black hole entropy analysis, the near-extremal case

- To ensure the validity of the classical gravity analysis, one should also send $N \to \infty$ while keeping ρ_0 and μ_c/L^2 finite. This is done if we scale $N \sim \epsilon^{-\beta},\ \beta \geq \frac{1}{2}$.
- The validity considerations does not fix β. As we will show, however, $\beta = 1$ is giving the appropriate choice,

$$N \sim \epsilon^{-1} \to \infty \ .$$

- In sum, we keep L, g_s, q_i/L^2 and ρ_0 finite while taking

$$l_s^4 \sim N^{-1} \sim \epsilon \to 0.$$

- In this case, as in the near-BPS case,

$$S_{BH} \sim N \to \infty.$$

The Black Hole Entropy, the $3d$ Analysis

- The rotating BTZ$\times S^3$ obtained in the near-horizon limit is also a solutions to $6d$ (super)gravity theory.
- One can further reduce this $6d$ theory on the S^3 to obtain a $3d$ gravity theory.
- The rotating BTZ solution is then a black hole solution to this $3d$ theory.
- What we are going to do here is to compute the BH entropy of this $3d$ black holes, which is obtained from

$$S_{BH}^{(3)} = \frac{A^{(3)}}{4G_N^{(3)}}$$

where $A^{(3)}$ is the area of horizon for the BTZ black hole.

The $3d$ black hole entropy analysis, the near-BPS case

- The $3d$ Newton constant is related to the $6d$ one as

$$G_N^{(3)} = \frac{G_N^{(6)}}{2\pi^2 R_S^3} = \frac{L^4}{16 R_S^3} \cdot \frac{1}{N^2\epsilon^2} \, \frac{1}{\mu_2^0 \mu_3^0} .$$

- The $3d$ entropy for any value of μ_2^0 and μ_3^0 is hence

$$s_{BH}^{(3)} = 8\pi \frac{\hat{\mu}_c}{L^2} \, \rho_h \, N^2 \epsilon^2 \, \mu_2^0 \mu_3^0 ,$$

with the ρ_h taking the same value as in the $5d$ case.

- The total entropy to be compared against the $5d$ entropy is integral of $s_{BH}^{(3)}$ over values of $\mu_2^0,\ \mu_3^0$, yielding

$$S_{BH}^{(3)} = \pi \frac{\hat{\mu}_c}{L^2} \, \rho_h \, N^2 \epsilon^2 \ ,$$

- This exactly matches the the entropy of the $5d$ black hole after taking the near-BPS decoupling limit.

The $3d$ black hole entropy analysis, the near-extremal case

- For the near-extremal case that is

$$A^{(3)} = 2\pi\epsilon R_{AdS_3}\ \rho_0 .$$

The $2\pi\epsilon$ comes from the fact that $\varphi \in [0, 2\pi\epsilon]$.

- The $3d$ Newton constant is

$$G_N^{(3)} = \frac{G_N^{(6)}}{2\pi^2 R_S^3} = \frac{L^4}{2R_S^3} \cdot \frac{1}{N^2} .$$

- Therefore,

$$S_{BH}^{(3)} = \pi \frac{R_{AdS} R_S^3}{L^4}\ \rho_0 N^2 \epsilon\ ,$$

- The above is the same as the $5d$ black hole entropy in the near-horizon near-extremal limit, recalling

$$R_{AdS} = R_S/\sqrt{f_0}\ , \qquad \mu_c = R_S^4/L^2 .$$

Dual Field Theory Descriptions

- So far we have shown that one can take specific near-horizon, near-extremal limits over $10d$ type IIB solutions which are asymptotically AdS_5.
- As such one would expect that these solutions, the limiting procedure and the resulting geometry after the limit should have a dual description via AdS_5/CFT_4.
- On the other hand, after the limit we obtain a space which contains $AdS_3 \times S^3$,
- and hence there should also be another dual description in terms of a $2d$ CFT.

Dual Field Theory Descriptions

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Dual Field Theory Descriptions, the $4d$ SYM

- Here we translate what taking the near-horizon limits on the gravity backgrounds corresponds to in the $\mathcal{N} = 4$, $d = 4$ $U(N)$ SYM theory.
- We argue that taking the near-horizon near-BPS and near-extremal limits correspond to focusing on specific sectors in the $\mathcal{N} = 4$ SYM which we identify.
- We argue that the decoupling in the gravity corresponds to the fact that these sectors are closed under SYM dynamics.
- The idea here is somewhat like that of BMN and almost-BPS operators there....

Dual Field Theory Descriptions, the $4d$ SYM

- The operators of $\mathcal{N}=4,\ d=4\ U(N)$ SYM theory are specified by their $SO(4,2)\times SO(6)$ quantum numbers.
- The scaling dimension of operators Δ and their R-charge J_i respectively correspond to the ADM mass and angular momentum of the objects in the gravity.
- Explicitly, for the two-charge case of our interest, with the perturbative addition of the third charge, the operators are specified by four quantum numbers

$$\Delta = L\cdot M_{ADM} = \frac{N^2}{2L^2}\,(\frac{3}{2}\mu + q_1 + q_2 + q_3)\ ,$$

$$J_i = \frac{\pi L}{4G_5}\tilde{q}_i = \frac{N^2}{2}\,\frac{\tilde{q}_i}{L^2}\ ,$$

- and are singlets of $SO(4) \in SO(4,2)$.
- If μ and q_i are finite, Δ and J_i scale as N^2.
- In both of the near-BPS and near-extremal limits we are taking the 't Hooft coupling, $\lambda = L^4/l_s^4$ to infinity.
- Despite of the large 't Hooft coupling, we may have a perturbative description.
- Recall the BMN case, where the effective expansion parameters of the $4d$ gauge theory is different in sectors of large R-charges and we have finite effective (or "dressed") 't Hooft coupling and the genus expansion parameter.

The $4d$ $\mathcal{N} = 4$ SYM description, the near-BPS case

- In the near-BPS limit case together with some of the coordinates we also scale μ and q_i as ϵ.
- Moreover, we need to also scale $N \sim \epsilon^{-2}$.
- Therefore, the sector of the $\mathcal{N} = 4\, U(N)$ SYM operators corresponding to the geometries in question have large scaling dimension and R-charge

$$\Delta = \frac{N^2 \epsilon}{2} \left(\hat{q}_2 + \hat{q}_3 + \mathcal{O}(\epsilon)\right)/L^2 \sim N^{3/2} \to \infty$$

$$J_i = \frac{N^2 \epsilon}{2} \left(\hat{q}_i + \mathcal{O}(\epsilon)\right)/L^2 \sim N^{3/2} \,.$$

The $4d$ $\mathcal{N} = 4$ SYM description, the near-BPS case

- In the same spirit as the BMN limit, one can find certain combinations of Δ and J_i which are finite and describe physics of the operators after the limit.
- In order that recall the way the limit was taken:

$$iL\frac{\partial}{\partial\tau} = iL\frac{\partial}{\partial t} + i\sum_{i=2,3}\frac{\partial}{\partial\phi_i} = \Delta - \sum_{i=2,3} J_i$$

$$-i\frac{\partial}{\partial\psi_i} = -i\frac{\partial}{\partial\phi_i} = J_i$$

- Up to leading order we have

$$\Delta - \sum_{i=2,3} J_i = \frac{N^2\epsilon^2}{4}\frac{\hat{\mu}}{L^2}\,, \qquad J_i = \frac{N^2\epsilon}{2}\frac{\hat{q}_i}{L^2}\,.$$

The $4d$ $\mathcal{N}=4$ SYM description, the near-BPS case

- $\Delta - \sum J_i \sim N^2 \cdot N^{-1} = N \to \infty$, while $J_i \sim N^{3/2}$.
- The “BPS deviation parameter”:
$$\eta_i \equiv \frac{\Delta - \sum_i J_i}{J_i} \sim \epsilon \sim N^{-1/2} \to 0 \ ,$$
and hence we are dealing with an “almost-BPS” sector.
- It is instructive to make parallels with the BMN sector, where we deal with operators with
$$\Delta \sim J \sim N^{1/2}, \qquad \text{while} \qquad \Delta - J \ = finite,$$
implying that, similarly to our case, $\eta_{BMN} \sim N^{-1/2} \to 0$.
- Note that, $\Delta - \sum J_i$ is linearly proportional to non-extremality parameter $\hat{\mu}$ and $S_{BH} \sim \Delta - \sum J_i \sim N$.

The $4d$ $\mathcal{N} = 4$ SYM description, the near-BPS case

- In sum, the sector we are dealing with is composed of "almost 1/4 BPS" operators of $U(N)$ SYM with

$$\Delta \sim J_i \sim N^{3/2}, \qquad \lambda = g_{YM}^2 N \sim N \to \infty$$

$$\frac{J_i}{N^{3/2}} \equiv \frac{\hat{q}_i}{L^2} = fixed, \qquad (\Delta - \sum_{i=2,3} J_i) \cdot \frac{1}{N} = \frac{\hat{\mu}}{L^2} = fixed.$$

- The dimensionless physical quantities that describe this sector are therefore $\hat{q}_i/L^2$, $\hat{\mu}/L^2$ and g_{YM}.
- To completely specify the sector, the *basis* used to contract $N \times N$ gauge indices should also be specified. This could be done by giving the (approximate) shape of the corresponding Young tableaux.

- To this end we recall the interpretation of the original $10d$ geometry in terms of the back-reaction of the intersecting giant gravitons and that giant gravitons and their open string fluctuations are described by (sub)determinant operators.
- Here we are dealing with a system of intersecting multi giants. The "number of giants" in each stack in the near-BPS, near-horizon limit is

$$N_i = N\epsilon \cdot \frac{\hat{q}_i}{L^2} = 2N^{1/2}\,\frac{\hat{q}_i}{L^2}\ ,$$

- Therefore, $\Delta - \sum_i J_i = \frac{N_2 N_3}{4}\frac{\hat{\mu}}{\hat{\mu}_c}$.

The $4d$ $\mathcal{N}=4$ SYM description, the near-BPS case

- Finally, let us consider addition of the third charge, where besides $J_2, \ J_3$ we have also turned on J_1,
$$J_1 = \frac{N^2\epsilon^2}{2} \cdot \frac{1}{L^2}\sqrt{\hat{q}_1(\hat{q}_1 + \hat{\mu})} \ .$$
- As we see $\Delta - \sum_{i=2,3} J_i \sim J_1 \sim N^2\epsilon^2 \sim N \to \infty$.
- In this case instead of $\Delta - \sum_{i=2,3} J_i$ it is more appropriate to define another positive definite quantity:
$$\Delta - \sum_{i=1}^{3} J_i = N \cdot \left(\frac{\hat{\mu} + 2\hat{q}_1 - \sqrt{(\hat{\mu} + 2\hat{q}_1)^2 - \hat{\mu}^2}}{L^2} \right) \ \geq 0 \ .$$

The $4d$ $\mathcal{N} = 4$ SYM description, the near-BPS case

- It is remarkable that the above BPS bound is exactly the same as the bound in which the generic rotating BTZ metric could be made sense of.
- This bound is more general than just the extremality bound of the rotating BTZ black hole $M_{BTZ} - J_{BTZ} \geq 0$.
- This bound besides the rotating black hole cases also includes the case in which we have a conical singularity which could be resolved in string theory.

End of the near-BPS case

The $4d$ $\mathcal{N}=4$ SYM description, the near-extremal case

- In the near-horizon, near-extremal limit we do not scale μ and q_i's. Therefore, we deal with a sector of $\mathcal{N}=4$ SYM in which $\Delta \sim J_i \sim N^2$ and, as noted $N \sim \epsilon^{-1}$.
- To deduce the correct "BMN-type" combination of Δ and J_i , we recall the way the limit has been taken:

$$\tau = \epsilon \frac{R_S}{R_{AdS_3}} \frac{t}{L}, \qquad \phi_i = \psi_i + \frac{\tilde{q}_i R_{AdS_3}}{q_i R_S} \frac{\tau}{\epsilon}, \ i = 2, 3 \ .$$

- Therefore, $-i\frac{\partial}{\partial \psi_i} = -i\frac{\partial}{\partial \phi_i} = J_i$ and

$$\mathcal{E} \equiv -i\frac{\partial}{\partial \tau} = -\frac{R_{AdS_3}}{\epsilon\, R_S}\left(iL\frac{\partial}{\partial t} + i\sum_{i=2,3} \frac{\tilde{q}_i}{q_i}\frac{\partial}{\partial \phi_i}\right)$$

$$= -\frac{R_{AdS_3}}{\epsilon\, R_S}\left(\Delta - \frac{2L^2}{N^2}\sum_{i=2,3}\frac{J_i^2}{q_i}\right)$$

The $4d\ \mathcal{N}=4$ *SYM description, the near-extremal case*

- Intuitive way of understanding $\mathcal{E}$:
  - In the near-extremal case we deal with massive giant gravitons which are far from being BPS
  - and hence are behaving like *non-relativistic* objects
  - which are rotating with angular momentum J_i over circles with radii R_i, $R_i^2 = \frac{L^2}{R_S^2} q_i$.
  - Therefore, the kinetic energy of this rotating branes is proportional to $\sum J_i^2/q_i$.
- In our limit $\epsilon \sim 1/N$ which for convenience we choose

$$\epsilon = \frac{4}{N}.$$

The $4d$ $\mathcal{N}=4$ SYM description, the near-extremal case

- Recalling that Δ is measuring the "total" energy of the system, n $\mathcal{E}$ should have two parts:
  - the rest mass of the system of giants and
  - the energy of "internal" excitations of the branes.
- To see this explicitly we note that

$$\mathcal{E} = \frac{R_{AdS_3}}{R_S} \cdot \frac{N^2}{4\epsilon} \cdot \frac{\mu}{L^2} = \mathcal{E}_0 + \frac{R_{AdS_3}}{R_S} \cdot (2\pi T_s^{(6)} M)$$

where have used $\mu = \mu_c + \epsilon^2 M$ (M is related to the mass of BTZ black hole), and

$$\mathcal{E}_0 = \frac{R_{AdS_3} R_S^3}{16 L^4} \cdot N^3.$$

The $4d$ $\mathcal{N}=4$ SYM description, the near-extremal case

- $\mathcal{E}_0$ which is basically $\mathcal{E}$ evaluated at $\mu = \mu_c$, is the rest mass of the brane system.
- $\mathcal{E} - \mathcal{E}_0$ corresponds to the fluctuations of the giants about the extremal point.
- $\mathcal{E} - \mathcal{E}_0$ is proportional to $T_s^{(6)} M$, indicating that it can be recognized as fluctuations of a $6d$ string.
- Recall also that from the $10d$ viewpoint, the $6d$ strings are uplifted to three-brane giants with two legs along the $\mathcal{M}_4$ directions.
- Therefore, $\mathcal{E} - \mathcal{E}_0$ corresponds to (three) brane-type fluctuations of the original “intersecting giants”.

De tour, The $4d$ $\mathcal{N}=4$ SYM description, the near-extremal case

- At the extremal point the system is not BPS and the "rest mass" of the giants system is not simply sum of the masses of individual stacks of giants and contains their "binding energy" (stored in the deformation of the giant shape from the spherical shape).
- Nonetheless, it should still be proportional to the number of giants times mass of a single giant.
- In the $6d$ language, as suggested previously, this corresponds to formation of a $6d$ (Q_e, Q_m)-string.

De tour, The $4d$ $\mathcal{N}=4$ SYM description, the near-extremal case

- Inspired by the expression for the $10d$ five-form flux and recalling that the IIB five-form is self-dual, the system of giants we start with, may also be interpreted as spherical three-branes wrapping $S^3 \in AdS_5$ while rotating on S^5, the dual giants.
- In terms of dual giants, after the limit, we are dealing with a system of dual giants wrapping $S^3 \in AdS_3 \times S^3$ which has radius R_S.

De tour, The $4d$ $\mathcal{N}=4$ SYM description, the near-extremal case

- The mass of a single such dual giant m_0 (as measured in R_{AdS_3} units and also noting the scaling of AdS_5 time with respect to AdS_3 time) is then

$$\frac{m_0}{R_{AdS_3}/\epsilon} = T_3(2\pi^2 R_S^3) = \frac{R_S^3}{L^4} \cdot N.$$

- The number of dual giants is again proportional to N and hence one expects the total "rest mass" of the system m_0 to be proportional to $N^3 R_S^3$.

End of De Tour to Dual Giants and their mass.

The $4d$ $\mathcal{N} = 4$ SYM description, the near-extremal case

- In sum, from the $U(N)$ SYM theory viewpoint the sector describing the near-extremal, near-horizon limit consists of operators specified with

$$\Delta \sim J_i \sim N^2, \qquad \lambda \sim N \to \infty,$$

$$\frac{J_i}{N^2} \equiv \frac{\tilde{q}_i}{2L^2} = fixed, \qquad \frac{\mathcal{E} - \mathcal{E}_0}{N} = fixed\ ,$$

where as discussed, $\mathcal{E}$, $\mathcal{E}_0$ are defined in terms of Δ, J_i.

The $4d$ $\mathcal{N}=4$ SYM description, the near-extremal case

- As discussed, one may obtain a rotating BTZ if we turn on the third R-charge in a perturbative manner.
- In the $4d$ gauge theory language this is considering the operators which besides the above $\mathcal{E}-\mathcal{E}_0$ and J_i carry the third R-charge J_1, $J_1 \sim N^2\epsilon^2 \sim 1$:

$$J_1 = \frac{N^2}{2L^2}\epsilon^2 \sqrt{\hat{q}_1 \mu_c}$$

- In terms of the AdS_3 parameters, since $\varphi = \epsilon\phi$, then

$$\mathcal{J} \equiv -i\frac{\partial}{\partial\varphi} = -i\frac{1}{\epsilon}\frac{\partial}{\partial\phi} = \frac{J_1}{\epsilon} = \frac{N^2\epsilon}{2}\,\frac{\mu_c}{L^2}\sqrt{\frac{\hat{q}_1}{\mu_c}}$$

The $4d$ $\mathcal{N} = 4$ SYM description, the near-extremal case

- As we see $\mathcal{J}$, similarly to $\mathcal{E} - \mathcal{E}_0$, is also scaling like $N^2\epsilon \sim N$ in our decoupling limit.
- When J_1 is turned on the expressions for Δ and hence $\mathcal{E}$ are modified, receiving contributions from q_1. These corrections, recalling that q_1 scales as ϵ^4, vanish in the leading order.
- However, one may still define physically interesting combinations like $\mathcal{E} - \mathcal{E}_0 \pm \mathcal{J}$.

End of the $4d$ SYM descriptions

Description in terms of $1+1$ *dim. dual theory*

- In either of the near-BPS or near-extremal near-horizon limits we obtain a space-time which has an $AdS_3 \times S^3$ factor.
- In both cases the AdS_3 factor is in global coordinates.
- This, within the AdS/CFT ideology, is suggesting that (type IIB) string theory on the corresponding geometries should have a dual $1+1$ CFT description.

Description in terms of $2d$ dual theory, the near-BPS case

- In the near-BPS case metric takes the same form as the near-horizon limit of a D1-D5 system, though the AdS_3 is obtained to be in *global* coordinates.
- This could be understood noting that the two-charge geometry corresponds to a system of *smeared* giant D3-branes intersecting on a circle.
- In the near-horizon limit we take the radius of the giants to be very large (or equivalently focus on a very small region on the worldvolume of the spherical brane) while keeping the radius of the intersection circle to be finite (in string units).

Description in terms of $2d$ dual theory, the near-BPS case

- Therefore, upon two T-dualities on the D3-branes along the $\mathcal{C}_4$ directions the system goes over to a D1-D5 system but now the D1 and D5 are lying on the circle (D5 has its other four directions along $\mathcal{C}_4$).
- Here we give the dictionary from our conventions and notations to that of the usual D1-D5 system, and discuss the similarities and difference.
- Number of D-strings Q_1 and number of D5-branes Q_5 are respectively equal to the number of giants in each stack N_2 and N_3.

Description in terms of $2d$ dual theory, the near-BPS case

- The degrees of freedom are coming from four DN modes of open strings stretched between intersecting giants which are in $(N_2, \bar{N}_3)$ representation of $U(N_2) \times U(N_3)$.
- In taking the near-horizon, near-BPS limit we are focusing on a narrow strip in μ_2, μ_3 directions and hence our BTZ$\times S^3 \times \mathcal{C}_4$ geometry and in this sense the corresponding $2d$ CFT description is only describing the narrow strips on the original $5d$ black hole.

Description in terms of $2d$ dual theory, the near-BPS case

- Therefore, our $5d$ black hole is described in terms of not a single $2d$ CFT, but a collection of (infinitely many of) them. The *only* property which is different among these $2d$ CFT's is their central charge.
- The "metric" on the space of these $2d$ CFT's is exactly the same as the metric on $\mathcal{C}_4$.
- As far as the entropy and the overall (total) number of degrees of freedom are concerned, one can define an *effective central charge* of the theory which is the integral over the central charge of the theory corresponding to each strip.

Description in terms of $2d$ dual theory, the near-BPS case

- For the central charge we use the Brown-Henneaux central charge formula,

$$c = \frac{3\, R_{AdS}}{2\, G_N^{(3)}}$$

and recall that for each strip

$$R_{AdS} = R_S, \qquad G_N^{(3)} = \frac{L^4}{16 R_S^3} \cdot \frac{1}{N^2 \epsilon^2}\, \mu_2^0 \mu_3^0$$

- The *effective* total central charge is obtained by integrating strip-wise c over the $\mathcal{C}_4$.
- Noting that

$$\int_{\mu_2^2+\mu_3^2 \leq 1} \mu_2 \mu_3 d\mu_2 d\mu_3 = \frac{1}{8},$$

Description in terms of $2d$ dual theory, the near-BPS case

- The *effective* central charge of the system is

$$c_L = c_R = c = 3N_2N_3 = 12N \cdot \frac{\hat{\mu}_c}{L^2}.$$

- Compare this with the central charge of the usual D1-D5 system is given by $6Q_1Q_5$.

- In near-BPS case $c \sim N \to \infty$, as opposed to N^2 because in our case the entropy scales as $N^2\epsilon^2$ and that $\epsilon^2 \sim 1/N$.

- The $2d$ CFT is described by $L_0, \bar{L}_0$ which are related to the BTZ black hole mass and angular momentum

$$L_0 = \frac{6}{c}N_L = \frac{1}{4}(M_{BTZ} - J_{BTZ}), \quad \bar{L}_0 = \frac{6}{c}N_R = \frac{1}{4}(M_{BTZ} + J_{BTZ}).$$

Description in terms of $2d$ dual theory, the near-BPS case

- Note that $L_0, \bar{L}_0$ are equal to the left and right excitation number of the $2d$ CFT N_L and N_R, divided by $N_2 N_3$.
- The above expressions for $L_0, \bar{L}_0$ are given for $M_{BTZ} - J_{BTZ} \geq 0$ when we have a black hole description.
- When $-1 \leq M_{BTZ} - J_{BTZ} < 0$, we need to replace them with $L_0 = -\frac{c}{24} a_+^2$, $\bar{L}_0 = -\frac{c}{24} a_-^2$.
- In the special case of *global* AdS_3 background, where $a_+ = a_- = 1/2$ formally corresponding to $M_{BTZ} = -1, J_{BTZ} = 0$, the ground state is describing an NSNS vacuum of the $2d$ CFT.

Description in terms of $2d$ dual theory, the near-BPS case

- With the above identification, the Cardy formula for the entropy of a $2d$ CFT gives

$$
\begin{aligned}
S_{2d\ CFT} &= 2\pi\left(\sqrt{cN_L/6}+\sqrt{cN_R/6}\right)\\
&= \frac{\pi}{6}\,c\left(\sqrt{M_{BTZ}-J_{BTZ}}+\sqrt{M_{BTZ}+J_{BTZ}}\right)
\end{aligned}
$$

- This exactly reproduces the expressions for the entropy we got in the $5d$ and $3d$ descriptions.
- Although the entropy and the energy of the system (which are both proportional to the central charge) grow like N and go to infinity the temperature and the horizon size remain finite.

Description in terms of $2d$ dual theory, the near-BPS case

- It is also instructive to directly connect the $4d$ and the $2d$ field theory descriptions. Comparing the expressions for M_{BTZ}, J_{BTZ} and $\Delta - \sum_{i=2,3} J_i$, J_1, we see that they match; explicitly

$$\Delta - \sum_{i=2,3} J_i = \frac{c}{12}(M_{BTZ} + 1), \qquad J_1 = \frac{c}{12} J_{BTZ} \ .$$

- The $4d$ gauge theory BPS bound, $\Delta - \sum_{i=1,2,3} J_i \geq 0$ now translates into the bound $M_{BTZ} - J_{BTZ} \geq -1$.
- This means that the $4d$ gauge theory, besides being able to describe the rotating BTZ black holes, can also describe the conical spaces.

Description in terms of $2d$ dual theory, the near-BPS case

- In other words, $\Delta - \sum_{i=1}^{3} J_i = 0$ and $N\frac{\hat{\mu}_c}{L^2}$ respectively correspond to global AdS_3 and massless BTZ cases
- and when

$$0 < \Delta - \sum_{i=1}^{3} J_i < \frac{c}{12} = N\frac{\hat{\mu}_c}{L^2} \ ,$$

$4d$ gauge theory describes a conical space, provided γ,

$$\gamma^2 \equiv \frac{12}{c}\left(\Delta - \sum_{i=1}^{3} J_i\right) - 1,$$

is a rational number.

Description in terms of $2d$ dual theory, the near-BPS case

- This is of course expected if the dual gauge theory description is indeed describing string theory on the conical space background.
- One should also keep in mind that entropy and temperature are sensible only when $\Delta - \sum_{i=1}^{3} J_i \geq \frac{c}{12}$;
- For smaller values the degeneracy of the operators in the $4d$ gauge theory is not large enough to form a horizon of finite size (in $3d$ Planck units).

End of the $2d$ CFT description of the near-BPS case.

Description in terms of $2d$ dual theory, the near-extremal case

- In the near-horizon limit of a near-extremal two-charge black hole we obtain an $AdS_3 \times S^3$ in which the AdS_3 and S^3 factors have different radii.
- Although locally AdS_3, the coordinate parameterizing $S^1 \in AdS_3$ is ranging over $[0, 2\pi\epsilon] = [0, 8\pi/N]$.
- As such, and recalling that the $AdS_3 \times S^3$ is not supersymmetric, one expects the dual $2d$ CFT description to have somewhat different properties than the standard D1-D5 system.

Description in terms of $2d$ dual theory, the near-extremal case

- Based on the analysis and results of previous sections we conjecture that

  there exists a $2d$ CFT which describes the $6d$ string theory on this $AdS_3 \times S^3$ geometry. This string theory could be embedded in the $10d$ IIB string theory on the background obtained in the near-horizon near-extremal limit.

- Here we just make some remarks about this conjectured $2d$ CFT and a full identification and analysis of this theory is still an open question.

Remarks on the conjectured $2d$ CFT dual to the near-extremal case

- This $2d$ CFT resides on the $R \times S^1$ causal boundary of the $AdS_3 \times S^3$ geometry.
- It is worth noting that in terms of the coordinates t and ϕ_1 of the original AdS_5 background, we have a space which looks like a (supersymmetric) null orbifold of AdS_3 , by $Z_{\epsilon^{-1}}$, that is an $AdS_3/Z_{N/4}$. It is desirable to understand our analysis from this orbifold viewpoint.
- One may use the Brown-Henneaux analysis to compute the central charge of this $2d$ CFT:

$$c = \frac{3R_{AdS_3}\epsilon}{2G_N^{(3)}} = 12\frac{\mu_c}{L^2\sqrt{f_0}}\, N\ .$$

Remarks on the conjectured $2d$ CFT dual to the near-extremal case

- In this case the expression for the central charge, except for the $1/\sqrt{f_0}$ factor, is the same as that of the near-BPS case, and scales like $N \to \infty$ in our limit.
- The $5d$ or $3d$ black hole entropies presented take exactly the same form obtained from counting the number of microstates of a $2d$ CFT, *i.e.* the Cardy formula, with the above central charge and M_{BTZ} and J_{BTZ} of the near-extremal case.
- As discussed, there is a sector of $\mathcal{N} = 4$, $d = 4$ SYM, characterized by $\mathcal{E} - \mathcal{E}_0$ and $\mathcal{J}$, which describes IIB string theory on the near-horizon near-extremal background.

Remarks on the conjectured $2d$ CFT dual to the near-extremal case

- One can readily express the $4d$ parameters in terms of $2d$ parameters, namely:
$$\mathcal{E} - \mathcal{E}_0 = \frac{c}{12} M_{BTZ} \,, \qquad \mathcal{J} = \frac{c}{12} J_{BTZ} \,,$$
where c, M_{BTZ} and J_{BTZ} are given in terms of μ and charges q_i.
- The above relations have of course the standard form of the usual D1-D5 system, and/or the near-BPS case discussed previously.
- Note, however, that in this case $\mathcal{E} - \mathcal{E}_0$ is measuring the mass of the BTZ with the zero point energy set at the massless BTZ case (rather than global AdS_3).

Remarks on the conjectured $2d$ CFT dual to the near-extremal case

- We expect the degrees of freedom of this $2d$ CFT to correspond to string states of the $6d$ gravity theory, which in turn from the $10d$ IIB theory viewpoint correspond to brane-like excitations about the extremal intersecting giant three-branes. It is of course desirable to make this picture precise and explicitly identify the corresponding $2d$ CFT.

Summary and Outlook

- We discussed the near-horizon decoupling limits of the near-extremal two-charge black holes of $U(1)^3$ $d=5$ gauged SUGRA.
- There are *two* such decoupling limits, one corresponding to *near-BPS* and the other to *near-extremal* black hole solutions.
- There were similarities and differences between the two cases. In both cases taking the limit over the uplift of the $5d$ black hole solution to $10d$ IIB theory, we obtain a geometry containing an $AdS_3 \times S^3$ factor.

Summary and Outlook

- Therefore, there should be $2d$ CFT dual descriptions.
- On the other hand, noting that the starting $5d$ (or $10d$) geometry is a solution in the AdS_5 (or $AdS_5 \times S^5$) background there is a description in terms of the dual $4d$ SYM theory.
- We identified central charge of the dual $2d$ CFT's in both cases and showed that B.-H. entropy of the original $5d$ solution, which is the same as the B.-H. entropy of the $3d$ BTZ black hole obtained after the limit, is reproduced by the Cardy formula of the $2d$ CFT.

Summary and Outlook

- We identified the $L_0, \bar{L}_0$ of the corresponding $2d$ CFT's in terms of the parameters of the original $5d$ black hole.
- Matching of the Bekenstein-Hawking entropy of the $5d$ and $3d$ black holes is a strong indication that the near-horizon limit we are taking is indeed a "decoupling" limit.
- For the near-BPS case, the $2d$ description is essentially the same as that of the D1-D5 system and the $2d$ CFT, modulo one complication.

Summary and Outlook

- The complication is that our background corresponds not to a single $2d$ CFT but a (continuous) collection of them, all of which have the same $L_0, \bar{L}_0$ but different central charges.
- Nonetheless, one can define an effective central charge for the system by summing over the "strip-wise" $2d$ CFT descriptions.

Summary and Outlook

- For the near-extremal case, however, we have a different situation; the conjectured $2d$ CFT description corresponds to a set of D3 giants which have a deformed shape and as a result only certain degrees of freedom on the giant theory survive our ("$\alpha' \to 0$") decoupling limit.
- In a sense, instead of intersecting giants of the near-BPS case, at the extremal point ($\mu = \mu_c$) we are dealing with a (non-marginal) bound state of giants.
- This may be traced in the $6d$ gravity theory obtained from reduction of $10d$ IIB theory.

Summary and Outlook

- As discussed, the two species of intersecting giants in $6d$ language appear as strings which are either electrically and/or magnetically charged under the three-form F_3.
- The bound state of giants in the $6d$ theory is expected to appear as a "(Q_e, Q_m)-string".
- The mass of this dyonic (Q_e, Q_m)-string state can be computed from the time-time component of the energy momentum tensor of the system T_0^0 for the $AdS_3 \times S^3$ configuration.

Summary and Outlook

- This has two parts, a cosmological constant piece and the part involving 2-form charges.
- The latter can be used to identify the mass squared of the (Q_e, Q_m)-string, which is
$$M^2_{(Q_e,Q_m)} = T_s^{(6)}\left(N_e^2\mathfrak{g}_s + N_m^2\mathfrak{g}_s^{-1}\right)$$
where $\mathfrak{g}_s = \langle X^{-2}\rangle$ is the "effective" $6d$ string coupling and $N_e,\ N_m$ are the number of electric and magnetic strings and are related to $Q_e,\ Q_m$.
- Note that in "Einstein frame" the mass of fundamental string mass squared is $T_s^{(6)}\mathfrak{g}_s$.

Summary and Outlook

- To complete this picture one should show the $6d$ (Q_e, Q_m)-string is a stable configuration in the corresponding gravity theory.
- We expect our $6d$ gravity description to be a part of a new type of $6d$ gauged supergravity.
- This $6d$ theory is expected to be a $U(1)^2$ $\mathcal{N} = (1, 1)$ gauged SUGRA with the matter content (in the language of $6d$ $\mathcal{N} = 1$):
  - one gravity multiplet,
  - one tensor multiplet and
  - two $U(1)$ vector multiplets.

Summary and Outlook

- This theory is a $6d$ version of the $d = 4,\ d = 5$ "gauged STU" models.
- It may be obtained from a suitable extension of the reduction we already discussed.
- The two $U(1)$ gauge fields A_i are coming from replacing $d\psi_i$ in reduction ansätz with $d\psi_i + LA_i$.
- The details of this reduction and construction and analysis of this "$6d$ gauged STU" supergravity will be discussed in an upcoming publication.

Summary and Outlook

- We gave a description of both the near-BPS and near-extremal cases in terms of specific sectors of large R-charge, large engineering dimension operators.
- We expect these sectors to be decoupled from the rest of the theory since they also have a description in terms of a unitary $2d$ CFT.

Summary and Outlook

- The near-BPS case has features similar to the BMN sector. In this case, however, the sector is identified with operators of $J_i \sim N^{3/2}$, as opposed to $J \sim N^{1/2}$ of BMN case.
- In the near-extremal case the operators we are dealing with are far from being BPS and their R-charge J_i $(i = 2, 3)$ scale as N^2.

Summary and Outlook

- Understanding these sectors in the $4d$ gauge theory and computing their effective 't Hooft expansion parameters,*i.e.* effective 't Hooft coupling and the planar-nonplanar expansion ratio, is an interesting open question.
- We expect there should be new "double scaling limits" similarly to the BMN case.
- To give another supportive evidence for the decoupling of these sectors one can count degeneracy of states in both of these sectors in $\mathcal{N} = 4$ SYM and match it with the B.-H. entropies computed here.

Summary and Outlook

- Here we focused on the two-charge $5d$ extremal black hole solutions of $U(1)^3$ $5d$ gauged SUGRA. The $U(1)^4$ $d=4$ gauged SUGRA has a similar set of black hole solutions.
- Among them there are three-charge extremal black holes of vanishing horizon size.
- One can take the near-horizon decoupling limits over these black holes to obtain $AdS_3 \times S^2$ geometries.

Summary and Outlook

- Again there are two possibilities, the near-BPS and near-extremal but non-BPS cases, very much the same as what we found here in the $5d$ case.
- This is under preparation......

Thanks for your attention.