2D Gravity on AdS_2 with Chern-Simons Correction

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Why 2D Gravity on AdS_2 ?!

- AdS_2 geometry is the factor which appears in the near horizon geometry of the extremal black holes in any dimension.

- Understanding quantum gravity on AdS_2 might ultimately help us understand the origin of the black hole entropy in other dimension.

- Being an AdS background it is natural to define the quantum gravity in terms of the dual CFT via AdS/CFT correspondence.

- Although AdS_{d+1}/CFT_d correspondence has been understood for $d > 1$ mainly due to explicit examples, little has been known for $d = 1$.

2D Maxwell-Dilaton Gravity

- Consider the theory of gravity with the action

$$S = \frac{1}{8G}\int d^2x\sqrt{-g}e^{\phi}\left(R + 2\partial_\mu\phi\,\partial^\mu\phi + \frac{2}{l^2}e^{2\phi} - \frac{l^2}{4}F_{\mu\nu}F^{\mu\nu}\right) \tag{1}$$

- Using asymptotic symmetries of AdS_2, it has been proposed that quantum gravity on the AdS background of this theory has a CFT dual which could be a chiral half of the 2D CFT [Hartman, Strominger, 08].

Adding Chern-Simons corrections

- The aim of this talk is to elaborate the above statement by adding higher order correction to the action.
- This correction is 2D Chern-Simons correction given by

$$S_{cs} = -\frac{1}{32G\mu}\int d^2x \left(lR\epsilon^{\mu\nu}F_{\mu\nu} + l^3\epsilon^{\mu\nu}F_{\mu\rho}F^{\rho\delta}F_{\delta\nu}\right) \tag{2}$$

- It is obtained by reducing 3D gravitational Chern-Simons along an S^1.

- We are looking for AdS_2 solutions of this theory. It is simply done by utilizing the entropy function formalism [Sen, 05].
- A generic solution preserving $SO(1,2)$ isometry of the AdS_2 is given by

$$ds^2 = v(-r^2dt^2 + \frac{dr^2}{r^2}), \qquad e^{\phi} = u, \qquad F_{01} = \frac{e}{l^2}. \tag{3}$$

- The parameters e, v and u can be obtained by extremizing the entropy function defined by

$$\mathcal{E} = 2\pi[qe - f(e,v,u)] \tag{4}$$

- q is the charge of the gauge field and $f(e, v, u)$ is the lagrangian density evaluated for the above ansatz.
- The entropy is given by the value of the entropy function evaluated at the extremum.
- For generic μ and l we find three different solutions:

$$
\begin{aligned}
1:\ v &= \frac{1+1/\mu l}{-16Gq}, \quad e^{2\phi} = \frac{-4Gql^2}{1+1/\mu l}, \quad \frac{e}{l} = -\sqrt{\frac{1+1/\mu l}{-16qG}}, \qquad q<0, \\
2: v &= \frac{1-1/\mu l}{16Gq}, \quad e^{2\phi} = \frac{4Gql^2}{1-1/\mu l}, \quad \frac{e}{l} = \sqrt{\frac{1-1/\mu l}{16qG}}, \qquad q>0, \\
3: v &= \frac{1}{8Gq\mu l}, \quad e^{2\phi} = \frac{72Gq\mu l^3}{\mu^2 l^2+27}, \quad \frac{e}{l} = \sqrt{\frac{\mu l}{2Gq(\mu^2 l^2+27)}}, \quad q>0.
\end{aligned}
\tag{5}
$$

- The entropy of the corresponding solutions written in a suggestive form is

$$\begin{aligned}
1: &\quad S = 2\pi\sqrt{\frac{-ql^2}{6}\frac{3}{2G}(1+\frac{1}{\mu l})},\\
2: &\quad S = 2\pi\sqrt{\frac{ql^2}{6}\frac{3}{2G}(1-\frac{1}{\mu l})},\\
3: &\quad S = 2\pi\sqrt{\frac{ql^2}{6}\frac{12\mu l}{G(\mu^2 l^2+27)}},
\end{aligned} \tag{6}$$

which may be compared with Cardy formula for the entropy $S = 2\pi\sqrt{\frac{L_0}{6}c}$.

- If we identify ql^2 with the eigenvalue of L_0 of the dual CFT then the central charges of the corresponding CFTs read

$$1: \ c_R = \frac{3}{2G}(1+\frac{1}{\mu l}), \ \ 2: \ c_L = \frac{3}{2G}(1-\frac{1}{\mu l}), \ \ 3: \ c_L = \frac{12\mu l}{G(\mu^2 l^2 + 27)}. \tag{7}$$

- If correct, this means that the 2D Maxwell-dilaton gravity on AdS_2 background is dual to chiral half of a 2D CFT characterized by the above central charges.
- The index L, R refer to the fact that whether the dual chiral CFT is left or right handed which in turn corresponds to the sign of the q.

Asymptotic symmetry and central charge

- Using Hartman-Strominger method (arXiv:0803.3621), we can do another calculation confirming our previous results.
- To proceed we choose a new coordinate $\sigma = \frac{1}{r}$, $t^{\pm} = t \pm \sigma$.
- Our AdS_2 solutions in this new coordinate can be recast to the form

$$ds^2 = -4v \frac{dt^+ dt^-}{(t^+ - t^-)^2}, \quad A_{\pm} = -\frac{e}{2\sigma l^2}, \quad u = \eta = \text{constant} \tag{8}$$

- We want to study the action of the 2D conformal group on this theory.
- To do so, we choose the conformal gauge for the metric and the Lorentz gauge for the gauge field:

$$ds^2 = -e^{2\rho} dt^+ dt^-, \quad \partial_+ A_- + \partial_- A_+ = 0 \tag{9}$$

- This gauge choice fixes the coordinate and $U(1)$ gauge transformation up to residual conformal and gauge transformation generated by

$$t^\pm \to t^\pm + \zeta^\pm(t^\pm)\,, \qquad a \to a + \theta(t^+) - \tilde{\theta}(t^-) \tag{10}$$

where $A_\pm = \pm\partial_\pm a$.

- On the other hand, in order to define the theory we must impose boundary conditions at $\sigma = 0$.
- Requiring no current flow out of the boundary imposes

$$j_\sigma \Big|_{\sigma=0} = 0 \tag{11}$$

where $j_\sigma = j_+ - j_-$ and

$$j_\pm = \mp \frac{l}{16G\mu} \partial_\pm (8e^{-2\rho} \partial_+ \partial_- \rho + 3l^2 F^2) \tag{12}$$

- As a result, the boundary terms in the variation of the action will vanish if

$$A_\sigma \Big|_{\sigma=0} = 0 \tag{13}$$

- In general the last condition is not preserved by the remaining allowed diffeomorphisms.
- In order to fix this, a diffeomorphism must be accompanied by a gauge transformation

$$\theta(t^+) = \frac{e}{2l^2}\partial_+\zeta^+ \,, \qquad \tilde{\theta}(t^-) = -\frac{e}{2l^2}\partial_-\zeta^- \tag{14}$$

- Therefore the improved conformal transformations are generated by the twisted energy momentum tensor

$$\tilde{T}_{\pm\pm} = T_{\pm\pm} \mp \frac{e}{2l^2}\partial_\pm \mathcal{G}_\pm, \tag{15}$$

where $\mathcal{G}_\pm$ is the current generates the gauge transformations (14).

- Using this twisted energy momentum tensor, the central charge of the model reads

$$c = 3k\frac{e^2}{l^4} \tag{16}$$

where k is the level of $U(1)$ current which parameterizes the gauge anomaly.

- We have fixed k using the known solutions. For the case of $\mu \to \infty$ the central charge is found to be $3/(2G)$ [Alishahiha, Ardalan, 08],[Larsen,et al, 08]. This determines k as

$$k = 8|q|l^2 \tag{17}$$

which results the same value (7) for the central charges.

Relation to 3D gravity

- Our two dimensional AdS_2 solutions can be uplifted to three dimension. The results are pure geometric with $SL(2,R) \times U(1)$ isometry.
- For our three solutions we get

$$
\begin{aligned}
1: ds^2 &= \frac{l^2}{4}\left(-r^2dt^2 + \frac{dr^2}{r^2} + (dz - rdt)^2\right), \\
2: ds^2 &= \frac{l^2}{4}\left(-r^2dt^2 + \frac{dr^2}{r^2} + (dz + rdt)^2\right), \\
3: ds^2 &= \frac{9l^2}{\mu^2 l^2 + 27}\left(-r^2dt^2 + \frac{dr^2}{r^2} + \frac{4\mu^2 l^2}{\mu^2 l^2 + 27}(dz + rdt)^2\right),
\end{aligned}
\tag{18}
$$

- In light of the recent terminology, the third metric is the *warped* AdS_3 [Strominger, et al, 08].
- It is notable that our third central charge is the same as left central charge proposed in Strominger's warped AdS_3 conjecture.
- It could be interesting to see why we can not read the right central charge of the warped AdS_3 gravity by this method.