M-flation:
Inflation from Matrix Valued Scalar Fields

Hassan Firouzjahi

IPM , Tehran

arXiv 0903.1481

In collaborations with

A. Ashoorioon and S. Sheikh-Jabbari

Outline

- Review of inflation
- Inflation from string theory
- M-flation set up
- Motivations from string theory
- Adiabatic and entropic modes and their power spectra
- Conclusion

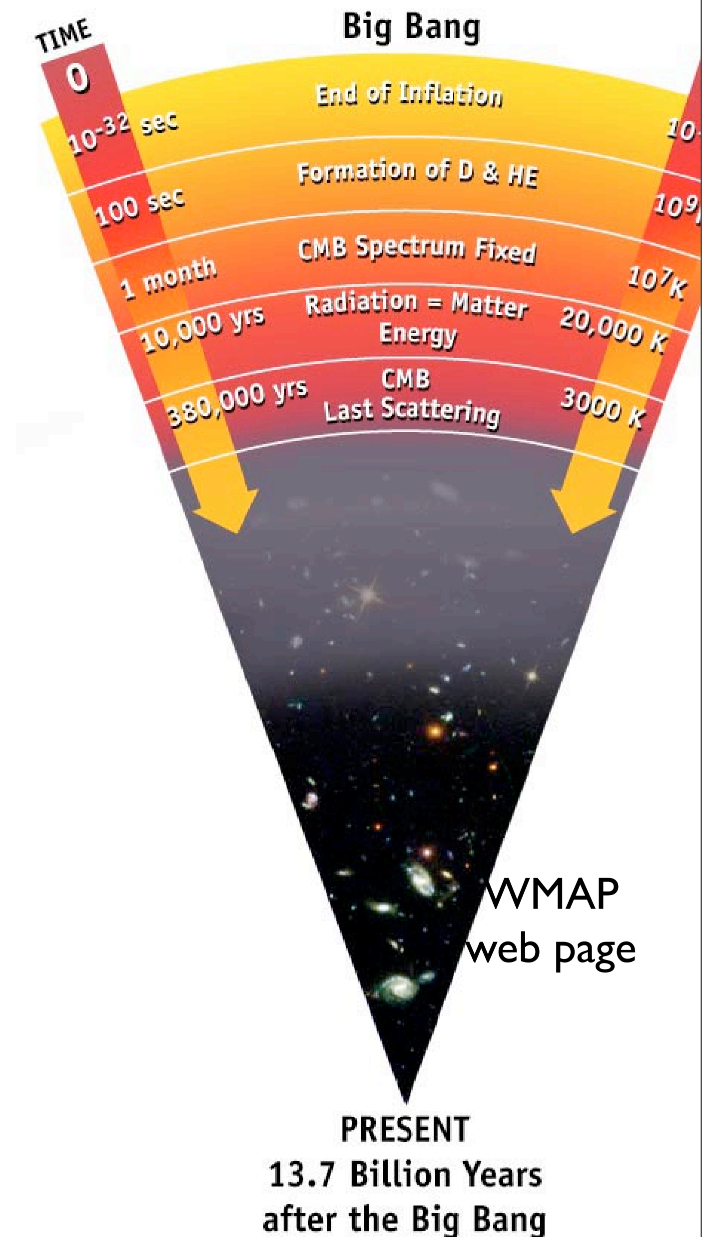
A Brief History of Universe

>Nearly 14 billion years ago the Universe was created from the big bang explosion.

> It was a hot soup filled with radiations and all sorts of free elementary particles. As it expanded, it cooled down.

>During the first few minutes the light elements(He, D,..) were formed.

> After 500,000 years atoms formed and CMB was released from plasma of electrons, protons and photons. Today CMB cooled down to 2.75 K.



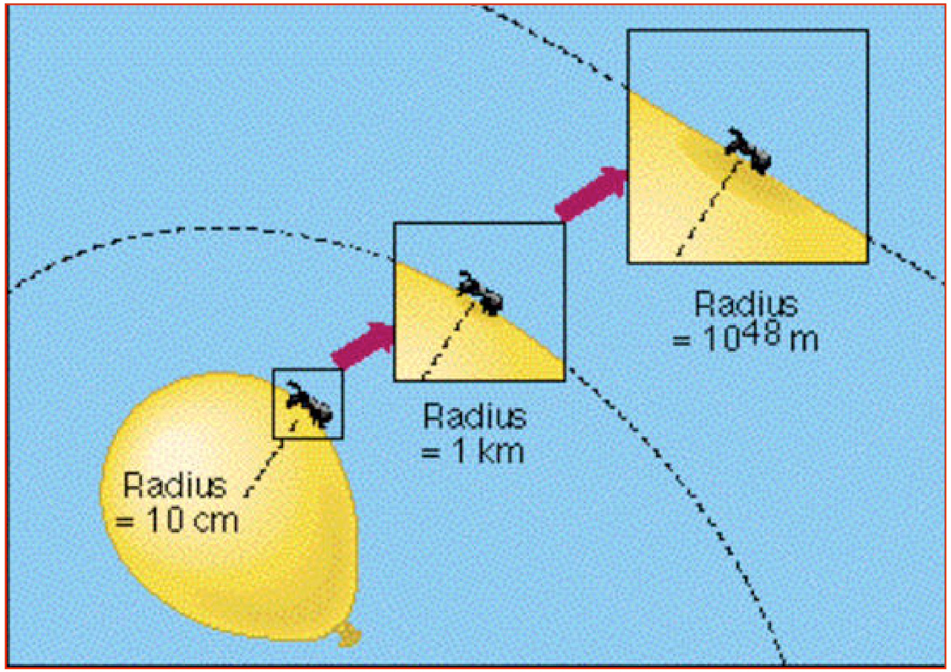
The Initial Conditions Puzzles

Despite the success of the big bang cosmology, there are initial conditions problems:

- The Horizon Problem: Why is the Universe so homogeneous and isotropic? During its evolution, the Universe did not have enough time to become so isotropic and homogeneous.
- The Flatness Problem: Why is the Universe so flat? If $\Omega \sim 1$ today, then extrapolating back to very early Universe at Planck time we find $|\Omega - 1| \sim 10^{-60}$.
- There are tiny fluctuations at the level of 10^{-5} on the smooth CMB background, which are almost scale invariant, adiabatic and Gaussian. What mechanism can create these perturbations ?

Inflation

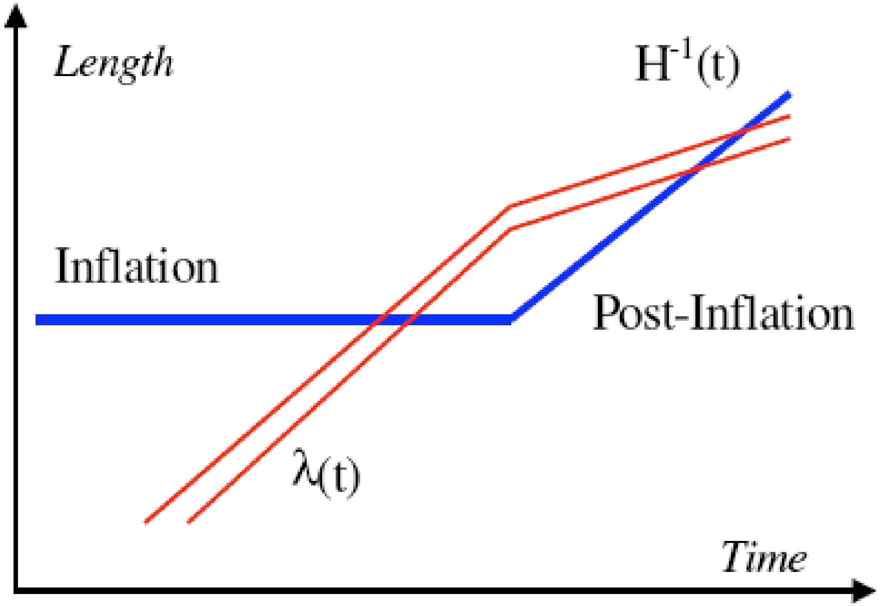
A short period of acceleration in very early Universe will provide all these necessary initial conditions and flattens the Universe.



www.astro.princeton.edu/~tremaine/ast541/das.ppt

- Primordial quantum fluctuations during inflation seeds the observed almost scale invariant Gaussian perturbations in CMB.

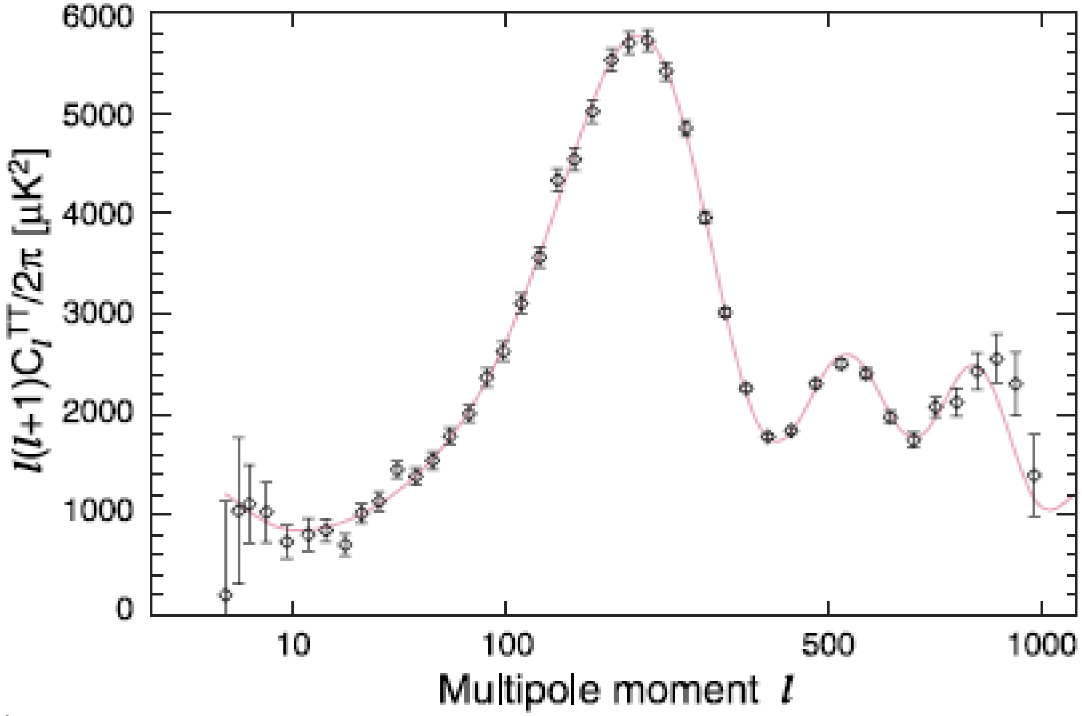
- Originally all of these modes were inside the horizon. Inflation stretches their wavelengths outside the horizon. While outside the horizon, they ``freeze out``. Later on they re-enter the horizon to form the observed structures.



WMAP 2003-08

- All observations, specially WMAP 2003-2008, strongly support inflation.

- Different inflationary models predict different values for cosmological parameters like the scalar spectrum index n_s which can be measured in CMB.



WMAP 08

- There is no compelling and theoretically well-motivated model of inflation. There have been many attempts to embed inflation within the context of string theory.

- If it works, this would provide a unique chance to test the relevance of string theory to the real world.

Inflation?

- What underlying physics drive inflation? What is the nature of inflaton field?

- Since the scale of inflation is very high, possibly GUT scale, it is natural to expect that physics beyond SM and effects of quantum gravity were important.

- String theory, on the other hand, is the best theory of quantum gravity. So far it did not make contact with the real world in a direct way.

- There have been many interesting models of inflation from string theory. Examples are Tachyon Inflation, Racetrack Inflation, DBI-Inflation, D3-D7 Inflation, Brane Inflation, Warped Brane Inflation,...

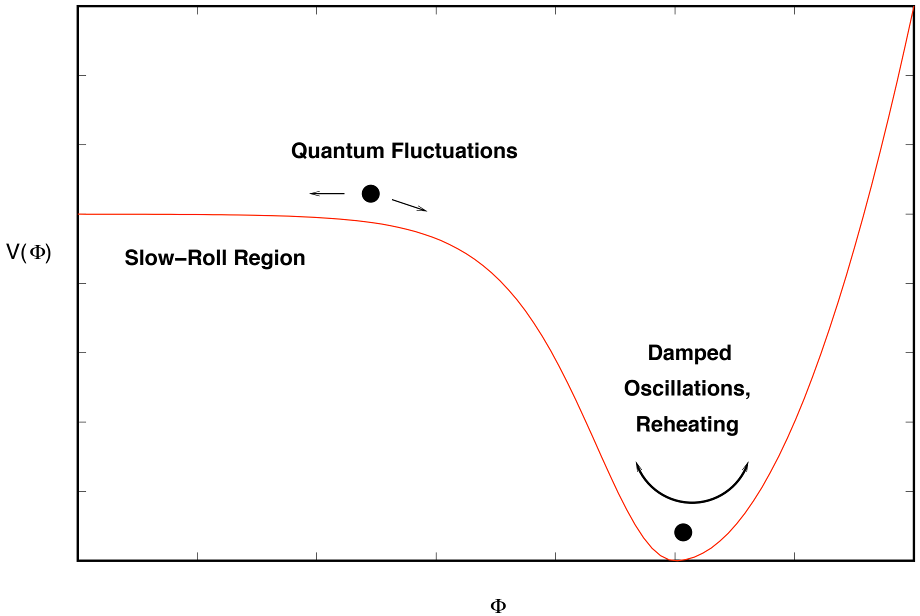
- There are mutual benefits in pursuing inflation in string theory :
  I. A unique chance to test string theory
  II. Explaining the nature of inflation from the first principles.

Slow Roll Inflation

In most models, inflation is derived by a scalar field, the inflaton.
This creates a negative pressure required for acceleration.

For a scalar field $$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$a(t) \sim e^{H\,t} \quad , \qquad H^2 = \frac{8\pi G}{3} V$$



- Simple models of chaotic inflation suffers from fine-tuning and issues with super-Planckian field values.

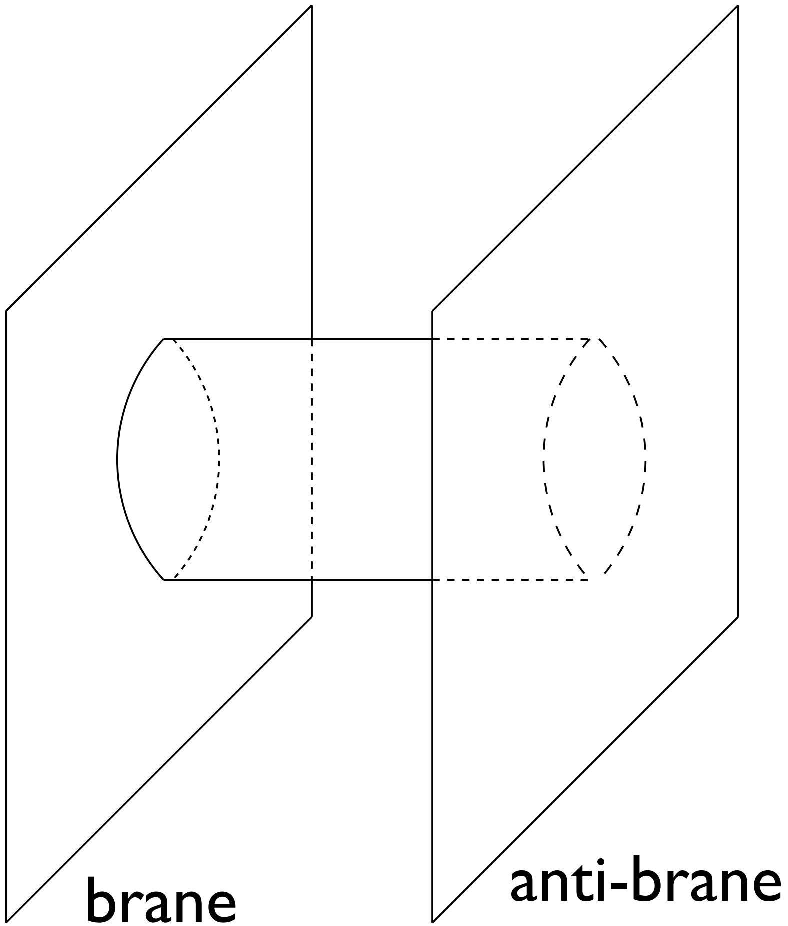
$$V = \frac{1}{2} m^2 \phi^2 \longrightarrow \phi_i \sim 15\, M_P$$

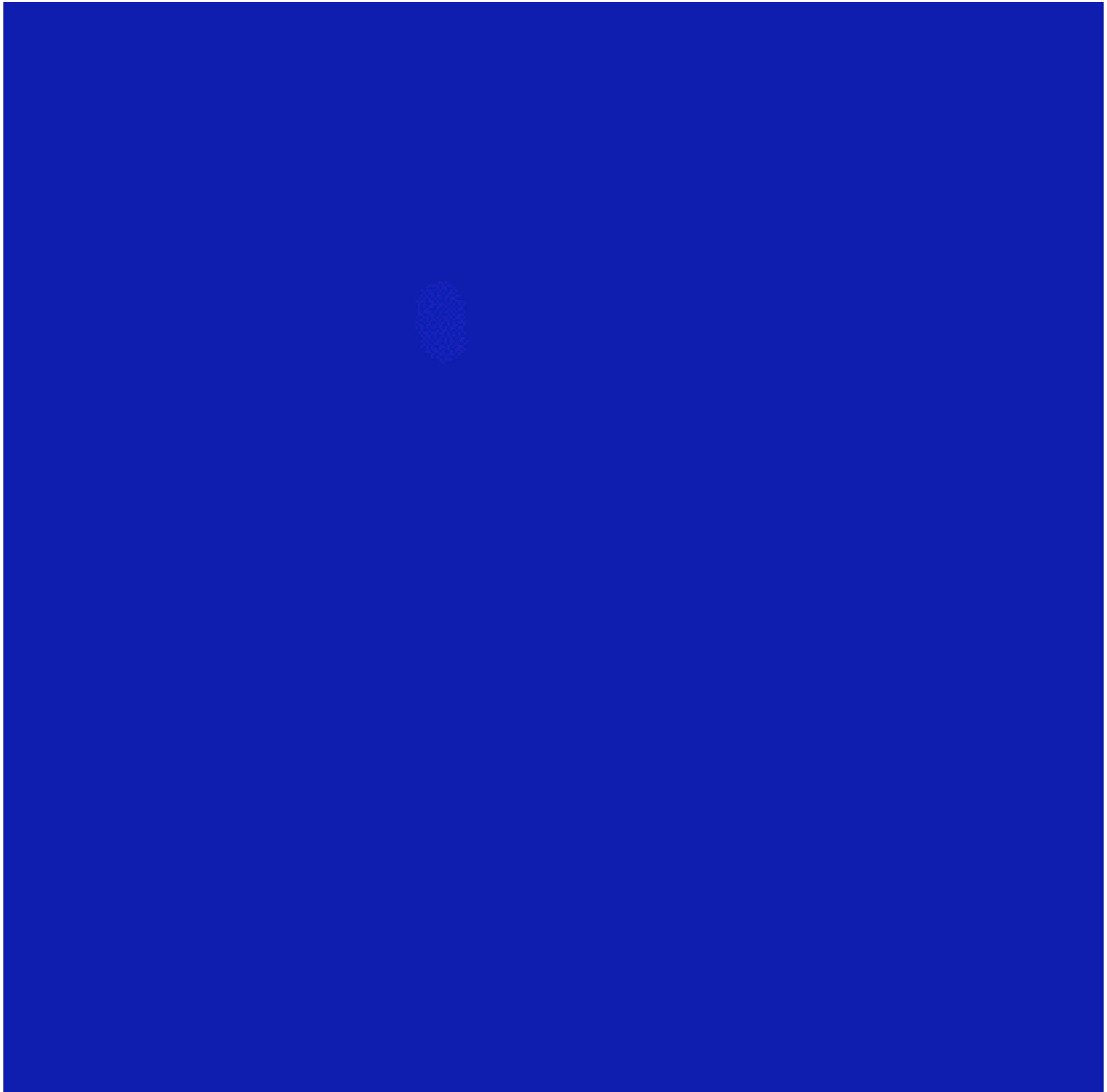
$$V = \frac{\lambda}{4}\phi^4 \longrightarrow \lambda \sim 10^{-14} \qquad \phi_i \sim 22\, M_P$$

Brane Inflation

G. Dvali and H. Tye, hep-ph/9812483

- In brane inflation the inflaton field is the distance between brane and anti-brane.
- There is an attractive force between brane and anti-brane. If the potential is flat enough one can get enough inflation.
- When the distance between brane and anti-brane is at the order of string scale, a tachyon develops. Inflation ends when brane and anti-brane collide.
- Problem: In flat CY, the potential is too steep to achieve the slow-roll conditions for inflation.

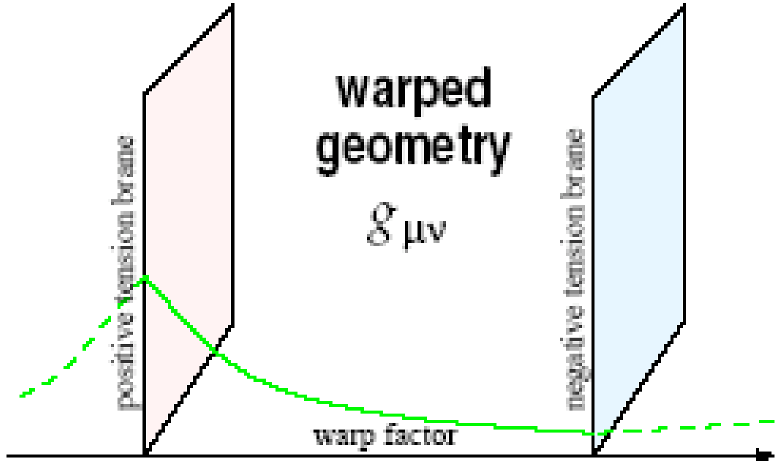




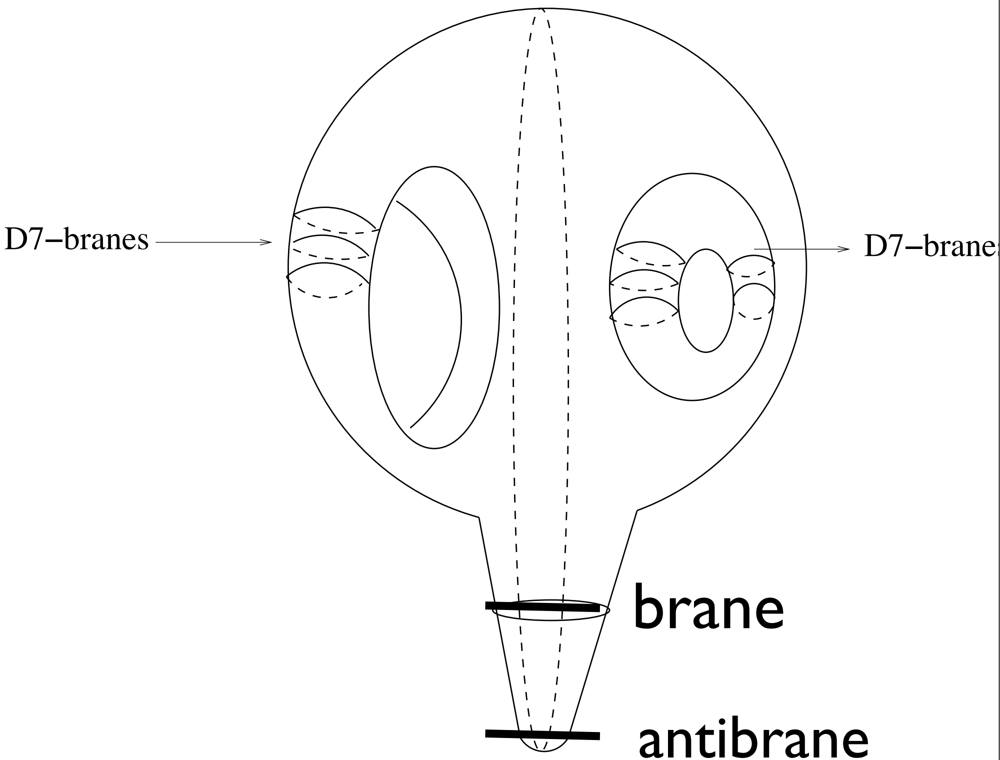
A. Miller

Warped Brane Inflation

- Warped Geometry is a method to flatten the potential between brane and anti-brane.

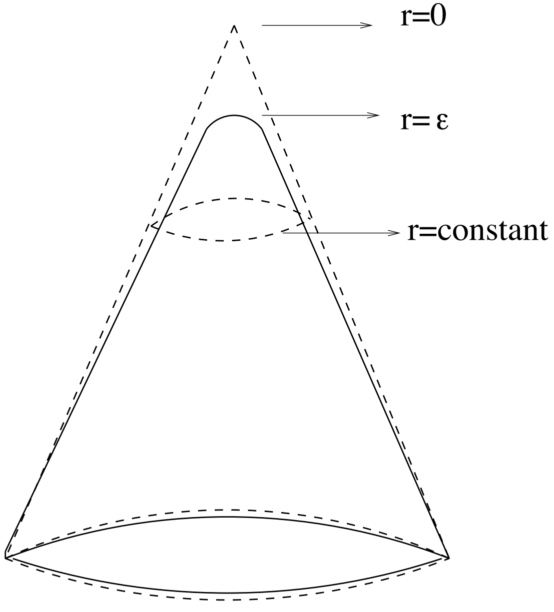


- There are localized regions in the bulk of the Calabi-Yau compactification which are highly warped. These regions are called the throats. Usually there are many of them.



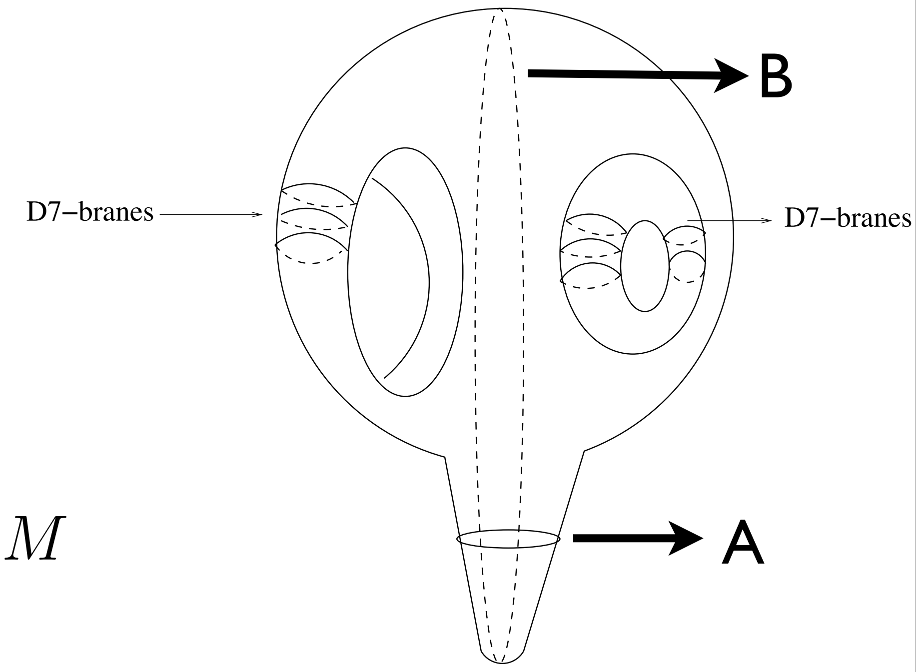
- By putting the brane and anti-brane in these throats the force between them becomes weaker and enough inflation can be obtained.
(KKLMMT: hep-th/0308055)

KS Throat



- Particular example studied carefully is a deformed conifold in IIB string theory(Kelebanov and Sttrassler, hep-th/0007191).

- A deformed conifold is defined by $$\sum_{i=1}^{4} w_i^2 = \epsilon^2$$



- There are two 3-cycles, called A and B. One can turn on M units of RR 3 forms F3 on A cycle and K units of NSNS 3 forms H3 on B cycles:

$$\frac{1}{4\pi^2 l_s^2}\int_B H_3 = -K, \qquad \frac{1}{4\pi^2 l_s^2}\int_A F_3 = M$$

The Warped Deformed Conifold

- By turning these fluxes one can create a warped geometry like Randall-Sundrum(RS) scenario(Kachru, Giddings and Polchiski, hep-th/0105097).
- The metric inside the conifold is almost an AdS metric:

$$ds^2 = h(r)^2 \left(-dt^2 + a(t)^2 d\vec{x}^2\right) + h(r)^{-2} dr^2 \qquad h(r) = \frac{r}{R}\,:$$

- Where R is the characteristic length scale of the AdS geometry

$$R^4 = \frac{27}{4}\pi g_s N \alpha'^2\,.$$

- N=MK is the effective background D3-brane charge. The warp factor at the end of the throat is given by

$$h_A = e^{-2\pi K/3g_s M}$$

M-Flation

Suppose inflation is driven by non-commutative matrices:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\sum_i \mathrm{Tr}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i\right) - V(\Phi_i,[\Phi_i,\Phi_j])\right)$$

Example :

$$V = \mathrm{Tr}\left(-\frac{\lambda}{4}[\Phi_i,\Phi_j][\Phi_i,\Phi_j] + \frac{i\kappa}{3}\epsilon_{jkl}[\Phi_k,\Phi_l]\Phi_j + \frac{m^2}{2}\Phi_i^2\right)$$

where Φ_i are $N\times N$ matrices.

The equations of motion are

$$H^2 = \frac{1}{3M_P^2}\left(-\frac{1}{2}\mathrm{Tr}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i\right) + V(\Phi_i,[\Phi_i,\Phi_j])\right)$$

$$\ddot{\Phi}_l + 3H\dot{\Phi}_l + \lambda\,[\Phi_j,\,[\Phi_l,\Phi_j]\,] - i\,\kappa\,\epsilon_{ljk}[\Phi_j,\Phi_k] + m^2\Phi_l = 0$$

Truncation to SU(2)sector

$$\Phi_i = \hat{\phi}(t) J_i \ , \qquad i = 1, 2, 3.$$

where J_i are the N-dimensional irreducible representation of SU(2) algebra.

$$[J_i, J_j] = i\, \epsilon_{ijk} J_k \ , \qquad \mathrm{Tr}(J_i\, J_j) = \frac{N}{12}(N^2 - 1)\, \delta_{ij} \ .$$

Plugging this ansatz into the action, we obtain

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \mathrm{Tr} J^2 \left(-\frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{\lambda}{2} \hat{\phi}^4 + \frac{2\kappa}{3} \hat{\phi}^3 - \frac{m^2}{2} \hat{\phi}^2 \right) \right]$$

where $\mathrm{Tr} J^2 = \mathrm{Tr}(J_i^2) = N(N^2 - 1)/4.$

Upon field re-definition

$$\hat{\phi} = \left(\mathrm{Tr} J^2\right)^{-1/2} \phi = \left[\frac{N}{4}(N^2-1)\right]^{-1/2} \phi$$

The effective potential is

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$

Where

$$\lambda_{eff} = \frac{2\lambda}{\mathrm{Tr} J^2} = \frac{8\lambda}{N(N^2-1)}\,, \quad \kappa_{eff} = \frac{\kappa}{\sqrt{\mathrm{Tr} J^2}} = \frac{2\,\kappa}{\sqrt{N(N^2-1)}}$$

Examples:

1- Chaotic Inflation $V = \frac{1}{4}\lambda_{eff}\,\phi^4$: $m = \kappa = 0$

To fit the CMB observation, we need

$$\lambda_{eff} \sim 10^{-15} \qquad \Delta\phi \sim 10 M_P.$$

On the other hand

$$\lambda_{eff} \sim \lambda N^{-3} \qquad \Delta\hat{\phi} \sim N^{-3/2}\Delta\phi$$

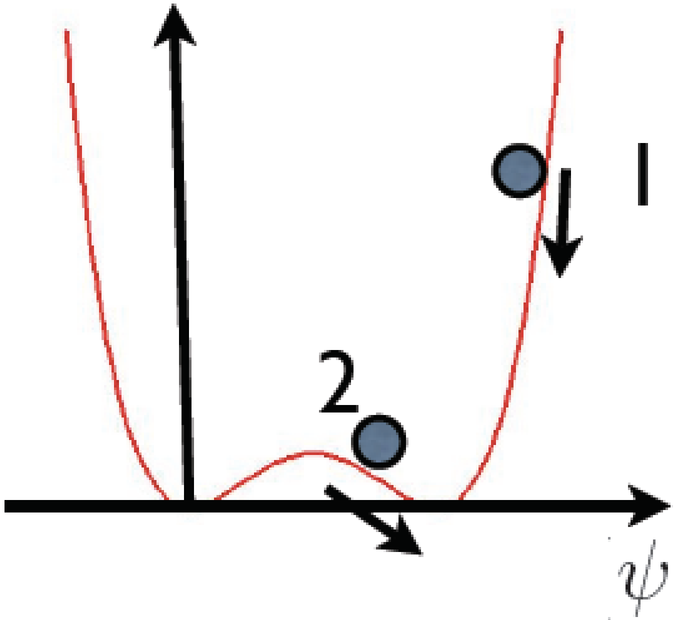
One obtains

$$N \sim 10^5 \qquad \Delta\hat{\phi} \sim 10^{-7} M_P$$

Due to large running of field values, a considerable amount of gravity waves can be produced.

2- Symmetry breaking potential:

$$V_0 = \frac{\lambda_{eff}}{4}\phi^2 (\phi - \mu)^2 \qquad \mu \equiv \sqrt{2}m/\sqrt{\lambda_{eff}}.$$



I: $\phi_i > \mu$

To fit the observational constraints

$$\phi_i \simeq 43.57 M_P \quad , \quad \phi_f \simeq 27.07 M_P \quad , \quad \mu \simeq 26 M_P.$$

$$\lambda_{eff} \simeq 4.91 \times 10^{-14}, \quad m \simeq 4.07 \times 10^{-6} M_P, \quad \kappa_{eff} \simeq 9.57 \times 10^{-13} M_P.$$

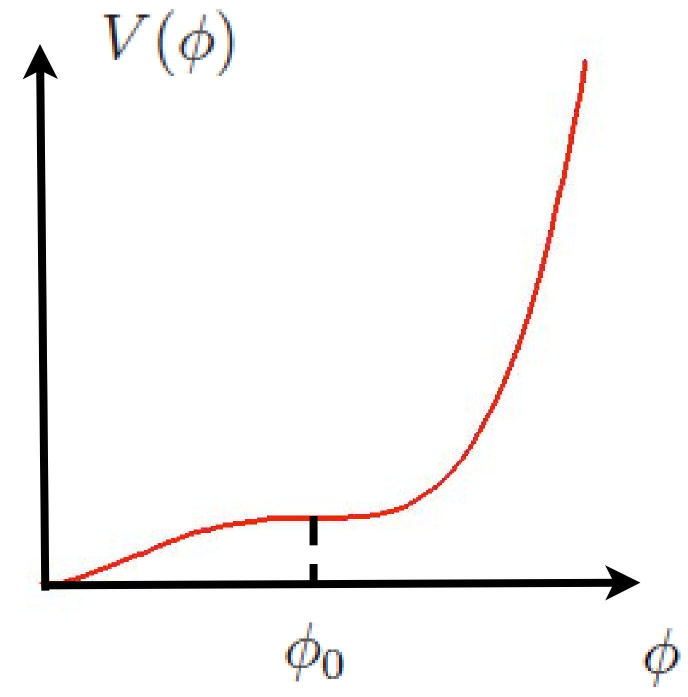
II: $\mu/2 < \phi_i < \mu$

$$\phi_i \simeq 23.5 M_P \quad , \quad \phi_f \simeq 35.03 M_P \quad , \quad \mu \simeq 36 M_P.$$

$$\lambda_{eff} \simeq 7.18 \times 10^{-14}, \quad m \simeq 6.82 \times 10^{-6} M_P, \quad \kappa_{eff} \simeq 1.94 \times 10^{-12} M_P.$$

One finds $N \sim 10^5.$ $\quad \Delta\hat{\phi} \sim 10^{-7} M_P$

3-Saddle-point Inflation $\kappa = \sqrt{2\lambda} m$



$$V(\phi) \simeq V(\phi_0) + \frac{1}{3!} V'''(\phi_0)(\phi - \phi_0)^3$$

$$V(\phi_0) = \frac{m^2}{12}\phi_0^2 \quad , \quad V'''(\phi_0) = \frac{2m^2}{\phi_0} \, .$$

The CMB observables are given by

$$n_s \simeq 1 - \frac{4}{N_e} \quad , \quad \delta_H \simeq \frac{2}{5\pi} \frac{\lambda_{eff} M_P}{m} N_e^2 \, .$$

$$\lambda_{eff} = (\frac{9\,r}{32})^{1/3} \left(\frac{5\pi}{8} \delta_H \right)^2 (1 - n_s)^{8/3} \, .$$

The upper bound r < 0.4 from WMAP5, and ns=0.96, gives

$$\lambda_{eff} \lesssim 10^{-13} \text{ and } N \gtrsim 10^5$$

Motivation from string theory

- When N D-branes are located on top of each other the gauge symmetry enhances to U(N)

$$A_a = A_a^{(n)} T_n \qquad , \qquad F_{ab} = \partial_a A_b - \partial_b A_a + i[A_a, A_b]$$

$$D_a \Phi^i = \partial_a \Phi^i + i[A_a, \Phi^i]$$

The action for N coincident brane is

$$S = -T_3 \int d^4x \, \mathrm{STr}\left(\sqrt{-|g_{ab}|}\sqrt{|Q^i_j|}\right) + \frac{\mu_3}{2} \int d^4x \, \mathrm{STr}\left([\Phi_i, \Phi_j] C^{(6)}_{ij0123}\right)$$

Where

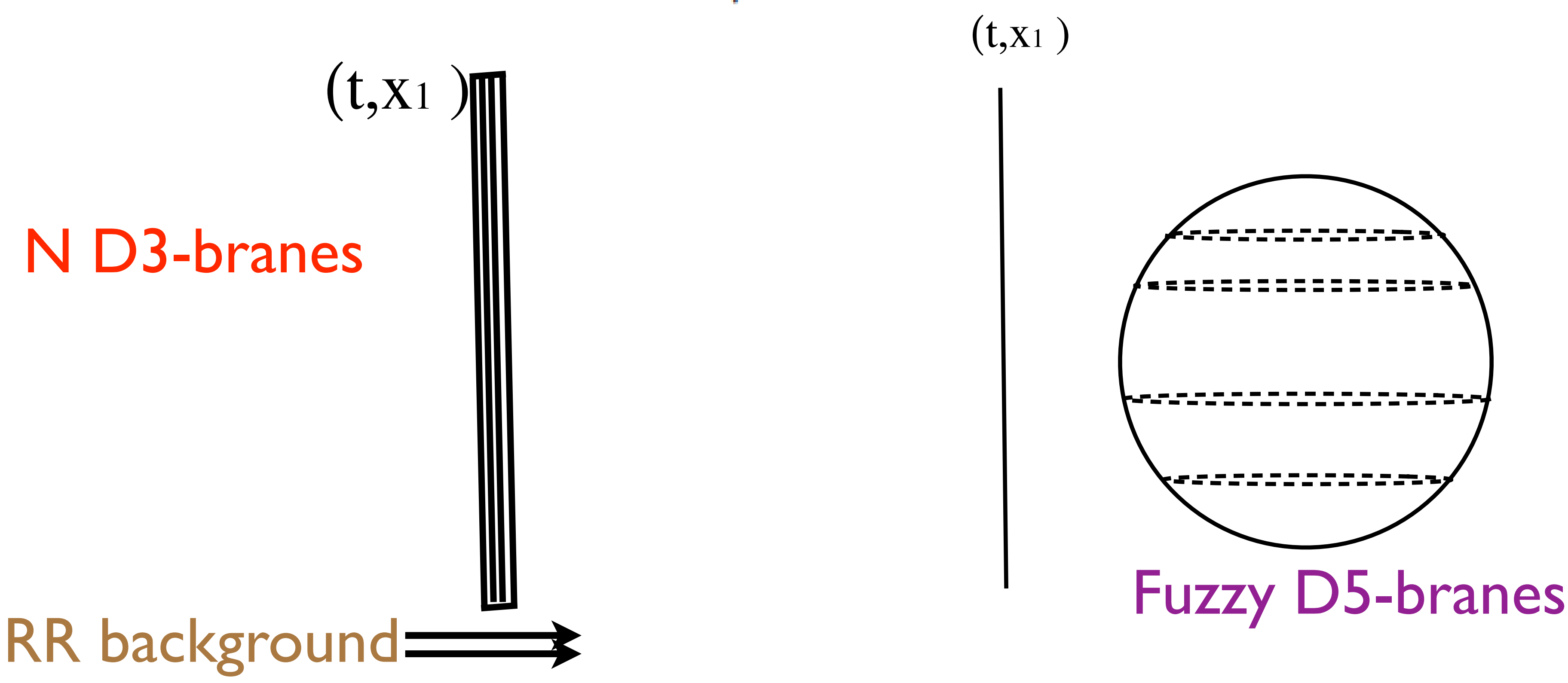
$$Q^j_k = \delta^i_j + 2\pi i \, \alpha' \, [\Phi_j, \Phi_k]$$

Consider the RR background $$C^{(6)}_{jk0123} = -\frac{2i}{3}\kappa\,\epsilon_{jkl}\,\Phi_l$$

Expanding the action up to leading terms, one obtains

$$S = -\frac{1}{2}\sum_i \mathrm{Tr}\,(\partial_\mu\Phi_i\partial^\mu\Phi_i) \;-\frac{\lambda}{4}[\Phi_i,\Phi_j][\Phi_i,\Phi_j] + \frac{i\kappa}{3}\epsilon_{jkl}[\Phi_k,\Phi_l]\Phi_j$$

with $\lambda = 2\pi g_s\ ,\qquad \hat{\kappa} = \dfrac{\kappa}{g_s\cdot\sqrt{2\pi g_s}}$

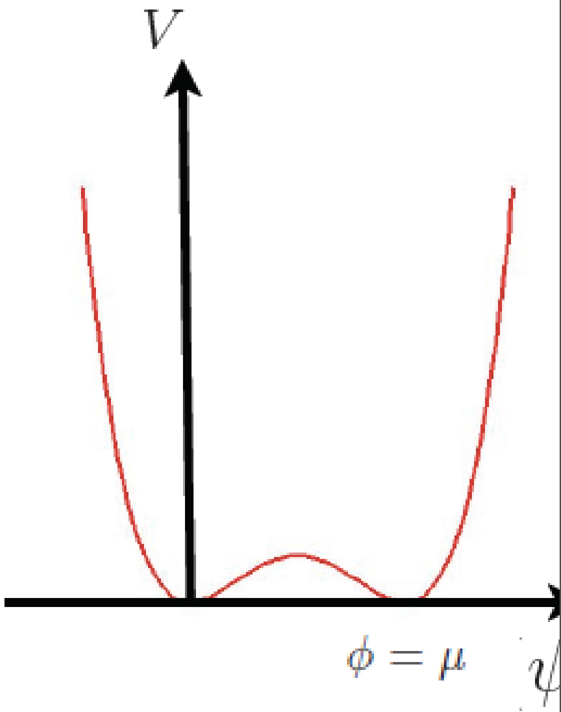


As mentioned the potential is

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2$$

The condition $\lambda m^2 = 4\kappa^2/9$ is required for background to be susy.

$$V_0 = \frac{\lambda_{eff}}{4}\phi^2\left(\phi - \mu\right)^2$$



The minimum $\phi = \mu$ is the susy vacuum.

This corresponds to the solution where N D-3 branes blow up into a fuzzy D5-branes.

Geometrically, ϕ is the radius of the fuzzy two-sphere.

Consistency of truncation to SU(2) sector

Φ_i are hermitian matrices, so we have $3N^2$ real scalar fields.

We have considered ϕ as the inflaton field and turned off the remaining $3N^2-1$ fields. How consistent is this truncation?

Suppose $\Psi_i = \Phi_i - \hat{\phi} J_i$ where $\hat{\phi} = \frac{4}{N(N^2-1)} \mathrm{Tr}(\Phi_i J_i)$

So $\mathrm{Tr}(\Psi_i J_i) = 0.$

Then $V = V_0(\hat{\phi}) + V_{(2)}(\hat{\phi}, \Psi_i)$

with $V_{(2)}(\hat{\phi}, \Psi_i = 0) = 0$, $\left(\frac{\delta V_{(2)}}{\delta \Psi_i}\right)_{\Psi_i=0} = 0.$

This leads to the important result that the ϕ field does not source the Ψ_i fields.

If we we turn off Ψ_i initially, they will always remain zero and will hence not contribute to the classical background inflationary dynamics at all .

Mass spectrum of the Ψ_i modes

Expanding the potential up to second order in Ψ_i one obtains

$$V_{(2)} = \left(\frac{\lambda_{eff}}{4} \phi^2 (\omega^2 - \omega) + \kappa_{eff}\, \omega\, \phi + \frac{m^2}{2} \right) \, \mathrm{Tr}\, \Psi_i \Psi_i \ .$$

where ω is classified in three forms

- *" The zero modes"* $\omega = -1$ $\qquad M^2 = \dfrac{V_0'}{\phi}\ .$
- *"The α modes"*: $\omega = -(l+1),\, l \in \mathbb{Z},\ 0 \le l < N$, with the mass

$$M_l^2 = \frac{\lambda_{eff}}{2}(l+1)(l+2)\phi^2 - 2\kappa_{eff}(l+1)\phi + m^2\,.$$

- *"The β modes"*: $\omega = l,\, l \in \mathbb{Z},\ 0 < l < N$, with the mass

$$M_l^2 = \frac{\lambda_{eff}}{2} l(l-1)\phi^2 - 2\kappa_{eff} l\phi + m^2\,.$$

Adiabatic and isocurvature power spectra

Our Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\psi^{(i)}_{mn}\partial^\mu\psi^{(i)}_{nm} - V_0(\phi) - \frac{1}{2}M^2(\phi)\psi^{(i)}_{mn}\psi^{(i)}_{nm}\,.$$

Define $$Q_\phi \equiv \delta\phi + \frac{\dot\phi}{H}\Phi\,.$$

The normalized curvature and isocurvature perturbations are

$$\mathcal{R} \equiv \frac{H}{\dot\phi}Q_\phi \quad , \quad \mathcal{S}^{(i)}_{mn} \equiv \frac{H}{\dot\phi}\psi^{(i)}_{mn}\,.$$

Using our equations of motion one obtains $$\dot{\mathcal{R}} = \frac{H}{\dot H}\frac{k^2}{a^2}\Phi\,.$$

Compare this to general multiple-field case

$$\dot{\mathcal{R}} = \frac{H}{\dot H}\frac{k^2}{a^2}\Phi + 2\sum_{\alpha=1}^{3N^2-1}\dot\theta_\alpha\mathcal{S}_\alpha\ .$$

At the time of Horizon crossing

$$P_{\mathcal{R}}|_\star \simeq \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_\star [1+(-2+6C)\epsilon - 2C\eta]_\star$$

$$P_{\mathcal{S}^{(i)}_{mn}}|_\star \simeq \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_\star [1+(-2+2C)\epsilon - 2C\eta_{ss}]_\star$$

when the mode leaves the horizon till Ne before the end of inflation

$$P_{\mathcal{R}}(N_e) \simeq P_{\mathcal{R}}|_\star$$

$$P_{\mathcal{S}^{(i)}_{mn}}(N_e) \simeq P_{\mathcal{S}^{(i)}_{mn}}|_* \exp\left[-2\int_0^{N_e} dN'_e B(N'_e)\right]$$

where

$$B(N_e) \simeq 2\epsilon + (2\omega+\omega^2)\eta - sgn(V'_0)\sqrt{2\epsilon}\frac{M_P}{\phi}(4\omega+3)(\omega+2) + 6\frac{M_P^2}{\phi^2}(\omega+1)(\omega+2)$$

I. Chaotic inflation $\frac{m^2}{2}\phi^2$

$$S = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\psi^{(i)}_{mn}\partial^\mu\psi^{(i)}_{nm} - \frac{1}{2}m^2\left[\phi^2 + \psi^{(i)}_{mn}\psi^{(i)}_{nm}\right] .$$

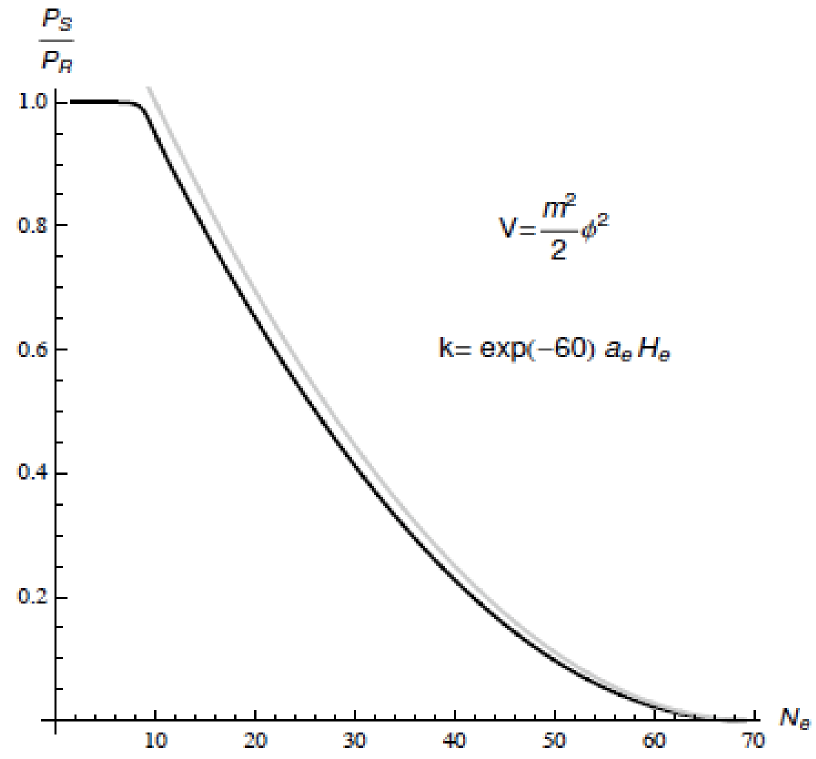
The system has SO($3N^2$) symmetry which is a specific realization of N-flation.

The spectral tilts are

$$n_R \simeq 0.966 \text{ and } n_\psi \simeq 0.9998.$$

Using our analytical formula

$$\frac{P_{S^{(i)}_{mn}}(N_e)}{P_{\mathcal{R}}|_*} \simeq (1 - N_e/60)^2$$



At the end of inflation Ps ~ 10^(-14) for mode of horizon scales.

II. Chaotic inflation: $\frac{\lambda_{eff}}{4}\phi^4$

The potential is
$$V = \frac{\lambda_{eff}}{4}\phi^4 + \frac{\lambda_{eff}}{4}(\omega^2 - \omega)\,\phi^2\,\psi^{(i)}_{mn}\psi^{(i)}_{nm}\,.$$

The mass of entropy modes are different:

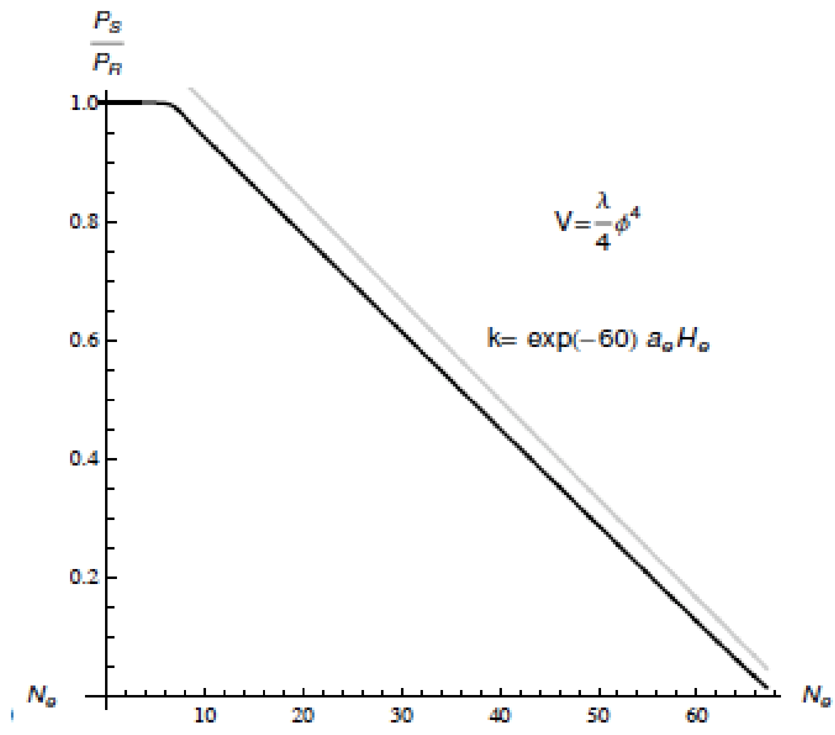
- *"The α modes"*: $M_l^2 = \frac{\lambda_{eff}}{2}(l+1)(l+2)\phi^2$ $\quad 0 \le l < N$
- *"The β modes"*: $M_l^2 = \frac{\lambda_{eff}}{2}l(l-1)\phi^2$ $\quad 0 < l < N$

The lowest mass states are

$l = 1$ $\beta-$mode	M=0
zero mode, $l = 0$ $\alpha-$mode and $l = 2$ $\beta-$mode	$\lambda_{eff}\phi^2$
$l = 1$ $\alpha-$mode $\quad l = 3$ $\beta-$mode	$3\lambda_{eff}\phi^2$

Numerically we find $n_R \simeq 0.949.$

l	M	Ps	ns
1	0	10^{-11}	0.966
2	$\lambda_{eff}\phi^2.$	10^{-13}	0.9828.
3	$3\lambda_{eff}\phi^2$	10^{-18}	1.016.



From our analytical solution

$$\frac{P_{S^{(i)}_{mn}}(N_e)}{P_{\mathcal{R}}|_*} \simeq (1 - N_e/60)^{1+\frac{\omega^2-\omega}{2}} = \begin{cases} (1 - N_e/60)^2 & \text{zero modes} \\ (1 - N_e/60)^{(l^2+3l+4)/2} & \alpha - \text{modes} \\ (1 - N_e/60)^{(l^2-l+2)/2} & \beta - \text{modes}, \end{cases}$$

III. Symmetry breaking potential:

$\phi > \mu$

l	M	Ps	ns
$l = 0\ \alpha$	$\lambda_{eff}\phi^2 - 2\kappa_{eff}\phi + m^2$	10^{-11}	0.981
$l = 1\ \alpha$	$3\lambda_{eff}\phi^2 - 4\kappa\phi + m^2$	10^{-15}	1.01
$l = 1\ \beta$	$2\kappa_{eff}\phi + m^2$	10^{-18}	1.002

Preheating

The preheating for $\lambda_{eff}\phi^4/4$ is studied by Greene et al, 1997

$$V_{\rm eff}(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

The structure of parametric resonance is completely determined by g^2/λ.

for our model $g^2/\lambda = n(n+1)/2$ $\quad n = 1, l, l-1$

As an estimate of Preheat temperature, suppose we have an instant preheating

$$N^2T^4 \sim 3H^2M_P^2$$

for large N one can get sufficiently small reheat temperature.

Reheating ?

We have not provided a mechanism of reheating where the energy from the ψ_{mn} particles are transferred into SM particles.

One scenario in M-flation in string theory: We may imagine that SM fields are localized on branes as open strings gauge fields $A_\mu^{(a)}$

This can naturally be embedded in model noting that

$$D_a\Phi^i = \partial_a\Phi^i + i[A_a, \Phi^i]$$

Non-Gaussianity?

Due to multiple-field nature of the model, there would be plenty of NG produced. It would be interesting to calculate primordial NGs and compare it with observation.

Conclusion

- All observations strongly support inflation as a theory of early Universe and structure formation. But there is no deep theoretical understanding of its origin.
- M-flation is an interesting realization of inflation which is strongly motivated from string theory. M-flation, like N-flation, can solve the fine-tunings associated with chaotic inflation and produce super-Planckian field during inflation.
- Due to Matrix nature of the fields there would be many scalar fields in the model. This leads to novel effects such as isocurvature productions, and no-Gaussianities which both are under intense observational investigations.
- M-flation has a natural built-in mechanism of preheating to end inflation. However, a mechanism of reheating has yet to be implemented.