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The RN/CFT Correspondence

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Abstract

Recently it has been shown in 0901.0931 [hep-th] that there are two extremal limits for the non-extremal Reissner-Nordstrom black hole: the extremal Reissner-Nordstrom black hole which carries no entropy and the "compactification solution" $AdS_2 \times S^2$ with flux which carries the macroscopic entropy of the non-extremal black hole. By uplifting the four dimensional solution to a five dimensional solution, we show that the "compactification solution" is dual to a CFT with central charge $c = 6Q_e(Q_e^2 + Q_m^2)$. The Cardy formula then shows that the microscopic entropy of the CFT is the same as the macroscopic entropy of the "compactification solution".

1 Introduction

One of the most exciting observation in the modern theoretical physics is the holographic dualities that relates a quantum gravity to a quantum field theory without gravity in fewer dimensions [1, 2]. The best understood holographic duality is the duality between the ten dimensional type IIB string theory on background $AdS_5 \times S^5$ with flux and the four dimensional $\mathcal{N} = 4$ super Yang-Mills theory at the boundary of AdS_5 [3–5]. Recently the idea of the holographic duality has been examined for the more interesting backgrounds using the Brown and Henneaux technique [6]. It has been shown in [7] that there is a two-dimensional CFT dual of quantum gravity on extreme Kerr background. Even though the structure of the CFT is not known, the central charge of the CFT can be found by studying the nontrivial asymptotic symmetry of the extreme Kerr solution. The Cardy formula then gives the microscopic entropy of the CFT to be exactly the same as the macroscopic entropy of the extreme Kerr background [7]. This duality has been extended to other backgrounds in [8–10], (see also [11]).

In this paper we would like to study the holographic duality for extreme Reissner-Nordstrom solution. It has been argued in [12] that the entropy of any extremal black hole of Einstein theory is zero even if its horizon area is non-zero. The reason is that the space outside of the horizon of a non-extremal black hole is a manifold with topology $R^2 \times S^2$ which has non-zero entropy, whereas, the space outside the horizon of an extremal black hole is a manifold with topology $R \times S^1 \times S^2$. This is resulted from the fact that the physical distance between an arbitrary point and the horizon in an extremal black hole is infinite. Therefore, entropy of any extrmal black hole is zero [12]! Where does the entropy of the non-extremal black hole have gone? A resolution for this puzzle has been proposed in [13] for RN black holes: There are two extremal limits. One is the usual extremal RN black hole and the other is another solution which has been called in [13] the "compactification solution". The entropy of the non-extremal black hole is argued to be carried by the latter solution [13].

In this paper we would like to find the CFT dual of the "compactification solution" by applying the Brown-Henneaux technique. It has been argued in [8] that the gauge symmetry of the extreme Kerr-Newman-AdS black hole can be combined with the geometry of the four dimensional extreme Kerr-Newman-AdS black hole to write a five dimensional metric from which the central charge of the extreme RN can be found in the limit $J \to 0$. Using this idea we find a five dimensional solution which reduces to the four dimensional "compactification solution" upon compactifying the 5-th dimension. The CFT dual of this five dimensional solution should be also dual to the four dimensional solution.

The paper is organized as follows. In the next section we review the non-extremal RN solution of Einstein-Maxwell theory in four dimensions. In section 3, we study the two extremal limits of the RN black hole. In section 4 we study the CFT dual of the compactification solution by uplifting the solution to five dimensional Einstein-Maxwell theory. In section 4.1, we study the asymptotic symmetry of the compactification solution and show that the $U(1)$ isometry of the compactification solution appears at the boundary as Virasoro algebra with the central charge which gives exactly the microscopic entropy after using the Cardy formula.

2 Review of non-extremal RN solution

In this section we review the non-extremal Reissner-Nordstrom solution of the Einstein-Maxwell theory in four dimensions. In the unit where $G_4 = 1$, the action is given by

$$S = \frac{1}{16\pi}\int d^4x\,\sqrt{-g}\Big\{R - \frac{1}{2}\frac{1}{2!}F_{(2)}^2\Big\}\,, \tag{1}$$

The non-extremal Reissner-Nordstrom solution with mass M, electric charge Q_e and magnetic charge Q_m is given by

$$\begin{aligned} ds^2 &= -\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)dt^2 + \frac{1}{\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)}dr^2 + r^2 d\Omega_2^2\,, \\ F_{(2)} &= \frac{Q_e}{r^2}dt\wedge dr + Q_m \sin\theta d\theta\wedge d\phi\,, \end{aligned} \tag{2}$$

There are two event horizons located at the coordinate singularities

$$r_\pm = M \pm \sqrt{M^2 - Q^2}\,, \tag{3}$$

where $Q = \sqrt{Q_e^2 + Q_m^2}$. There are different types of patches

$$\begin{aligned} \text{RegionI}: &\quad r_+ < r < \infty\,, &\quad -\infty < t < \infty\,, \\ \text{RegionII}: &\quad r_- < r < r_+\,, &\quad -\infty < t < \infty\,, \\ \text{RegionIII}: &\quad 0 < r < r_-\,, &\quad -\infty < t < \infty\,. \end{aligned} \tag{4}$$

The distance between an arbitrary point and the outer horizon is finite, hence, entropy of this solution can be found from the semi-classical method to be

$$S = \pi r_+^2. \tag{5}$$

The Hawking temperature of the black hole which is given by $2\pi T = \sqrt{g^{rr}}\partial_r\sqrt{g_{tt}}$ at the outer horizon is

$$T = \frac{1}{2\pi r_+^2}(r_+ - r_-)\,. \tag{6}$$

The Hawking temperature is zero when $r_+ = r_-$, however, the entropy remains non-zero.

3 Extremal limits

It was shown in [13] that there are two different extremal limits. The usual extremal limit $r_+ = r_- = Q$ without necessary going to the near horizon, and the simultaneous limit of extremal and near horizon. In the first case the solution becomes

$$\begin{aligned}
ds^2 &= -(1-\frac{Q}{r})^2dt^2 + \frac{1}{(1-\frac{Q}{r})^2}dr^2 + r^2d\Omega_2^2\,,\\
F_{(2)} &= \frac{Q_e}{r^2}dt\wedge dr + Q_m\sin\theta d\theta\wedge d\phi\,,
\end{aligned} \tag{7}$$

which is an extremal black hole with event horizon at $r = Q$. There are two regions I, III for this solution. The region II disappears in this limit. This solution carries no entropy [13] because the physical distance between an arbitrary point and the horizon is infinite[1]. To go to the near horizon, one introduces the new spacelike coordinate $0 < \lambda < \infty$ and timelike coordinate $-\infty < \sigma < \infty$ as

$$\lambda = \frac{r-Q}{Q}\,,\quad \sigma = -\frac{t}{Q}\,. \tag{8}$$

The solution for arbitrary λ becomes

$$\begin{aligned}
ds^2 &= -\frac{Q^2\lambda^2}{(1+\lambda)^2}d\sigma^2 + \frac{Q^2(1+\lambda)^2}{\lambda^2}d\lambda^2 + Q^2(1+\lambda)^2d\Omega_2^2\\
F_{(2)} &= -\frac{Q_e}{(1+\lambda)^2}d\sigma\wedge d\lambda + Q_m\sin\theta d\theta\wedge d\phi\,,
\end{aligned} \tag{9}$$

[1]If one consider the Reissner-Nordstrom solution as a solution of the effective theory of the string theory, the situation will change. In that case, it has been argued in [14] that near the horizon, the length of periodic time coordinate approaches to zero and hence the string winding modes become massless or even tachyonic. So one must include these modes to the effective action. It has been speculated in [14] that in the presence of these modes the physical distance between an arbitrary point and the horizon remains finite, hence, the macroscopic entropy of extremal solution of the string theory effective action is non-zero which should be the same as the microscopic entropy of string microstate counting [15].

Only at the horizon, $\lambda \to 0$, it becomes

$$
\begin{aligned}
ds^2 &= Q^2\left(-\lambda^2 d\sigma^2 + \frac{1}{\lambda^2}d\lambda^2 + d\Omega_2^2\right) \\
F_{(2)} &= -Q_e d\sigma \wedge d\lambda + Q_m \sin\theta d\theta \wedge d\phi\,, \qquad (10)
\end{aligned}
$$

which is locally $AdS_2 \times S^2$.

In the second case, the solution has the three regions I, II, and III. In fact the physical distance between the inner and outer horizons of the non-extremal solution remains non-zero in this case [13]. By appropriate coordinate transformation, the metric of the three regions can be mapped to a global $AdS_2 \times S^2$ solution [13]. For instance, in region II using the new timelike coordinate $0 < \chi < \pi$ and spacelike coordinate $-\infty < \psi < \infty$ via the following coordinate transformation:

$$
r = Q - \epsilon\cos\chi\,, \quad \psi = \frac{\epsilon}{Q^2}t\,, \qquad (11)
$$

where $\epsilon = \sqrt{M^2 - Q^2}$, one finds the metric and the field strength map to

$$
\begin{aligned}
ds^2 &= Q^2(-d\chi^2 + \sin^2\chi d\psi^2 + d\Omega_2^2)\,, \\
F_{(2)} &= Q_e \sin\chi d\psi \wedge d\chi + Q_m \sin\theta d\theta \wedge d\phi\,, \qquad (12)
\end{aligned}
$$

where we have sent $\epsilon \to 0$. Note that in this limit $r_+ = r_- = Q$ and at the same time $r \to Q$. Moreover, the physical distance between the outer and the inner horizons remains non zero at this limit. Using the coordinate transformation

$$
\cos\chi = \frac{\cos\tau}{\cos\vartheta}\,, \quad \tanh\psi = \frac{\sin\vartheta}{\sin\tau}\,, \qquad (13)
$$

the metric (12) transforms to [13]

$$
ds^2 = \frac{Q^2}{\cos^2\vartheta}(-d\tau^2 + d\vartheta^2) + Q^2 d\Omega_2^2\,, \qquad (14)
$$

which is $AdS_2 \times S^2$. The fluxes are mapped to

$$
F_{(2)} = -\frac{Q_e}{\cos^2\vartheta}d\tau \wedge d\vartheta + Q_m \sin\theta d\theta \wedge d\phi\,. \qquad (15)
$$

The metric (14) covers a portion of the global AdS_2. The other portions of the entire manifold are covered by the metric in regions I and III [13]. The boundaries of the global

AdS_2 are at $\vartheta = \pm\pi/2$. In terms of new coordinate $u = 1/\cos\vartheta$, the boundaries are at $u \to \infty$. The solution in terms of the u-coordinate is

$$\begin{aligned} ds^2 &= Q^2\left(-u^2 d\tau^2 + \frac{du^2}{u^2-1} + d\Omega_2^2\right) , \\ F_{(2)} &= -\frac{Q_e u}{\sqrt{u^2-1}} d\tau \wedge du + Q_m \sin\theta d\theta \wedge d\phi \, . \end{aligned} \tag{16}$$

Near the boundary, $u \to \infty$, it behaves as

$$\begin{aligned} ds^2 &= Q^2\left(-u^2 d\tau^2 + \frac{du^2}{u^2} + d\Omega_2^2\right) , \\ F_{(2)} &= -Q_e d\tau \wedge du + Q_m \sin\theta d\theta \wedge d\phi \, . \end{aligned} \tag{17}$$

which is similar to the near horizon of extremal black hole (10). However, we note that the equation (10) is not valid for $\lambda \to \infty$.

At the extremal limit the entropy of the non-extremal black hole remains non-zero. On the other hand, the extremal black hole solution (7) carries no entropy, hence, one expects that the above $AdS_2 \times S^2$ solution carries the following macroscopic entropy [13]:

$$S_{\text{macro}} = \pi Q^2 = \pi(Q_e^2 + Q_m^2) \, . \tag{18}$$

Note that the metric (14) is a solution of the four dimensional equations of motion for any value of Q^2, similarly, the fluxes in (15) are valid solution for any Q_e and Q_m. However, they are the simultaneous limit of the non-extremal RN black hole only for $Q^2 = Q_e^2 + Q_m^2$.

The Hawking temperature (6) is zero in the simultaneous limit, however, there is another temperature which is conjugate to the electric charge and is defined by $T_e dS = dQ_e$. This temperature is

$$T_e = \frac{1}{2\pi Q_e} \, . \tag{19}$$

The macroscopic entropy (18) should be extract also from microstates counting. In the next section we would like to show that there is a CFT dual of the above solution whose microstates counting gives its macroscopic entropy.

4 The CFT dual

To study the CFT dual of the compactification solution using the Brown-Henneaux's technique [6] that has been used for the extreme Kerr solution in [7], one should write the metric in a form which has isometry $SL(2,R) \times U(1)$ with off-diagonal metric in the $U(1)$ part. Using this idea, the $U(1)$ gauge symmetry of the extreme Kerr-Newman-AdS black hole has been combined in [8] with the geometry of the four dimensional extreme Kerr-Newman-AdS black hole to write a five dimensional metric with off-diagonal component in the 5th-direction. We note that the new metric must satisfy the equations of motion in order to use the formula for the 5-dimensional on-shell generators [16]. So we should combine the $U(1)$ gauge symmetry (15) with the metric (14) to write a 5-dimensional metric with off-diagonal component in the 5th direction. Moreover, the new metric must satisfy the 5-dimensional equations of motion. To this end, we first uplift the compactification solution to a five dimensional solution of the Einstein-Maxwell theory for a specific value of the magnetic charge Q_m, and then find the CFT dual of the five dimensional solution.

Consider the five dimensional Einstein-Maxwell theory:

$$S = \frac{1}{16\pi G^{(5)}} \int d^5x \sqrt{-g} \Big\{ R - \frac{1}{2}\frac{1}{2!} F_{(2)}^2 \Big\} , \tag{20}$$

The equations of motion are

$$\begin{aligned} R^{\mu}{}_{\nu} &= \frac{1}{4}\left(2F^{\mu\alpha}F_{\nu\alpha} - \frac{1}{3}\delta^{\mu}{}_{\nu}F_{(2)}^2\right) \\ \partial_\mu\left(\sqrt{g}F^{\mu\nu}\right) &= 0 \end{aligned} \tag{21}$$

One can show that the above equations are satisfied by the following solution:

$$\begin{aligned} ds_5^2 &= \frac{\rho^2}{\cos^2\vartheta}(-d\tau^2 + d\vartheta^2) + \rho^2 d\Omega_2^2 + (dy + Q_e \tan\vartheta d\tau)^2 , \\ F &= Q_m \sin\theta d\theta \wedge d\phi , \quad Q_m = \sqrt{3} Q_e , \end{aligned} \tag{22}$$

where y is a fiber coordinate with period 2π, and ρ, Q_e are arbitrary constants. Upon dimensionally reducing the y coordinate as

$$ds_5^2 = ds_4^2 + (dy + A)^2 , \tag{23}$$

the action (20) reduces to (1) and the five dimensional solution reduces to the following solution:

$$\begin{aligned} ds_4^2 &= \frac{\rho^2}{\cos^2\vartheta}(-d\tau^2 + d\vartheta^2) + \rho^2 d\Omega_2^2 , \\ A &= Q_e \tan\vartheta d\tau - Q_m \cos\theta d\phi , \end{aligned} \tag{24}$$

which is the four dimensional compactification solution when ρ in above solution is $\rho^2 = Q_m^2 + Q_e^2$ and Q_m in the compactification solution (15) is $Q_m = \sqrt{3} Q_e$.

Using the coordinate transformation $\cos\vartheta = 1/u$ where $1 \leq u \leq \infty$, the five dimensional solution (22) becomes

$$\begin{aligned} ds_5^2 &= \rho^2 \left\{ -u^2 d\tau^2 + \frac{du^2}{u^2-1} + d\Omega_2^2 \right\} + (dy + Q_e\sqrt{u^2-1}d\tau)^2 \,, \\ A &= -Q_m \cos\theta d\phi \,. \end{aligned} \tag{25}$$

In this coordinate the boundaries are at $u \to \infty$. Note that the curvature of the above metric is finite everywhere, *i.e.,* $R = Q_e^2/(2\rho^4)$, so there is no singularity at $u = 1$.

There is a $U(1)$ gauge transformation under which $\delta_\Lambda = d\Lambda$, for arbitrary scalar field Λ, as well as the isometry group of $SL(2,R) \times SO(3) \times U(1)$. The Killing vector that generates the rotational $U(1)$ isometry group is

$$\zeta^{(y)} = -\partial_y \,, \tag{26}$$

the Killing vectors that generate the $SO(3)$ isometry group are the followings:

$$\begin{aligned} \hat{\zeta}_1 &= \sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi \,, \\ \hat{\zeta}_2 &= -\cos\phi\,\partial_\theta + \cot\theta\sin\phi\,\partial_\phi \,, \\ \hat{\zeta}_3 &= -\partial_\phi \,, \end{aligned} \tag{27}$$

and the Killing vectors that generate the $SL(2,R)$ isometry group are the followings:

$$\begin{aligned} \zeta_1 &= \frac{2\sin\tau\sqrt{u^2-1}}{u}\partial_\tau - 2\cos\tau\sqrt{u^2-1}\partial_u + \frac{2Q_e\sin\tau}{u}\partial_y \,, \\ \zeta_2 &= \frac{2\cos\tau\sqrt{u^2-1}}{u}\partial_\tau + 2\sin\tau\sqrt{u^2-1}\partial_u + \frac{2Q_e\cos\tau}{u}\partial_y \,, \\ \zeta_3 &= 2Q_e\partial_\tau \,. \end{aligned} \tag{28}$$

At the boundary, $u \to \infty$, the above Killing vectors become

$$\zeta_\eta = \eta(\tau)\partial_\tau - \partial_\tau(\eta(\tau))u\partial_u \tag{29}$$

for $\eta(\tau) = 2\sin\tau, 2\cos\tau, 2Q_e$. If one perturbs the background (25), then the Killing vectors will change and hence their values at the boundary will be modified.

4.1 The Asymptotic Symmetry Group

The asymptotic symmetry group (ASG) of a spacetime is the group of non-trivial allowed symmetries. A non-trivial allowed symmetry is the one which generates a transformation that obeys the boundary conditions and its associated charge is non-vanishing [7].

Since ∂_τ is the generator whose conjugate conserved charge measures the deviation of the solution from extremality [7], we consider the perturbations that their associated conserved charges commute with ∂_τ. For the fluctuations of the metric and gauge field (25) we choose the following boundary condition:

$$h_{\mu\nu} \sim \mathcal{O}\begin{pmatrix} u^2 & u & u & 1/u^2 & 1 \\ & 1 & 1 & 1/u & 1 \\ & & 1 & 1/u & 1 \\ & & & 1/u^3 & 1/u \\ & & & & 1 \end{pmatrix}, \quad a_\mu \sim \mathcal{O}(u, 1, 1, 1/u^2, 1/u) \tag{30}$$

in the basis $(\tau, \phi, \theta, u, y)$. At leading order, the diffeomorphisms which preserve the above boundary condition are

$$\begin{aligned} \zeta_\epsilon &= \epsilon(y)\partial_y - u\epsilon'(y)\partial_u \,, \\ \zeta^{(\tau)} &= \partial_\tau \,, \\ \hat{\zeta}_1 &= \sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi \,, \\ \hat{\zeta}_2 &= -\cos\phi\,\partial_\theta + \cot\theta\sin\phi\,\partial_\phi \,, \\ \hat{\zeta}_3 &= -\partial_\phi \,, \end{aligned} \tag{31}$$

where $\epsilon(y)$ is an arbitrary smooth function. The Lie derivative of metric (25) with respect to $\zeta^{(\tau)}$ and $\hat{\zeta}$'s are zero, and with respect to ζ_ϵ is

$$\begin{aligned} \delta_\epsilon ds^2 &= 2\Big((\rho^2 - Q_e^2)u^2\epsilon'(y)d\tau^2 + \frac{\rho^2}{(u^2-1)^2}\epsilon'(y)du^2 + \epsilon'(y)dy^2 \\ &- \frac{Q_e}{\sqrt{u^2-1}}\epsilon'(y)d\tau dy - \frac{\rho^2 u}{u^2-1}\epsilon''(y)dudy\Big)\,, \end{aligned} \tag{32}$$

which is consistent with the boundary condition (30). The Lie derivative of gauge field (25) with respect to (31), however, does not satisfy the boundary condition (30). One must add a compensating $U(1)$ gauge transformation such that the combined gauge transformation and the diffeomorphism transformation, *i.e.*, $\delta_\Lambda A + \mathcal{L}_\zeta A$, satisfies the boundary condition [8]. The appropriate gauge transformation is

$$\Lambda = Q_m \cos\theta\epsilon(y) \tag{33}$$

The combined transformation is

$$\delta A = Q_m \sin\theta\epsilon(y)d\theta \tag{34}$$

which satisfies the boundary condition (30).

Using the periodicity of the y coordinate, one can expand $\epsilon(y)$ in terms of the basis $\epsilon_n(y) = -e^{-iny}$. Defining the generators $\zeta_n \equiv \zeta_{\epsilon_n}$, $\Lambda_n \equiv \Lambda_{\epsilon_n}$, one finds they satisfy the following Virasoro algebra:

$$\begin{aligned} i[\zeta_m, \zeta_n]_{L.B.} &= (m-n)\zeta_{m+n}\,, \\ i[\Lambda_m, \Lambda_n]_{\zeta} &= (m-n)\Lambda_{m+n}\,, \end{aligned} \tag{35}$$

where $[\Lambda_m, \Lambda_n]_\zeta = \zeta_m^\mu \partial_\mu \Lambda_n - \zeta_n^\mu \partial_\mu \Lambda_m$. They have zero central charge. To evaluate the central term of the above algebra, one needs to construct the surface charges which generate the asymptotic symmetry (31). For asymptotically AdS spacetimes, the charge differences between $(g_{\mu\nu}, A_\mu)$ and $(g_{\mu\nu} + h_{\mu\nu}, A_\mu + a_\mu)$ are given by [16] (see [10] for a review)

$$Q_{\zeta,\Lambda}[g, A] = \frac{1}{8\pi G}\int_{\partial\Sigma}\left(k_\zeta^{grav}[h; g] + k_{\zeta,\Lambda}^{gauge}[h, a; g, A]\right)\,, \tag{36}$$

where the integral is over the boundary and

$$\begin{aligned} k_\zeta^{grav}[h, g] &= -\frac{1}{2}\frac{1}{3!}\epsilon_{\alpha\beta\gamma\mu\nu}\Big[\zeta^\nu D^\mu h_\sigma^\sigma - \zeta^\nu D_\sigma h^{\mu\sigma} + \zeta_\sigma D^\nu h^{\mu\sigma} + \frac{1}{2}h_\sigma^\sigma D^\nu \zeta^\mu \\ &\quad - h^{\nu\sigma} D_\sigma \zeta^\mu + \frac{1}{2}h^{\sigma\nu}(D^\mu \zeta_\sigma + D_\sigma \zeta^\mu)\Big] dx^\alpha \wedge dx^\beta \wedge dx^\gamma\,, \\ k_{\zeta,\Lambda}^{gauge}[h, a; g, A] &= \frac{1}{4}\frac{1}{3!}\epsilon_{\alpha\beta\gamma\mu\nu}\Big[(-\frac{1}{2}h_\sigma^\sigma F^{\mu\nu} + 2F^{\mu\sigma}h_\sigma^\nu - \delta F^{\mu\nu})(\zeta^\rho A_\rho + \Lambda) \\ &\quad - F^{\mu\nu}\zeta^\rho a_\rho - 2F^{\sigma\mu}\zeta^\nu a_\sigma + \frac{1}{4}g^{\nu\sigma}a^\mu(\mathcal{L}_\zeta A_\sigma + \partial_\sigma \Lambda)\Big] dx^\alpha \wedge dx^\beta \wedge dx^\gamma \end{aligned} \tag{37}$$

where $\delta F^{\mu\nu} = g^{\mu\sigma}g^{\nu\lambda}(\partial_\sigma a_\lambda - \partial_\lambda a_\sigma)$. The covariant derivatives and raised indices are computed using the metric $g_{\mu\nu}$. For the magnetic gauge field (25) and the perturbation (34), one finds $k_{\zeta,\Lambda}^{gauge}[h, a; g, A] = 0$. Hence, the charge (36) simplifies to

$$Q_{\zeta,\Lambda}[g, A] = \frac{1}{8\pi G}\int_{\partial\Sigma} k_\zeta^{grav}[h; g]\,, \tag{38}$$

$$\equiv Q_\zeta[g] \tag{39}$$

For the diffeomorphism (31), one finds[2]

$$\begin{aligned} k_{\zeta_\epsilon}^{grav} &= -\frac{Q_e \sin\theta}{4u^2}\Big[2\epsilon(y)u^3\partial_y h_{uy} + \frac{\rho^2 + Q_e^2}{\rho^2}u^2\epsilon(y)h_{yy} \\ &\quad - 2\epsilon'(y)u^3 h_{uy} + \frac{1}{\rho^2}\epsilon(y)h_{\tau\tau}\Big] d\theta \wedge d\phi \wedge dy\,, \end{aligned}$$

[2]Note that the Lie derivative of metric with respect to the diffeomorphisms $\zeta^{(\tau)} = \partial_\tau$ and $\hat{\zeta}_1$, $\hat{\zeta}_2$, $\hat{\zeta}_3$ are zero, hence, their corresponding charges are zero too.

where we have discarded total ϕ derivative terms and keep only terms that are non-zero at the boundary $u \to \infty$. We have also included only the terms that are tangent to $\partial\Sigma$.

The algebra of the non trivial asymptotic symmetries is the Poisson bracket algebra of the charges [16]

$$\{Q_{\zeta_m}, Q_{\zeta_n}\}_{P.B.} = Q_{[\zeta_m,\zeta_n]} + \frac{1}{8\pi G}\int_{\partial\Sigma} k^{grav}_{\zeta_m}[\mathcal{L}_{\zeta_n} g, g]\,. \tag{40}$$

The last term has the structure

$$\frac{1}{8\pi G}\int_{\partial\Sigma} k^{grav}_{\zeta_m}[\mathcal{L}_{\zeta_n}, g] = -iA(m^3 + Bm)\delta_{m+n,0}\,. \tag{41}$$

If one defines the quantum version of the Q's by

$$L_n \equiv Q_{\zeta_n} + \frac{1}{2}(AB + A)\delta_{n,0}\,, \tag{42}$$

plus the usual rule of $\{.,.\}_{P.B.} \to -i[.,.]$, then the algebra becomes the standard Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + Am(m^2-1)\delta_{m+n,0}\,, \tag{43}$$

with central charge $c = 12A$.

The Lie derivatives of metric (25) at the boundary are

$$\begin{aligned}
\mathcal{L}_{\zeta_n} g_{\tau\tau} &= 2i(\rho^2 - Q_e^2)u^2 n e^{-iny}\,,\\
\mathcal{L}_{\zeta_n} g_{\tau y} &= -\frac{iQ_e}{u} n e^{-iny}\,,\\
\mathcal{L}_{\zeta_n} g_{uy} &= -\frac{\rho^2}{u} n^2 e^{-iny}\,,\\
\mathcal{L}_{\zeta_n} g_{yy} &= 2ine^{-iny}\,,\\
\mathcal{L}_{\zeta_n} g_{uu} &= \frac{2i\rho^2}{u^4} n e^{-iny}\,.
\end{aligned} \tag{44}$$

Replacing above perturbation into the central term of (40), one finds

$$\frac{1}{8\pi G^{(5)}}\int_{\partial\Sigma} k^{grav}_{\zeta_m}[\mathcal{L}_{\zeta_n} g, g] = -\frac{i}{2} Q_e(m^3\rho^2 + m)\delta_{m+n,0} \tag{45}$$

where we have used the fact that in five dimension $G^{(5)} = 2\pi$. Therefore, the central charge is

$$c = 6Q_e\rho^2\,, \tag{46}$$

This is the central charge of the CFT dual of the background (25) in which the parameter ρ is arbitrary. For the specific value of $\rho^2 = Q_e^2 + Q_m^2$ the solution is the 4-dimensional compactification solution. So the central charge of the CFT dual of the compactification solution, at least for $Q_m = \sqrt{3}Q_e$, is

$$c = 6Q_e(Q_e^2 + Q_m^2)\,, \tag{47}$$

This central charge has been also found in [8] by combining the gauge field of extremal Kerr-Newman-AdS black hole with the 4-dimensional metric and taking $J \to 0$.

The Cardy formula gives the microscopic entropy of a unitary CFT at large T_e to be

$$S_{\text{micro}} = \frac{\pi^2}{3} c T_e\,, \tag{48}$$

Using (19) and (46), one finds

$$S_{\text{micro}} = \pi Q^2 = \pi(Q_e^2 + Q_m^2)\,, \tag{49}$$

This exactly reproduces the macroscopic entropy (18).

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