Aspects of non-BPS Black Holes in D=4

A series of three lectures

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Content of Lecture 1

- ***Simple*** BPS/non-BPS ***brane configurations***

  sets the scene the non-BPS solutions in Lecture 2+3, and for the general non-extreme solutions.

- Some ***general non-extremal black holes***

- Some ***quantization conditions*** satisfied by the general non-extreme black holes

- ***Extremal limits***, BPS and otherwise.

Lecture 2+3

Focus on more recent work:

- ***The extreme non-BPS seed solution***
- ***Constituent Model for non-BPS black holes***
  motivated by some remarkable cancellations
- ***Attractor behavior*** and more surprising cancellations in the classical moduli space.

References for the full lecture series:

M. Cvetic and FL: hep-th 9705192, 9708090, 9712112

FL: hep-th 9702153, 9806071, 9909102, 0002166

E. Gimon, FL, J. Simon: arXiv: 0710.4264, 0903.0719.

Omitted Aspects of non-BPS BHs

- ***Higher Derivative Corrections.***

  Entropy extremization, c-extremization, and anomalies are some techniques for these studies.

- ***AdS_2/CFT, Kerr/CFT.***

  Several approaches to the entropy counting are under very active investigation.

- ***Interesting Compactifications***

  We just consider tori here.

The Setting

- We mostly work in maximally supersymmetric SUGRA: $N = 8$ in $D = 4$.

- The discussion for $N = 4$ in $D = 4$ is almost identical - no details will be given.

- Often useful to lift the solutions to $N = IIA$ in $D = 10$ or $N = 1$ SUGRA in $D = 11$.

- Alternative view point: truncate to the STU model. This is $N = 2$ SUGRA in $D = 4$ with prepotential
$$F = \frac{X^1 X^2 X^3}{X^0}$$
This model has an obvious embedding in $N = 8$, but it is also a good starting point for generalization to other $N = 2$ models.

The D4-brane Solution

The 10D IIA SUGRA solution representing a $D4$-brane at low energy:

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H}}(-dt^2 + dz_{\parallel}^2) + \sqrt{H} dz_{\perp}^2 \ , \\ e^{-2\Phi} &= \sqrt{H} \\ A^{(M)}_{t1234} &= H^{-1} \quad (\text{the magnetic five} - \text{form } {}^*A^{(M)} = F_4) \end{aligned}$$

Among the transverse $z_{\perp}$, single out two compact coordinates $y_{\perp}$ and write remaining spatial dimensions in radial coordinates

$$dz_{\perp}^2 = dy_{\perp}^2 + (dr^2 + r^2 d\Omega_2^2)$$

Take the harmonic function on the transverse space:

$$H = 1 + \frac{Q}{r} \ .$$

H is independent of $y_{\perp}$ even though those are transverse to the $D4$ — $D4$ is smeared in those two directions. This gives the 4D interpretation of the 10D solution.

The D0-brane Solution

The 10D IIA SUGRA solution representing a $D0$-brane at low energy:

$$\begin{aligned} ds^2 &= -\frac{1}{\sqrt{H}}dt^2 + \sqrt{H}dz_\perp^2 \, , \\ e^{-2\Phi} &= \frac{1}{H^{3/2}} \\ A_t &= H^{-1} \end{aligned}$$

Single out six compact coordinates $y_\perp$ and write remaining spatial dimensions in radial coordinates

$$dz_\perp^2 = dy_\perp^2 + (dr^2 + r^2 d\Omega_2^2)$$

Take the harmonic function on transverse space that smears $D0$'s over six compact directions $y_\perp$:

$$H = 1 + \frac{Q}{r} \, .$$

The Canonical Black Hole

	0	1	2	3	4	5	6	7	8	9
D4	x			x	x	x	x			
D4	x	x	x			x	x			
D4	x	x	x	x	x					
D0	x									

The basic solutions lend themselves to harmonic superposition. The configuration above gives

$$ds^2 = -\frac{1}{\sqrt{H_0 H_1 H_2 H_3}} dt^2 + \sqrt{H_0 H_1 H_2 H_3} (dr^2 + r^2 d\Omega_2^2)$$

$$+ \sqrt{\frac{H_0 H_1}{H_2 H_3}} dz^1 d\bar{z}^1 + \sqrt{\frac{H_0 H_2}{H_3 H_1}} dz^2 d\bar{z}^2 + \sqrt{\frac{H_0 H_3}{H_1 H_2}} dz^3 d\bar{z}^3 \,,$$

$$e^{-2\Phi} = \sqrt{\frac{H_1 H_2 H_3}{H_0^3}} \,.$$

The Solution in $4D$

The 4D dilaton is simply

$$e^{-2\Phi_4} = e^{-2\Phi}\mathrm{Vol}_6 = 1 \ ,$$

so the 4D Einstein metric can be read off immediately

$$ds_4^2 = -\frac{1}{\sqrt{H_0 H_1 H_2 H_3}} dt^2 + \sqrt{H_0 H_1 H_2 H_3}(dr^2 + r^2 d\Omega_2^2) \ .$$

The black hole entropy computed from the area is

$$S = \frac{A}{4G_4} = \frac{\pi}{G_4}\sqrt{Q_0 Q_1 Q_2 Q_3} \ .$$

Quantized Charges

The Q_i are "physical" charges that depend on moduli. The quantized charges are related by conversion factors

$$C_0 = \sqrt{\frac{2G_4}{v_6}} \,,$$
$$C^i = \sqrt{2G_4 v_6} \cdot \frac{1}{v_i} \,,$$

where v_i are volumes of T^2 measured in string units $v_i = V_i/(2\pi l_s)^2$ and the overall volume is $v_6 = v_1 v_2 v_3$.

The dependence of charges on moduli is such that entropy in fact depends on the quantized charges alone:

$$S = 2\pi\sqrt{n_0 n_1 n_2 n_3}$$

This is one aspect of the attractor mechanism (see later).

BPS and Non-BPS

The solution discussed above makes sense only when $Q_0, Q_1, Q_2, Q_3 > 0$, or else the metric is singular when the harmonic functions vanish.

But: the field strengths appear only quadratically in the action so there are also solutions with $Q_i \to -Q_i$ in the field strengths, but $Q_i \to Q_i$ in the harmonic functions. Alternatively: take Q_i's of any sign, but insert $|Q_i|$ in the harmonic functions.

Convention: take $Q_1, Q_2, Q_3 > 0$, and consider Q_0 of either sign.

The sign is extremely important: $Q_0 > 0$ is the BPS solution, and $Q_0 < 0$ is the non-BPS solution.

The two branches have many ***qualitative*** differences, exhibited in the coming lectures.

Supersymmetry

Type IIA SUSY has two supersymmetry generators, related by the Dirichlet boundary conditions on the D-branes. The resulting relations between the super-translations become

$$\begin{aligned}
\tilde{\epsilon} &= \Gamma^{\hat{3}\hat{4}\hat{5}\hat{6}}\epsilon \\
\tilde{\epsilon} &= \Gamma^{\hat{1}\hat{2}\hat{5}\hat{6}}\epsilon \\
\tilde{\epsilon} &= \Gamma^{\hat{1}\hat{2}\hat{3}\hat{4}}\epsilon \\
\tilde{\epsilon} &= \mp\epsilon
\end{aligned}$$

where the choices in the last relation refers to the sign of Q_0. Consistency of the first three relations give

$$\tilde{\epsilon} = -\epsilon$$

so that only $Q_0 > 0$ is consistent with supersymmetry, as claimed.

M-theory Interpretation

We can lift the $D4 - D4 - D4 - D0$ configuration to M-theory with the result:

	0	1	2	3	4	5	6	7	8	9	10
M5	x			x	x	x	x				x
M5	x	x	x			x	x				x
M5	x	x	x	x	x						x
KK											x

In this duality frame there are three $M5$-branes that intersect over a line, denoted x_{10}. The fourth charge is momentum along that line.

The change from BPS to non-BPS is just the sign of the momentum along x_{10}.

However, $M5$-branes are chiral so such a change is not a symmetry.

Microscopics: the Upshot

The relation to microscopics is invariably through Cardy's formula for the asymptotic density of states in a unitary CFT

$$S = 2\pi\sqrt{\frac{ch}{6}}$$

This is the entropy of a 1D gas with $c = 6n_1n_2n_3$ degrees of freedom and energy (in units of the box size) $h = n_0$. It gives agreement with the area law.

The central charge can be derived in many different ways including:

- Analysis of D-brane bound states.
- Anomaly inflow on $M5$'s.
- Exploiting near horizon AdS$_3$ symmetry.

We will not need the details of these derivations.

Some Non-Extremal Solutions

Ultimately we would like to understand the entropy of black holes arbitrarily far from extremality, including Scwarzchild black holes.

The natural generalization of the extremal four charge solutions above are

$$ds_4^2 = \frac{-1}{\sqrt{H_0 H_1 H_2 H_3}}(1 - \frac{2\mu}{r})dt^2 + \sqrt{H_0 H_1 H_2 H_3}(\frac{1}{1 - \frac{2\mu}{r}}dr^2 + r^2 d\Omega_2^2)$$

$$H_i = 1 + \frac{2\mu \sinh^2 \delta_i}{r}$$

The gauge fields are essentially the inverse harmonic functions, but there is an overall factor such that

$$Q_i = \mu \sinh 2\delta_i \; , \quad i = 0, 1, 2, 3$$

Note: the entire solution is in parametric form: it is written in terms of δ_i, μ, which encode the four charges and the total mass.

Physical Thermodynamics

We can extract the physical mass from the solution

$$2G_4M \;=\; \frac{1}{2}\mu\sum_{i=0}^{3}\cosh 2\delta_i$$

and also find the thermodynamic entropy in parametric variables

$$S = \frac{4\pi\mu^2}{G_4}\prod_{i=0}^{3}\cosh\delta_i$$

The general black hole entropy is a complicated function of the four charges and the mass.

The BPS limit: $\delta_i \to \infty$ for $i = 0, 1, 2, 3$.

The non-BPS extremal limit is $\delta_i \to \infty, i = 1, 2, 3$, $\delta_0 \to -\infty$.

The Dilute Gas Limit

It is useful to relax the extremal limit to the dilute gas limit:
$\delta_i \to \infty, i = 1, 2, 3$ with δ_0 *fixed*.

In this limit the parametric formula for the entropy can be inverted

$$S = 2\pi \left[\sqrt{n_1 n_2 n_3 \frac{\epsilon + p}{2}} + \sqrt{n_1 n_2 n_3 \frac{\epsilon - p}{2}} \right]$$

Here p is usefully thought of as the momentum quantum number (the terminology of the M-theory duality frame, whereas in type IIA duality frame $D0$-charge $p = n_0$).

The dimensionless energy above extremality is

$$\epsilon \ = \ (M - M_{\text{ext}}) R_{10}$$

Microscopic interpretation of entropy : $c = 6n_1n_2n_3$ weakly interacting massless particles in a box of length R_{10}. The left- and right-moving momenta are

$$h_L = \frac{\epsilon + p}{2} \ , \quad h_R = \frac{\epsilon - p}{2} \ ,$$

and Cardy's formula applies for right and left movers independently

$$S = 2\pi \left(\sqrt{\frac{c_L h_L}{6}} + \sqrt{\frac{c_R h_R}{6}} \right) \ .$$

The independent left and right moving temperature are formed from the true temperature (dual to energy) and the chemical potential dual to momentum:

$$\beta_L = \frac{1}{2}(\beta - \mu) \ , \quad \beta_R = \frac{1}{2}(\beta + \mu) \ .$$

In the extremal limit $\beta \to \infty$, $\mu \to \infty$, such that β_L remains finite.

The physical significance of β_L: it controls fluctuations of the L-movers, those that give rise to the entropy of the black hole in the extremal limit.

Fractionation

An important aspect of the dilute gas model: the momenta of the individual quanta are quantized in units of $1/(n_1 n_2 n_3 R_{10})$.

So wave functions of individual quanta only close on the cover, but the total wave function does not: the *total* momentum is quantized in standard units of $1/R_{10}$.

A convenient summary: there is a ***level matching rule*** on the total charges

$$N_R - N_L = \frac{c_R h_R}{6} - \frac{c_L h_L}{6} = \text{integer} \; ,$$

that is much more stringent than the implied quantization of a 1D gas

$$S = \frac{\pi(c_R + c_L)}{12} T\mathcal{L} = \frac{\pi c}{6} T\mathcal{L} \; ,$$

where $\mathcal{L} = n_1 n_2 n_3 R_{10}$ is the length of the box.

Effective Levels

Proposal: the physics of the general class of black holes (including Schwarzchild) is that of an ”effective string”: there are L and R sectors, each with the modes of a chiral string.

The total level $N = \frac{ch}{6}$ (conformal weight, in units of fractionation) on each side are quantized.

The general entropy of the black hole should be given by Cardy's formula

$$S = 2\pi \left[\sqrt{N_L} + \sqrt{N_R} \right]$$

One way to find find levels: take the dilute gas expressions for $N_{L,R}$ and then symmetrize with respect to the four charges.

The proposed effective levels of the effective string are

$$N_L = \frac{c_L}{6} h_L = \frac{\mu^4}{G_4^2} \left(\prod_{i=0}^{3} \cosh \delta_i + \prod_{i=0}^{3} \sinh \delta_i \right)^2$$

$$N_R = \frac{c_R}{6} h_R = \frac{\mu^4}{G_4^2} \left(\prod_{i=0}^{3} \cosh \delta_i - \prod_{i=0}^{3} \sinh \delta_i \right)^2$$

They give the correct entropy arbitrarily away from extremality

$$S = 2\pi \left[\sqrt{N_L} + \sqrt{N_R} \right] = \frac{4\pi\mu^2}{G_4} \prod_{i=0}^{3} \cosh \delta_i$$

A Test

A test of the model:

$$\begin{aligned} N_L - N_R &= \frac{4\mu^4}{G_4^2} \prod_{i=0}^{3} \cosh\delta_i \sinh\delta_i \\ &= \frac{1}{4G_4^2} \prod_{i=0}^{3} Q_i \\ &= \prod_{i=0}^{3} n_i \end{aligned}$$

The final line is the rewriting known from the black hole entropy.

The test: the difference $N_L - N_R$ is independent of moduli, and it is an integer. These are facts for the entire class of black holes considered here.

Rotation

The black holes we consider can be further generalized to include angular momentum, so that a special case is the Kerr black hole. The resulting solutions are very complicated.

The entropy from the area of these black holes

$$S = 2\pi \left(\sqrt{N_R} + \sqrt{N_L} \right)$$

with

$$N_L = \frac{c_L}{6} h_L = \frac{\mu^2}{G_4} \left(\prod_{i=0}^{3} \cosh \delta_i + \prod_{i=0}^{3} \sinh \delta_i \right)^2$$

$$N_R = \frac{c_R}{6} h_R = \frac{\mu^2}{G_4} \left(\prod_{i=0}^{3} \cosh \delta_i - \prod_{i=0}^{3} \sinh \delta_i \right)^2 - J^2$$

Extremal Limits

The quantization condition remains valid in the presence of angular momentum

$$N_L - N_R = \prod_{i=0}^{3} n_i + J^2$$

The extremal limit $T \to 0$ can be taken in two very different ways:

$$N_L \;\to\; 0 \;:\quad N_R = |n_0| \prod_{i=1}^{3} n_i - J^2 \;,$$

$$N_R \;\to\; 0 \;:\quad N_L = \prod_{i=0}^{3} n_i + J^2 \quad (\text{BPS branch})$$

Either case corresponds to AdS$_2 \times S^2$ geometry. The BPS branch is only supersymmetric when $J = 0$.

Extremal Kerr: BPS branch with no charges and so $S = 2\pi\sqrt{N_R} = 2\pi|J|$.

Significance of the Inner Horizon

The proposal in general involves two gasses that a free, except for the coupling due to the matching condition.

There is a geometric interpretation of the resulting ***two*** entropies in terms of the areas of the ***outer*** and ***inner*** horizons

$$S_R = 2\pi\sqrt{N_R} = \frac{1}{2}\left(\frac{A_+}{4G_4} - \frac{A_-}{4G_4}\right)$$

$$S_L = 2\pi\sqrt{N_L} = \frac{1}{2}\left(\frac{A_+}{4G_4} + \frac{A_-}{4G_4}\right)$$

The temperatures of the gasses are related to the ***two*** surface accelerations of the ***outer*** and ***inner*** horizons

$$\beta_R = 2\pi\left(\frac{1}{\kappa_+} + \frac{1}{\kappa_-}\right)$$

$$\beta_L = 2\pi\left(\frac{1}{\kappa_+} - \frac{1}{\kappa_-}\right)$$

Summary

- Introduced the canonical D4-D4-D4-D0 brane configurations.
- Emphasized that the ***sign*** of Q_0 determines whether the confugation is BPS or non-BPS. This will be the starting point of the next lecture.
- Introduced a much more general family of four-charge solutions that includes Schwarzchild, Kerr, and Reissner-Nordström black holes as special cases.
- Hypothesized of general microscopic structure underlying all these black holes: it is a direct product of a BPS black hole theory and a non-BPS black hole theory, coupled by a level matching condition.
- Tested this hypothesis by showing the the natural level matching condition is independent of moduli and integer quantized.