Aspects of non-BPS Black Holes in D=4

Lecture 3

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Outline of Lecture 3

Further studies of the general non-BPS black hole solutions:

- ***Attractor Behavior*** of non-BPS black holes uncovers surprising flat directions in moduli space.

- ***Duality groups*** shed light on the attractor behavior.

- Some other approaches to $D0 - D6$ ***bound states***

- ***A first order phase transition*** relates the non-BPS branch to a BPS branch.

- Towards a ***microscopic description*** of non-BPS black holes. First do not harm: beyond analytical continuation.

Attractors

Previous lecture: lessons from the geometry of non-BPS solutions, specifically the mass and entropy.

Now: consider the scalar fields in the solution.

Qualitative structure: scalar fields flow radially, towards some attractor-value at the horizon of the black hole.

The attractor value of the scalar fields depend on the black hole charges, but not the asymptotic value of the scalars (the moduli).

Thus the moduli decouple from the near horizon behavior.

Example: $D0-D4-D4-D4$ with no B

10D geometry of the canonical solution from lecture 1:

$$ds^2 = -\frac{1}{\sqrt{H_0H_1H_2H_3}}dt^2 + \sqrt{H_0H_1H_2H_3}(dr^2 + r^2 d\Omega_2^2)$$
$$+ \sqrt{\frac{H_0H_1}{H_2H_3}}dz^1 d\bar{z}^1 + \sqrt{\frac{H_0H_2}{H_3H_1}}dz^2 d\bar{z}^2 + \sqrt{\frac{H_0H_3}{H_1H_2}}dz^3 d\bar{z}^3 \ ,$$

The volume of the first T^2 near the horizon is independent of the asymptotic volume

$$\frac{V_1}{(2\pi l_s)^2} = v_1\sqrt{\frac{Q_0Q_1}{Q_2Q_3}} = v_1\sqrt{\frac{\frac{n_0}{\sqrt{v_6}}\frac{n_1\sqrt{v_6}}{v_1}}{\frac{n_2\sqrt{v_6}}{v_2}\frac{n_3\sqrt{v_6}}{v_3}}} = \sqrt{\frac{n_0n_1}{n_2n_3}}$$

Interpretation: attractor behavior is an equilibrium between branes squeezing the cycles they wrap, and blowing up transverse cycles.

Flat Directions

Consider BPS black holes in $N = 2$ SUGRA with vector- and hyper-multiplets.

In this case the attractor mechanism applies to scalars in vector multiplets.

The attractor mechanism does not apply to scalars in hyper multiplets. Those decouple from the flow and so keep their (arbitrary) asymptotic value.

The hyper-multiplets parametrize flat directions of the effective potential for scalars in the black hole background.

BPS Flat Directions in $N = 8$ SUGRA

The duality group $G = E_{7(7)}$ has maximal compact subgroup $H = SU(8)$.

$N = 8$ SUGRA has 70 scalars parametrizing $G/H = E_{7(7)}/SU(8)$.

BPS charge vectors are left invariant by a $g = E_{6(2)}$ subgroup of $G = E_{7(7)}$. Among these dualities, the compact subgroup $h = SU(2) \times SU(6)$ leave the scalars invariant as well.

The coset $g/h = E_{6(2)}/SU(2) \times SU(6)$ parametrize $78 - 3 - 35 = 40$ flat directions of the scalar potential.

Decomposing $N = 8$ SUGRA into $N = 2$ multiplets we find that, indeed, 40 scalars belong to hyper-multiplets.

Non-BPS Flat Directions

Duality group $G = E_{7(7)}$ has maximal compact subgroup $H = SU(8)$.

$N = 8$ SUGRA has 70 scalars parametrizing $G/H = E_{7(7)}/SU(8)$.

Non-BPS charge vectors are left invariant by a $g = E_{6(6)}$ subgroup of $G = E_{7(7)}$. Among these dualities, the compact subgroup $h = USp(8)$ leave the scalars invariant as well.

The coset $g/h = E_{6(6)}/USp(8)$ parametrize $78 - 36 = 42$ flat directions of the scalar potential.

Decomposing $N = 8$ SUGRA into $N = 2$ multiplets we find that 40 scalars belong to hyper-multiplets.

So: ***two scalars parametrize flat directions of the potential even though they belong to vector-multiplets*** . These two scalars are not fixed by the attractor mechanism.

Interpretation of Flat Directions

The $\overline{D0} - D4 - D4 - D4$ duality frame makes the physical interpretation of the mass formula optimally clear: non-BPS black holes are marginal bound states of 1/2-BPS constituents.

The origin of flat directions is clearest in the $D0 - D6$ duality frame.

Attractor behavior is due to branes squeezing the cycles they wrap, expanding transverse cycles.

$D6$ on $T^6 = T^2 \times T^2 \times T^2$ squeeze the overall T^6, and $D0$ blows it up. ***Both are indifferent to the volumes of each T^2 component*** by themselves.

The two flat directions in the $D0 - D6$ duality frame are the ratios of T^2 volumes!

The two flat directions in other duality frames are generally much more complicated, but they are determined by the duality transformation from $D0-D6$.

In $\overline{D0}-D4-D4-D4$ duality frame the potential

$$V_{\rm BH}=\frac{Q_0^2}{2}\frac{1}{y_1y_2y_3}+Q_0\frac{P^3x_1x_2+P^2x_1x_3+P^1x_2x_3}{y_1y_2y_3}$$

$$+\ \frac{1}{2}y_1y_2y_3\sum_{i=1}^{3}(P^i)^2y_i^{-2}+\frac{1}{2y_1y_2y_3}(P^3x_1x_2+P^2x_1x_3+P^1x_2x_3)^2$$

$$+\ \frac{1}{2y_1y_2y_3}\sum_{j\neq k\neq i=1}^{3}(y_i)^2(P^kx_j+P^jx_k)^2$$

has flat directions

$$x_j-iy_j\rightarrow\frac{\cosh 2\alpha_j\,x_j+\frac{1}{2}\sinh 2\alpha_j[\rho_j+\rho_j^{-1}(x_j^2+y_j^2)]-iy_j}{\cosh^2\alpha_j+\rho_j^{-2}\sinh^2\alpha_l(x_j^2+y_j^2)+\rho_j^{-1}\sinh 2\alpha_j\,x_j}$$

with $\rho_i=\sqrt{\frac{-Q_0P^i}{\frac{1}{2}s_{ijk}P^jP^k}}$ and the generators α_i satisfy $\sum_i\alpha_i=0$.

Aside on Flux Vacua

Flux vacua are semi-realistic models that stabilize moduli by turning on many fluxes that squeeze and expand different combinations of moduli.

It is thought that, generically, all moduli are stabilized if only there are fluxes enough.

The counting of fluxes and moduli is the same for SUSY and for non-SUSY vacua, so moduli stabilization should work at least as well for non-SUSY vacua, as for SUSY vacua.

The STU model is an example where there is a symmetry only for non-SUSY vacua and this symmetry gives rise to flat directions, in conflict with generic expectations.

This mechanism can give light scalars in the spectrum.

$D0 - D6$ Revisited

So far: we have analyzed the $D0 - D6$ black hole with those asymptotic charges.

In the previous lecture we found an interpretation as a marginal bound state of four $D6$-branes with certain fluxes turned on.

The next step: compare with other results on $D0 - D6$.

$D0 - D6$ Supersymmetry?

The SUSY-projections due to Dirichlet conditions on the $D0$ and the $D6$ branes are

$$\begin{aligned} \tilde{\epsilon} &= \Gamma^{\hat{1}}\Gamma^{\hat{2}}\Gamma^{\hat{3}}\Gamma^{\hat{4}}\Gamma^{\hat{5}}\Gamma^{\hat{6}}\epsilon \\ \tilde{\epsilon} &= \pm\epsilon \end{aligned}$$

There are no solutions because $(\Gamma^{\hat{1}}\Gamma^{\hat{2}}\Gamma^{\hat{3}}\Gamma^{\hat{4}}\Gamma^{\hat{5}}\Gamma^{\hat{6}})^2 = -1$.

Background B-fields rotate the $D6$ condition by a factor

$$\prod_{i=1}^{3} \frac{1+iB_i}{1-iB_i}$$

There may be SUSY in the presence of B-fields if

$$\sum_{i<j} B^i B^j = 1\,.$$

$D0 - D6$ Bound States

It is simple to compute the spectrum of ***open strings stretching between the $D0$ and the $D6$***.

The spectrum is ***supersymmetric*** if

$$\sum_{i<j} B^i B^j = 1 \, .$$

For

$$\sum_{i<j} B^i B^j > 1 \, .$$

there is ***a tachyon*** in the spectrum.

The tachyon may condense into a ***supersymmetric ground state***, interpreted as ***a genuine bound state*** of the $D0 - D6$-system (the Higgs-branch).

The Multi-Center BPS solutions

There are no ***single center*** BPS black holes with $D0 - D6$ charges but there are ***BPS multicenter solutions***. Some of their properties:

- The simplest multi-center configuration: two centers, one $D0$, the other $D6$.

  (Single-center non-BPS $D0 - D6$: four $1/2$-BPS constituents, all $D6$'s with fluxes.)

- The charge vectors of the 1/2-BPS ***constituents are mutually non-local***, *i.e.* they have non-zero intersection number.

  (The four constituents of the non-BPS black holes are mutually local.)

The Wall of Marginal Stability

- BPS configurations of $D0 - D6$ branes exist only for

$$\sum_{i<j} B^i B^j \geq 1 \, .$$

  This is a co-dimension one wall of marginal stability in moduli space

  (The non-BPS black holes exist everywhere in moduli space.)

- BPS multicenter solutions exist in the same range, with ***a specified separation scale***

$$R = |\vec{x}_1 - \vec{x}_2| = \frac{|Q_0 + iP^0 \prod_{i=1}^{3}(1 + iB^i)|}{\sum_{i<j} B^i B^j - 1}$$

  (Constituents of the non-BPS black holes can move freely in the supergravity approximation.)

The $D0 - D6$ constituents move apart as the wall of marginal stability is approached; ***they are removed from the spectrum***.

First Order Phase Transition

- The BPS solutions ***cannot be continuously connected*** to the non-BPS solution through the wall of marginal stability.

- There ***can be decay*** from the non-BPS branch to the BPS branch on the part of moduli space where BPS solutions exist.

- The BPS mass is always ***strictly smaller*** than the non-BPS mass with otherwise identical quantum numbers.

- So the ***transition will release energy***, entropy and generally also angular momentum.

- This indicates ***a first order transition*** between the two branches.

Is $D0 - D6$ Unstable?

What is the faith of $D0 - D6$ on the part of moduli space where BPS states cannot exist?

The non-rotating solution with canonical asymptotic moduli has mass formula ($Q_0 = Q$, $P = P^0$):

$$M_{D0-D6} = \frac{1}{2G_4}\left[Q^{2/3} + P^{2/3}\right]^{3/2}$$

This formula applies even when there is angular momentum, as long as $J < PQ/2$.

The energetics allows spontaneous decay into widely separated $D0$'s and $D6$'s:

$$M_{D0-D6} > \frac{1}{2G_4}\left[Q + P\right] = M_{D0} + M_{D6}$$

But: ***this process is forbidden*** by angular momentum conservation: widely separated $D0$'s and $D6$'s have $J \geq PQ/2$!

Candidate final states consistent with conservation laws must have at least three bodies.

The decay we know is important for the system is the marginal one: spontaneous separation into four constituents. No energy is released in this process.

It is not known what the dominant decay mode of $D0 - D6$ is.

Microscopics of non-BPS black holes

- Classically, the entropy of BPS and non-BPS black holes are related by analytical continuation: $S_{\rm BPS} = 2\pi\sqrt{|J_4|}$

- This suggests that their microscopic origins are virtually identical, *i.e* related by analytical continuation.

- The problem: ***there are significant differences between the two branches***.

- For example, ***the classical moduli spaces are completely different***: they have different dimension.

- Also, ***the mass formulae*** on the two branches are ***not related by analytical continuation***.

- These distinctions are relevant: ***precise counting of BPS states involves choosing a favorable point in moduli space***.

- Specifically, one may want to turn on a small B-field to avoid bound states at threshold. ***This is not possible for the non-BPS states***.

- Conclusion: ***the corresponding microstates are not be related by analytical continuation***.

Presumably there is a simple understanding of extremal non-BPS entropy anyway. But the significant differences between the two branches must be addressed by a more detailed understanding of the microscopics.

Indeed, these differences may give guidance towards a microscopic description.

Summary

The non-BPS extremal black holes exhibit some surprising properties:

- ***Mass Formula:*** the extremal non-BPS mass is the sum of four primitive $1/2$ BPS constituent masses. This property applies everywhere in moduli space, although the specific four-part split changes.

- ***Flat Directions:*** some of the scalars in the theory experience a flat potential when all forces are taken into account.

- ***Forces on Probes:*** the proposed constituents do not experience any forces from the black hole anywhere in moduli space.

- ***Phase Transition:*** the non-BPS mass is strictly greater than the BPS bound, even in regions of moduli space where BPS multi-center solutions exist. The two branches are related by a first order phase transition.