On New Massive Gravity and Holography Renormalization

Mohsen.Alishahiha A.Naseh

In honor of Prof.Ardalan

IPM School of Physics, Tehran
April 2010

Introduction:

Three dimensional gravity offers an interesting arena to investigate quantization of gravitational theories.

However Einstein gravity in three dimensions has no propagating degrees of freedom

$$g_{\mu\nu} \to \frac{D(D+1)}{2} - D(e.o.m) - D(diff) = 0 \qquad trivial$$

$$I = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\,(R-2\Lambda)$$

$$ds^2 = (M - \frac{r^2}{l^2})dt^2 + \frac{1}{(M + \frac{r^2}{l^2} + \frac{J^2}{4r^2})}dr^2 - Jdt\,d\varphi + r^2 d\phi^2$$

BTZ black hole !!

$$r_{\pm} = \frac{Ml^2}{2}\{1 \pm [1 - (\frac{J}{Ml})^2]^{1/2}\}$$

Topologically massive gravity is obtained by adding to the Einstein gravity the gravitational CS term,

$$S=\int d^3x\left(\sqrt{-g}\,(R-2\Lambda)+\frac{1}{2\mu}\varepsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\sigma\lambda}(\partial_{\mu}\Gamma^{\sigma}_{\rho\nu}+\frac{2}{3}\Gamma^{\sigma}_{\kappa\mu}\Gamma^{\kappa}_{\sigma\nu})\right)$$

where Γ is the 1-form Christoffel symbol. parity is not preserved
This theory admits asymptotically AdS solutions, for example the BTZ black hole, and has perturbative massive modes.

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\frac{1}{l^2}g_{\mu\nu}+\frac{1}{\mu}C_{\mu\nu}=0$$

Were $C_{\mu\nu}$ is the Cotton tensor

$$C_{\mu\nu}=\varepsilon_{\mu}^{\kappa\sigma}\nabla_{\kappa}(R_{\sigma\nu}-\frac{1}{4}g_{\sigma\nu}R)$$

Holography renormalization of TMG :[K.Skenderis, M.Taylor and B. C. van Rees(2009)]

In this massive gravity, higher derivative terms are added to the Einstein Hilbert action and unlike in topological massive gravity, parity is preserved in this new massive gravity.

$$S=\int d^3x\sqrt{-g}\left(R-2\Lambda-\frac{1}{2m^2}K\right) \qquad K=R^{\mu\nu}R_{\mu\nu}-\frac{3}{8}R^2$$

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}-\frac{1}{2m^2}K_{\mu\nu}=0$$

$$K_{\mu\nu}=-\frac{1}{2}\nabla^2 Rg_{\mu\nu}-\frac{1}{2}\nabla_\mu\nabla_\nu R+2\nabla^2 R_{\mu\nu}+4R_{\mu\alpha\nu\beta}R^{\alpha\beta}-\frac{3}{2}RR_{\mu\nu}-R_{\alpha\beta}R^{\alpha\beta}g_{\mu\nu}+\frac{3}{8}R^2 g_{\mu\nu}$$

It was also shown that NMG model on an asymptotically AdS3 geometry may have a dual CFT whose central charges are given by

$$c_L=c_R=\frac{3l}{2G}\left(1-\frac{1}{2m^2l^2}\right)$$

Variation of a gravitational action with respect to the metric schematically is

$$\delta S = \frac{1}{16\pi G}\int_M d^{d+1}x\sqrt{-G}\left[(...)\delta G_{\mu\nu}\right] + \frac{1}{16\pi G}\int_{\partial M} d^d x\sqrt{-\gamma}\left[(...)\delta G_{\mu\nu} + (...)\delta G_{\mu\nu,\sigma}\right]$$

As usual setting the first term (the volume term) to zero one finds the equations of motion while the boundary terms must be set to zero by a proper boundary condition. Indeed the second term is set to zero due to the fact that at the boundary we impose Dirichlet boundary condition. On the other hand to have a well-posed variational principle one has to add a boundary term to the action to remove the last term. Boundary term, known as the Gibbons-Hawking term.

NMG, at a special coupling don’t need to Gibbons-Hawking term and has a good variational principle for AlAdS spacetimes.

An AlAdS spacetime admits the following metric in a finite neighborhood of the conformal boundary, located at $\rho = 0$:
[Fefferman-Graham (1985)]

$$ds^2 = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho} g_{ij}(x,\rho) dx^i dx^j$$

For the Asymptotically locally AdS3 solution

$$g_{ij} = g_{(0)ij} + (b_{(2)ij} \log(\rho) + g_{(2)ij})\rho + \ldots$$

$g_{(2)ij}$ is only partially determined by asymptotics. This coefficient is related via AdS/CFT to the 1-point function of T_{ij} and thus to bulk conserved charges.

$b_{(2)ij}$ is related to the Weyl anomaly of the boundary theory, [Henningson, Skenderis(1998)]

The precise form of the expansion $g_{ij}(x,\rho)$ is determined by solving the bulk field equations asymptotically.

At the special value of $m^2 = \frac{1}{2}$ the model admits a vacuum solution which is not asymptotically locally AdS3

$$g_{ij} = b_{(0)ij}\log(\rho) + g_{(0)ij} + (b_{(2)ij}\log(\rho) + g_{(2)ij})\rho + \ldots$$

1-Brown-Henneaux boundary condition is $b_{(0)ij} = b_{(2)ij} = 0,\ g_{(0)ij} = \delta_{ij}$

2-Modified boundary conditions are determined with the solutions of equation of motion not with the traditional way.

- Traditional way: Change the boundary condition by hand up to the level that the value of charge will not be zero.

In the AdS/CFT correspondence the on-shell action treat as the generating function of CFT:

$$< T_{ij} >= \frac{\delta S_{onshell}}{\delta g_{(0)}^{ij}}$$

In general the on-shell action is divergent and one needs to regularized it by adding a proper counterterms [Henningson,Skenderis (1998)].

One then obtains a finite 1-point function for T_{ij} for general AlAdS spacetimes [de Haro, Solodukhin, Skenderis (2000)]

Using Wald's covariant phase space method proved that the holographic charges are the correct gravitational conserved charges [Papadimitriou, Skenderis (2005)].

The holographic methodology is:

1- Derive the most general linearized solution of the bulk equations with general Dirichlet boundary conditions for all fields.

$$d^2s = \frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}(\eta_{ij} + h_{ij})dx^i dx^j$$

2-Evaluate the on-shell action up to quadratic terms. We note,however, that in general the on-shell action is divergent and one needs to regularized it by adding a proper local boundary covariant counterterms.

3-One-point functions and Two-point functions will be evaluated from Finite Onshell action.

One vary the action with respect to the sources to find the on-shell one point function

$$< T_{ij} >= \frac{1}{4G}[-8b_{(2)ij} + 4\eta_{im}(\partial_j \partial_n b^{mn}_{(0)}) + 2\eta^{nl}\eta_{ij}(\partial_n \partial^m b_{(0)ml})],$$

$$< t_{ij} >= \frac{1}{4G}[-8g_{(2)ij} - 4\eta_{im}(\partial_j \partial_n g^{mn}_{(0)})].$$

and are expressed in terms of coefficients in the asymptotic expansions.

From the general solution of the linearized equations of motion we extract the following 2-point functions:

$$< T_{zz} T_{zz} >= 0 \qquad < T_{zz} t_{zz} >= 0 \qquad < t_{zz} t_{zz} >= 0$$

Consider the most general OPE's of a LCFT[(V. Gurarie and A. W. W. Ludwig(1999)),(S. Moghimi-Araghi, S. Rouhani and M. Saadat(2000))]

$$T(z)T(0) = \frac{cI_{(0)}}{2z^4} + \ldots,$$

$$T(z)t(0) = \frac{cI_{(1)} + bI_{(0)}}{2z^4} + \ldots,$$

$$t(z)t(0) = -\frac{\log(z)[(c\log(z) + 2b)I_{(0)} + 2cI_{(1)}]}{2z^4} + \ldots$$

$< I_{(0)} >= 1 \quad and \quad < I_{(1)} >= 0$

Daniel Grumiller, Olaf Hohm
hep-th :0911.4274

$$< T(z)T(0) >= \frac{c}{2z^4} + ...,$$

$$b = \frac{12l}{G}$$

$$< T(z)t(0) >= \frac{b}{2z^4} + ...,$$

$$< t(z)t(0) >= -\frac{\log(z)[c\log(z) + 2b]}{2z^4} + ...$$

$< I_{(0)} >= 0 \quad and \quad < I_{(1)} >= 1$

$$b \neq 0$$

↓

Three point functions

$$< T(z)T(0) >= 0,$$

$$< T(z)t(0) >= \frac{c}{2z^4} + ...,$$

$$< t(z)t(0) >= -\frac{2c\log(z)}{2z^4} + ...$$

$$\delta S = \frac{1}{16\pi G}\int_M d^{d+1}x\sqrt{-G}\left[(\ldots)\delta G_{\mu\nu}\right] + \frac{1}{16\pi G}\int_{\partial M} d^d x\sqrt{-\gamma}\left[(\ldots)\delta G_{\mu\nu} + (\ldots)\delta G_{\mu\nu,\sigma}\right]$$

$$[\alpha\frac{1}{\rho}g^{ij} + \ldots]\delta g_{ij}$$

Make divegencies $\longrightarrow$ Add counterterm $I_{ct} = \frac{1}{8\pi G}\int d^2x\sqrt{-\gamma}[-1+\frac{1}{4}R[\gamma]\log(\rho_{(0)})]$

Under subgroup of bulk diffeomorphisms which preserve Fefferman-Graham form of metric and which on the boundary reduce to a Weyl transformation

$$\delta I \neq 0 \qquad \frac{\delta I}{\delta\alpha(x)} = A = Weyl\ anomaly = \frac{c}{12}R = <T^i{}_i>$$

$$b \quad \underrightarrow{??} \quad <t^i{}_i>$$

Thank you