Extremal BTZ Quasi-Normal Modes for tensor perturbations in TMG

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Gravitational Quasi-Normal Modes (QNM's)

- Definition
- Determination
- Application

Topologically Massive Gravity (TMG)

- Definition
- BTZ black holes
- Gravitons

QNM’s definition

- Everything around us has characteristic modes of vibration characterizing the structure and composition of it.
- Normal modes: closed physical systems, frequencies are real.
- Quasi normal modes: dissipative physical systems, frequencies are complex.
- Perturbed BH spacetimes are intrinsically dissipative due to the presence of an event horizon.

A recent rev. on QNM’s:0905.2975 Berti, Cardoso, Starinets

QNM’s determination

- Asymptotically flat BH’s: By requiring the solution to be incoming at the horizon and outgoing at infinity
- Asymptotically AdS BH’s: By requiring the solution to be incoming at the horizon and vanishing at infinity
- Incoming at the horizon as it doesn’t radiate classically
- Outgoing/ Vanishing at the boundary to discard unphysical waves entering the space time from infinity.

QNM's application

- The inverse relaxation time to equilibrium is the imaginary part of the lowest QNM frequency.

AdS/CFT applications:

- According to the AdS/CFT~ BH in AdS corresponds to a thermal state in the CFT, perturbing the BH corresponds to perturbing the thermal state, and the decay of the perturbation describes the return to thermal equilibrium.

- They are identical to the momentum space poles of the retarded corelator in the strongly coupled dual CFT.

G. T. Horowitz & V. E. Hubeny 9909056, D. T. Son & A. O. Starinets 0205051

Euclidean AdS/CFT

- The bulk field ϕ is coupled to an operator $\hat{\mathcal{O}}$ on the boundary in such a way that the interaction Lagrangian is $\phi\hat{\mathcal{O}}$.

- AdS/CFT correspondence: $$\langle e^{\int_{\partial M} \phi_0 \hat{\mathcal{O}}} \rangle = e^{-S_{\mathrm{cl}}[\phi]}$$

  and the exponent on the right hand side is the action of the classical solution to the equation of motion for ϕ in the bulk metric with the boundary condition:
  $$\phi\big|_{z=0} = \phi_0$$

Retarded Green Function

$$G^R(k) = -i\int d^4x\, e^{-ik\cdot x}\,\theta(t)\,\langle[\hat{\mathcal{O}}(x),\hat{\mathcal{O}}(0)]\rangle$$

$$\phi(z,x) = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x} f_k(z)\phi_0(k)$$

$$\phi(z_B,x) = \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot x}\phi_0(k)$$

$$f_k(z) = A(k)z^{\Delta_-}(1+\cdots) + B(k)z^{\Delta_+}(1+\cdots)$$

- Where Δ_+, Δ_- are the exponents of the ODE satisfied by $f_k(z)$
- Applying the gauge-gravity duality recipe for Minkowski correlators, for the retarded two-point function of the operator $\hat{\mathcal{O}}$:

$$G^R(k) \sim \frac{B}{A}$$

QNM’s for BTZ’s

- In pure gravity the BTZ black holes have no gravitational QNM's because there are no local propagating degrees of freedom in the bulk. Therefore in such theories, the focus has only been on scalar, fermion or vector perturbations on BTZ black holes. D. Birmingham 0101194 ,…

- However, in TMG, as well as other higher derivative three dimensional gravities, gravitons can propagate and hence gravitational QNM's become important. I. Sachs & S. N. Solodukhin 0806.1788

QNM's for BTZ's

The retarded two-point function of the scalar operator of conformal dimension $\Delta = 2$ in a 2d CFT dual to BTZ background at finite temperature is given by

$$G^R \sim \frac{\omega^2 - q^2}{4\pi^2}\left[\psi\left(1 - i\frac{\omega - q}{4\pi T}\right) + \psi\left(1 - i\frac{\omega + q}{4\pi T}\right)\right] \qquad \psi(z) = \Gamma'(z)/\Gamma(z)$$

$$\omega_n = \pm q - i\,4\pi T(n+1)\,, \qquad n = 0, 1, 2, \ldots$$

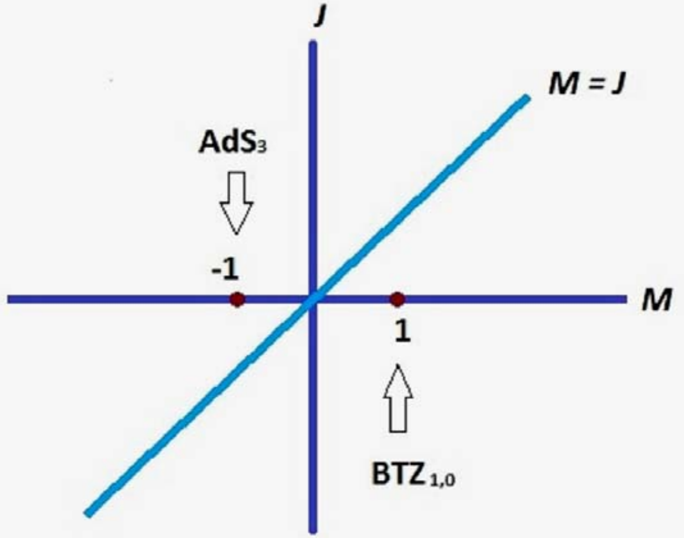
BTZ black holes

$$ds^2 = -N^2 dt^2 + \frac{dr^2}{N^2} + r^2(d\phi + N^\phi dt)^2$$

$$N^2 = -8MG + \frac{r^2}{\ell^2} + \frac{16G^2J^2}{r^2} \qquad N^\phi = -\frac{4GJ}{r^2}$$

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2\ell^2} dt^2 + \frac{r^2\ell^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2(d\phi - \frac{r_+ r_-}{r^2 \ell} dt)^2$$

$$M = \frac{r_+^2 + r_-^2}{8G\ell^2} \qquad J = \frac{r_+ r_-}{4G\ell}$$



Gaussian Normal Coordinate

$$u = t/\ell - \phi,\ v = t/\ell + \phi$$

$$r^2 = r_+^2 \cosh^2(\rho - \rho_0) - r_-^2 \sinh^2(\rho - \rho_0), \qquad e^{2\rho_0} = \frac{r_+^2 - r_-^2}{4\ell^2}$$

$$ds^2 = \ell^2[L^+ du^2 + L^- dv^2 + d\rho^2 - \left(e^{2\rho} + L^+ L^- e^{-2\rho}\right) du dv]$$

$$ds^2 = \ell^2[L^+ du^2 + L^- dv^2 + d\hat{\rho}^2 - 2\sqrt{L^+ L^-} \cosh(2\hat{\rho}) du dv]$$

$$L^\pm = \frac{(r_+ \pm r_-)^2}{4\ell^2} \qquad \frac{e^{2\rho}}{\sqrt{L^+ L^-}} = e^{2\hat{\rho}}$$

S. Carlip 0503022, M. Banados 9901148

Extremal BTZ

$$r_+ = r_- = r_{ex}$$

$$ds^2 = -\frac{(r^2 - r_{ex}^2)^2}{r^2\ell^2}dt^2 + \frac{r^2\ell^2}{(r^2 - r_{ex}^2)^2}dr^2 + r^2(d\phi - \frac{r_{ex}^2}{r^2\ell}dt)^2$$

$$e^{2\rho} = \frac{r^2 - r_{ex}^2}{\ell^2} \qquad u = t/\ell - \phi,\ v = t/\ell + \phi$$

$$ds^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = r_{ex}^2 du^2 - \ell^2 e^{2\rho} du dv + \ell^2 d\rho^2$$

TMG definition

$$I_{TMG} = \frac{1}{16\pi G}\left(I_{EH} + \frac{1}{\mu} I_{CS}\right)$$

$$I_{EH} = \int d^3x \sqrt{-g}\left(R + \frac{2}{\ell^2}\right),$$

$$I_{CS} = \frac{1}{2}\int d^3x \sqrt{-g}\,\epsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left(\partial_\mu \Gamma^{\rho}_{\lambda\sigma} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right)$$

$$R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} + \Lambda g_{\alpha\beta} + \frac{1}{\mu} C_{\alpha\beta} = 0$$

$$C_{\alpha\beta} = \epsilon_{\alpha}{}^{\mu\nu}\nabla_\mu\left(R_{\nu\beta} + \frac{1}{4} R g_{\nu\beta}\right)$$

For any conformally flat 3d metric, Cotton tensor is zero

TMG solutions

- AdS

$$ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = \ell^2 \left(-\cosh^2 \rho d\tau^2 + \sinh^2 \rho d\phi^2 + d\rho^2\right)$$

- AdS waves, Warped AdS

  Classification of TMG exact solutions: D. D. K. Chow, C. N. Pope and E. Sezgin 0906.3559

- BTZ black holes, Log Black holes, Warped Black holes

Gravitons

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{G}^{(1)}_{\alpha\beta} + \frac{1}{\mu}\epsilon_\alpha{}^{\sigma\rho}\bar{\nabla}_\sigma \mathcal{G}^{(1)}_{\rho\beta} = 0$$

$$\bar{\nabla}_\mu h^{\mu\nu} = 0, \qquad h = 0$$

$$\mathcal{G}^{(1)}_{\alpha\beta} = \frac{1}{2}\left(-\bar{\nabla}^2 h_{\alpha\beta} - \bar{\nabla}_\alpha \bar{\nabla}_\beta h + \bar{\nabla}^\sigma \bar{\nabla}_\beta h_{\sigma\alpha} + \bar{\nabla}^\sigma \bar{\nabla}_\alpha h_{\sigma\beta}\right) + \frac{2}{\ell^2} h_{\alpha\beta}$$

$$\left(\mathcal{D}^{L/R}\right)_\alpha{}^\beta = \delta_\alpha{}^\beta \pm \ell \epsilon_\alpha{}^{\sigma\beta}\bar{\nabla}_\sigma, \qquad \left(\mathcal{D}^M\right)_\alpha{}^\beta = \delta_\alpha{}^\beta + \frac{1}{\mu}\epsilon_\alpha{}^{\sigma\beta}\bar{\nabla}_\sigma$$

$$\left(\mathcal{D}^L \mathcal{D}^R \mathcal{D}^M h\right)_{\alpha\beta} = 0$$

$$\left(\mathcal{D}^L h^L\right)_{\alpha\beta} = 0, \qquad \left(\mathcal{D}^R h^R\right)_{\alpha\beta} = 0, \qquad \left(\mathcal{D}^M h^M\right)_{\alpha\beta} = 0$$

W. Li, W. Song & A. Strominger 0801.4566

New Mode

$$\left(\mathcal{D}^L \mathcal{D}^L h^{new}\right)_{\alpha\beta} = 0, \qquad \left(\mathcal{D}^L h^{new}\right)_{\alpha\beta} \neq 0$$

$$h_{\mu\nu} = \Re \psi_{\mu\nu}$$

$$\psi^{new} = \frac{d\psi}{d(\mu l)}\Big|_{\mu l = 1}$$

$$\psi_{\mu\nu}(h, \bar{h}) = e^{-ihu - i\bar{h}v} F_{\mu\nu}(\rho)$$

D. Grumiller & N. Johansson 0805.2610

Whittaker Equation

$$z = 2i\bar{h}r_{ex}e^{-2\rho}$$

$$\frac{d^2W}{dz^2} + \left[-\frac{1}{4} + \frac{\lambda}{z} + \frac{\frac{1}{4} - m^2}{z^2}\right] W = 0$$

$$W(z) = F_{vv}, \qquad \lambda = \frac{h}{2ir_{ex}}, \qquad m^2 = \left[\frac{\mu}{2} + 1\right]^2$$

$$W(z) = C_1 W_{\lambda,m}(z) + C_2 W_{-\lambda,m}(-z)$$

$$W_{\lambda,m}(z) = \frac{\Gamma(-2m)}{\Gamma(\frac{1}{2} - m - \lambda)} M_{\lambda,m}(z) + \frac{\Gamma(2m)}{\Gamma(\frac{1}{2} + m - \lambda)} M_{\lambda,-m}(z)$$

$$M_{\lambda,m}(z) = z^{m+1/2} e^{-z/2} {}_1F_1(m - \lambda + \frac{1}{2}, 1 + 2m; z)$$

QNM's B.C.

1- Ingoing wave at the horizon
2- Vanishing at the boundary

$$\begin{aligned} W_{\lambda,m}(z) &\sim e^{-z/2} z^{\lambda} \\ W_{-\lambda,m}(-z) &\sim e^{z/2} z^{-\lambda} \end{aligned}$$

$1+h/\bar{h} > 0$

$$W_{-\lambda,m}(-z) \sim z^{\frac{1}{2}+m} \frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-m+\lambda)} + z^{\frac{1}{2}-m} \frac{\Gamma(2m)}{\Gamma(\frac{1}{2}+m+\lambda)}$$

$$F_{uu} \sim z^{-\frac{3}{2}+m} \frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-m+\lambda)} + z^{-\frac{3}{2}-m} \frac{\Gamma(2m)}{\Gamma(\frac{1}{2}+m+\lambda)} \qquad \mu \neq \pm 1$$

$$F_{uu} \sim z^{-\frac{1}{2}+m} \frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-m+\lambda)} + z^{-\frac{1}{2}-m} \frac{\Gamma(2m)}{\Gamma(\frac{1}{2}+m+\lambda)} \qquad \mu = \pm 1$$

QNM's Frequencies

$$\frac{1}{2}+m+\lambda=0,-1,-2,\cdots$$

$$\{u,v\}\sim\{u+2\pi,v-2\pi\}$$

$$h-\bar{h}=k\in\mathbb{Z}$$

$$\omega_n=h+\bar{h}=k-4ir_{ex}\left(n+\left|\frac{\mu}{2}+1\right|-\frac{1}{2}\right)$$

$$\psi_{\mu\nu}=e^{-ik(t+\phi)-4r_{ex}\left(n+\left|\frac{\mu}{2}+1\right|-\frac{1}{2}\right)t}F_{\mu\nu}(\rho)$$

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