A D2-brane in the Penrose limits of AdS_4xCP^3

In honor of Prof. Farhad Ardalan

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Outline

1. Review AdS/CFT, BLG and ABJM theories.
2. Pp-wave backgrounds of AdS_4xCP^3.
3. Light-cone gauge fixing for a D2-brane.
4. BPS configuration: Giant-like solution

AdS/CFT correspondence

Type IIB string theory on AdS_5xS^5 background

Strong/weak coupling

N=4 superconformal YM theory

Penrose limit

Corresponds to Penrose limit

pp-wave

BMN sector [hep-th/0202021]

String theory on pp-wave is solvable and we know string spectrum. [hep-th/0202109]

$$\boxed{\mathrm{AdS}_4/\mathrm{CFT}_3}$$

BLG model
SU(2)xSU(2) Chern-Simon theory
[0712.3738]

ABJM model
U(N)xU(N) Chern-Simon theory
(low energy limit of N M2-brane probing a C^4/Z_k)
[0806.1218]

$k^5 \ll N$

M-theory on
$AdS_4 \times S^7/Z_k$

$N^{1/5} \ll k \ll N$

Type IIA string theory
on AdS_4xCP^3

Penrose limit

pp-wave

pp-wave backgrounds

General form

$$ds^2 = -4dx^+dx^- + \sum_{\hat{i}=1}^{4}\left(du_{\hat{i}}^2 - u_{\hat{i}}^2(dx^+)^2\right) + \sum_{a=1}^{2}\Big[dx_a^2 + dy_a^2$$

$$+ (\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2)(dx^+)^2 + 2\Big((\xi_a - 2C_a)x_a dy_a - (\xi_a + 2C_a)y_a dx_a\Big)dx^+\Big]$$

Three pp-waves backgrounds

$$\text{no flat direction} \leftrightarrow \xi_a = C_a = 0$$

$$\text{one flat direction} \leftrightarrow \xi_1 = \frac{1}{2}, \xi_2 = b + \frac{1}{2}, C_1 = \frac{1}{4}, C_2 = 0$$

$$\text{two flat directions} \leftrightarrow \xi_a = \frac{1}{2}, C_a = \frac{1}{4}$$

Form-fields $\quad C_{+ij} = -\frac{1}{g_s}\epsilon_{ijk}u_k, \quad C_+ = -\frac{1}{g_s}u_4$

Light-cone gauge fixing

Low energy action for a D2-brane

$$S = \int d\tau d^2\sigma \sqrt{-\det N} + \int C^{(3)} + \int C^{(1)} \wedge F$$

$$N_{\hat{\mu}\hat{\nu}} = g_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}}$$

where

$$\begin{aligned} g_{\hat{\mu}\hat{\nu}} =& -4\partial_{\hat{\mu}} x^- \partial_{\hat{\nu}} x^+ + \Big[(\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - u_{\hat{i}}^2\Big] \partial_{\hat{\mu}} x^+ \partial_{\hat{\nu}} x^+ + \partial_{\hat{\mu}} x^I \partial_{\hat{\nu}} x^I \\ & + 2(\xi_a - 2C_a) x_a \partial_{\hat{\mu}} y_a \partial_{\hat{\nu}} x^+ - 2(\xi_a + 2C_a) y_a \partial_{\hat{\mu}} x_a \partial_{\hat{\nu}} x^+ \\ F_{\hat{\mu}\hat{\nu}} =& \partial_{\hat{\mu}} A_{\hat{\nu}} - \partial_{\hat{\nu}} A_{\hat{\mu}} \\ x^I =& \{u^{\hat{i}}, x^a, y^a\} \end{aligned}$$

L.C. gauge $x^{+} = \tau,\ \sigma^{\hat{\mu}} = (\tau = \sigma^0, \sigma^r),\ r = 1, 2$

Polyakov action

$$S' = -\frac{T'}{g_s} \int d^{p+1}\sigma \det(hg)^{\frac{1}{4}} \left[h^{\hat{\mu}\hat{\nu}} (g - F^2)_{\hat{\mu}\hat{\nu}} + \Lambda \right]$$

$$\Lambda = \frac{3-p}{p+1} h^{\hat{\mu}\hat{\nu}} (g - F^2)_{\hat{\mu}\hat{\nu}}$$

L.C. gauge fixing condition

$$h_{0r} = 0$$

$$\frac{\partial \mathcal{L}}{\partial h_{0r}} = 0 \rightarrow G^{0r} = 0$$

L.C. gauge fixing condition

$$N^{0r} + N^{r0} \equiv G^{0r} = G_{0r} = (g - FgF)_{0r} = 0$$

$$p^+ = \frac{\partial \mathcal{L}}{\partial(\partial_\tau x^-)} = -\frac{2}{g_s}\sqrt{-\det N}N^{00}$$

L.C. Hamiltonian

$$\begin{aligned}\mathcal{H}_{lc} = p^- &= \frac{\partial \mathcal{L}}{\partial(\partial_\tau x^+)} \\ &= p^+ \Bigg(\partial_\tau x^- - \frac{1}{2}\big[(\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - u_{\hat{i}}^2\big] \\ &\qquad - \frac{1}{2}(\xi_a - 2C_a)x_a\dot{y}_a + \frac{1}{2}(\xi_a + 2C_a)y_a\dot{x}_a \Bigg)\end{aligned}$$

$$\det N = \det(N_{rs})(N_{00} - N_{0r}N^{rs}N_{s0})$$

$$N^{00} = \frac{\det(N_{rs})}{\det N}$$

$$p^+ = \frac{\partial \mathcal{L}}{\partial(\partial_\tau x^-)} = -\frac{2}{g_s}\sqrt{-\det N}N^{00}$$

$$N_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} N_{00} & N_{0r} \\ N_{r0} & N_{rs} \end{pmatrix}$$

$$N_{00} = -(\frac{2}{p^+ g_s})^2 \det(N_{rs}) + N_{0r}N^{rs}N_{s0}$$

$$N_{00} = -4\partial_\tau x^- + (\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - (u^{\hat{i}})^2 + (\dot{x}^I)^2 + 2(\xi_a - 2C_a)x_a\dot{y}_a - 2(\xi_a + 2C_a)y_a\dot{x}_a$$

$$\mathcal{H}_{lc} = p^+ \Bigg((\frac{1}{p^+ g_s})^2 \det N_{rs} - \frac{1}{4} N_{0r} N^{rs} N_{s0} + \frac{1}{4} (\dot{x}^I)^2$$
$$- \frac{1}{4} \big[(\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - (u^{\hat{i}})^2 \big] - \frac{2}{p^+ g_s} \epsilon^{ijk} u^i \{u^j, u^k\} - \frac{8 B u_4}{3 p^+ g_s} \Bigg)$$

The first term in Hamiltonian

$$\det N_{rs} = \det g_{rs} + \det F_{rs}$$
$$= \frac{1}{2} \{x^I, x^J\}^2 + B^2, \;\; B = F_{12} \, ,$$
$$\{F, G\} = \epsilon^{rs} \partial_r F \partial_s G$$

Momentum conjugate to gauge field

$$P_E^r = \frac{\partial \mathcal{L}}{\partial F_{0r}} = \frac{2}{g_s} \sqrt{-\det N} N^{0r}$$

The second term in Hamiltonian

$$-p^+ N_{0r} N^{rs} N_{s0} = \frac{1}{p^+} P_E^r g_{rs} P_E^s$$
$$= \frac{1}{p^+} P_E^r \partial_r X^I P_E^s \partial_s X^I = \frac{(P_E^I)^2}{p^+}$$

The final L.C. Hamiltonian is

$$\mathcal{H}_{lc} = \frac{(P_E^I)^2}{4p^+} + \frac{p^+}{4}(\dot{x}^I)^2 + \frac{1}{2p^+g_s^2}\{x^I, x^J\}^2 + \frac{B^2}{p^+g_s^2}$$
$$- \frac{p^+}{4}\left[(\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - (u^{\hat{i}})^2\right] - \frac{2}{g_s}\epsilon^{ijk}u^i\{u^j, u^k\} - \frac{8Bu_4}{3g_s}$$

Giant gravitom

Light-cone hamiltonian with magnetic field

$$\mathcal{H}_{lc} = \frac{p^+}{4}\left(u^i - \frac{1}{p^+ g_s}\epsilon^{ijk}\{u^j, u^k\}\right)^2 + \frac{B^2}{p^+ g_s^2}$$

Ansatz

$$u^i = \frac{\alpha}{2} p^+ g_s J^i \ , \ \{J^i, J^j\} = \epsilon^{ijk} J^k$$

A 2-sphere with radius one

Perfect square trick

$$\mathcal{H}_{lc} = \frac{1}{p^+ g_s^2}\left(\frac{(p^+ g_s)^4}{4}\alpha^2(1-\alpha)^2 + B^2\right)$$
$$= \frac{1}{p^+ g_s^2}\left(\frac{(p^+ g_s)^2}{2}\alpha(1-\alpha) \pm B\right)^2 \mp p^+\alpha(1-\alpha)B$$

where $\alpha = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{8B}{(p^+ g_s)^2}}\right).$

$$B = 0(\mathcal{H}_{lc} = 0) \begin{cases} \alpha = 0 \to R = 0 & \text{Graviton} \\ \alpha = 1 \to R = \frac{1}{2}p^+ g_s & \text{Giant graviton} \end{cases}$$

$$B_{max}(\alpha = \frac{1}{2}) = \frac{1}{8}(p^+ g_s)^2 \to \mathcal{H}_{lc} = \frac{1}{4}p^+ g_s$$

Thanks for attention